

Subsurface Flow Module

User's Guide

Subsurface Flow Module User's Guide

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Introduction

This guide describes the Subsurface Flow Module, an optional add-on package that extends the COMSOL Multiphysics® modeling environment with customized physics interfaces for modeling fluid flow below the ground level or in porous media in general. It allows to combine this flow with other phenomena, such as poroelasticity, heat transfer, chemistry, and electromagnetic fields.

This chapter introduces you to the capabilities of this module. A summary of the physics interfaces and where you can find documentation and model examples is also included. The last section is a brief overview with links to each chapter in this guide.

- [About the Subsurface Flow Module](#)
- [Overview of the User's Guide](#)

About the Subsurface Flow Module

In this section:

- [How the Subsurface Flow Module Helps Improve Your Modeling](#)
- [The Subsurface Flow Module Physics Interface Guide](#)
- [Common Physics Interface and Feature Settings and Nodes](#)
- [Subsurface Flow Module Study Availability](#)
- [The Liquids and Gases Materials Database](#)
- [Where Do I Access the Documentation and Application Libraries?](#)



[The Physics Interfaces and Building a COMSOL Multiphysics Model in the *COMSOL Multiphysics Reference Manual*](#)

How the Subsurface Flow Module Helps Improve Your Modeling

The earth and surrounding planets are a giant laboratory filled with an unlimited array of basic physics and multiphysics interactions. Whether in concert or alone, these physical phenomena alter our access to important resources, affect the quality of the environment, shape the ground beneath our feet on earth, and form other planets.

The Subsurface Flow Module is used in the quantitative investigation of geophysical and environmental phenomena, especially within the area of subsurface flow. The module combines physics interfaces for fundamental processes and links to COMSOL Multiphysics and the other modules for structural mechanics and electromagnetics analyses. New physics represented include heating from radiogenic decay that produces the geotherm, which is the increase in background temperature with depth. You can use the variably saturated flow interfaces to analyze unsaturated zone processes (important to environmental engineers) and two-phase flow (of particular interest in the petroleum industry as well as steam-liquid systems). Important in subsurface flow, the heat transfer and chemical transport interfaces explicitly account for physics in the liquid, solid, and gas phases.

The physics interfaces, options, and functions are tailored to account for subsurface flow and geologic process terminology. The Heat Transfer interfaces, for example, include features to superpose a background geotherm without incorporating it as a

boundary condition. These physics interfaces also include options to automate the calculation of effective thermal properties for multicomponent systems.

The Fluid Flow branch represent a wide range of possibilities. The Richards' Equation interface describes nonlinear flow in variably saturated porous media. The options for saturated porous media include the Darcy's Law interface for slow flow and the Brinkman Equations interface where shear is nonnegligible. The Laminar Flow interface uses the Navier–Stokes equations to cover free flows and the Fracture Flow interface is used for modeling flow in fractures.

The module also treats the transport of chemicals and their reactions with the Chemical Species Transport branch. The Transport of Diluted Species in Porous Media interface account for chemical transfer in solid, liquid, and gas phases for free, saturated, and variably saturated fluid flows. The Subsurface Flow Module Application Library has a number examples linking these physics interfaces together.

The Subsurface Flow Module Physics Interface Guide

The table below lists all the physics interfaces specifically available with this module in addition to the core physics interfaces included in COMSOL Multiphysics. The Subsurface Flow Module enhances the COMSOL Multiphysics basic interfaces regarding the “Transport of Diluted Species” interface and adds the Poroelasticity interface to structural mechanics.

The Subsurface Flow Module has a set of physics interfaces adopted for a broad category of simulations important in earth investigations. These physics interface and models can serve equally well as tools for research, applied investigations, and classroom education.

The predefined physics interfaces available in this module cover three main application areas. The Subsurface Flow Module interfaces are available in 2D, 2D axisymmetry, and 3D. The physics interfaces for heat transfer and species transport are also available in 1D and 1D axisymmetry. All physics interfaces are available for both stationary and time-dependent analyses. The physics interfaces can be combined to model, for

example, reacting systems and nonisothermal flow. Alternatively, predefined multiphysics interfaces can be used.

	In the <i>COMSOL Multiphysics Reference Manual</i>: • Studies and Solvers • The Physics Interfaces • For a list of all the core physics interfaces included with a COMSOL Multiphysics license, see Physics Interface Guide.

PHYSICS INTERFACE	ICON	TAG	SPACE DIMENSION	AVAILABLE STUDY TYPE
 Chemical Species Transport				
Transport of Diluted Species ¹		tds	all dimensions	stationary; time dependent
Transport of Diluted Species in Porous Media		tds	all dimensions	stationary; time dependent
Transport of Diluted Species in Fractures		dsf	3D, 2D, 2D axisymmetric	stationary; time dependent
 Reacting Flow				
Laminar Flow, Diluted Species ¹		—	3D, 2D, 2D axisymmetric	stationary; time dependent
 Reacting Flow in Porous Media				
Transport of Diluted Species		—	3D, 2D, 2D axisymmetric	stationary; time dependent
 Fluid Flow				
 Single-Phase Flow				
Creeping Flow		spf	3D, 2D, 2D axisymmetric	stationary; time dependent

PHYSICS INTERFACE	ICON	TAG	SPACE DIMENSION	AVAILABLE STUDY TYPE
Laminar Flow ¹		spf	3D, 2D, 2D axisymmetric	stationary; time dependent
 Multiphase Flow				
 <i>Phase Transport</i>				
Phase Transport		phtr	all dimensions	stationary; time dependent
Phase Transport in Porous Media		phtr	all dimensions	stationary; time dependent
Phase Transport in Free and Porous Media Flow		phtr	all dimensions	stationary; time dependent
 Porous Media and Subsurface Flow				
Brinkman Equations		br	3D, 2D, 2D axisymmetric	stationary; time dependent
Darcy's Law		dl	all dimensions	stationary; time dependent
Fracture Flow		ff	3D, 2D, 2D axisymmetric	stationary; time dependent
Richards' Equation		dl	all dimensions	stationary; time dependent
Multiphase Flow in Porous Media		—	all dimensions	stationary; time dependent
Two-Phase Darcy's Law		tpdl	3D, 2D, 2D axisymmetric	stationary; time dependent
Free and Porous Media Flow, Brinkman		fp	3D, 2D, 2D axisymmetric	stationary; time dependent
Free and Porous Media Flow, Darcy		—	3D, 2D, 2D axisymmetric	stationary; time dependent

PHYSICS INTERFACE	ICON	TAG	SPACE DIMENSION	AVAILABLE STUDY TYPE
 Nonisothermal Flow				
Brinkman Equations		—	3D, 2D, 2D axisymmetric	stationary; time dependent; stationary, one-way NITF; time dependent, one-way NITF
 Shallow Water Equations				
Shallow Water Equations, Time Explicit		swe	1D and 2D	time dependent
 Heat Transfer				
Heat Transfer in Solids		ht	all dimensions	stationary; time dependent
Heat Transfer in Fluids ¹		ht	all dimensions	stationary; time dependent
Heat Transfer in Solids and Fluids ¹		ht	all dimensions	stationary; time dependent
Heat Transfer in Fractures		htlsh	3D, 2D, 2D axisymmetric	stationary; time dependent; thermal perturbation, frequency domain
Heat Transfer in Porous Media		ht	all dimensions	stationary; time dependent
 Structural Mechanics				
 Poroelasticity				
Poroelasticity, Solid ²		—	3D, 2D, 2D axisymmetric	stationary; time dependent
¹ This physics interface is included with the core COMSOL Multiphysics software but has added functionality for this module.				
² This physics interface is a predefined multiphysics coupling that automatically adds all the physics interfaces and coupling features required.				

Common Physics Interface and Feature Settings and Nodes

There are several common settings and sections available for the physics interfaces and feature nodes. Some of these sections also have similar settings or are implemented in the same way no matter the physics interface or feature being used. There are also some physics feature nodes that display in COMSOL Multiphysics.

In each module's documentation, only unique or extra information is included; standard information and procedures are centralized in the *COMSOL Multiphysics Reference Manual*.



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Subsurface Flow Module Study Availability

All the physics interfaces included with the module have the Stationary and Time Dependent preset studies available. Custom studies are also available based on the physics interface.



[Studies and Solvers](#) and [The Physics Interfaces](#) in the *COMSOL Multiphysics Reference Manual*

The Liquids and Gases Materials Database

The Liquids and Gases material database provides temperature-dependent fluid dynamic and thermal properties such as dynamic viscosity, heat capacity at constant pressure, density, and thermal conductivity.



For detailed information, see [Liquids and Gases Material Library](#) and [Materials](#) in the *COMSOL Multiphysics Reference Manual*.

Where Do I Access the Documentation and Application Libraries?

A number of online resources have more information about COMSOL, including licensing and technical information. The electronic documentation, topic-based (or context-based) help, and the Application Libraries are all accessed through the COMSOL Desktop.



If you are reading the documentation as a PDF file on your computer, the [blue links](#) do not work to open an application or content referenced in a different guide. However, if you are using the Help system in COMSOL Multiphysics, these links work to open other modules, application examples, and documentation sets.

THE DOCUMENTATION AND ONLINE HELP

The *COMSOL Multiphysics Reference Manual* describes the core physics interfaces and functionality included with the COMSOL Multiphysics license. This book also has instructions on how to use COMSOL Multiphysics and how to access the electronic Documentation and Help content.

Opening Topic-Based Help

The Help window is useful as it is connected to the features in the COMSOL Desktop. To learn more about a node in the Model Builder, or a window on the Desktop, click to highlight a node or window, then press F1 to open the Help window, which then displays information about that feature (or click a node in the Model Builder followed by the **Help** button (). This is called *topic-based* (or *context*) *help*.

Win

To open the **Help** window:

- In the **Model Builder**, **Application Builder**, or **Physics Builder**, click a node or window and then press F1.
- On any toolbar (for example, **Home**, **Definitions**, or **Geometry**), hover the mouse over a button (for example, **Add Physics** or **Build All**) and then press F1.
- From the **File** menu, click **Help** ().
- In the upper-right corner of the COMSOL Desktop, click the **Help** () button.

Mac Linux	To open the Help window: • In the Model Builder or Physics Builder, click a node or window and then press F1. • In the main toolbar, click the Help () button. • From the main menu, select Help > Help.

Opening the Documentation Window

Mac Linux	To open the Documentation window: • Press Ctrl+F1. • In the main toolbar, click the Documentation () button. • From the main menu, select Help > Documentation.

THE APPLICATION LIBRARIES WINDOW

Each model or application includes documentation with the theoretical background and step-by-step instructions to create a model or application. The models and applications are available in COMSOL Multiphysics as MPH files that you can open for further investigation. You can use the step-by-step instructions and the actual models as templates for your own modeling. In most models, SI units are used to describe the relevant properties, parameters, and dimensions, but other unit systems are available.

Once the Application Libraries window is opened, you can search by name or browse under a module folder name. Click to view a summary of the model or application and its properties, including options to open it or its associated PDF document.

Opening the Application Libraries Window

To open the **Application Libraries** window ():

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Learning Center	www.comsol.com/support/learning-center
Support Knowledge Base	www.comsol.com/support/knowledgebase

Overview of the User's Guide

The *Subsurface Flow Module User's Guide* gets you started with modeling using COMSOL Multiphysics. The information in this guide is specific to this module. Instructions how to use COMSOL in general are included with the *COMSOL Multiphysics Reference Manual*.



As detailed in the section [Where Do I Access the Documentation and Application Libraries?](#) this information can also be searched from the COMSOL Multiphysics software **Help** menu.

TABLE OF CONTENTS AND INDEX

To help you navigate through this guide, see the [Contents](#) and [Index](#).

THE SINGLE-PHASE FLOW BRANCH

The single-phase flow interfaces Laminar Flow and Creeping Flow are described in Chapter [Single- and Multiphase Flow Interfaces](#).

THE MULTIPHASE FLOW BRANCH

[The Phase Transport Interfaces](#) section describes the transport of multiple immiscible phases in free and porous media flow.

THE POROUS MEDIA FLOW BRANCH

[Porous Media and Subsurface Flow Interfaces](#) chapter describes the following physics interfaces and includes the underlying theory for each physics interface at the end of the chapter.

- The Porous Media and Subsurface Flow group of interfaces estimates the pressure and velocity of fluids flowing in free systems and within the interstices of a porous medium. [Theory for the Darcy's Law Interface](#) helps you choose the right physics interface to use.
- [The Darcy's Law Interface](#) is for flow in porous media that is relatively slow.
- [The Two-Phase Darcy's Law Interface](#) to simulate two-phase slow flow through interstices in a porous medium.
- [The Richards' Equation Interface](#) is for variably saturated systems.

- [The Fracture Flow Interface](#) is a special application of Darcy’s law for modeling flow in fractures.
- [The Brinkman Equations Interface](#) is for fast flow in porous media flow.
- [The Free and Porous Media Flow, Brinkman Interface](#) and [The Free and Porous Media Flow, Darcy Interface](#) are for modeling fluid flow in systems with both free and porous media flow.

THE SHALLOW WATER EQUATIONS BRANCH

The [Shallow Water Equations Interface](#) chapter describes [The Shallow Water Equations, Time Explicit Interface](#) and includes the underlying theory. It is suitable in cases where the horizontal length scale is much greater than the vertical length scale.

THE CHEMICAL SPECIES TRANSPORT BRANCH

[Chemical Species Transport Interfaces](#) chapter describes the physics interfaces found under the Chemical Species Transport branch when adding a physics interface. The [Transport of Diluted Species](#) interface is used to compute the concentration field of a dilute solute in a solvent. Transport and reactions of the species dissolved in a gas, liquid, or solid can be computed.

[The Transport of Diluted Species in Porous Media Interface](#) characterizes the fate and transport of individual or multiple and interacting chemical species for systems containing fluids, solids, and gases. Theory for the physics interfaces is included at the end of the chapter.

[The Transport of Diluted Species in Fractures Interface](#) is used to model the transport of a solute species along thin fractures. The interface takes into account diffusion, dispersion, convection, and chemical reactions in fractures. The fractures are defined by boundaries in 2D and 3D, and the solute species is assumed to be diluted in a solvent. The mass transport equation solved along the fractures is the tangential differential form of the convection–diffusion–reaction equation.

THE HEAT TRANSFER BRANCH

[Heat Transfer Interfaces](#) chapter describe the group of interfaces that estimate the temperature distribution in solids, fluids, and fluid–solid systems. [The Mechanisms for Heat Transfer](#) helps you choose the physics interface to use. It includes physics interfaces to estimate effective properties in multicomponent systems. All heat transfer interfaces come with interfaces to account for a geotherm brought about through radiogenic decay.

The Heat Transfer Interface models heat transfer by conduction and convection. Surface-to-ambient radiation effects around edges and boundaries can also be included. The physics interfaces are available in 1D, 2D, and 3D and for axisymmetric models with cylindrical coordinates in 1D and 2D.

[The Heat Transfer in Porous Media Interface](#) lets you describe heat transferred both with and without flowing fluids. You can define the velocity in the convective term with any of the flow equations just mentioned or set it with an arbitrary expression. With convective heat transfer, the effective thermal properties also include an option to estimate the dispersion or spreading of heat from small-scale velocity variations.



When you have the Subsurface Flow Module plus the Chemical Reaction Engineering and/or Heat Transfer Modules, the Heat Transfer interface also has extended features available.

THE MULTIPHYSICS COUPLING NODES

In the [Multiphysics Interfaces and Couplings](#) chapter the predefined multiphysics interfaces are introduced.

[The Poroelasticity, Solid Interface](#) chapter describes the physics interface for Biot's poroelasticity, and combines Darcy's law with solid mechanics to provide suitable settings to describe the interaction between porous media and fluids.

[The Multiphase Flow in Porous Media Interface](#) combines the functionality of the Darcy's Law and Phase Transport in Porous Media interfaces.

[The Reacting Flow in Porous Media Interface](#) combines the Brinkman Equations and the Transport of Diluted Species in Porous Media interfaces.

[The Nonisothermal Flow, Brinkman Equations Interface](#) couples porous media flow and heat transfer by combining the Brinkman Equations and the Heat Transfer in Porous Media interfaces.

Subsurface Flow Modeling

A variety of modeling techniques are discussed in the following sections:

- [Where Do I Start with Subsurface Flow Modeling?](#)
- [Selecting the Right Fluid Flow Interface](#)
- [Note About the Groundwater Flow Equation](#)
- [Dual Porosity & Dual Permeability Modeling](#)
- [Pressure, Pressure Head, and Hydraulic Head Formulation](#)
- [Gravity Effects](#)

Where Do I Start with Subsurface Flow Modeling?

Before beginning with modeling, it is important to be aware of the following questions:

- Which goal do I pursue with the simulation?
- What accuracy is required from the results?
- Which physical effects play a role?

The first question refers to the results that the simulation should deliver. The system under consideration is often very complex in structure and the physical processes involved. Not everything is relevant for the specific task. Together with the second and third question, the simulation model can then be reduced to the most essential.

Often a 2D or 2D axial symmetric model, sometimes even a 1D model is sufficient. Especially in subsurface flow problems, one has to deal with very large areas. The question is to what extent small-scaled structures are relevant for the result and therefore have to be represented in the geometry. The physical effects to be considered also depend on how accurate the results should be: Does gravity play a role or do I have to take temperature fluctuations into account? These questions should be answered before starting the simulation.

The accuracy of the results also depends on the accuracy of the input quantities. Especially in geophysical questions the exact structure, the material and its properties are not exactly known. Therefore the result cannot be more exact than the accuracy of these and it is not necessary to represent every smallest detail in the model.

Regardless of the above answers, it is always advisable to make the models successively more complex. Complex 3D models are always significantly more computationally demanding and often make high demands on the meshing, whereas in 2D models the focus can be placed on the physical processes and effects. This implies using a simplified model as a starting point in the project. Complexities can then be introduced gradually so that the effect of each refinement of the model description is well understood before introducing new complexities.

Selecting the Right Fluid Flow Interface

The Fluid Flow physics features included with the Subsurface Flow Module are used to characterize movements of liquids, gases, and other flowing media, such as molten rock. The physics interfaces are set up using terms like pressure and hydraulic heads, that are familiar to hydrologists.

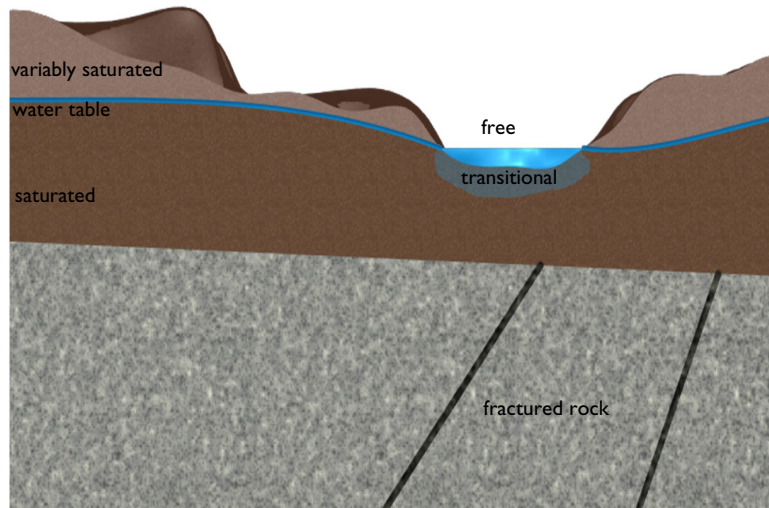


Figure 2-1: Different areas of subsurface flow regimes.

Saturated porous media flow can be modeled through either Darcy's Law or Brinkman's extension to Darcy's Law, depending on the size of the pores involved. If the pore is such that viscous effects to the fluid flow can be ignored, then Darcy's Law can be used, and flow is described exclusively by the pressure variable. If the size of the pores is such that the fluid can impose momentum changes through shear effects, then the Brinkman equations are suitable. They solve for the same variables as the Navier–

Stokes equations, but include terms that consider the porosity of the media the fluid is flowing through.



Read more [About Darcian and Non-Darcian Flow](#) and [Permeability Models](#) in the *Porous Media Flow User's Guide*.

In variably saturated porous media flow (usually above the water table), hydraulic properties may change as fluids move through the medium, filling some pores and draining others. Richards' equation is employed to model this type of flow, and the van Genuchten and Brooks and Corey formulations can be applied to take retention in the pores into account. Density, dynamic viscosity, saturated and residual liquid fraction, hydraulic conductivities, and storage models can be provided.

The Fracture Flow interface solves for pressure on internal (2D) boundaries within a 3D matrix and is automatically coupled to the physics describing the porous media flow in the surrounding matrix. This approximation saves you from meshing the actual fractures and the computational resources this entails. If a fluid happens to flow from one medium to the other (and back) in the same model, all of the porous media physics are automatically coupled to descriptions of free flow within the Subsurface Flow Module.

The Navier–Stokes equation describes free flow of liquid. In the above picture it is valid to model the flow in the river bank but it can also be used to model flow through the porous medium on the microscopic scale.

If more than one phase is present in the porous medium, such as in oil reservoirs, where also the Richard's Equation is not valid the flow can be modeled using the Multiphase flow in Porous Media interface which couples Darcy's Law with a transport equation for the phases.

TABLE 2-1: POROUS MEDIA FLOW INTERFACES.

PHYSICS INTERFACE	FLOW REGIME	EQUATION
Laminar Flow	Free Flow	Navier–Stokes Equation
Free and Porous Media Flow, Brinkman	Free flow and fast flow in porous media	Navier–Stokes Equation in free flow Brinkman equations in porous media
Free and Porous Media Flow, Darcy	Free flow and slow flow in porous media	Navier–Stokes Equation in free flow Darcy's Law in porous media

TABLE 2-1: POROUS MEDIA FLOW INTERFACES.

PHYSICS INTERFACE	FLOW REGIME	EQUATION
Brinkman Equations	Fast flow in porous media	Brinkman equations
Darcy's Law	Slow flow in porous media	Darcy's Law
Richards' Equation	Variably saturated porous media	Richards' equation
Fracture Flow	Flow along surfaces of fractures	Darcy's Law
Multiphase Flow in Porous Media	Flow of more than one fluid through a porous medium	Darcy's Law Transport equation for each phase
Two-Phase Darcy's Law	Two-phase flow in porous media	Darcy's law Transport equation

Note About the Groundwater Flow Equation

In the area of hydrogeology, the groundwater flow equation is commonly used to describe the groundwater flow through an aquifer. In COMSOL Multiphysics, equation-based models of physical processes are solved by ensuring a balance of modeled quantities, such as mass or energy.

The derivation of the groundwater flow equations starts with the mass balance in a representative elementary volume (REV). This entails ensuring that the fluxes into and out of the REV are in equilibrium with the mass stored within the volume. This relationship can be expressed as follows:

$$q_{\text{in}} = q_{\text{out}} + \Delta S$$

with S denoting the storage within the volume.

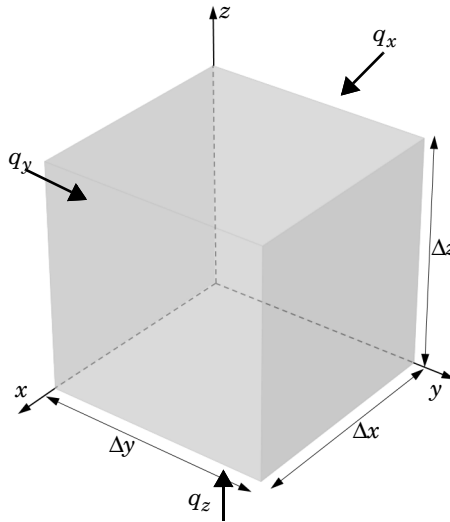


Figure 2-2: Fluxes through an elementary volume.

The flux in the x direction is:

$$(q_{x, \text{out}} - q_{x, \text{in}}) \Delta y \Delta z = \frac{(q_{x, \text{out}} - q_{x, \text{in}})}{\Delta x} \Delta x \Delta y \Delta z$$

or in differential form

$$\frac{\partial q_x}{\partial x} \Delta x \Delta y \Delta z$$

The flux balance in the y and z directions are defined accordingly and the total flux balance in the volume is

$$\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) \Delta x \Delta y \Delta z = \Delta S \quad (2-1)$$

The specific storage S_s is defined as

$$S_s = \frac{\Delta V_w}{\Delta h \Delta x \Delta y \Delta z}$$

where Δh denotes the change in hydraulic head, a specific measure of fluid pressure. The rate of change in volume of water V_w then is

$$\frac{\Delta V_w}{\Delta t} = -S_s \frac{\Delta h}{\Delta t} \Delta x \Delta y \Delta z$$

together with [Equation 2-1](#), the definition of the flux $\mathbf{q} = \rho \mathbf{u}$, and accounting for source/sink terms Q_m this leads to the formulation of the groundwater flow equation

$$S_s \frac{\partial h}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (2-2)$$

In the COMSOL Multiphysics® software [Equation 2-2](#) is formulated in terms of pressure within the Darcy's Law interface (See [Pressure, Pressure Head, and Hydraulic Head Formulation](#)). Use the Storage Model to include the Storage term on the left hand side of the equation and add a mass source to include the sink term on the right hand side.

HYDRAULIC CONDUCTIVITY

The velocity $\mathbf{u}$ in [Equation 2-2](#) is commonly described by the formulation of Darcy law in terms of the gradient of the hydraulic head as follows:

$$\mathbf{u} = -K \nabla h \quad (2-3)$$

where $K(\text{m/s})$ is the hydraulic conductivity which is primarily used in hydrology applications where the fluid is water. It is related to the permeability according to

$$K = \frac{k \rho g}{\mu}$$

Note that the hydraulic conductivity is a property of both the porous matrix and the fluid, whereas the permeability describes solely the properties of the fluid.

Dual Porosity & Dual Permeability Modeling

The concept of dual porosity and dual permeability is a valuable framework used to describe the flow of fluids within porous media, taking into account the existence of two distinct types of pores. Typically, one type of pore is significantly smaller and is referred to as “micropores”, while the other type is notably larger and is known as “macropores”. This concept finds widespread application in various fields, including subsurface hydrology, petroleum engineering, environmental science, and geology, aiding in the comprehension of fluid movement within porous materials. For instance, it is particularly relevant in scenarios involving fissured rocks or the upper soil layer, where macropores may resemble wormholes.

While the terms “dual porosity” and “dual permeability” are occasionally employed interchangeably and inconsistently in the literature, within COMSOL Multiphysics, three distinct features are provided under the following names:

- **Dual Porosity Medium:** Use this feature when the flow through macropores is governed by Darcy’s law and the micropores do not significantly contribute to the overall flow in the porous medium.
- **Dual Permeability Medium:** Use this feature when both macro- and micropores significantly influence the flow field, with both following Darcy’s law.
- **Unsaturated Dual Permeability Medium:** Use this feature for unsaturated porous media, where both macro- and micropores play a role in fluid flow, and their behavior is governed by Richards’ Equation.

DUAL POROSITY MEDIUM

Here, the flow through macropores is typically governed by Darcy’s law where the micropores do not significantly contribute to the overall flow in the porous medium. This assumption can be made because the macropores, due to their larger size and higher permeability, dominate the flow behavior. While micropores may not actively participate in the flow, they are considered in terms of fluid exchange with the macropores.

In practical terms, the **Dual Porosity Medium** is employed when you have a porous medium where most of the flow occurs through a well-connected network of macropores, and the contribution of the smaller micropores can be safely neglected.

This simplifies the modeling process while still allowing you to capture the essential features of the system's behavior. It is a valuable tool in various fields, including hydrogeology, where it can be used to simulate the movement of groundwater in complex geological formations.

DUAL PERMEABILITY MEDIUM

This feature can be applied in scenarios where the porous medium is saturated with a fluid. One key difference between dual permeability and dual porosity is that, in a dual permeability medium, both macro- and micropores contribute significantly to the flow field. This means that Darcy's law, which governs fluid flow through porous media, is applied to both pore types. In other words, the flow rates through both macropores and micropores are considered. It can be applied to situations where both pore types have varying permeabilities. This is in contrast to the dual porosity approach where it is assumed that one material is highly permeable and the other is not.

In practical terms, the **Dual Permeability Medium** is used when modeling saturated fluid flow in a porous medium with a complex internal structure that includes both micropores and macropores, and where both pore types significantly influence the overall flow behavior. This modeling approach is particularly valuable in fields such as subsurface hydrology, petroleum reservoir engineering, and geology, where reservoirs or aquifers exhibit dual permeability characteristics.

UNSATURATED DUAL PERMEABILITY MEDIUM

This feature extends the concept of having two types of pores within the porous medium. Fluid flow through macro- and micropores is described by Richards' Equation and is solved separately for both the macropores and micropores, accounting for the differing flow dynamics in each. This accounts for the heterogeneous nature of unsaturated porous media which is particularly valuable for simulating real-world subsurface environments where the presence of both matrix and fractures/channels significantly affects fluid flow, such as in soil profiles or fractured rock formations. It provides a more realistic representation of fluid movement in these environments compared to models that assume homogeneous behavior.

In summary, the **Unsatuated Dual Permeability Medium** is designed to address the complexities of fluid flow in unsaturated porous media by considering the presence of both macro- and micropores, each governed by Richards' Equation. This approach enhances the accuracy and realism of subsurface simulations in situations where unsaturated conditions are prevalent, leading to more informed decisions in various scientific and engineering applications.

Pressure, Pressure Head, and Hydraulic Head Formulation

Within COMSOL Multiphysics, all equations are formulated in terms of pressure. When working with the Subsurface Flow Module, you have the flexibility to define pressure, hydraulic head, or pressure head as boundary conditions for your model. Similarly, you can employ any of these three variables throughout your analysis and postprocessing. It is important to note that these formulations are interchangeable, and the software internally handles the conversion between them. For instance, if you specify a boundary condition in terms of pressure or hydraulic head, the software seamlessly translates it into the corresponding pressure variable for computation.

Hydraulic head (also known as piezometric head), $H(\text{m})$, and pressure head, $H_p(\text{m})$, relate to pressure p as

$$\begin{aligned}H_p &= \frac{p}{\rho g} \\H &= H_p + D\end{aligned}$$

where ρ is the fluid density, g denotes the acceleration of gravity, and D is the elevation or elevation head. When defined as vertical elevation, the horizontal gradients in D equal zero and vertical gradients in D equal one. Turning off gravity effects sets the elevation D to zero, and the hydraulic head equals the pressure head.

Gravity Effects

For large-scale applications it might be necessary to take gravity effects into account. Darcy's law then applies when the gradient in hydraulic potential drives fluid movement in the porous medium. In this case Darcy's law can be written as follows:

$$\mathbf{u} = -\frac{\kappa}{\mu}(\nabla p - \rho \mathbf{g})$$

In this equation $\mathbf{g}$ is the vector of gravitational acceleration. The hydraulic potential in the equation comes from the pore pressure, p , and gravity, $\rho \mathbf{g}$.

Furthermore, in the *Subsurface Flow Module*, $\mathbf{g}$ can be defined as using the elevation D according to

$$\mathbf{g} = -g \nabla D$$

where g is the magnitude of gravitational acceleration. The direction of the elevation is defined as

$$\mathbf{e}_D = \frac{\mathbf{g}}{\|\mathbf{g}\|}$$

If a reference position $\mathbf{r}_{\text{ref}}$ is specified the elevation is determined using

$$D = \mathbf{e}_D \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}})$$

The choice of D has a significant impact on results and the physics involved. For example, if D is the vertical coordinate z and if the flow is entirely horizontal within the x - y -plane, then the gradient in D vanishes and the driving force is caused by pressure gradients alone.

Single- and Multiphase Flow Interfaces

This chapter describes the interfaces found under the **Fluid Flow > Single-Phase Flow** branch ()

- [The Laminar Flow and Creeping Flow Interfaces](#)
- [Theory for the Single-Phase Flow Interfaces.](#)

Under the **Fluid Flow > Multiphase Flow** branch () the following interfaces are available:

- The Phase Transport Interface
- The Phase Transport in Porous Media Interface
- The Phase Transport in Free and Porous Media Flow Interface

They are fully described in Section [The Phase Transport Interfaces](#) of the *CFD Module User's Guide*.

The Laminar Flow and Creeping Flow Interfaces

In this section:

- [The Creeping Flow Interface](#)
- [The Laminar Flow Interface](#)
- [Domain, Boundary, Pair, and Point Nodes for Single-Phase Flow](#)



- [Theory for the Single-Phase Flow Interfaces](#)
-

In the *COMSOL Multiphysics Reference Manual*, see [Table 2-4](#) for links to common sections such as **Discretization**, **Consistent Stabilization**, **Inconsistent Stabilization**, and **Advanced Settings** sections, some of them accessed by clicking the **Show** button () and choosing the applicable option. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

The Creeping Flow Interface

The **Creeping Flow (spf)** interface () is used for simulating fluid flows at very low Reynolds numbers for which the inertial term in the Navier–Stokes equations can be neglected. Creeping flow, also referred to as Stokes flow, occurs in systems with high viscosity or small geometrical length scales (for example, in microfluidics and MEMS devices). The fluid can be compressible or incompressible, as well as Newtonian or non-Newtonian.

The equations solved by the Creeping Flow interface are the Stokes equations for conservation of momentum and the continuity equation for conservation of mass.

The Creeping Flow interface can be used for stationary and time-dependent analyses.

The main feature is the **Fluid Properties** node, which adds the Stokes equations and provides an interface for defining the fluid material and its properties. Except where noted below, see [The Laminar Flow Interface](#) for all other settings.

PHYSICAL MODEL

By default, the **Neglect inertial term (Stokes flow)** checkbox is selected. If cleared, the inertial terms are included in the computations.

DISCRETIZATION

By default, the Creeping Flow interface uses **P2+P1** elements. Contrary to general laminar and turbulent single-phase flow simulations employing purely linear P1+P1 elements, P2+P1 elements are well suited for most creeping flow simulations but are not in general fully conservative. Full conservation properties and high accuracy is obtained by selecting P2+P2 or P3+P3 elements.

Note that P1+P1, P2+P2, and P3+P3 discretizations (the so-called equal order interpolations) require streamline diffusion to be active.

	• The Laminar Flow Interface • Theory for the Single-Phase Flow InterfacesTheory for the Single-Phase Flow Interfaces
	<i>Pore-Scale Flow</i> : Application Library path Subsurface_Flow_Module/Fluid_Flow/pore_scale_flow

The Laminar Flow Interface

The **Laminar Flow (spf)** interface () is used to compute the velocity and pressure fields for the flow of a single-phase fluid in the laminar flow regime. A flow remains laminar as long as the Reynolds number is below a certain critical value. At higher Reynolds numbers, disturbances have a tendency to grow and cause transition to turbulence. This critical Reynolds number depends on the model, but a classical example is pipe flow, where the critical Reynolds number is known to be approximately 2000.

The physics interface supports incompressible flow, weakly compressible flow (the density depends on temperature but not on pressure), and compressible flow at low Mach numbers (typically less than 0.3). It also supports flow of non-Newtonian fluids.

The equations solved by the Laminar Flow interface are the Navier–Stokes equations for conservation of momentum and the continuity equation for conservation of mass.

The Laminar Flow interface can be used for stationary and time-dependent analyses. Time-dependent studies should be used in the high-Reynolds number regime as these flows tend to become inherently unsteady.

When the Laminar Flow interface is added, the following default nodes are also added in the **Model Builder: Fluid Properties**, **Wall** (the default boundary condition is **No slip**), and **Initial Values**. Other nodes that implement, for example, boundary conditions and volume forces, can be added from the **Physics** toolbar or from the context menu displayed when right-clicking **Laminar Flow**.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Physics interface variables can be referred to using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `spf`.

About the Physics Interface Label Names

The label for a single-phase flow interface (the node name in the Model Builder) is dynamic and is reset according to the turbulence model and the neglect inertial term (Stokes flow) property set at the physics interface level. The availability also depends on the software license.

If the **Neglect inertial term (Stokes flow)** checkbox is selected, then the Label changes to **Creeping Flow**, which is the same Label that displays when that interface is added from the Model Wizard or Add Physics window.

PHYSICAL MODEL

Compressibility

Depending of the fluid properties and the flow regime, three options are available for the **Compressibility** option. In general the computational complexity increases from **Incompressible flow** to **Weakly compressible flow** to **Compressible flow (Ma<0.3)** but the underlying hypotheses are increasingly more restrictive in the opposite direction.

When the **Incompressible flow** option (default) is selected, the incompressible form of the Navier–Stokes and continuity equations is applied. In addition, the fluid density is

evaluated at the **Reference pressure level** defined in this section. The **Reference temperature** is set to 293.15 K.

The **Weakly compressible flow** option models compressible flow when the pressure dependency of the density can be neglected. When selected, the compressible form of the Navier–Stokes and continuity equations is applied. In addition, the fluid density is evaluated at the **Reference pressure level** defined in this section.

When the **Compressible flow (Ma<0.3)** option is selected, the compressible form of the Navier–Stokes and continuity equations is applied. $Ma < 0.3$ indicates that the inlet and outlet conditions, as well as the stabilization, may not be suitable for transonic and supersonic flow. For more information, see [The Mach Number Limit](#).

Neglect Inertial Term (Stokes Flow)

For low Reynolds number flows, the inertial terms in the Navier–Stokes equations may be neglected.

Porous Media Domains

With the addition of various modules, the **Enable porous media domains** checkbox is available. Selecting this option, a **Fluid and Matrix Properties** node, a **Mass Source** node, and a **Forchheimer Drag** subnode are added to the physics interface. These are described for the **Brinkman Equations** interface in the respective module’s documentation. The **Fluid and Matrix Properties** can be applied on all domains or on a subset of the domains.

Porous Treatment of No Slip Condition

Porous treatment of no slip condition is available when **Enable porous media domains** is activated. It controls how the **No Slip** condition on **Wall** boundaries and **Interior Wall** boundaries adjacent to **Porous Medium** should be treated. The options are **Standard no slip formulation** (default) and **Porous slip**. The latter option provides a unified treatment when the porous matrix is fully resolved as well as when it is under resolved ensuring a smooth transition between regions with different resolutions; see No Slip under [Wall](#).

Include Gravity

When the **Include gravity** checkbox is selected, a global **Gravity** feature is shown in the interface model tree, and the buoyancy force is included in the Navier–Stokes equations.

Also, when the **Include gravity** checkbox is selected, the **Use reduced pressure** option changes the pressure formulation from using the total pressure (default) to using the reduced pressure. This option is suitable for configurations where the density changes are very small; otherwise, the default formulation can be used. For more information, see [Gravity](#)

Reference Values

Reference values are global quantities used to evaluate the density of the fluid when the **Incompressible flow** or the **Weakly compressible flow** option is selected.

Reference pressure level There are generally two ways to include the pressure in fluid flow computations: either to use the absolute pressure $p_A = p + p_{\text{ref}}$, or the gauge pressure p . When p_{ref} is nonzero, the physics interface solves for the gauge pressure whereas material properties are evaluated using the absolute pressure. The reference pressure level is also used to define the reference density.

Reference temperature The reference temperature is used to define the reference density.

Reference position When **Include gravity** or **Rotating frame** is selected, the reference position can be defined. It corresponds to the location where the total pressure (that includes the hydrostatic pressure) is equal to the **Reference pressure level**.

DEPENDENT VARIABLES

The following dependent variables (fields) are defined for this physics interface — the **Velocity field $\mathbf{u}$** and its components, and the **Pressure p** .

If required, the names of the field, component, and dependent variable can be edited. Editing the name of a scalar dependent variable changes both its field name and the dependent variable name. If a new field name coincides with the name of another field of the same type, the fields share degrees of freedom and dependent variable names. A new field name must not coincide with the name of a field of another type or with a component name belonging to some other field. Component names must be unique within a model except when two fields share a common field name.

CONSISTENT STABILIZATION

To enable this section, click the **Show More Options** button () and select **Stabilization** in the **Show More Options** dialog.

There are two *consistent stabilization methods*: **Streamline diffusion** and **Crosswind diffusion**. Usually, both checkboxes for these methods are selected by default and should remain selected for optimal performance. Consistent stabilization methods do not perturb the original transport equation. Streamline diffusion must be selected when using equal-order interpolation for pressure and velocity.

Select the **Use dynamic subgrid time scale** checkbox to estimate the influence of temporal variations in the velocity field, instead of using the inverse time step, in the consistent stabilization terms. This checkbox is selected by default

Select the **Limit small time steps effect on stabilization time scale** checkbox to prevent loss of stabilization at small time steps.

INCONSISTENT STABILIZATION

To enable this section, click the **Show More Options** button () and select **Stabilization** in the **Show More Options** dialog.

There is usually just one *inconsistent stabilization method* — **Isotropic diffusion**. This method is equivalent to adding a term to the diffusion coefficient in order to dampen the effect of oscillations by making the system somewhat less dominated by convection. If possible, minimize the use of the inconsistent stabilization method because by using it you no longer solve the original problem. By default, the **Isotropic diffusion** checkbox is not selected because this type of stabilization adds artificial diffusion and affects the accuracy of the original problem. However, this option can be used to get a good initial guess for underresolved problems.

If required, select the **Isotropic diffusion** checkbox and enter a **Tuning parameter** δ_{id} as a scalar positive value. The default value is 0.25 (a reasonable value to start with is roughly 0.5 divided by the element order). A higher value adds more isotropic diffusion.



- [Numerical Stability — Stabilization Techniques for Fluid Flow](#)
-

ADVANCED SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. Normally these settings do not need to be changed.

The **Use pseudo time stepping for stationary equation form** is per default set to **Automatic from physics**. This option can add pseudo time derivatives to the equation when the **Stationary equation** form is used in order to speed up convergence. Pseudo time stepping is triggered when the Laminar Flow interface is selected in some multiphysics coupling features. Set **Automatic from physics** to **On** to apply pseudo time stepping also for laminar flows. Set it to **Off** to disable pseudo time stepping completely.

When **Use pseudo time stepping for stationary equation form** is set to **Automatic from physics** or **On**, a **CFL number expression** should also be defined. For the default **Automatic** option, the local CFL number (from the Courant–Friedrichs–Lewy condition) is determined by a PID regulator.

The **Use Block Navier–Stokes preconditioner** checkbox under **Linear solver** is available when the **Compressibility** option is set to **Incompressible flow**. When this checkbox is selected, the default solver for time-dependent study steps will use the Block Navier–Stokes preconditioner in iterative solvers for the velocity and pressure. Using this preconditioner may result in shorter solution times for large time-dependent problems with high Reynolds numbers.



- [Pseudo Time Stepping for Laminar Flow Models](#)
- [Domain, Boundary, Pair, and Point Nodes for Single-Phase Flow](#)
- [Theory for the Single-Phase Flow Interfaces](#)
- [Block Navier–Stokes](#) in the *COMSOL Multiphysics Reference Manual*.

DISCRETIZATION

The default discretization for Laminar Flow is **P1+P1** elements — that is, piecewise linear interpolation for velocity and pressure. This is suitable for most flow problems.

Some higher-order interpolations are also available, and they can be cost-effective options to obtain high accuracy for flows with low Reynolds numbers.

The **P2+P2** and **P3+P3** options, the equal-order interpolation options, are the preferred higher-order options because they have higher numerical accuracy than the mixed-order options **P2+P1** and **P3+P2**. The equal-order interpolation options do, however, require streamline diffusion to be active.



Flow Past a Cylinder: Application Library path
COMSOL_Multiphysics/Fluid_Dynamics/cylinder_flow

Domain, Boundary, Pair, and Point Nodes for Single-Phase Flow

The following nodes, listed in alphabetical order, are available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or by right-clicking to access the context menu (all users).

- Average Pressure
- Boundary Stress
- Flow Continuity
- Fluid Properties
- Gravity
- Initial Values
- Inlet
- Line Mass Source¹
- Open Boundary
- Outlet
- Periodic Flow Condition
- Point Mass Source¹
- Porous Interface
- Pressure Point Constraint
- Rotating Frame
- Symmetry
- Volume Force
- Wall

¹ A feature that may require an additional license



For 2D axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and adds an **Axial Symmetry** node that is valid on the axial symmetry boundaries only.

In the *COMSOL Multiphysics Reference Manual*, see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.



For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

Fluid Properties

The **Fluid Properties** node adds the momentum and continuity equations solved by the physics interface, except for volume forces, which are added by the [Volume Force](#)

feature. The node also provides an interface for defining the material properties of the fluid.

MODEL INPUTS

Fluid properties, such as density and viscosity, can be defined through user inputs, variables, or by selecting a material. For the latter option, additional inputs, for example temperature or pressure, may be required to define these properties.

Temperature

By default, the **Temperature** model input is set to **Common model input**, and the temperature is controlled from [Default Model Inputs](#) under **Global Definitions** or by a locally defined [Model Input](#). If a Heat Transfer interface is included in the component, it controls the temperature **Common model input**. Alternatively, the temperature field can be selected from another physics interface. All physics interfaces have their own tags (**Name**). For example, if a Heat Transfer in Fluids interface is included in the component, the **Temperature (ht)** option is available for T .

You can also select **User defined** from the **Temperature** model input in order to manually prescribe T .

Absolute Pressure

This input appears when a material requires the absolute pressure as a model input. The absolute pressure is used to evaluate material properties, but it also relates to the value of the calculated pressure field. There are generally two ways to calculate the pressure when describing fluid flow: either to solve for the absolute pressure or for a pressure (often denoted gauge pressure) that relates to the absolute pressure through a reference pressure.

The choice of pressure variable depends on the system of equations being solved. For example, in a unidirectional incompressible flow problem, the pressure drop over the modeled domain is probably many orders of magnitude smaller than the atmospheric pressure, which, when included, may reduce the stability and convergence properties of the solver. In other cases, such as when the pressure is part of an expression for the gas volume or the diffusion coefficients, it may be more convenient to solve for the absolute pressure.

The default **Absolute pressure** p_A is $p + p_{\text{ref}}$, where p is the dependent pressure variable from the Navier–Stokes or RANS equations, and p_{ref} is from the user input defined at the physics interface level. When p_{ref} is nonzero, the physics interface solves for a gauge pressure. If the pressure field instead is an absolute pressure field, p_{ref} should be set to 0.

The **Absolute pressure** field can be edited by clicking **Make All Model Inputs Editable** () and entering the desired value in the input field.



FLUID PROPERTIES

Density

The density can either be specified by a material, or by a **User defined** expression. The density in a material can depend on temperature and pressure, and these dependencies are automatically replaced by p_{ref} for weakly compressible flows and p_{ref} and T_{ref} for incompressible flows (as specified by the **Compressibility** setting at the physics interface level). If density variations with respect to pressure are to be included in the computations, **Compressibility** must be set to compressible. Any dependencies in the density on quantities other than temperature and pressure must be consistent with the **Compressibility** setting at the interface level.

Dynamic Viscosity

The **Dynamic viscosity** μ describes the relationship between the shear rate and the shear stresses in a fluid. Intuitively, water and air have low viscosities, and substances often described as thick (such as oil) have higher viscosities.

Volume Force

The **Volume Force** node specifies the volume force $\mathbf{F}$ on the right-hand side of the momentum equation.

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot \left[-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right] + \mathbf{F}$$

If several volume-force nodes are added to the same domain, then the sum of all contributions are added to the momentum equation.

Initial Values

The initial values serve as initial conditions for a transient simulation or as an initial guess for a nonlinear solver in a stationary simulation. Note that for a transient compressible-flow simulation employing a material for which the density depends on

the pressure (such as air), discontinuities in the initial values trigger pressure waves even when the Mach number is small. The pressure waves must be resolved and this puts a restriction on the time step.

INITIAL VALUES

Initial values or expressions should be specified for the **Velocity field $\mathbf{u}$** and the **Pressure p** .

When **Include gravity** is selected and **Use reduced pressure** not selected in the interface **Physical model** section, the **Compensate for hydrostatic pressure approximation** (named **Compensate for hydrostatic pressure** for compressible flows) checkbox is available and selected by default. When it is selected, the hydrostatic pressure is automatically added to the pressure entered in p user input.

Wall

The **Wall** node includes a set of boundary conditions describing fluid-flow conditions at stationary, moving, and leaking walls. For turbulent flow, the description may involve wall functions and asymptotic expressions for certain turbulence variables.

BOUNDARY CONDITION

Select a **Boundary condition** for the wall.

- [No Slip](#)
- [Slip](#)
- [Leaking Wall](#)

No Slip

No slip is the default boundary condition to model solid walls. A no-slip wall is a wall where the fluid velocity relative to the wall velocity is zero. For a stationary wall that means that $\mathbf{u} = 0$.

The option for **Porous treatment of no slip condition** is available when **Enable porous media domains** is activated in the **Physical Model** section in the settings for the main physics interface node. It specifies how **Wall** boundaries and **Interior Wall** boundaries adjacent to porous domains are treated. When **Standard no slip formulation** is chosen, a common no slip condition is applied on all solid walls. When the default **Porous slip** is chosen, a blending analytic expression is instead applied on the corresponding wall boundaries adjacent to the porous medium domain. It results in a no slip condition in case the porous length scale is fully resolved by the mesh and a slip condition in the

opposite limit when the mesh is much coarser than the porous scale. A smooth transition between these limits is ensured. Note that the interpretation and usage of nonzero slip at the wall is the same as in the **Navier slip** boundary condition. By default, **Velocity formulation** is on and the treatment is based on an approximate reconstruction of the far-field pressure gradient using the slip velocity at the wall. If the **Pressure-gradient formulation** is chosen, the local pressure gradient at the wall is employed.

Slip

The **Slip** option prescribes a no-penetration condition, $\mathbf{u} \cdot \mathbf{n} = 0$. It is implicitly assumed that there are no viscous effects at the slip wall and hence, no boundary layer develops. From a modeling point of view, this can be a reasonable approximation if the main effect of the wall is to prevent fluid from leaving the domain.

Leaking Wall

This boundary condition may be used to simulate a wall where fluid is leaking into or leaving the domain with the velocity $\mathbf{u} = \mathbf{u}_l$ through a perforated wall. The components of the **Fluid velocity** $\mathbf{u}_l$ on the leaking wall should be specified.

WALL MOVEMENT

This section contains controls to describe the wall movement relative to the lab (or spatial) frame.

The **Translational velocity** setting controls the translational wall velocity, $\mathbf{u}_{tr}$. The list is per default set to **Automatic from frame**. The physics automatically detects if the spatial frame moves. This can for example happen if an ALE interface is present in the model component. If there is no movement $\mathbf{u}_{tr} = \mathbf{0}$. If the frame moves, $\mathbf{u}_{tr}$ becomes equal to the frame movement. $\mathbf{u}_{tr}$ is accounted for in the actual boundary condition prescribed in the **Boundary condition** section.

Select **Zero (Fixed wall)** from **Translational velocity** selection list to prescribe $\mathbf{u}_{tr} = \mathbf{0}$.

Select **Manual** from **Translational velocity** selection list in order to manually prescribe **Velocity of moving wall**, $\mathbf{u}_{tr}$. This can for example be used to model an oscillating wall where the magnitude of the oscillations are very small compared to the rest of the model. Specifying translational velocity manually does not automatically cause the associated wall to move. An additional Moving Mesh node needs to be added from Definitions to physically track the wall movement in the spatial reference frame.

The **Sliding wall** option is appropriate if the wall behaves like a conveyor belt with the surface sliding in a tangential direction. A velocity is prescribed at the wall and the boundary itself does not have to actually move relative to the reference frame.

- For 3D components, values or expressions for the **Velocity of sliding wall** $\mathbf{u}_w$ should be specified. If the velocity vector entered is not in the plane of the wall, COMSOL Multiphysics projects it onto the tangential direction. Its magnitude is adjusted to be the same as the magnitude of the vector entered.
- For 2D components, the tangential direction is unambiguously defined by the direction of the boundary. For this reason, the sliding wall boundary condition has different definitions in different space dimensions. A single entry for the **Velocity of the tangentially moving wall** U_w should be specified in 2D.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show More Options** button () and selecting **Advanced Physics Options** in the **Show More Options** dialog. The **Constraints** settings can be set to **Automatic**, **Pointwise constraints**, **Nitsche constraints**, or **Weak constraints**. **Mixed constraints** can be selected when imposing a no slip condition exactly.

Depending on the constraint method selected, the following settings are available:

- **Apply reaction terms on** can be set to **Individual dependent variables** (default) or **All physics (symmetric)**. This setting is not available when **Nitsche constraints** is selected.
- Select **Elemental** (default) or **Nodal** under **Constraint method**. This setting is not available for **Nitsche constraints** or **Weak constraints**.



- [Theory for the Wall Boundary Condition](#)
- [The Moving Mesh Interface](#) in the *COMSOL Multiphysics Reference Manual*

Inlet

This condition should be used on boundaries for which there is a net flow into the domain. To obtain a numerically well-posed problem, it is advisable to also consider the Outlet conditions when specifying an Inlet condition. For example, if the pressure is specified at the outlet, the velocity may be specified at the inlet, and vice versa. Specifying the velocity vector at both the inlet and the outlet may cause convergence difficulties.

BOUNDARY CONDITION

The available **Boundary condition** options for an inlet are **Velocity**, **Fully developed flow**, and **Pressure**. After selecting a **Boundary Condition** from the list, a section with the same or a similar name displays underneath. For example, if **Velocity** is selected, a **Velocity** section, where further settings are defined, is displayed.

VELOCITY

The **Normal inflow velocity** is specified as $\mathbf{u} = -\mathbf{n}U_0$, where $\mathbf{n}$ is the boundary normal pointing out of the domain and U_0 is the normal inflow speed.

The **Velocity field** option sets the velocity vector to $\mathbf{u} = \mathbf{u}_0$. The components of the inlet velocity vector $\mathbf{u}_0$ should be defined for this choice.

PRESSURE CONDITIONS

This option specifies the normal stress, which in most cases is approximately equal to the pressure.

- For incompressible flow, the **Pressure** list has two options, **Static** and **Total**. For weakly compressible and compressible flow, the static pressure should be specified in the text field.
 - If **Pressure** is **Static**, and the reference pressure p_{ref} , defined at the physics interface level, is equal to 0, the value of the pressure p_0 , at the boundary, is the absolute pressure. Otherwise, p_0 is the relative pressure at the boundary.
 - If **Pressure** is **Total**, the **Average** checkbox is available and cleared by default to prescribe the total pressure pointwise. If it is selected, the averaged total pressure is imposed in the weak forms instead.

When **Include gravity** is selected and **Use reduced pressure** not selected in the interface **Physical model** section, the **Compensate for hydrostatic pressure approximation** (named **Compensate for hydrostatic pressure** for incompressible flows) checkbox is available and selected by default. When it is selected, the hydrostatic pressure is automatically added to the pressure entered in p_0 user input.

- The **Suppress backflow** option adjusts the inlet pressure locally in order to reduce the amount of fluid exiting the domain through the boundary. If you clear the suppress backflow option, the inlet boundary can become an outlet depending on the pressure field in the rest of the domain.

- **Flow direction** controls in which direction the fluid enters the domain.
 - For **Normal flow**, it prescribes zero tangential velocity component.
 - For **User defined**, an **Inflow velocity direction** $\mathbf{d_u}$ (dimensionless) should be specified. The magnitude of $\mathbf{d_u}$ does not matter, only the direction. $\mathbf{d_u}$ must point into the domain.

FULLY DEVELOPED FLOW

The **Fully developed flow** option adds contributions to the inflow boundary, which force the flow toward the solution for a fully developed channel flow. The channel can be thought of as a virtual extrusion of the inlet cross section. The inlet boundary must hence be flat in order for the fully developed flow condition to work properly. In 2D axisymmetric models, the inlet normal must be parallel to the symmetry axis.

Select an option to control the flow rate at the inlet:

- **Average velocity**, U_{av} .
- **Flow rate**, V_0 . Two-dimensional models also require an **Entrance thickness**, D_z , which is the out-of-plane thickness of the extruded entrance channel.
- **Average pressure**, P_{av} . Note that P_{av} is the average pressure on the inflow boundary.

The **Apply condition on each disjoint selection separately** checkbox is selected per default. When this setting is selected, the fully developed flow condition is applied separately on each disjoint selection. If this option is not selected, the condition is applied over the whole feature selection. The **Apply condition on each disjoint selection separately** should be disabled only if the flow conditions are known to be identical on each disjoint boundary.

The fully developed flow condition requires any volume force to be approximately aligned with the normal of the inlet boundary. The exception is gravity when the **Include gravity** setting is selected in the physics interface settings. Unless **Use reduced pressure** is also selected, an option to **Compensate for hydrostatic pressure** or **Compensate for hydrostatic pressure approximation** becomes available. It is selected per default and should only be deselected if the inlet normal is aligned with the gravity force and you want to specify an average pressure that includes the hydrostatic pressure.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show** button () and selecting **Advanced Physics Options**.

	• Prescribing Inlet and Outlet Conditions • Normal Stress Boundary Condition
	Fully Developed Flow (Inlet)

Outlet

This condition should be used on boundaries for which there is a net outflow from the domain. To obtain a numerically well-posed problem, it is advisable to also consider the Inlet conditions when specifying an Outlet condition. For example, if the velocity is specified at the inlet, the pressure may be specified at the outlet, and vice versa. Specifying the velocity vector at both the inlet and the outlet may cause convergence difficulties. Selecting appropriate outlet conditions for the Navier–Stokes equations is a nontrivial task. Generally, if there is something interesting happening at an outflow boundary, the computational domain should be extended to include this phenomenon.

BOUNDARY CONDITION

The available **Boundary condition** options for an outlet are **Pressure**, **Velocity**, **Fully developed flow**, and **Mass flow**.

PRESSURE CONDITIONS

This option specifies the normal stress, which in most cases is approximately equal to the pressure. The tangential stress component is set to zero.

- For incompressible flow, the **Pressure** list has two options, **Static** and **Total**. For weakly compressible and compressible flow, the static pressure should be specified in the text field.
 - If **Pressure** is **Static**, and the reference pressure p_{ref} defined at the physics interface level, is equal to 0, the value of the pressure p_0 , at the boundary, is the absolute pressure. Otherwise, p_0 is the relative pressure at the boundary.
 - If **Pressure** is **Total**, the **Average** checkbox is available and selected by default to prescribe the averaged total pressure in the weak forms. If it is cleared, the total pressure is imposed pointwise.

When **Include gravity** is selected and **Use reduced pressure** not selected in the interface **Physical model** section, the **Compensate for hydrostatic pressure approximation** (named **Compensate for hydrostatic pressure** for incompressible flows) checkbox is available and selected by default. When it is selected, the hydrostatic pressure is automatically added to the pressure entered in p_0 user input.

- The **Normal flow** option changes the no tangential stress condition to a no tangential velocity condition. This forces the flow to exit (or enter) the domain perpendicularly to the outlet boundary.
- The **Suppress backflow** checkbox is selected by default. This option adjusts the outlet pressure in order to reduce the amount of fluid entering the domain through the boundary.

VELOCITY

See the **Inlet** node [Velocity](#) section for the settings.

FULLY DEVELOPED FLOW

This boundary condition is applicable when the flow exits the domain into a long pipe or channel, at the end of which a flow profile is fully developed.

The channel can be thought of as a virtual extrusion of the outlet cross section. The outlet boundary must hence be flat in order for the fully developed flow condition to work properly. In 2D axisymmetric models, the outlet normal must be parallel to the symmetry axis.

Select an option to control the flow rate at the outlet:

- **Average velocity**, U_{av} .
- **Flow rate**, V_0 . Two-dimensional models also require an **Entrance thickness**, D_z , which is the out-of-plane thickness of the extruded entrance channel.
- **Average pressure**, P_{av} . Observe that P_{av} is the average pressure on the outflow.

The **Apply condition on each disjoint selection separately** checkbox is selected per default. When this setting is selected, the fully developed flow condition is applied separately on each disjoint selection. If this option is not selected, the condition is applied over the whole feature selection. The **Apply condition on each disjoint selection separately** should be disabled only if the flow conditions are known to be identical on each disjoint boundary.

The fully developed flow condition requires any volume force to be approximately aligned with the normal of the outlet boundary. The exception is gravity when the **Include gravity** setting is selected in the physics interface settings. Unless **Use reduced pressure** is also selected, an option to **Compensate for hydrostatic pressure** or **Compensate for hydrostatic pressure approximation** becomes available. It is selected per default and should only be deselected if the outlet normal is aligned with the gravity force and you want to specify an average pressure that includes the hydrostatic pressure.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show** button () and selecting **Advanced Physics Options**.

	Prescribing Inlet and Outlet Conditions
	Fully Developed Flow (Outlet)

Symmetry

This node provides a boundary condition for symmetry boundaries. It should only be used when the geometry and expected solution have mirror symmetry. By using

symmetries in a model its size can be reduced by one-half or more, making this an efficient tool for solving large problems.

The **Symmetry** boundary condition prescribes no penetration and vanishing shear stresses. The boundary condition is a combination of a Dirichlet condition and a Neumann condition:

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \left(-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) \mathbf{n} = \mathbf{0}$$

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad (-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))\mathbf{n} = \mathbf{0}$$

for the compressible and incompressible formulations. The Dirichlet condition takes precedence over the Neumann condition, and the above equations are equivalent to the following equation for both the compressible and incompressible formulations:

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \mathbf{K} - (\mathbf{K} \cdot \mathbf{n})\mathbf{n} = \mathbf{0}$$

$$\mathbf{K} = \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)\mathbf{n}$$

BOUNDARY SELECTION

For 2D axial symmetry, a boundary condition does not need to be defined for the symmetry axis at $r = 0$. The software automatically provides a condition that prescribes $u_r = 0$ and vanishing stresses in the z direction and adds an **Axial Symmetry** node that implements these conditions on the axial symmetry boundaries only.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show** button () and selecting **Advanced Physics Options**.

Open Boundary

The **Open Boundary** condition describes boundaries in contact with a large volume of fluid. Fluid can both enter and leave the domain on boundaries with this type of condition.

BOUNDARY CONDITIONS

The **Boundary condition** options for open boundaries are **Normal stress** and **No viscous stress**.

Normal Stress

The **Normal stress** f_0 condition implicitly imposes $p \approx f_0$.

When **Include gravity** is selected and **Use reduced pressure** not selected in the interface **Physical model** section, the **Compensate for hydrostatic pressure approximation** (named **Compensate for hydrostatic pressure** for incompressible flows) checkbox is available and selected by default. When it is selected, the hydrostatic pressure is automatically added to the pressure entered in f_0 user input.

No Viscous Stress

The **No Viscous Stress** condition specifies vanishing viscous stress on the boundary. This condition does not provide sufficient information to fully specify the flow at the open boundary and must at least be combined with pressure constraints at adjacent points.

The **No viscous stress** condition prescribes:

$$\left(\mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) \mathbf{n} = \mathbf{0}$$

$$\mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \mathbf{n} = \mathbf{0}$$

for the compressible and the incompressible formulations. This condition can be useful in some situations because it does not impose any constraint on the pressure. A typical example is a model with volume forces that give rise to pressure gradients that are hard to prescribe in advance. To make the model numerically stable, this boundary condition should be combined with a point constraint on the pressure.

Boundary Stress

The **Boundary Stress** node adds a boundary condition that represents a general class of conditions also known as traction boundary conditions.

BOUNDARY CONDITION

The **Boundary condition** options for the boundary stress are **General stress**, **Normal stress**, and **Normal stress, normal flow**.

General Stress

When **General stress** is selected, the components for the **Stress F** should be specified. The total stress on the boundary is set equal to the given stress **F**:

$$\left(-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) \mathbf{n} = \mathbf{F}$$

$$(-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} = \mathbf{F}$$

for the compressible and the incompressible formulations.

This boundary condition implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu \frac{\partial u_n}{\partial n} - \mathbf{n} \cdot \mathbf{F} \quad (3-1)$$

If $\partial u_n / \partial n$ is small, Equation 3-1 states that $p \approx -\mathbf{n} \cdot \mathbf{F}$.

Normal Stress

Normal Stress is described for the Open Boundary node.

Normal Stress, Normal Flow

For **Normal stress**, **normal flow**, the magnitude of the **Normal stress** f_0 should be specified. The tangential velocity is set to zero on the boundary:

$$\mathbf{n}^T \left(-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) \mathbf{n} = -f_0, \quad \mathbf{t} \cdot \mathbf{u} = 0$$

$$\mathbf{n}^T (-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} = -f_0, \quad \mathbf{t} \cdot \mathbf{u} = 0$$

for the compressible and the incompressible formulations.

This boundary condition implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu \frac{\partial u_n}{\partial n} + f_0 \quad (3-2)$$

If $\partial u_n / \partial n$ is small, Equation 3-2 states that $p \approx f_0$.

When **Include gravity** is selected and **Use reduced pressure** not selected in the interface **Physical model** section, the **Compensate for hydrostatic pressure approximation** (named **Compensate for hydrostatic pressure** for incompressible flows) checkbox is available and selected by default. When it is selected, the hydrostatic pressure is automatically added to the pressure entered in f_0 or p_{hydro} $\mathbf{n}$ is added to $\mathbf{F}$ depending on the selected option.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show** button () and selecting **Advanced Physics Options**.

If **Normal Stress**, **Normal Flow** is selected as the **Boundary condition**, then to **Apply reaction terms on** all dependent variables, the **All physics (symmetric)** option should be

selected. Alternatively, the **Individual dependent variables** could be selected to restrict the reaction terms as needed.

Periodic Flow Condition

The **Periodic Flow Condition** splits its selection into a source group and a destination group (the **Destination Selection** section is optional). Fluid that leaves the domain through one of the destination boundaries enters the domain through the corresponding source boundary. This corresponds to a situation where the geometry is a periodic part of a larger geometry. If the boundaries are not parallel to each other, the velocity vector is automatically transformed.

If the boundaries are curved, the orientation of the source must be specified manually (see [Orientation of Source](#)).

No input is required when **Compressible flow (Ma<0.3)** is selected for **Compressibility** under the **Physical Model** section for the physics interface. Typically when a periodic boundary condition is used with a compressible flow, the pressure is the same at both boundaries and the flow is driven by a volume force.

BOUNDARY SELECTION



The software usually automatically identifies the boundaries as either source boundaries or destination boundaries, as indicated in the selection list. This works fine for cases like opposing parallel boundaries. In other cases, right-click **Periodic Flow Condition** and select **Manual Destination Selection** to control the destination. By default it contains the selection that COMSOL Multiphysics identifies.

DESTINATION SELECTION

This section is available for specifying the destination boundaries, if needed, when the **Manual Destination Selection** option is selected in the context menu for the **Periodic Flow Condition** node. You can only select destination boundaries from the union of all source and destination boundaries.

FLOW CONDITION

This section is available when **Incompressible flow** is selected for **Compressibility** under the **Physical Model** section for the physics interface.

The **Flow Condition** at the boundary is specified through a value or expression for either the **Pressure difference**, $p_{\text{src}} - p_{\text{dst}}$, or the **Mass flow**, $\dot{m}$. The mass flow option ensures

a pressure difference is achieved across the source and destination boundaries. This pressure difference can, for example, drive the fully developed flow in a channel.

To set up a periodic boundary condition, both boundaries must be selected in the **Periodic Flow Condition** node. COMSOL Multiphysics automatically assigns one boundary as the source and the other as the destination. To manually set the destination selection, a **Destination Selection** subnode is available from the context menu (by right-clicking the parent node) or from the **Physics** toolbar, **Attributes** menu. All destination sides must be connected.

ORIENTATION OF SOURCE

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. For information about the **Orientation of Source** section, see [Orientation of Source and Destination](#) in the *COMSOL Multiphysics Reference Manual*.

ORIENTATION OF DESTINATION

This section appears if the setting for **Transform to intermediate map** in the **Orientation of Source** section is changed from the default value, **Automatic**, and **Advanced Physics Options** is selected in the **Show More Options** dialog. For information about the **Orientation of Destination** section, see [Orientation of Source and Destination](#) in the *COMSOL Multiphysics Reference Manual*.

MAPPING BETWEEN SOURCE AND DESTINATION

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. For information about the **Mapping Between Source and Destination** section, see [Mapping Between Source and Destination](#) in the *COMSOL Multiphysics Reference Manual*.

Porous Interface

The **Porous Interface** condition is designed to properly account for a sudden jump in porosity, ϵ_p , across a boundary between different domains. It describes possible discontinuity of stress across the interface, and also reveals modified behavior of tangential velocity due to boundary layer on the upstream side of the interface. Choose among the following options under the **Continuity conditions**:

- **Velocity and stress** — stress continuity is invoked;

- **Velocity and porosity-corrected stress** (default)— the sum of stress and a contribution due to the inertial term being constant across the interface;
- **Velocity and porosity-corrected stress with loss** — the sum of the stress and a contribution due to the inertial term is lost in the downstream direction.

All three options give continuity of the velocity across the interface in constant-density flows, while in case of a jump in density:

- 1 give continuity of the mass flux (product of density and normal velocity) and tangential velocity;
- 2 account for the density-jump correction in the stress condition.

When **Velocity and porosity-corrected stress with loss** is chosen, a **Pressure loss model** needs to be specified. The options are:

- **User-defined loss coefficients** (default). Enter values for **Expansion loss coefficient** and **Contraction loss coefficient**;
- **Borda–Carnot**. Specify **Vena contracta model**.

Vena contracta model allows two options:

- **User defined** (default) — enter value of **Vena contracta coefficient**;
- **Weisbach** — enter values of **Lower limit** and **Exponent**.

Notice that the notions ‘expansion’ and ‘contraction’ used in this section correspond not to geometrical changes, but to the variations of the microscopic structure across the interface in the flow direction — increasing or decreasing porosity, ϵ_p , respectively.

Flow Continuity

The **Flow Continuity** condition is suitable for pairs where the boundaries match; it prescribes that the flow field is continuous across the pair.

The parts of the boundary where a source boundary lacks a corresponding destination boundary and vice versa will get conditions from other boundary conditions under the physics interface acting as fallback boundary conditions on the nonoverlapping parts. By default, the [Wall](#) node is used as fallback unless another feature has been selected for the boundary. When the condition is set for pairs adjacent to domains with different porosity ϵ_p , it imposes conditions identical to the **Porous Interface** feature, which is designed to properly account for a sudden jump in porosity. See the section [Porous](#)

[Interface](#) above for further details (note that the **Flow Continuity** does not account for the density-jump corrections),



About Pairs

Pressure Point Constraint

The **Pressure Point Constraint** condition can be used to specify the pressure level. If it is not possible to specify the pressure level using a boundary condition, the pressure level must be set in some other way, for example, by specifying a fixed pressure at a point.

PRESSURE CONSTRAINT

The relative pressure value is set by specifying the **Pressure** p_0 . If the reference pressure p_{ref} defined at the physics interface level is equal to zero, p_0 represents the absolute pressure.

When **Include gravity** is selected and **Use reduced pressure** not selected in the interface **Physical model** section, the **Compensate for hydrostatic pressure approximation** (named **Compensate for hydrostatic pressure** for incompressible flows) checkbox is available and selected by default. When it is selected, the hydrostatic pressure is automatically added to the pressure entered in p_0 user input.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show** button () and selecting **Advanced Physics Options**.

The **Apply reaction terms on** setting is set per default to **Individual dependent variables**. **All physics (symmetric)** cannot be used together with a segregated solver when fluid flow is coupled with Moving Mesh, Level Set, or Phase Field in Fluids.

Average Pressure

The **Average Pressure** condition can be used to specify the pressure level. If it is not possible to specify the pressure level using a boundary condition, the pressure level must be set in some other way, for example, by specifying an average pressure in all flow domains.

AVERAGE PRESSURE CONSTRAINT

The relative average pressure is set by specifying the **Average pressure** p_{ave} . If the reference pressure p_{ref} defined at the physics interface level is equal to zero, p_{ave} represents the average absolute pressure.

When **Include gravity** and **Use reduced pressure** are selected, the average reduced pressure is constrained to p_{ave} .



The **Average Pressure** constraint couples all the degrees of freedom for the pressure. This could lead to large memory usage and ill-conditioned systems. In such cases, consider using a **Pressure Point Constraint** instead.

Point Mass Source

This feature requires at least one of the following licenses: Battery Design Module, CFD Module, Chemical Reaction Engineering Module, Corrosion Module, Electrochemistry Module, Electrodeposition Module, Fuel Cell & Electrolyzer Module, Microfluidics Module, Pipe Flow Module, or Subsurface Flow Module.

The **Point Mass Source** feature models mass flow originating from an infinitely small domain centered around a point.

For the Reacting Flow in Porous Media, Diluted Species interface, which is available with the CFD Module, Chemical Reaction Engineering Module, or Battery Design Module, there are two nodes: one for the fluid flow (**Fluid Point Source**) and one for the species (**Species Point Source**).

SOURCE STRENGTH

The source **Mass flux**, $\dot{q}_p$, should be specified. A positive value results in mass being ejected from the point into the computational domain. A negative value results in mass being removed from the computational domain.

Point sources located on a boundary or on an edge affect the adjacent computational domains. This has the effect, for example, that a point source located on a symmetry plane has twice the given strength.



[Mass Sources for Fluid Flow](#) in the *COMSOL Multiphysics Reference Manual*

Line Mass Source

This feature requires at least one of the following licenses: Battery Design Module, CFD Module, Chemical Reaction Engineering Module, Corrosion Module, Electrochemistry Module, Electrodeposition Module, Fuel Cell & Electrolyzer Module, Microfluidics Module, Pipe Flow Module, or Subsurface Flow Module.

The **Line Mass Source** feature models mass flow originating from a tube region with infinitely small radius.

For the Reacting Flow in Porous Media, Diluted Species interface, which is available with the CFD Module, Chemical Reaction Engineering Module, or Battery Design Module, there are two nodes, one for the fluid flow (**Fluid Line Source**) and one for the species (**Species Line Source**).

SELECTION

The **Line Mass Source** feature is available for all dimensions, but the applicable selection differs between the dimensions.

MODEL DIMENSION	APPLICABLE GEOMETRICAL ENTITY
2D	Points
2D Axisymmetry	Points not on the axis of symmetry
3D	Edges

SOURCE STRENGTH

The source **Mass flux**, $\dot{q}_1$, should be specified. A positive value results in mass being ejected from the line into the computational domain and a negative value means that mass is removed from the computational domain.

Line sources located on a boundary affect the adjacent computational domains. This, for example, has the effect that a line source located on a symmetry plane has twice the given strength.

	Mass Sources for Fluid Flow in the <i>COMSOL Multiphysics Reference Manual</i>

Gravity

The **Gravity** global feature is automatically added when **Include gravity** is selected at the interface level in the **Physical Model** settings. It defines the gravity forces from the **Acceleration of gravity** value.

ACCELERATION OF GRAVITY

The **Acceleration of gravity** (SI unit m/s, default value $-g_{\text{const}}\mathbf{e}_z$ in 2D axial symmetry and 3D and $-g_{\text{const}}\mathbf{e}_y$ in 2D) is used to define the gravity forces. It should be a global quantity.

When in addition to **Include gravity** also **Rotating frame** is selected (only possible in 3D and 2D axial symmetry), the **Project gravity vector on rotation axis** checkbox is available (by default selected).

Rotating Frame

The **Rotating Frame** global feature is automatically added when **Rotating frame** is selected at the interface level in the **Physical Model** settings. It defines the fictitious Coriolis, centrifugal and Euler forces from the settings in the **Rotating Frame** section.

ROTATING FRAME

In 2D, the axis of rotation is an out-of-plane axis. In this case, select a **Rotation axis base point** $\mathbf{r}_{\text{bp}}$.

In 2D axial symmetry, the axis of rotation is the symmetry axis.

In 3D, select an **Axis of rotation** — **x-axis**, **y-axis**, **z-axis** (the default), or **User defined**. For **User defined** enter a **Rotation axis direction** $\mathbf{e}_{\text{ax}}$ and a **Rotation axis base point** $\mathbf{r}_{\text{bp}}$.

Select a **Rotation speed** — **Angular velocity**, **Revolutions per time**, or **User defined**.

- For **Angular velocity**, enter a value for the angular velocity magnitude Ω_f .
- For **Revolutions per time**, enter an RPM value.
- For **User defined**, enter the **Rotation angle** α in radians as function of time (the variable τ).

In case **Angular velocity** or **Revolutions per time** is selected, also select a **Rotational direction** — **Counterclockwise** (the default) or **Clockwise**.

Theory for the Single-Phase Flow Interfaces

The theory for the Single-Phase Flow, Laminar Flow interface is described in this section:

- [General Single-Phase Flow Theory](#)
- [Compressible Flow](#)
- [Weakly Compressible Flow](#)
- [The Mach Number Limit](#)
- [Incompressible Flow](#)
- [The Reynolds Number](#)
- [Gravity](#)
- [Rotating Frame](#)
- [Theory for the Wall Boundary Condition](#)
- [Prescribing Inlet and Outlet Conditions](#)
- [Fully Developed Flow \(Inlet\)](#)
- [Fully Developed Flow \(Outlet\)](#)
- [No Viscous Stress](#)
- [Normal Stress Boundary Condition](#)
- [Porous Interface](#)
- [Mass Sources for Fluid Flow](#)
- [Numerical Stability — Stabilization Techniques for Fluid Flow](#)
- [Solvers for Laminar Flow](#)
- [Pseudo Time Stepping for Laminar Flow Models](#)
- [Nitsche Constraints](#)
- [Particle Tracing in Fluid Flow](#)
- [References for the Single-Phase Flow, Laminar Flow Interfaces](#)



The theory about most boundary conditions is found in [Ref. 2](#).

The Single-Phase Fluid Flow interfaces are based on the Navier–Stokes equations, which in their most general form read

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (3-3)$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot [-p \mathbf{I} + \mathbf{K}] + \mathbf{F} \quad (3-4)$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right) = -(\nabla \cdot \mathbf{q}) + \mathbf{K} : \mathbf{S} - \frac{T}{\rho} \frac{\partial \rho}{\partial T} \left(\frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla) p \right) + Q \quad (3-5)$$

where

- ρ is the density (SI unit: kg/m³)
- $\mathbf{u}$ is the velocity vector (SI unit: m/s)
- p is pressure (SI unit: Pa)
- $\mathbf{I}$ is the identity matrix (unitless)
- $\mathbf{K}$ is the viscous stress tensor (SI unit: Pa)
- $\mathbf{F}$ is the volume force vector (SI unit: N/m³)
- C_p is the specific heat capacity at constant pressure (SI unit: J/(kg·K))
- T is the absolute temperature (SI unit: K)
- $\mathbf{q}$ is the heat flux vector (SI unit: W/m²)
- Q contains the heat sources (SI unit: W/m³)
- $\mathbf{S}$ is the strain-rate tensor:

$$\mathbf{S} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

The operation “:” denotes a contraction between tensors defined by

$$\mathbf{a} : \mathbf{b} = \sum_n \sum_m a_{nm} b_{nm} \quad (3-6)$$

This is sometimes referred to as the double dot product.

Equation 3-3 is the continuity equation and represents conservation of mass.

Equation 3-4 is a vector equation which represents conservation of momentum.

Equation 3-5 describes the conservation of energy, formulated in terms of temperature. This is an intuitive formulation that facilitates boundary condition specifications.

To close the equation system, Equation 3-3 through Equation 3-5, constitutive relations are needed.

For a Newtonian fluid, which has a linear relationship between stress and strain, Stokes (Ref. 1) deduced the following expression:

$$\mathbf{K} = 2\mu\mathbf{S} - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \quad (3-7)$$

The dynamic viscosity, μ (SI unit: Pa·s), for a Newtonian fluid is allowed to depend on the thermodynamic state but not on the velocity field. All gases and many liquids can be considered Newtonian.

For an inelastic non-Newtonian fluid, the relationship between stress and strain rate is nonlinear, and an apparent viscosity is introduced instead of the dynamic viscosity. Examples of non-Newtonian fluids are honey, mud, blood, liquid metals, and most polymer solutions.

In theory, the same equations describe both laminar and turbulent flows. In practice, however, the mesh resolution required to simulate turbulence with the Laminar Flow interface makes such an approach impractical.



There are several books where derivations of the Navier–Stokes equations and detailed explanations of concepts such as Newtonian fluids can be found. See, for example, the classical text by Batchelor (Ref. 3) and the more recent work by Panton (Ref. 4).

Many applications describe isothermal flows for which Equation 3-5 is decoupled from Equation 3-3 and Equation 3-4.

2D AXISYMMETRIC FORMULATIONS

A 2D axisymmetric formulation of Equation 3-3 and Equation 3-4 requires $\partial/\partial\phi$ to be zero. That is, there must be no gradients in the azimuthal direction. A common additional assumption is, however, that $u_\phi = 0$. In such cases, the ϕ -equation can be removed from Equation 3-4. The resulting system of equations is both easier to converge and computationally less expensive compared to retaining the ϕ -equation.

The default 2D axisymmetric formulation of [Equation 3-3](#) and [Equation 3-4](#) therefore assumes that

$$\begin{aligned}\partial/\partial\phi &= 0 \\ u_\phi &= 0\end{aligned}$$

Compressible Flow

The equations of motion for a single-phase fluid are the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (3-8)$$

and the momentum equation:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \left(\mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right) + \mathbf{F} \quad (3-9)$$

These equations are applicable for incompressible as well as for compressible flow with density and viscosity variations.

Weakly Compressible Flow

The same equations as for [Compressible Flow](#) are applied for weakly compressible flow. The only difference is that the density is evaluated at the reference pressure. The density may be a function of other quantities, in particular it may be temperature dependent.

The weakly compressible flow equations are valid for incompressible as well as compressible flow with density variations independent of the pressure.

Provided that the densities dependency pressure is specified through model inputs, the density is automatically evaluated at the reference pressure level.

The Mach Number Limit

An important dimensionless number in fluid dynamics is the Mach number, Ma , defined by

$$\text{Ma} = \frac{|\mathbf{u}|}{a}$$

where a is the speed of sound. A flow is formally incompressible when $\text{Ma} = 0$. This is theoretically achieved by letting the speed of sound tend to infinity. The Navier–Stokes equations then have the mathematical property that pressure disturbances are instantaneously propagated throughout the entire domain. This results in a parabolic equation system.

The momentum equation, Equation 3-9, is parabolic for unsteady flow and elliptic for steady flow, whereas the continuity equation, Equation 3-8, is hyperbolic for both steady and unsteady flow. The combined system of equations is thus hybrid parabolic-hyperbolic for unsteady flow and hybrid elliptic-hyperbolic for steady flow. An exception occurs when the viscous term in Equation 3-9 becomes vanishingly small, such as at an outflow boundary, in which case the momentum equation becomes locally hyperbolic. The number of boundary conditions to apply on the boundary then depends on the number of characteristics propagating into the computational domain. For the purely hyperbolic system, the number of characteristics propagating from the boundary into the domain changes as the Mach number passes through unity. Hence, the number of boundary conditions required to obtain a numerically well-posed system must also change. The compressible formulation of the laminar and turbulent interfaces uses the same boundary conditions as the incompressible formulation, which implies that the compressible interfaces are not suitable for flows with a Mach number larger than or equal to one. Yet, the practical Mach number limit is lower than one. The main reason is that the numerical scheme (stabilization and boundary conditions) of the Laminar Flow interface does not recognize the direction and speed of pressure waves. The fully compressible Navier–Stokes equations do, for example, start to display very sharp gradients already at moderate Mach numbers. But the stabilization for the single-phase flow interface does not necessarily capture these gradients. It is impossible to give an exact limit where the low Mach number regime ends and the moderate Mach number regime begins, but a rule of thumb is that the Mach number effects start to appear at $\text{Ma} = 0.3$. For this reason, the compressible formulation is referred to as *Compressible flow* ($\text{Ma} < 0.3$) in COMSOL Multiphysics.

Incompressible Flow

When the temperature variations in the flow are small, a single-phase fluid can often be assumed incompressible; that is, ρ is constant or nearly constant. This is the case for all liquids under normal conditions and also for gases at low velocities. For constant ρ , Equation 3-8 reduces to

$$\rho \nabla \cdot \mathbf{u} = 0 \quad (3-10)$$

and Equation 3-9 becomes

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot [-p \mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \mathbf{F} \quad (3-11)$$

Provided that the densities dependency on temperature and pressure is specified through model inputs, the density is evaluated at the reference pressure level and at the reference temperature. However, if the density is a function of other quantities such as a concentration field, or if the density is specified by a user defined expression, the user has to make sure that the density is defined as constant when the incompressible flow formulation is used.

The Reynolds Number

A fundamental characteristic in analyses of fluid flow is the Reynolds number:

$$\text{Re} = \frac{\rho U L}{\mu}$$

where U denotes a velocity scale, and L denotes a representative length. The Reynolds number represents the ratio between inertial and viscous forces. At low Reynolds numbers, viscous forces dominate and tend to damp out all disturbances, which leads to laminar flow. At high Reynolds numbers, the damping in the system is very low, giving small disturbances the possibility to grow by nonlinear interactions. If the Reynolds number is high enough, the flow field eventually ends up in a chaotic state called turbulence.

Note that the Reynolds number can have different meanings depending on the length scale and velocity scale. To be able to compare two Reynolds numbers, they must be based on equivalent length and velocity scales.

The Fluid Flow interfaces automatically calculate the local cell Reynolds number $\text{Re}^c = \rho |\mathbf{u}| h / (2\mu)$ using the element length h for L and the magnitude of the velocity vector $\mathbf{u}$ for the velocity scale U . This Reynolds number is not related to the character of the flow field, but to the stability of the numerical discretization. The risk for numerical oscillations in the solution increases as Re^c grows. The cell Reynolds number is a predefined quantity available for visualization and evaluation (typically it is available as: `spf.cellRe`).

DEFINITION

The gravity force is defined from the acceleration of gravity vector, $\mathbf{g}$, and the density, ρ . Under usual conditions and in Cartesian coordinates with the z -axis in the vertical direction,

$$\mathbf{g} = \begin{pmatrix} 0 \\ 0 \\ -g_{\text{const}} \end{pmatrix}.$$

When gravity is considered, a volume force equal to $\rho\mathbf{g}$ is included in the momentum equation. For example, for laminar weakly compressible flow, it reads:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \left(-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) + \mathbf{F} + \rho \mathbf{g} \quad (3-12)$$

Introducing a constant reference density ρ_{ref} , and assuming that $\mathbf{g}$ is homogeneous, this equation is equivalently written:

$$\begin{aligned} \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = & \nabla \cdot \left(-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} + \rho_{\text{ref}}\mathbf{g} \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}})\mathbf{I} \right) + \mathbf{F} + (\rho - \rho_{\text{ref}})\mathbf{g} \end{aligned} \quad (3-13)$$

where $\mathbf{r}$ is the position vector and $\mathbf{r}_{\text{ref}}$ is an arbitrary reference position vector.

From this equation, it is convenient to define the reduced pressure which accounts for the hydrostatic pressure, $\tilde{p} = p - \rho_{\text{ref}}\mathbf{g} \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}})$

$$\begin{aligned} \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = & \nabla \cdot \left(-\tilde{p}\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) + \\ & \mathbf{F} + (\rho - \rho_{\text{ref}})\mathbf{g} \end{aligned} \quad (3-14)$$

In Equation 3-14, the gravity force is written $(\rho - \rho_{\text{ref}})\mathbf{g}$.

PRESSURE FORMULATION

When the relative pressure is used (default option) the interface dependent variable represents the relative pressure and the absolute pressure is defined as $p_A = p_{\text{ref}} + p$.

When the pressure is used to define a boundary condition (for example, when p_0

defines the pressure condition at an outlet), it represents the relative pressure. Hence defining the outlet pressure as $p_{\text{hydro,approx}} = -\rho_{\text{ref}} \mathbf{g} \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}})$ compensates for the gravity force for an ambient reference pressure of 0 Pa when the density is constant, there is no external force, and provided p_{ref} , $\mathbf{g}$, and $\mathbf{r}_0$ are defined consistently.

When the reduced pressure is used, the interface dependent variable (named p by default) represents the reduced pressure. The absolute pressure is then defined as $p_A = p_{\text{ref}} - \rho_{\text{ref}} \mathbf{g} \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}}) + p$. In this case when the pressure is used to define a boundary condition (for example, to define a pressure condition at an outlet), its value corresponds to the reduced pressure. Hence, the prescribed pressure compensates for an approximate hydrostatic pressure, $p_{\text{hydro,approx}} = -\rho_{\text{ref}} \mathbf{g} \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}})$, which is exact only when the density is constant and there is no external force.

PRESSURE BOUNDARY CONDITION

For an immobile fluid the momentum equation simplifies to $\nabla \cdot (p\mathbf{I}) = \mathbf{F} + \rho\mathbf{g}$ or

$$\nabla \cdot (-\tilde{p}\mathbf{I}) = \mathbf{F} + (\rho - \rho_{\text{ref}})\mathbf{g} \text{ depending on the pressure formulation.}$$

For incompressible flow, assuming there are no external forces, this leads respectively to $p = -\rho_{\text{ref}} \mathbf{g} \cdot (\mathbf{r} - \mathbf{r}_{\text{ref}}) + p_0$ or $\tilde{p} = p_0$.

Rotating Frame

The fictitious Coriolis, centrifugal and Euler forces, present in a rotating frame of reference, are stemming from the change to a non-inertial frame of reference, and are defined using the vector of angular velocity, Ω_f , and the density, ρ . This amounts to adding to the momentum equation a volume force equal to

$$\mathbf{F}_{\text{rf}} = -\rho \left(\Omega_f \times (\Omega_f \times \mathbf{r}_p) + 2\Omega_f \times \mathbf{u} + \frac{\partial \Omega_f}{\partial t} \times \mathbf{r}_p \right) \quad (3-15)$$

where $\mathbf{r}_p = \mathbf{r} - \mathbf{r}_{\text{bp}}$, with $\mathbf{r}_{\text{bp}}$ a position vector on the axis of rotation. The first term on the right-hand side is the centrifugal force, the second term the Coriolis force, and the third term the Euler force. With these forces, the momentum equation reads:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot (-p\mathbf{I} + \mathbf{K}) - \rho \left(\Omega_f \times (\Omega_f \times \mathbf{r}_p) + 2\Omega_f \times \mathbf{u} + \frac{\partial \Omega_f}{\partial t} \times \mathbf{r}_p \right) \quad (3-16)$$

Introducing a constant reference density ρ_{ref} , this equation is equivalently written:

$$\begin{aligned} \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \left(-p \mathbf{I} + \mathbf{K} + \frac{1}{2} \rho_{\text{ref}} (|\Omega_f \times \mathbf{r}_p|^2 - |\Omega_f \times (\mathbf{r}_{\text{ref}} - \mathbf{r}_{\text{bp}})|^2) \mathbf{I} \right) \\ - (\rho - \rho_{\text{ref}}) (\Omega_f \times (\Omega_f \times \mathbf{r}_p)) + \rho \left(2\Omega_f \times \mathbf{u} + \frac{\partial \Omega_f}{\partial t} \times \mathbf{r}_p \right) \end{aligned} \quad (3-17)$$

where $\mathbf{r}$ is the position vector and $\mathbf{r}_{\text{ref}}$ is an arbitrary reference position vector. From this equation, it is convenient to define the reduced pressure which accounts for the hydrostatic pressure stemming from the centrifugal force as

$$\tilde{p} = p - \frac{1}{2} \rho_{\text{ref}} (|\Omega_f \times \mathbf{r}_p|^2 - |\Omega_f \times (\mathbf{r}_{\text{ref}} - \mathbf{r}_{\text{bp}})|^2) \quad (3-18)$$

When the relative pressure formulation is used, also the hydrostatic pressure, $p_{\text{hydro}} = \frac{1}{2} \rho_{\text{ref}} (|\Omega_f \times \mathbf{r}_p|^2 - |\Omega_f \times (\mathbf{r}_{\text{ref}} - \mathbf{r}_{\text{bp}})|^2)$, stemming from the centrifugal force, is taken into account when compensating for the hydrostatic pressure at a pressure boundary conditions. When a reduced pressure formulation is used, p_{hydro} is used to define the absolute pressure, see also the section on [Gravity](#).

CORIOLIS FORCE IN POROUS DOMAINS

In a porous domain, the Coriolis force should be divided by the porosity ϵ_p , such that the volume force added to the momentum equation becomes

$$\mathbf{F}_{\text{rf}} = -\rho \left(\Omega_f \times (\Omega_f \times \mathbf{r}_p) + \frac{2}{\epsilon_p} \Omega_f \times \mathbf{u} + \frac{\partial \Omega_f}{\partial t} \times \mathbf{r}_p \right) \quad (3-19)$$

Theory for the Wall Boundary Condition

See [Wall](#) for the node settings.

SLIP

The [Slip](#) condition assumes that there are no viscous effects at the slip wall and hence, no boundary layer develops. From a modeling point of view, this is a reasonable approximation if the important effect of the wall is to prevent fluid from leaving the domain. Mathematically, the constraint can be formulated as:

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad (-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} = \mathbf{0}$$

The no-penetration term takes precedence over the Neumann part of the condition and the above expression is therefore equivalent to

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \mathbf{K}_n - (\mathbf{K}_n \cdot \mathbf{n})\mathbf{n} = \mathbf{0}$$

$$\mathbf{K}_n = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)\mathbf{n}$$

expressing that there is no flow across the boundary and no viscous stress in the tangential direction.

For a moving wall with translational velocity $\mathbf{u}_{\text{tr}}$, $\mathbf{u}$ in the above equations is replaced by the relative velocity $\mathbf{u}_{\text{rel}} = \mathbf{u} - \mathbf{u}_{\text{tr}}$.

POROUS SLIP

The **Porous slip** option, which can be chosen in the **Porous treatment of no slip condition** list when **Enable porous media domains** is activated, results in a special treatment of **Wall** boundaries and **Interior Wall** boundaries adjacent to porous domains (**Porous Medium** feature). Namely, similar to the **Navier slip** boundary condition, no penetration and tangential stress conditions are applied at the wall:

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \mathbf{K}_n - (\mathbf{K}_n \cdot \mathbf{n})\mathbf{n} = -\frac{\mu}{\varepsilon_p} \frac{\mathbf{u}_{\text{slip}}}{y_{\text{ps}}},$$

$$\mathbf{u}_{\text{slip}} = \mathbf{u} - (\mathbf{u} \cdot \mathbf{n})\mathbf{n}, \quad \mathbf{K}_n = \frac{\mu}{\varepsilon_p} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)\mathbf{n}$$

Here, $\mathbf{K}_n$ is the viscous wall traction, $\mathbf{n}$ is the wall normal, $\mathbf{u}_{\text{slip}}$ is the tangential velocity at the wall while real no slip is assumed to be applied at a distance d_w (half-height of the first cell adjacent to the wall) outside the wall, and y_{ps} is the porous slip length. An analytical derivation of the velocity profile in the boundary layer where the pressure gradient is balanced by the sum of the Darcy term, the Forchheimer drag, and the viscous term (neglecting convective terms) leads to the following expression for y_{ps} :

$$y_{\text{ps}} = \tilde{l} \left(\frac{1 + X_p e^{-\xi_p}}{1 - X_p e^{-\xi_p}} \right) \left[\left(\frac{1 + X_p e^{-\xi_p}}{1 + X_p} \right)^2 e^{\xi_p} - 1 \right], \quad \xi_p = \frac{d_w}{\tilde{l}}, \quad \tilde{l} = \frac{l_{\text{pore}}}{\sqrt{1 + 2c_\beta}},$$

$$X_p = \frac{\sqrt{1 + 2c_\beta} - \sqrt{1 + \frac{4}{3}c_\beta}}{\sqrt{1 + 2c_\beta} + \sqrt{1 + \frac{4}{3}c_\beta}}, \quad c_\beta = \frac{\sqrt{1 + 4c_D} - 1}{2}, \quad c_D = \frac{\beta_{\text{ND}}}{\mu} \frac{\kappa^2}{\mu} |\nabla p - \rho \mathbf{g}|$$

where $l_{\text{pore}} = \sqrt{\kappa/\varepsilon_p}$ is the porous length scale, $\beta_{\text{ND}} = \rho c_F/\sqrt{\kappa}$ is the non-Darcian coefficient (c_F is the Forchheimer coefficient), $\mathbf{g}$ is the gravity vector, and $\xi_p, l, X_p, c_\beta, c_D$, are intermediate variables. This formula is used when the **Pressure-gradient**

formulation is chosen and it uses the pressure gradient at the wall. By default, the **Velocity formulation** is activated and the corresponding formula is:

$$y_{ps} = \tilde{l} (e^{\xi_p} - 1), \quad \xi_p = \frac{d_w}{\tilde{l}}, \quad \tilde{l} = \frac{l_{\text{pore}}}{\sqrt{1 + 2c_\beta}},$$

$$c_\beta = \left[\left(\frac{\beta_{\text{ND}} \kappa |\mathbf{u}_{\text{slip}}|}{\mu} \frac{l_{\text{pore}}}{d_w} \right)^2 / 2 \right]^{1/3}$$

Although this formulation is an approximation, since it uses the slip velocity at the wall to reconstruct the pressure gradient, it is rather accurate. Moreover, the influence of the convective terms is partially accounted for in this formulation.

SLIDING WALL

The sliding wall option is appropriate if the wall behaves like a conveyor belt; that is, the surface is sliding in its tangential direction. The wall does not have to actually move in the coordinate system.

- In 2D, the tangential direction is unambiguously defined by the direction of the boundary, but the situation becomes more complicated in 3D. For this reason, this boundary condition has slightly different definitions in the different space dimensions.
- For 2D and 2D axisymmetric components, the velocity is given as a scalar U_w and the condition prescribes

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \mathbf{u} \cdot \mathbf{t} = U_w$$

where $\mathbf{t} = (n_y, -n_x)$ for 2D and $\mathbf{t} = (n_z, -n_r)$ for axial symmetry.

- For 3D components, the velocity is set equal to a given vector $\mathbf{u}_w$ projected onto the boundary plane:

$$\mathbf{u} = \frac{\mathbf{u}_w - (\mathbf{n} \cdot \mathbf{u}_w) \mathbf{n}}{\|\mathbf{u}_w - (\mathbf{n} \cdot \mathbf{u}_w) \mathbf{n}\|} \|\mathbf{u}_w\|$$

The normalization makes $\mathbf{u}$ have the same magnitude as $\mathbf{u}_w$ even if $\mathbf{u}_w$ is not exactly parallel to the wall.

CONSTRAINT SETTINGS

The wall feature uses three different techniques to constrain the velocity field:

- **Pointwise constraints** is the standard technique to enforce strong constraints in the finite element method. The desired value of the velocity is prescribed at each node

point in the mesh. Since the constraint is enforced locally at each node, only local values are affected by the constraint and the constraints are independent of each other. The solvers can therefore eliminate both the constrained degrees of freedom and the constraint force terms, effectively reducing the number of degrees of freedom being solved for.

The main advantage of pointwise constraints is that they enforce the constraint exactly pointwise. This means that they do not introduce any leakage of the velocity across the wall, unless specified. The main disadvantage of pointwise constraints is that they introduce locking effects when trying to impose a no-penetration condition for the velocity, $\mathbf{u} \cdot \mathbf{n} = 0$, on curved walls or walls with sharp corners.

- **Weak constraints** is an alternative method to prescribe the velocity. It consists on enforcing the boundary condition for the velocity via *Lagrange multipliers*. Their main advantage is that the Lagrange multiplier can provide an accurate representation of the reaction flux at the wall. Their main disadvantage is that they introduce extra unknowns, and are usually difficult to combine with other constraint methods on adjacent boundaries. Moreover, they may require extra constraints for the Lagrange multipliers. For more information, see [Weak Constraints](#) in the *COMSOL Multiphysics Reference Manual*.
- **Nitsche constraints** use a numerical flux to prescribe the velocity at the wall. They impose the constraint in an integral sense rather than pointwise, and do not suffer from the locking effects introduced by pointwise constraints when trying to prescribe a no penetration condition for the velocity. They are also better behaved when prescribing nonlinear constraints. Their main disadvantage is that the constraint is only imposed approximately, and may produce small leaks. For more information, see [Nitsche Constraints](#).

The following combination of Constraint techniques can be selected in the **Constraint Setting** sections of Wall boundary conditions:

- **Automatic** (default) use different constraint methods depending on whether only the normal component of the velocity is prescribed, such as in the no penetration condition, $\mathbf{u} \cdot \mathbf{n} = 0$, imposed, for example, in Slip walls or No Slip walls using Wall Functions or Automatic Wall Treatment, or both tangential and normal components are prescribed, as is the case of No Slip walls in laminar flow.

Nitsche constraints are used to impose the no penetration condition for Slip walls. When a No Slip condition is prescribed, pointwise constraints are used except for moving walls where Nitsche constraints are used.

- **Pointwise constraints**

- **Nitsche constraints**
- **Weak constraints** — these are not available on Interior Walls.
- **Mixed constraints** — this option is only available when both the tangential and normal components of the velocity need to be prescribed. The velocity on the wall normal direction is imposed via pointwise constraints. The constraint for the tangential directions is relaxed, and Nitsche constraints are used instead. This provides improved accuracy and performance when working with coarse boundary layer meshes. For more information, see [Ref. 18](#).

Prescribing Inlet and Outlet Conditions

The Navier–Stokes equations can show large variations in mathematical behavior, ranging from almost completely elliptic to almost completely hyperbolic. This has implications when it comes to prescribing admissible boundary conditions. There is also a discrepancy between mathematically valid boundary conditions and practically useful boundary conditions. See [Inlet](#) and [Outlet](#) for the node settings.

INLET CONDITIONS

An inlet requires specification of the velocity components. The most robust way to do this is to prescribe a velocity field using a Velocity condition.

A common alternative to prescribing the complete velocity field is to prescribe a pressure and all but one velocity component. The pressure cannot be specified pointwise because this is mathematically over-constraining. Instead the pressure can be specified via a stress condition:

$$-p + 2\mu \frac{\partial u_n}{\partial n} = F_n \quad (3-20)$$

where $\partial u_n / \partial n$ is the normal derivative of the normal velocity component.

[Equation 3-20](#) is prescribed by the Pressure condition in the Inlet and Outlet features and the Normal stress condition in the Open Boundary and Boundary Stress features. [Equation 3-20](#) is mathematically more stringent compared to specifying the pressure pointwise and at the same time cannot guarantee that p obtains the desired value. In practice, p is close to F_n , except for low Reynolds number flows where viscous effects are the only effects that balance the pressure. In addition to [Equation 3-20](#), all but one velocity component must be specified. For low Reynolds numbers, this can be specified by a vanishing tangential stress condition:

$$\mu \frac{\partial u_t}{\partial n} = 0$$

which is what the Normal stress condition does. Vanishing tangential stress becomes a less well-posed inlet condition as the Reynolds number increases. The Pressure condition in the Inlet feature therefore requires a flow direction to be prescribed, which provides a well-posed condition independent of Reynolds number.

OUTLET CONDITIONS

The most common approach is to prescribe a pressure via a normal stress condition on the outlet. This is often accompanied by a vanishing tangential stress condition:

$$\mu \frac{\partial u_t}{\partial n} = 0$$

where $\partial u_t / \partial n$ is the normal derivative of the tangential velocity field. It is also possible to prescribe u_t to be zero. The latter option should be used with care since it can have a significant effect on the upstream solution.

The elliptic character of the Navier–Stokes equations mathematically permits specifying a complete velocity field at an outlet. This can, however, be difficult to apply in practice. The reason being that it is hard to prescribe the outlet velocity so that it is consistent with the interior solution at each point. The adjustment to the specified velocity then occurs across an outlet boundary layer. The thickness of this boundary layer depends on the Reynolds number; the higher the Reynolds number, the thinner the boundary layer.

ALTERNATIVE FORMULATIONS

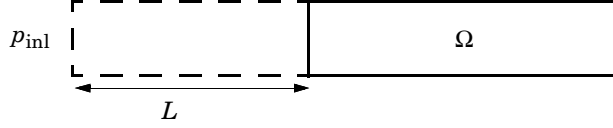
COMSOL Multiphysics provides several specialized boundary conditions that either provide detailed control over the flow at the boundary or that simulate specific devices. In practice they often prescribe a velocity or a pressure but calculate the prescribed values using, for example, ODEs.



Normal Stress Boundary Condition

Fully Developed Flow (Inlet)

The **Fully developed flow** boundary condition can be understood from the following figure:



The flow to the domain Ω is assumed to enter through a straight channel of length L . The channel is a virtual extrusion of the inlet cross section and a pressure constant pressure P_{inl} is applied on the inlet of the virtual channel.

The Fully Developed Flow boundary condition prescribes that the tangential flow component on the boundary is zero:

$$\mathbf{u} - (\mathbf{u} \cdot \mathbf{n})\mathbf{n} = \mathbf{0}$$

The momentum equation for a fully developed flow in the virtual extrusion of the inlet cross section can then be projected onto the inlet boundary with the following weak equation as the result:

$$\int_{\partial\Omega} \left(\left(L \left(-\mu (\nabla_{\mathbf{t}} \mathbf{u} + (\nabla_{\mathbf{t}} \mathbf{u})^T) + \frac{(p + P_{\text{inl}})}{2} \mathbf{I} \right) : (\nabla_{\mathbf{t}} \hat{\mathbf{u}}) \right) - P_{\text{inl}} (\mathbf{n} \cdot \hat{\mathbf{u}}) \right)$$

The exact value of L is somewhat arbitrary as long as it is not too high or too low. L is therefore set to ten times the inlet edge length in 2D and to ten times the square root of the inlet area in 2D axisymmetry and in 3D.

The fact that the velocity profile is not prescribed, but rather the solution of a projected weak contribution, means that the actual velocity profile that is obtained on the inlet can deviate from the analytical fully developed flow profile (in cases such an analytical solution exists) if required by the solution inside the computational domain, Ω . This is most notably if the flow is strongly curved just downstream of the inlet. This can, for example, be the case if an object is positioned just downstream of the inlet or if the adjacent boundaries are not orthogonal to the inlet. The analytical solution can in these cases be recovered by including a little bit of the inlet channel in the computational domain.

The inlet pressure, P_{inl} , must be solved for and its equation is a discrete algebraic equation (DAE) for P_{inl} . When, for example, the average velocity is specified, the DAE reads

$$(\langle \mathbf{u} \cdot \mathbf{n} \rangle + U_{\text{av}}) \hat{P}_{\text{inl}}$$

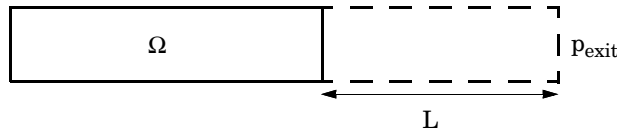
where $\langle \cdot \rangle$ denotes the average over the inlet. Since the equation for P_{inl} is a DAE (the equation for P_{inl} does not contain P_{inl}), it must be solved coupled to Navier–Stokes and it must be treated by a Vanka presmoothing and postsmoothing if iterative solvers are used.

The boundary conditions for the virtual inlet channel are inherited from the boundaries adjacent to the inlet channel. Virtual boundaries adjacent to walls (except slip walls) are treated as no-slip walls. Virtual boundaries adjacent to any other type of boundary are treated as slip-walls (or equivalently as symmetry boundaries). So in the figure above, if the lower boundary of Ω is a no-slip wall and the top boundary is a symmetry boundary, the lower boundary of the virtually extruded channel becomes a no-slip wall and the upper boundary becomes a symmetry boundary.

Algebraic turbulence models need additional equations or constraints. For all other turbulence models, their weak equations in the virtual inlet channel are projected onto the inlet of the computational domain. The projections are regularized to prevent the production to fall to zero, so the solutions of the projects can show slight deviation from the correct solutions close to no-slip walls.

Fully Developed Flow (Outlet)

The **Fully developed flow** boundary condition can be understood from the following figure:



The flow to the domain Ω is assumed to enter through a straight channel of length L . The channel is a virtual extrusion of the inlet cross section and a pressure constant pressure P_{exit} is applied on the inlet of the virtual channel.

The Fully Developed Flow boundary condition prescribes that the tangential flow component on the boundary is zero:

$$\mathbf{u} - (\mathbf{u} \cdot \mathbf{n})\mathbf{n} = \mathbf{0}$$

The momentum equation for a fully developed flow in the virtual extrusion of the inlet cross section can then be projected onto the inlet boundary with the following weak equation as the result:

$$\int_{\partial\Omega} \left(\left(L \left(-\mu(\nabla_{\mathbf{t}}\mathbf{u} + (\nabla_{\mathbf{t}}\mathbf{u})^T) + \frac{(p + P_{\text{exit}})}{2} \mathbf{I} \right) : (\nabla_{\mathbf{t}}\hat{\mathbf{u}}) \right) - P_{\text{exit}}(\mathbf{n} \cdot \hat{\mathbf{u}}) \right)$$

The exact value of L is somewhat arbitrary as long as it is not too high or too low. L is therefore set to ten times the inlet edge length in 2D and to ten times the square root of the inlet area in 2D axisymmetry and in 3D.

The fact that the velocity profile is not prescribed, but rather the solution of a projected weak contribution, means that the actual velocity profile that is obtained on the outlet can deviate from the analytical fully developed flow profile (in cases such an analytical solution exists) if required by the solution inside the computational domain, Ω . The inlet pressure, P_{exit} , must be solved for and its equation is a discrete algebraic equation (DAE) for P_{exit} . When, for example, the average velocity is specified, the DAE reads

$$(\langle \mathbf{u} \cdot \mathbf{n} \rangle + U_{\text{av}})\dot{P}_{\text{exit}}$$

where $\langle \cdot \rangle$ denotes the average over the inlet. Since the equation for P_{ext} is a DAE, it must be solved coupled to Navier–Stokes and it must be treated by a Vanka presmoothing and postsmoothing if iterative solvers are used.

The boundary conditions for the virtual outlet channel are inherited from the boundaries adjacent to the outlet channel. Virtual boundaries adjacent to walls (except slip walls) are treated as no-slip walls. Virtual boundaries adjacent to any other type of boundary are treated as slip-walls (or equivalently as symmetry boundaries).

No Viscous Stress

For this module, and in addition to the Pressure, No Viscous Stress boundary condition, the viscous stress condition sets the viscous stress to zero:

$$\left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) \mathbf{n} = \mathbf{0}$$

$$(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))\mathbf{n} = \mathbf{0}$$

using the compressible/weakly compressible and the incompressible formulation, respectively.

The condition is not a sufficient outlet condition since it lacks information about the outlet pressure. It must hence be combined with pressure point constraints on one or several points or lines surrounding the outlet.

This boundary condition is numerically the least stable outlet condition, but can still be beneficial if the outlet pressure is nonconstant due to, for example, a nonlinear volume force.

Normal Stress Boundary Condition

The total stress on the boundary is set equal to a stress vector of magnitude f_0 , oriented in the negative normal direction:

$$\left(-p\mathbf{I} + \left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = -f_0\mathbf{n}$$

$$(-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))\mathbf{n} = -f_0\mathbf{n}$$

using the compressible/weakly compressible and the incompressible formulation, respectively.

This implies that the total stress in the tangential direction is zero. This boundary condition implicitly sets a constraint on the pressure which for 2D flows is

$$p = 2\mu \frac{\partial u_n}{\partial n} + f_0 \quad (3-21)$$

If $\partial u_n / \partial n$ is small, [Equation 3-21](#) states that $p \approx f_0$.

The Normal Stress condition is the mathematically correct version of the [Pressure Conditions](#) condition ([Ref. 4](#)), but it is numerically less stable.

Pressure Boundary Condition

For single-phase flow, a mathematically correct natural boundary condition for outlets is

$$\left(-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I}\right)\mathbf{n} = -p_0\mathbf{n} \quad (3-22)$$

$$(-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))\mathbf{n} = -p_0\mathbf{n} \quad (3-23)$$

using the compressible/weakly compressible and the incompressible formulation, respectively.

This is a normal stress condition together with a no-tangential-stress condition. When $\mu > 0$, Equation 3-22 or Equation 3-23 can be supplemented with a tangential velocity condition

$$\mathbf{u} \cdot \mathbf{t} = 0 \quad (3-24)$$

If so, the no-tangential-stress condition is overridden. An issue with Equation 3-22 or Equation 3-23 is that it does not strongly enforce unidirectional flow on the boundary. If the prescribed pressure on an outlet is too high, parts of the outlet can actually have inflow. This is not as much of an issue for the Navier–Stokes equations as it is an issue for scalar transport equations solved along with the Navier–Stokes equations. Hence, when applying the **Pressure** boundary condition at an outlet or inlet you can further constrain the flow. With the **Suppress backflow** option

$$\begin{aligned} \left(-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) \mathbf{n} &= -\hat{p}_0\mathbf{n} \\ (-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))\mathbf{n} &= -\hat{p}_0\mathbf{n} \\ \hat{p}_0 &\leq p_0 \end{aligned} \quad , \quad (3-25)$$

the normal stress is adjusted to keep

$$\mathbf{u} \cdot \mathbf{n} \geq 0 \quad (3-26)$$

Equation 3-25 effectively means that the prescribed pressure is p_0 if $\mathbf{u} \cdot \mathbf{n} \geq 0$, but smaller at locations where $\mathbf{u} \cdot \mathbf{n} < 0$. This means that Equation 3-25 does not completely prevent backflow, but the backflow is substantially reduced. Backflow is suppressed also when external forces are acting on the fluid, provided the magnitude of these forces are of the same order as the dynamic pressure at the outlet.

A pressure condition can also be applied at an inlet. In this case, either the normal stress is prescribed

$$\begin{aligned}
\mathbf{n}^T \left(-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \mathbf{n} &= -\hat{p}_0 \\
\mathbf{n}^T (-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} &= -\hat{p}_0 \\
\hat{p}_0 &\geq p_0
\end{aligned} \tag{3-27}$$

together with the tangential condition in [Equation 3-24](#), or, a general flow direction is prescribed.

$$\begin{aligned}
\mathbf{r}_{\mathbf{u}}^T \left(-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \mathbf{n} &= -\hat{p}_0 (\mathbf{r}_{\mathbf{u}} \cdot \mathbf{n}) \\
\mathbf{r}_{\mathbf{u}}^T (-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} &= -\hat{p}_0 (\mathbf{r}_{\mathbf{u}} \cdot \mathbf{n}) \\
\hat{p}_0 &\geq p_0 \\
\mathbf{u} - (\mathbf{u} \cdot \mathbf{r}_{\mathbf{u}}) \mathbf{r}_{\mathbf{u}} &= \mathbf{0}, \quad \mathbf{r}_{\mathbf{u}} = \frac{\mathbf{d}_{\mathbf{u}}}{\|\mathbf{d}_{\mathbf{u}}\|}
\end{aligned} \tag{3-28}$$

The “>” option is used with suppress backflow to have $\mathbf{u} \cdot \mathbf{n} \leq 0$ or $\mathbf{u} \cdot \mathbf{r}_{\mathbf{u}} \geq 0$.

For incompressible single-phase flow, it is also allowed to specify the total pressure, p_{tot} , instead of the static pressure, p_{stat} , on inlet and outlet boundaries. It is more useful, for example, in pump applications. The pressure is then prescribed at the boundaries using Bernoulli’s principle,

$$p = p_{\text{stat}} = p_{\text{tot}} - \frac{1}{2} \rho |\mathbf{u}|^2 \tag{3-29}$$

The equation is imposed with two options: **Average** and **Pointwise**.

In the first option, p_{stat} is prescribed by:

$$\mathbf{n}^T (-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} = \mathbf{n}^T (-p_{\text{stat}} \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) \mathbf{n} \tag{3-30}$$

where p_{tot} and $|\mathbf{u}|^2$ are averaged over the boundaries using the aveop operator:

$$p_{\text{stat}} = \text{aveop}(p_{\text{tot}}) - \frac{1}{2} \rho \cdot \text{aveop}(|\mathbf{u}|^2)$$

For the second option, [Equation 3-29](#) is prescribed pointwise.

See [Inlet](#), [Outlet](#), [Open Boundary](#), and [No Viscous Stress](#) for the individual node settings. Note that some modules have additional theory sections describing options available with that module.

Porous Interface

This condition properly accounts for a sudden jump in porosity ε_p across boundary between different domains, giving the corresponding jump in stress across the interface. The emergence of a boundary layer on the upstream side of the interface is also revealed, which provides modifications to the behavior of the tangential velocity near the interface. The feature helps to avoid the need to add a very thin transition region, in which $\varepsilon_{p,\text{upstream}}$ changes to $\varepsilon_{p,\text{downstream}}$ continuously, while solving the Brinkman equation in such situations. Below, ‘ $-$ ’ and ‘ $+$ ’ (used as subscripts and superscripts) refer to the upstream and downstream side of the interface, respectively.

Whatever the option for **Continuity conditions** is chosen, the velocity conditions are

$$[\rho(\mathbf{u} \cdot \mathbf{n})]_{-}^{+} = 0, \quad \mathbf{u}_t \Big|_{-}^{+} \equiv [\mathbf{u} - (\mathbf{u} \cdot \mathbf{n})\mathbf{n}] \Big|_{-}^{+} = 0$$

That is, the mass flux and tangential velocity across the interface are constant, and $\mathbf{n}$ is assumed to be a normal pointing from ‘ $-$ ’ to ‘ $+$ ’. **Velocity and stress** option just imposes usual stress continuity adding a correction for a possible jump in density:

$$\left[p\mathbf{n} - \mathbf{K}\mathbf{n} + \rho(\mathbf{u} \cdot \mathbf{n}) \left(\left[\frac{1}{\varepsilon_p} \right]_{\text{av}} \right)^2 \mathbf{u} \right]_{-}^{+} = 0$$

The option **Velocity and porosity-corrected stress** (default) adds a contribution due to the inertial term, which is nontrivial (gives nonzero effect) when ε_p jumps across the interface:

$$\left[p\mathbf{n} - \mathbf{K}\mathbf{n} + \rho(\mathbf{u} \cdot \mathbf{n}) \left(\left[\frac{1}{\varepsilon_p} \right]_{\text{av}} \right)^2 \mathbf{u} + \rho(\mathbf{u} \cdot \mathbf{n}) \left(\frac{1}{2\varepsilon_p^2} \right) \mathbf{u}_{\text{av}} \right]_{-}^{+} = 0$$

Notice that $\left[\frac{1}{\varepsilon_p} \right]_{\text{av}} = \left(\left[\frac{1}{\varepsilon_p} \right]_{-} + \left[\frac{1}{\varepsilon_p} \right]_{+} \right) / 2$, $\mathbf{u}_{\text{av}} = (\mathbf{u}_{-} + \mathbf{u}_{+}) / 2$ and so on.

Then, the **Velocity and porosity-corrected stress with loss** option has a ‘pressure loss’ term:

$$\left[p\mathbf{n} - K\mathbf{n} + \rho(\mathbf{u} \cdot \mathbf{n}) \left(\left[\frac{1}{\epsilon_p} \right]_{\text{av}} \right)^2 \mathbf{u} + \rho(\mathbf{u} \cdot \mathbf{n}) \left(\frac{1}{2\epsilon_p^2} \right) \mathbf{u}_{\text{av}} \right] \Bigg|_{-}^{+} + p^{(\text{loss})} \mathbf{n} = 0$$

This loss occurs in the downstream direction. Its mechanism is similar to the mechanism of common engineering ‘total pressure loss’. Indeed, ‘expansion’ and ‘contraction’ across the interface experienced by the flow on the microscopic level (due to increasing or decreasing porosity, ϵ_p , respectively), are roughly equivalent to expansion and contraction of flow in a pipe with geometrical variations. Accordingly, it is assumed that only the normal velocity component is responsible for the loss. In the case of normal flow and negligible viscous term the pipe-analogy is quite exact.

Subsequently, **Pressure loss model** should be specified. Choosing **User-defined loss coefficients** requires values of **Expansion loss coefficient** and **Contraction loss coefficient** ($K_{\text{exp}} = 1$ and $K_{\text{contr}} = 0.5$ are default values). The resulting losses on the ‘expansion’ and ‘contraction’ boundaries are:

$$p^{(\text{loss,expansion})} = K_{\text{exp}} \left(\left(\frac{\rho(\mathbf{u} \cdot \mathbf{n})^2}{2\epsilon_p^2} \right) \right)_{-},$$

$$p^{(\text{loss,contraction})} = K_{\text{contr}} \left(\frac{\rho(\mathbf{u} \cdot \mathbf{n})^2}{2\epsilon_p^2} \right)_{+}$$

Borda–Carnot option results in the following losses:

$$p^{(\text{loss,expansion})} = \frac{\rho_{\text{av}}}{2} \left(\left[\frac{(\mathbf{u} \cdot \mathbf{n})}{\epsilon_p} \right]_{+} - \left[\frac{(\mathbf{u} \cdot \mathbf{n})}{\epsilon_p} \right]_{-} \right)^2,$$

$$p^{(\text{loss,contraction})} = \frac{1}{2} \left(\frac{\rho(\mathbf{u} \cdot \mathbf{n})^2}{\epsilon_p^2} \right)_{+} \left(\frac{1}{c_{\text{vc}}} - 1 \right)^2$$

and requires to choose **Vena contracta model**. The **User defined** option asks for value of **Vena contracta coefficient** ($c_{\text{vc}} = 0.5$ by default). **Weisbach** option computes the coefficient based on the ratio of porosities across the interface:

$$c_{\text{vc}} = c_{\text{vc,ll}} + (1 - c_{\text{vc,ll}}) \left(\frac{\epsilon_{\text{p},+}}{\epsilon_{\text{p},-}} \right)^{n_{\text{vc}}}$$

and requires inputs for **Lower limit** and **Exponent** ($c_{\text{vc,ll}} = 0.63$ and $n_{\text{vc}} = 3$ are default values).

Although the velocity conditions formally look the same for all **Continuity conditions** options, the latter two (containing the inertial correction due to the porosity jump) lead to an extra nontrivial effect. Namely, the tangential velocity exponentially evolves over a boundary layer formed on the upstream side of the interface boundary. This boundary layer has a characteristic length equal to

$$l_- = \frac{\mu \varepsilon_{p,-}}{\rho(\mathbf{u} \cdot \mathbf{n})}$$

The ratio of the tangential velocities right after the interface (‘downstream’) and outside the boundary layer (‘upstream infinity’) is

$$\frac{|\mathbf{u}_{t,+}|}{|\mathbf{u}_{t,\text{upstream infinity}}|} = \frac{2\varepsilon_{p,+}/\varepsilon_{p,-}}{\varepsilon_{p,+}/\varepsilon_{p,-} + \varepsilon_{p,-}/\varepsilon_{p,+}}$$

In terms of the interstitial velocity, $\mathbf{u}^{(i)} = \mathbf{u}/\varepsilon_p$, this results in:

$$\frac{|\mathbf{u}_{t,+}^{(i)}|}{|\mathbf{u}_{t,\text{upstream infinity}}^{(i)}|} = \frac{2}{\varepsilon_{p,+}/\varepsilon_{p,-} + \varepsilon_{p,-}/\varepsilon_{p,+}} \leq 1$$

Obviously, tangential component of the interstitial kinetic energy is lost across the interface. For low viscosity and high normal-flow component, when l_u is much lower than l_{geom} (the characteristic geometry length) there are apparent variations in $\mathbf{u}_t$ and $\mathbf{u}_t^{(i)}$ in a thin boundary layer on the upstream side of the interface. However, for the cases with high viscosity or nearly interface-parallel flow, l_u is much larger than l_{geom} , and any variations in $\mathbf{u}_t$ and $\mathbf{u}_t^{(i)}$ are virtually invisible.

Notice that the feature does not take into account possible underresolved porous scale, as is described in [Theory for the Free and Porous Media Flow, Darcy Interface](#).

Mass Sources for Fluid Flow

There are two types of mass sources in a Single-Phase Flow interface: point sources and line sources.



These features require at least one of the following licenses: Battery Design Module, CFD Module, Chemical Reaction Engineering Module, Corrosion Module, Electrochemistry Module, Electrodeposition Module, Fuel Cell & Electrolyzer Module, Microfluidics Module, Pipe Flow Module, Polymer Flow Module, or Subsurface Flow Module.

POINT SOURCE

A point source is theoretically formed by taking a mass injection/ejection, $\dot{Q}$ (SI unit: $\text{kg}/(\text{m}^3 \cdot \text{s})$), in a small volume δV and then letting the size of the volume tend to zero while keeping the total mass flux constant. Given a point source strength, $\dot{q}_p$ (SI unit: kg/s), this can be expressed as

$$\lim_{\delta V \rightarrow 0} \int_{\delta V} \dot{Q} = \dot{q}_p \quad (3-31)$$

An alternative way to form a point source/sink is to assume that mass is injected/extracted through the surface of a small object. Letting the object surface area tend to zero while keeping the mass flux constant, results in the same point source. For this alternative approach, effects resulting from the physical object volume, such as drag and fluid displacement, need to be neglected.

The weak contribution

$$\dot{q}_p \text{test}(p)$$

is added to a point in the geometry. As can be seen from [Equation 3-31](#), $\dot{Q}$ must tend to plus or minus infinity as δV tends to zero. This means that in theory the pressure also tends to plus or minus infinity.

Observe that “point” refers to the physical representation of the source. A point source can therefore only be added to points in 3D components and to points on the symmetry axis in 2D axisymmetry components. Other geometrical points in 2D components represent physical lines.

The finite element representation of [Equation 3-31](#) corresponds to a finite pressure in a point with the effect of the point source spread out over a region around the point.

The size of the region depends on the mesh and on the strength of the source. A finer mesh gives a smaller affected region, but also a more extreme pressure value. It is important not to mesh too finely around a point source since the resulting pressure can result in unphysical values for the density, for example. It can also have a negative effect on the condition number for the equation system.

LINE SOURCE

A line source can theoretically be formed by assuming a source of strength $\dot{Q}$ (SI unit: $\text{kg}/(\text{m}^3 \cdot \text{s})$), located within a tube with cross-sectional area δS and then letting δS tend to zero, while keeping the total mass flux per unit length constant. Given a line source strength, $\dot{q}_l$ (SI unit: $\text{kg}/(\text{m} \cdot \text{s})$), this can be expressed as

$$\lim_{\delta S \rightarrow 0} \int_{\delta S} \dot{Q} = \dot{q}_l \quad (3-32)$$

As in the point source case, an alternative approach is to assume that mass is injected/extracted through the surface of a small object. This results in the same mass source, but requires that effects on the fluid resulting from the physical object volume are neglected.

The weak contribution

$$\dot{q}_{l\text{test}}(p)$$

is added to lines in 3D or to points in 2D (which represent cut-through views of lines). Line sources can also be added to the axisymmetry line in 2D axisymmetry components. It cannot, however, be added to geometrical lines in 2D because they represent physical planes.

As with a point source, it is important not to mesh too finely around the line source.



For feature node information, see [Line Mass Source](#) and [Point Mass Source](#) in the *COMSOL Multiphysics Reference Manual*.



For the Reacting Flow in Porous Media, Diluted Species interface, which is available with the CFD Module, Chemical Reaction Engineering Module, or Battery Design Module, these shared physics nodes are renamed as follows:

- The **Line Mass Source** node is available as two nodes, one for the fluid flow (**Fluid Line Source**) and one for the species (**Species Line Source**).
- The **Point Mass Source** node is available as two nodes, one for the fluid flow (**Fluid Point Source**) and one for the species (**Species Point Source**).

Numerical Stability — Stabilization Techniques for Fluid Flow

The momentum equation ([Equation 3-9](#) or [Equation 3-11](#)) is a (nonlinear) convection–diffusion equation. Such equations can easily become unstable if discretized using the Galerkin finite element method. Stabilized finite element methods are usually necessary in order to obtain physical solutions. The stabilization settings are found in the main Fluid Flow interface features. To display this section, click the **Show More Options** button () and select **Stabilization** in the **Show More Options** dialog.

There are three types of stabilization methods available for Navier–Stokes — *streamline diffusion*, *crosswind diffusion*, and *isotropic diffusion*. Streamline diffusion and crosswind diffusion are consistent stabilization methods, whereas isotropic diffusion is an inconsistent stabilization method.

For optimal functionality, the exact weak formulations of and constants in the streamline diffusion and crosswind diffusion methods depend on the order of the shape functions (basis functions) for the elements. The values of constants in the streamline diffusion and crosswind diffusion methods follow [Ref. 5](#) and [Ref. 6](#).

STREAMLINE DIFFUSION

For strongly coupled systems of equations, the streamline diffusion method must be applied to the system as a whole rather than to each equation separately. These ideas were first explored by Hughes and Mallet ([Ref. 8](#)) and were later extended to Galerkin least-squares (GLS) applied to the Navier–Stokes equations ([Ref. 9](#)). This is the streamline diffusion formulation that COMSOL Multiphysics supports.

Streamline diffusion is active by default because it is necessary when convection is dominating the flow.

The governing equations for incompressible flow are subject to the Babuška–Brezzi condition, which states that the shape functions (basis functions) for pressure must be of lower order than the shape functions for velocity. If the incompressible Navier–Stokes equations are stabilized by streamline diffusion, it is possible to use equal-order interpolation. Hence, streamline diffusion is necessary when using first-order elements for both velocity and pressure. This applies also if the model is solved using geometric multigrid (either as a solver or as a preconditioner) and at least one multigrid hierarchy level uses linear Lagrange elements.

CROSSWIND DIFFUSION

Crosswind diffusion can also be formulated for systems of equations, and when applied to the Navier–Stokes equations it becomes a shock-capturing operator. COMSOL Multiphysics supports the formulation in [Ref. 9](#) with a shock-capturing viscosity of the Hughes–Mallet type [Ref. 8](#).

Incompressible flows do not contain shock waves, but crosswind diffusion is still useful for introducing extra diffusion in sharp boundary layers and shear layers that otherwise would require a very fine mesh to resolve.

Crosswind diffusion is active by default as it makes it easier to obtain a solution even if the problem is fully resolved by the mesh. Crosswind diffusion also enables the iterative solvers to use inexpensive presmoothers. If crosswind diffusion is deactivated, more expensive preconditioners must be used instead.

STABILIZATION TIME SCALES

The time-scale tensor for time-dependent problems should in theory depend on a time-scale that many references just set to the time-step taken by the time-solver, Δt (see, for example, [Ref. 9](#) and [Ref. 10](#)). This simple approach does not, however, necessarily reflect on the actual time-scales in the physics. A typical example is reacting flows where the time step is often guided by fast reactions, while the flow develops relatively slowly. The COMSOL Multiphysics software can therefore replace Δt^2 in the time-scale tensor with measures of type $(1/(\Delta \tilde{t})^2)^{-1}$, which are calculated from projections of weak expressions in a fashion similar to those in [Ref. 11](#). These measures of the time scale are used when **Use dynamic subgrid time scale** is activated.

The **Limit small time steps effect on stabilization time scale** option can be used to ramp down the time-step contribution in the stabilization when the time step becomes much smaller than the convective and diffusive time scales. This option prevents loss of pressure stabilization at such small time steps.

ISOTROPIC DIFFUSION

Isotropic diffusion adds diffusion to the Navier–Stokes equations. Isotropic diffusion significantly reduces the accuracy of the solution but does a very good job at reducing oscillations. The stability of the continuity equation is not improved.



[Numerical Stabilization](#) and [Iterative](#) in the *COMSOL Multiphysics Reference Manual*

Solvers for Laminar Flow

The Navier–Stokes equations constitute a nonlinear equation system. A nonlinear solver must hence be applied to solve the problem. The nonlinear solver iterates to reach the final solution. In each iteration, a linearized version of the nonlinear system is solved using a linear solver. In the time-dependent case, a time marching method must also be applied. The default suggestions for each of these solver elements are discussed below.

NONLINEAR SOLVER

The nonlinear solver method depends on if the model solves a stationary or a time-dependent problem.

Stationary Solver

In the stationary case, a fully coupled, damped Newton method is applied. The initial damping factor is low since a full Newton step can be harmful unless the initial values are close to the final solution. The nonlinear solver algorithm automatically regulates the damping factor in order to reach a converged solution.

For advanced models, the automatically damped Newton method might not be robust enough. A pseudo time-stepping algorithm can then be invoked. See [Pseudo Time Stepping for Laminar Flow Models](#).

Time-Dependent Solver

In the time-dependent case, the initial guess for each time step is (loosely speaking) the previous time step, which is a very good initial value for the nonlinear solver. The automatic damping algorithm is then not necessary. The damping factor in the Newton method is instead set to a constant value slightly smaller than one. Also, for the same reason, it suffices to update the Jacobian once per time step.

It is seldom worth the extra computational cost to update the Jacobian more than once per time step. For most models it is more efficient to restrict the maximum time step or possibly lower the damping factor in the Newton method.

LINEAR SOLVER

The linearized Navier–Stokes equation system has saddle point character, unless the density depends on the pressure. This means that the Jacobian matrix has zeros on the diagonal. Even when the density depends on the pressure, the equation system effectively shares many numerical properties with a saddle point system.

For small 2D and 3D models, the default solver suggestion is a direct solver. Direct solvers can handle most nonsingular systems and are very robust and also very fast for small models. Unfortunately, they become slow for large models and their memory requirement scales as somewhere between $N^{1.5}$ and N^2 , where N is the number of degrees of freedom in the model. The default suggestion for large 2D and 3D models is therefore the iterative GMRES solver. The memory requirement for an iterative solver optimally scales as N .

GMRES is accelerated by a multigrid method, per default the smoothed aggregation algebraic multigrid (SAAMG) method. The cost of SAAMG is typically very low compared to the number of GMRES iterations necessary if no multigrid method is used. As the name implies, SAAMG builds its coarser meshes algebraically, so the application requires no additional meshes in order to employ SAAMG. In contrast, the geometric multigrid (GMG) method requires actual meshes. If a sufficient number of multigrid levels can be constructed, GMG is often faster than SAAMG. GMG is also superior for cluster computations and for shared-memory computations with many cores. When the default linear solver is GMRES, an optional, but deactivated, linear solver node is available where GMRES is accelerated by GMG.

Multigrid methods need smoothers, but the saddle point character of the linear system restricts the number of applicable smoothers. The choices are further restricted by the anisotropic meshes frequently encountered in fluid-flow problems. The efficiency of the smoothers is highly dependent on the numerical stabilization. Iterative solvers perform at their best when both [Streamline Diffusion](#) and [Crosswind Diffusion](#) are active.

The default smoother for P1+P1 elements is SCGS. This is an efficient and robust smoother specially designed to solve saddle point systems on meshes that contain anisotropic elements. The SCGS smoother works well even without crosswind diffusion. SCGS can sometimes work for higher-order elements, especially if **Method** in the SCGS settings is set to **Mesh element lines**. But there is no guarantee for this, so the

default smoother for higher order elements is an SOR Line smoother. SOR Line handles mesh anisotropy but does not formally address the saddle point character. It does, however, function in practice provided that streamline diffusion and crosswind diffusion are both active.

A different kind of saddle point character can arise if the equation system contains ODE variables. Some advanced boundary conditions can add equations with such variables. These variables must be treated with the Vanka algorithm. SCGS includes an option to invoke Vanka. Models with higher-order elements must apply SCGS or use the Vanka smoother. The latter is the default suggestion for higher-order elements, but it does not work optimally for anisotropic meshes.

TIME-DEPENDENT SOLVERS

The default time-dependent solver for Navier–Stokes is the BDF method with maximum order set to two. Higher BDF orders are not stable for transport problems in general nor for Navier–Stokes in particular.

BDF methods have been used for a long time and are known for their stability. However, they can have severe damping effects, especially the lower-order methods. Hence, if robustness is not an issue, a model can benefit from using the generalized- α method instead. Generalized- α is a solver that has properties similar to those of the second-order BDF solver but it is much less diffusive.

Both BDF and generalized- α are per default set to automatically adjust the time step. While this works well for many models, extra efficiency and accuracy can often be gained by specifying a maximum time step. It is also often beneficial to specify an initial time step to make the solver progress smoothly in the beginning of the time series.



In the *COMSOL Multiphysics Reference Manual*:

- [Time-Dependent Solver](#)
- [Multigrid, Direct, Iterative, SCGS, SOR Line, and Vanka](#)
- [Stationary Solver](#)

Pseudo Time Stepping for Laminar Flow Models

A stationary formulation has per definition no time derivatives and [Equation 3-11](#) reduces to:

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)] + \mathbf{F} \quad (3-33)$$

Solving Equation 3-33 requires a starting guess that is close enough to the final solution. If no such guess is at hand, the fully transient problem can be solved instead. This is, however, a rather costly approach in terms of computational time. An intermediate approach is to add a fictitious time derivative to Equation 3-33:

$$\rho \frac{\mathbf{u} - \text{nojac}(\mathbf{u})}{\Delta \tilde{t}} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)] + \mathbf{F}$$

where $\Delta \tilde{t}$ is a *pseudo time step*. Since $\mathbf{u} - \text{nojac}(\mathbf{u})$ is always zero, this term does not affect the final solution. It does, however, affect the discrete equation system and effectively transforms a nonlinear iteration into a step of size $\Delta \tilde{t}$ of a time-dependent solver.

Pseudo time stepping is not active per default. The pseudo time step $\Delta \tilde{t}$ can be chosen individually for each element based on the local CFL number:

$$\Delta \tilde{t} = \text{CFL}_{\text{loc}} \frac{h}{|\mathbf{u}|}$$

where h is the mesh cell size. A small CFL number means a small time step. It is practical to start with a small CFL number and gradually increase it as the solution approaches steady state.

If the automatic expression for CFL_{loc} is set to the built-in variable CFLCMP , then the automatic setting suggests a PID regulator for the pseudo time step in the default solver. The PID regulator starts with a small CFL number and increases CFL_{loc} as the solution comes closer to convergence.

The default manual expression is

$$\begin{aligned} & 1.3^{\min(\text{niterCMP}, 9)} + \\ & \text{if}(\text{niterCMP} > 20, 9 \cdot 1.3^{\min(\text{niterCMP} - 20, 9)}, 0) + \\ & \text{if}(\text{niterCMP} > 40, 90 \cdot 1.3^{\min(\text{niterCMP} - 40, 9)}, 0) \end{aligned} \quad (3-34)$$

The variable niterCMP is the nonlinear iteration number. It is equal to one for the first nonlinear iteration. CFL_{loc} starts at 1.3 and increases by 30% each iteration until it reaches $1.3^9 \approx 10.6$. It remains there until iteration number 20 at which it starts to increase until it reaches approximately 106. A final increase after iteration number 40

then takes it to 1060. [Equation 3-34](#) can, for some advanced flows, increase CFL_{loc} too slowly or too quickly. CFL_{loc} can then be tuned for the specific application.



For details about the CFL regulator, see [Pseudo Time Stepping](#) in the *COMSOL Multiphysics Reference Manual*.

Nitsche Constraints

Some boundary conditions are implemented using Nitsche constraints. These boundary conditions include

- [Wall](#) — Slip, sliding walls, and moving walls (that is, walls with nonzero translational velocity).
- [Periodic Flow Condition](#)
- [Flow Continuity](#)

The formulation used in the Fluid Flow interfaces in single-phase flow interfaces corresponds to the Symmetric Interior Penalty Galerkin method (SIPG). The SIPG method can be regarded to satisfy the boundary conditions in an integral sense rather than pointwise. More information on SIPG can be found in [Ref. 15](#).

In particular, the SIPG formulation includes a penalty parameter that must be large enough for the formulation to be coercive. The higher the value, the better the boundary condition is fulfilled, but a value that is too high results in an ill-conditioned equation system. The penalty parameter is implemented according to [Ref. 16](#).

Particle Tracing in Fluid Flow

The Particle Tracing Module is available to assist with these types of modeling problems.

It is possible to model particle tracing with COMSOL Multiphysics provided that the impact of the particles on the flow field is negligible. First compute the flow field, and then, as an analysis step, calculate the motion of the particles. The motion of a particle is defined by Newton's second law

$$m \frac{d^2 \mathbf{x}}{dt^2} = \mathbf{F}\left(t, \mathbf{x}, \frac{d\mathbf{x}}{dt}\right)$$

where $\mathbf{x}$ is the position of the particle, m the particle mass, and $\mathbf{F}$ is the sum of all forces acting on the particle. Examples of forces acting on a particle in a fluid are the drag force, the buoyancy force, and the gravity force. The drag force represents the force that a fluid exerts on a particle due to a difference in velocity between the fluid and the particle. It includes the viscous drag, the added mass, and the Basset history term. Several empirical expressions have been suggested for the drag force. One of those is the one proposed by Khan and Richardson (Ref. 13). That expression is valid for spherical particles for a wide range of particle Reynolds numbers. The particle Reynolds number is defined as

$$\text{Re}_p = \frac{|\mathbf{u} - \mathbf{u}_p| 2r\rho}{\mu}$$

where $\mathbf{u}$ is the velocity of the fluid, $\mathbf{u}_p$ the particle velocity, r the particle radius, ρ the fluid density, and μ the dynamic viscosity of the fluid. The empirical expression for the drag force according to Khan and Richardson is

$$\mathbf{F} = \pi r^2 \rho |\mathbf{u} - \mathbf{u}_p| (\mathbf{u} - \mathbf{u}_p) [1.84 \text{Re}_p^{-0.31} + 0.293 \text{Re}_p^{0.06}]^{3.45}$$



The model *Flow Past a Cylinder* (Application Library path **COMSOL_Multiphysics/Fluid_Dynamics/cylinder_flow**) demonstrates how to add and set up particle tracing in a plot group using the **Particle Tracing with Mass** node. It uses the predefined Khan–Richardson model for the drag force and neglects gravity and buoyancy forces.

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Porous Media and Subsurface Flow Interfaces

This chapter has information about the physics interfaces found under the **Fluid Flow > Porous Media and Subsurface Flow** branch (). It contains physics interfaces for the following sets of equations that apply under different conditions:

- [The Darcy's Law Interface](#) describes low-velocity flows of one or more fluids.
- [The Richards' Equation Interface](#) governs the wetting and drying of rocks or soils.
- [The Two-Phase Darcy's Law Interface](#) describes two-phase fluid movement through interstices in a porous medium. An extra equation is computed to calculate the saturation transport.
- [The Multiphase Flow in Porous Media Interface](#) which combines low velocity flow with phase transport of multiple immiscible phases in a porous medium is described in Chapter [Multiphysics Interfaces and Couplings](#).
- [The Fracture Flow Interface](#) is a variant of Darcy's law that defines the flow along the interior boundaries representing fractures within a porous or solid medium.
- [The Brinkman Equations Interface](#) is for porous media high-velocity flows.
- [The Free and Porous Media Flow, Brinkman Interface](#) and [The Free and Porous Media Flow, Darcy Interface](#) combine free flow adjacent to porous domains.
- [The Laminar Flow and Creeping Flow Interfaces](#) use the *Navier–Stokes equations* (in which the inertia term is neglected in case of Creeping Flow) to

describe free flows within a river or well, for example. The Laminar Flow Interface is available with all COMSOL licenses.

With Darcy's law and Richards' equation, COMSOL Multiphysics solves for pressure and has physics interfaces for pressure head and hydraulic head. Set up these equations for single or multiple fluids. The physics interfaces can be coupled to other physics interfaces to describe poroelasticity, convection of heat- and density-driven flows, solute transfer, and electromagnetic forces, to name a few.

In this chapter you will find the following sections:

- Darcy's Law:
 - [Theory for the Darcy's Law Interface](#)
 - [The Darcy's Law Interface](#)
- Richards' Equation:
 - [Theory for the Richards' Equation Interface](#)
 - [The Richards' Equation Interface](#)
- Two-Phase Darcy's Law:
 - [Theory for the Two-Phase Darcy's Law Interface](#)
 - [The Two-Phase Darcy's Law Interface](#)
- Fracture Flow:
 - [Theory for the Fracture Flow Interface](#)
 - [The Fracture Flow Interface](#)
- Brinkman Equations:
 - [Theory for the Brinkman Equations Interface](#)
 - [The Brinkman Equations Interface](#)
- Free and Porous Media Flow, Brinkman:
 - [Theory for the Free and Porous Media Flow, Brinkman Interface](#)
 - [The Free and Porous Media Flow, Brinkman Interface](#)
- Free and Porous Media Flow, Darcy:
 - [Theory for the Free and Porous Media Flow, Darcy Interface](#)
 - [The Free and Porous Media Flow, Darcy Interface](#)

Theory for the Darcy's Law Interface

The Darcy's Law Interface theory is described in this section:

- [About Darcy's Law](#)
- [Darcy's Law — Equation Formulation](#)
- [Storage Models](#)
- [Average Linear Velocity](#)
- [Dual Porosity and Dual Permeability Theory](#)
- [References for the Darcy's Law Interface](#)

About Darcy's Law

In a porous medium, the global transport of momentum by shear stresses in the fluid is often negligible, because the pore walls impede momentum transport to the fluid outside the individual pores. A detailed description, down to the resolution of every pore, is not practical in most applications. A homogenization of the porous and fluid media into a single medium is a common alternative approach. Darcy's law together with the continuity equation and equation of state for the pore fluid (or gas) provide a complete mathematical model suitable for a wide variety of applications involving porous media flows, for which the pressure gradient is the major driving force.

Darcy's law describes fluid movement through interstices in a porous medium. Because the fluid loses considerable energy to frictional resistance within pores, flow velocities in porous media are very low. The Darcy's Law interface can be applied to small-scale tasks in chemical engineering as well as to large-scale geophysical or hydrological tasks such as water moving in an aquifer or stream bank, oil migrating to a well, and even magma rising through the earth to a chamber in a volcano (see [Ref. 1](#), [Ref. 2](#), [Ref. 3](#), and [Ref. 4](#)). Also set up multiple Darcy's Law interfaces to model multiphase flows involving more than one mobile phase.

Darcy's law describes flow in porous media driven by gradients in the hydraulic potential field, which has units of pressure. For many applications it is convenient to represent the total hydraulic potential or the pressure and the gravitational components with equivalent heights of fluid or head. Division of potential by the fluid weight can simplify modeling because units of length make it straightforward to compare to many physical data. Consider, for example, fluid levels in wells, stream heights, topography, and velocities. The physics interface also supports specifying

boundary conditions and result evaluation using hydraulic head and pressure head. In the physics interface, pressure is always the dependent variable.

Darcy's Law — Equation Formulation

Darcy's law states that the velocity field is determined by the pressure gradient, the fluid viscosity, and the structure of the porous medium:

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p \quad (4-1)$$

In this equation, $\mathbf{u}$ is the Darcy's velocity or specific discharge vector (SI unit: m/s); κ is the permeability of the porous medium (SI unit: m²); μ is the fluid's dynamic viscosity (SI unit: Pa·s); p is the pore pressure (SI unit: Pa) and ρ is the density of the fluid (SI unit: kg/m³).



- The above equation formulation is called linear Darcy's law. The applicability of this formulation is discussed in the chapter [About Darcian and Non-Darcian Flow](#) in the *Porous Media Flow Module User's Guide*.
- Permeability can be defined in different ways, either directly or based on predefined permeability models. These are described in detail in the chapter [Permeability Models](#).
- For a detailed overview of the permeability models available in each product visit www.comsol.com/products/specifications/.

The Darcy's Law interface combines Darcy's law with the continuity equation

$$\frac{\partial}{\partial t}(\rho \epsilon_p) + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (4-2)$$

In the above equation, ρ is the fluid density (SI unit: kg/m³), ϵ_p is the porosity, and Q_m is a mass source term (SI unit: kg/(m³·s)). Porosity is defined as the fraction of the control volume that is occupied by pores. Thus, the porosity can vary from zero for pure solid regions to unity for domains of free flow.

For large-scale applications it might be necessary to take gravity effects into account. Darcy's law then applies when the gradient in hydraulic potential drives fluid movement in the porous medium. Darcy's law then has the form:

$$\mathbf{u} = -\frac{\kappa}{\mu}(\nabla p - \rho \mathbf{g}) \quad (4-3)$$



- See the section [Gravity Effects](#) in the *Subsurface Flow Module User's Guide*.
- Note that gravity effects are only included in some modules. For a detailed overview of the functionality available in each product visit www.comsol.com/products/specifications/.
- Gravity effects are not active by default. Select the checkbox **Include gravity** to activate the acceleration of gravity. Setting the elevation D to zero also turns off gravity effects.

Storage Models

The continuity equation ([Equation 4-2](#)) allows flexible specification of relationships between parameters like density or permeability with respect to variables such as pressure, temperature, concentration, and more.

Expanding the time-derivative term and defining the porosity and density as functions of the pressure and using the chain rule results in

$$\frac{\partial}{\partial t}(\rho \epsilon_p) = \epsilon_p \frac{\partial \rho}{\partial t} + \rho \frac{\partial \epsilon_p}{\partial t} = \epsilon_p \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} + \rho \frac{\partial \epsilon_p}{\partial p} \frac{\partial p}{\partial t}$$

By inserting the definition of fluid compressibility $\chi_f = (1/\rho)(\partial \rho / \partial p)$ to the right-hand side and rearrange the following expression is obtained

$$\frac{\partial(\rho \epsilon_p)}{\partial t} = \rho \left(\epsilon_p \chi_f + \frac{\partial \epsilon_p}{\partial p} \right) \frac{\partial p}{\partial t} = \rho S_p \frac{\partial p}{\partial t}$$

Using this relation, the generalized governing equation ([Equation 4-2](#)) takes the following form:

$$\rho S_p \frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (4-4)$$

In this equation, S_p is the storage coefficient (SI unit: 1/Pa), which can be interpreted as the weighted compressibility of the porous material and the fluid. The storage can be an expression involving results from a solid-deformation equation or an expression involving temperatures and concentrations from other analyses. The Darcy's Law

interface implements Equation 4-4 in the **Porous Medium** node, which explicitly includes different storage models to choose from:

- **From density and porosity**, where the storage directly depends on changes in porosity or fluid density
- **Linearized storage**, which defines S_p using the compressibility of the fluid χ_f and of the porous matrix χ_p according to

$$S_p = \varepsilon_p \chi_f + (1 - \varepsilon_p) \chi_p$$

- **Poroelastic storage** where

$$S_p = \varepsilon_p \chi_f.$$



Storage models are only available for the Subsurface Flow Module and the Porous Media Flow Module. For a detailed overview of the functionality available in each product visit www.comsol.com/products/specifications/.

Average Linear Velocity

Because fluids typically occupy only 10% to 50% of a porous medium, it follows that velocities within the pore channels exceed the Darcy velocity, $\mathbf{u}$, on the order of two to ten times. For clarity, the physics interface includes the average linear velocity within the pores, $\mathbf{u}_\alpha$, (also termed the seepage velocity) defined as $\mathbf{u}_\alpha = \mathbf{u}/\varepsilon_p$, where ε_p is the porosity.

Dual Porosity and Dual Permeability Theory

The first well known publication on this topic is “The Behavior of Naturally Fractured Reservoirs” by J.E. Warren and P.J. Root from 1963. Since then the researchers further developed the idea of a dual porosity/permeability type of a porous medium. One of the well established models is based on the findings of H. Gerke and M. van Genuchten (Ref. 10). Basically the idea behind is to solve for each part of the porous medium (macro- and micropores) the relevant equations and connect them via an exchange term.

DUAL POROSITY MEDIUM

Darcy’s Law is solved for the macropores and taking into account their volume fraction θ_M the equation reads:

$$\theta_M \left(\rho S_{p,M} \frac{\partial p_M}{\partial t} + \nabla \cdot (\rho \mathbf{u}_M) \right) = \theta_M Q_M - Q_{ip} \quad (4-5)$$

The term Q_{ip} describes the mass source from the [Interporosity Flow](#) which is the mass transfer from the macropores to the micropores system. An additional ODE for the (passive) micropores is solved which only act as additional storage volume:

$$(1 - \theta_M) \left(\rho S_{p,m} \frac{\partial p_m}{\partial t} \right) = (1 - \theta_M) Q_M + Q_{ip}.$$

The indices M and m stand for macropores and micropores, respectively.

DUAL PERMEABILITY MEDIUM

Darcy's law has to be solved for both the macroscale system and the microscale system:

$$\begin{aligned} \theta_M \left(\frac{\partial}{\partial t} (\varepsilon_{p,M} \rho) + \nabla \cdot (\rho \mathbf{u}_M) \right) &= -Q_{ip} + \theta_M Q_m \\ (1 - \theta_M) \left(\frac{\partial}{\partial t} (\varepsilon_{p,m} \rho) + \nabla \cdot (\rho \mathbf{u}_m) \right) &= Q_{ip} + (1 - \theta_M) Q_m \\ p &= \theta_M p_M + (1 - \theta_M) p_m \end{aligned} \quad (4-6)$$

θ_M is the volume fraction of the macropores, Q_{ip} denotes the interporosity flow, signifying the mass transfer from the macropores (identified by index M) to the micropores (identified by index m).

Solving Darcy's law for both systems entails solving two equations, each associated with dependent pressure variables p_m and p_M . These equations are coupled via Q_{ip} . From this, average quantities for the dual permeability medium as a whole are computed. The average pressure of the dual permeability medium is defined as:

$$p = \theta_M p_M + (1 - \theta_M) p_m$$

Likewise, average values for permeability, storage, velocity and other relevant parameters are computed.

UNSATURATED DUAL PERMEABILITY MEDIUM

Use the **Unsatuated Dual Permeability Medium** feature to model the transport in an unsaturated dual permeability medium where Richards' Equation ([Equation 4-26](#)) describes the transport in each system.

$$\begin{aligned}
\theta_M \left(\rho \left(S_{e,M} S_{p,M} + \frac{C_{m,M}}{\rho g} \right) \frac{\partial p_M}{\partial t} + \nabla(\rho \mathbf{u}_M) \right) &= -Q_{ip} + \theta_M Q_m \\
(1 - \theta_M) \left(\rho \left(S_{e,m} S_{p,m} + \frac{C_{m,m}}{\rho g} \right) \frac{\partial p_m}{\partial t} + \nabla(\rho \mathbf{u}_m) \right) &= Q_{ip} + (1 - \theta_M) Q_m \\
p &= \theta_M p_M + (1 - \theta_M) p_m
\end{aligned} \tag{4-7}$$

θ_M is the volume fraction of the macropores, Q_{ip} is the interporosity flow which is the mass transfer from the macroscale to the microscale system. The indices **M** and **m** stand for macropores and micropores, respectively.

INTERPOROSITY FLOW

Specify the **Fluid Transfer Function** α_w which is used to calculate the interporosity flow, which is the mass exchange between macro- and microscale system:

$$Q_{ip} = \alpha_w (p_M - p_m) \tag{4-8}$$

with p_M being the pressure within the macroscale system and p_m the pressure within the microscale system

References for the Darcy's Law Interface

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10. H. Gerke and M. van Genuchten, “A dual-porosity model for simulating the preferential movement of water and solutes in structured porous media,” *Water Resour. Res.*, vol. 29, pp. 305–319, 1993.

The Darcy's Law Interface

The **Darcy's Law (dl)** interface (), found under the **Porous Media and Subsurface Flow** branch () when adding a physics interface, is used to simulate fluid flow through interstices in a porous medium. It can be used to model low-velocity flows or media where the permeability and porosity are very small, and for which the pressure gradient is the major driving force and the flow is mostly influenced by the frictional resistance within the pores. Set up multiple **Darcy's Law** interfaces to model multiphase flows involving more than one mobile phase. The Darcy's Law interface can be used for stationary and time-dependent analyses.

The main feature is the **Porous Medium** Properties node, which provides interfaces for defining the fluid material and the porous matrix properties.

When this physics interface is added, the following default nodes are also added in the **Model Builder — Porous Medium, No Flow** (the default boundary condition), and **Initial Values**. Then, from the **Physics** toolbar, add other nodes that implement, for example, boundary conditions and mass sources. You can also right-click **Darcy's Law** to select physics features from the context menu.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `dl`.

PHYSICAL MODEL

Enter a **Reference pressure level** p_{ref} (SI unit: Pa). The default value is `1 [atm]`.

GRAVITY EFFECTS

By default there are no gravity effects added to Darcy's Law interface. Select the checkbox **Include gravity** to activate the acceleration of gravity. When this checkbox is selected, a global **Gravity** feature node is shown in the interface model tree.

Enter a value for the acceleration of gravity. The default value `g_const` is the predefined standard acceleration of gravity on earth.



For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/.

DISCRETIZATION

To display all settings available in this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog.

You can choose the order of the shape functions used for the pressure variable solved by the Darcy's Law interface. The default shape functions are **Quadratic** Lagrange.

The **Compute boundary fluxes** checkbox is not activated by default. When this option is selected, the solver computes variables storing accurate boundary fluxes from each boundary into the adjacent domain.

If the checkbox is cleared, the COMSOL Multiphysics software instead computes the flux variables from the dependent variables using extrapolation, which is less accurate in postprocessing results but does not create extra dependent variables on the boundaries for the fluxes.

Also, the **Apply smoothing to boundary fluxes** checkbox is available if the previous checkbox is selected. The smoothing can provide a better behaved flux value close to singularities.

For details about the boundary fluxes settings, see [Computing Accurate Fluxes](#) in the *COMSOL Multiphysics Reference Manual*.

The **Value type when using splitting of complex variables** setting should in most pure mass transport problems be set to **Real** which is the default. It makes sure that the dependent variable does not get affected by small imaginary contributions, which can occur, for example, when combining a Time Dependent or Stationary study with a frequency-domain study. For more information, see [Splitting Complex-Valued Variables](#) in the *COMSOL Multiphysics Reference Manual*.

DEPENDENT VARIABLES

The dependent variable (field variable) is the **Pressure**. The name can be changed but the names of fields and dependent variables must be unique within a model.

	• Domain, Boundary, Edge, Point, and Pair Nodes for the Darcy's Law Interface • Theory for the Darcy's Law Interface • Physical Constants in the <i>COMSOL Multiphysics Reference Manual</i>
	• <i>Species Transport in the Gas Diffusion Layers of a PEM</i>: Application Library path Fuel_Cell_and_Electrolyzer_Module/Fuel_Cells/pem_gdl_species_transport_2d • <i>Steam Reformer</i>: Application Library path Chemical_Reaction_Engineering_Module/Reactors_with_Porous_Catalysts/steam_reformer • <i>Terzaghi Compaction</i>: Application Library path Subsurface_Flow_Module/Poromechanics/terzaghi_compaction

Domain, Boundary, Edge, Point, and Pair Nodes for the Darcy's Law Interface

The [Darcy's Law Interface](#) has the following domain, boundary, edge, point, and pair nodes, these nodes are available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

DOMAIN

- Porous Medium
- Unsaturated Porous Medium
- Dual Porosity Medium
- Dual Permeability Medium
- Unsaturated Dual Permeability Medium
- Mass Source
- Cross Section
- Initial Values
- Unsaturated Porous Medium
- ThicknessContributing Velocity
- Porous Electrode Coupling

BOUNDARY, EDGE, AND POINT

The following nodes (listed in alphabetical order) are available on exterior boundaries:

- Atmosphere/Gauge
- Electrode Surface Coupling
- Flux Discontinuity
- Fracture
- Hydraulic Head
- Inlet
- Interior Wall
- Line Mass Source
- Mass Flux
- No Flow
- Outlet
- Periodic Condition
- Pervious Layer
- Point Mass Source
- Precipitation
- Pressure
- Pressure Head
- Reaction Coefficients¹
- Symmetry
- Barrier

¹ This node is described for the Transport of Diluted Species Interface

The relevant physics interface condition at interior boundaries is continuity:

$$\mathbf{n} \cdot (\rho_1 \mathbf{u}_1 - \rho_2 \mathbf{u}_2) = 0$$

The continuity boundary condition ensures that the pressure and mass flux are continuous. In addition, the [Pressure](#) boundary condition is available on interior boundaries.

The continuity boundary condition provides continuity for the pressure and mass flux. In addition, the following boundary conditions are available on interior boundaries:

- Pressure
- Pressure Head
- Hydraulic Head
- Interior Wall
- Pervious Layer
- Flux Discontinuity
- Fracture
- Barrier



Note that some of the above-mentioned domain or boundary conditions are only available with some products. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/



In general, to add a node, go to the **Physics** toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the **Attributes** menu.



For axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** node that is valid on the axial symmetry boundaries only.



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Porous Medium

The **Porous Medium** node adds the equations for modeling flow through porous domains. By default these are [Equation 4-9](#) and [Equation 4-10](#) (excluding any mass sources).

$$\frac{\partial}{\partial t}(\varepsilon_p \rho) + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (4-9)$$

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p \quad (4-10)$$

For a steady-state problem the first term in [Equation 4-9](#) disappears. When [Mass Source](#) is active [Equation 4-10](#) is of the following form:

$$\mathbf{u} = -\frac{\kappa}{\mu} (\nabla p - \rho \mathbf{g}) \quad (4-11)$$

The **Porous Medium** feature has two default subfeatures — the [Fluid](#) subfeature where the fluid density ρ (SI unit: kg/m^3) and dynamic viscosity μ (SI unit: $\text{Pa}\cdot\text{s}$) are defined and the [Porous Matrix](#) subfeature where the permeability κ (SI unit: m^2) and porosity ϵ_p (dimensionless) are specified.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the **Definitions** node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** or **Hydraulic conductivity** (in the [Porous Matrix](#) subnode) should be set according to the coordinate system selected in this section.

FLOW MODEL

Define if the flow follows a linear or nonlinear pressure-velocity relationship.

- **Darcian flow** (default) defines a linear relationship using [Equation 4-9](#) and [Equation 4-10](#).
- **Non-Darcian flow** defines a nonlinear relationship using [Equation 4-9](#) and

$$\nabla p = -\frac{\mu}{\kappa} \mathbf{u} - \beta \rho |\mathbf{u}| \mathbf{u}$$

where the nonlinear parameter β (SI unit: $1/\text{m}$) is further specified in the [Porous Matrix](#) subfeature.

STORAGE MODEL

Define the time dependent storage term by selecting one of the following options from the list:

- **From density and porosity** (default), uses the formulation of [Equation 4-9](#)

- **Linearized storage**, where the following linear equation is used to define the storage:

$$S_p = \varepsilon_p \chi_f + (1 - \varepsilon_p) \chi_p$$

- **Poroelastic storage**, where $S_p = \varepsilon_p \chi_f$
- **Quasistatic**, where the storage term is set to zero
- **User defined** to directly specify the storage coefficient S_p

	• Porous Material in the <i>COMSOL Multiphysics Reference Manual</i> • About Darcian and Non-Darcian Flow in the <i>Porous Media Flow Module User's Guide</i> • Storage Models in the <i>Subsurface Flow Module User's Guide</i>
	Note that some options are only available in some modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Porous Medium

Richards' Equation > Porous Medium

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > interface > Porous Medium

Fluid

This node defines the material properties of the fluid of the [Porous Medium](#), the [Unsaturated Porous Medium](#), the [Dual Porosity Medium](#), the [Dual Permeability Medium](#), or the [Unsaturated Porous Medium](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

FLUID PROPERTIES

The available options for the **Fluid type** are **Incompressible**, **Compressible**, **Compressible, linearized**, or **Ideal gas**. The selection determines how the density and compressibility of the fluid are set.

Incompressible

The default **Reference density** ρ_{ref} uses values **From material**. For **User defined** enter another value or expression.

Compressible

The default **Density** ρ uses values **From material**. For **User defined** enter another value or expression as a function of the absolute pressure. The compressibility of the fluid is computed from it.

Compressible, Linearized

The default **Reference density** ρ_{ref} and the **Fluid compressibility** χ_f use values **From material**. For **User defined** enter another value or expression. The fluid density increases linearly as a function of the fluid pressure.

Ideal Gas

For **Ideal gas** it uses the ideal gas law to describe the gas density. In this case, specify the thermodynamics properties. Select a **Gas constant type** — **Specific gas constant** R_s (the default) or **Mean molar mass** M_n (SI unit: J/(mol·K)). For **Mean molar mass** the universal gas constant $R = 8.314$ J/(mol·K) is used as the built-in physical constant. For both properties, the defaults use values **From material**. For **User defined** enter another value or expression.

Dynamic Viscosity

For all fluid types specify the **Dynamic viscosity** μ (SI unit: Pa·s) uses values **From material**. For **User defined** enter another value or expression.

Constitutive Relation

The Polymer Flow Module allows the use of inelastic non-Newtonian models. For Darcian Flow, you can choose between two options: **Specify dynamic viscosity** (default) or **Inelastic non-Newtonian**. Use the default option to either get the dynamic viscosity from a material, or to add a user defined expression. The default option lets you either use the dynamic viscosity from a material or define it with a custom expression. The **Inelastic non-Newtonian** option provides several models and defines the **Apparent shear rate**.

Inelastic Non-Newtonian

The following inelastic non-Newtonian Models are available for the **Fluid** feature under the **Porous Medium** feature:

- Power Law
- Carreau
- Carreau–Yasuda
- Cross
- Cross–Williamson
- Sisko

The setup of these models in porous media is similar to their definition under the [Fluid Properties](#) feature.

Apparent Shear Rate

To define the apparent shear rate $\dot{\gamma}_{app}$, either enter a user-defined expression for the correction factor (α) or choose the **Capillary bundle approach** from the list and provide a value for the Tortuosity factor (C).



- Read more about the theory of [Non-Newtonian Fluids in Porous Media](#) in the *Polymer Flow Module User's Guide*
-

LOCATION IN USER INTERFACE

A default **Fluid** node is automatically added to the [Porous Medium](#), the [Unsaturated Porous Medium](#), the [Dual Porosity Medium](#), the [Dual Permeability Medium](#), and the [Unsaturated Dual Permeability Medium](#) node.

Context Menus

Darcy's Law > *porous medium feature* > **Fluid**

Richards' Equation > *porous medium feature* > **Fluid**

Porous Matrix

This node defines the porosity and material properties of the solid matrix of the [Porous Medium](#) or [Unsaturated Porous Medium](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

MATRIX PROPERTIES

The default **Porosity** ϵ_p of the solid matrix is taken **From material**. In a **Porous Material** node, the porosity is defined as follows:

$$\epsilon_p = 1 - \sum_i \theta_{si} - \sum_i \theta_{imfi}$$

where θ_{si} and θ_{imfi} are the porosities of the **Solid** and **Immobile Fluids** subnodes under the **Porous Material** node. See [Porous Material](#) in the *COMSOL Multiphysics Reference Manual* for more information.

Select **User defined** to enter a value or expression for ϵ_p .

If required by a **Storage model** in the parent node, specify the **Effective compressibility** of the porous matrix. The default value is $\chi_p = 10^{-4} \text{ 1/Pa}$.

Select a **Permeability model** to specify the capacity of the porous material to transmit flow. Which options are available depends on the **Flow model** setting in the parent node. Note that for the [Unsaturated Porous Medium](#) parent node only the Darcian Flow models are available.

Darcian Flow

If **Darcian flow** is selected in the parent **Porous medium** node, select between the following options for the **Permeability model**:

- Select **Permeability** (default) to define the permeability of the porous matrix. The default **Permeability** κ (SI unit: m^2) uses the value **From material**, as defined by the **Porous material**. For **User defined** select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value.
- Select **Hydraulic conductivity** or define a combination of fluid permeability and dynamic viscosity. For **Hydraulic conductivity** K (SI unit: m/s) select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value. The default is $2.94 \cdot 10^{-4} \text{ m/s}$.

- Select **Kozeny–Carman** to define the permeability from the porosity and mean particle diameter. Enter the mean **Particle diameter** d_p (SI unit: m), the default value is 0.5 mm.
- Select **Power law** to define the permeability as an exponential function of the porosity: $\kappa = b_{PI} \epsilon_p^n$, where b_{PI} and n are empirical parameters. Enter the proportionality factor b_{PI} (SI unit: m^2) and the exponent n (dimensionless).

Non-Darcian Flow

If **Non-Darcian flow** is selected in the parent node, select between the following options for the **Permeability model**:

- For **Forchheimer**, the default **Permeability** κ (SI unit: m^2) uses the value **From material**. Furthermore, the dimensionless **Forchheimer parameter** c_F can be defined. The default value is 0.55.
- If **Ergun** is selected, enter the mean **Particle diameter** d_p (default value: 0.5 mm).
- For **Burke–Plummer**, specify the mean **Particle diameter** d_p (default value: 0.5 mm).
- For **Klinkenberg**, the default **Permeability** κ_∞ (SI unit: m^2) uses the value **From material**. Enter the **Klinkenberg parameter** b_K . Its default value is 10^3 Pa.
- For **User defined**, the default **Permeability** κ (SI unit: m^2) uses the value **From material**. In addition, the **Inertial resistance coefficient** β (SI unit: 1/m) can be defined. Its default value is zero.

	• About Darcian and Non-Darcian Flow in the <i>Porous Media Flow Module User's Guide</i> • Permeability Models in the <i>Porous Media Flow Module User's Guide</i>
	Note that some options are only available in some modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

RETENTION MODEL

For an **Unsaturated Porous Medium** select between the following options for the retention model:

- **van Genuchten**, to specify the retention model using the van Genuchten relationship. Enter the constitutive relation constants α (the default value is 1, and the SI unit is

$1/m$), n (default value is 2), and l (default value is 0.5). The constitutive parameter m is equal to $1 - 1/n$.

- **Brooks and Corey** to use the retention model according to the Brooks and Corey relationships.
- **User defined** to use a different retention model. Enter user defined expressions for the **Effective saturation** S_e , the **Liquid volume fraction** θ_l . The default is ϵ_p (the porosity variable). Specify the **Specific moisture capacity** C_m (SI unit: $1/m$). The default value is 0 ($1/m$). Enter an expression for the **Relative permeability** κ_r . The default value is 1.

If there is a residual volume of liquid that cannot move through the pore network, also enter a value between 0 and 1 for the **Residual liquid volume fraction** θ_r . The default value is 0.



[Retention and Permeability Relationships](#) in the *Subsurface Flow Module User's Guide*

LOCATION IN USER INTERFACE

A default **Porous Matrix** node is automatically added to [Porous Medium](#) or the [Unsaturated Porous Medium](#) node.

Context Menus

Darcy's Law > *porous medium feature* > **Fluid**

Richards' Equation > *porous medium feature* > **Fluid**

Unsaturated Porous Medium

The **Unsaturated Porous Medium** node adds Richards' equation ([Equation 4-12](#)) for flow in variably saturated porous media.

$$\rho \left(\frac{C_m}{\rho g} + S_e S_p \right) \frac{\partial p}{\partial t} + \nabla \cdot \rho \left(-\frac{\kappa_r \kappa}{\mu} (\nabla p + \rho g \nabla D) \right) = Q_m \quad (4-12)$$

The Unsaturated Porous Medium feature has two default subfeatures: the [Fluid](#) subfeature, where the fluid density ρ (SI unit: kg/m^3) and viscosity μ (SI unit: $\text{Pa}\cdot\text{s}$) are defined, and the [Porous Matrix](#) subfeature, where the permeability κ (SI unit: m^2) and porosity ϵ_p (dimensionless) are specified as well as the retention model.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the **Definitions** node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** or **Hydraulic conductivity** (in the **Porous Matrix** subnode) should be set according to the coordinate system selected in this section.

STORAGE MODEL

With this option, define the first term in [Equation 4-12](#). Therefore, choose from a list one of the following formulations:

- From retention model
- Linearized storage (default)
- From liquid content
- User defined

If **From retention model** is chosen, the first term of [Equation 4-12](#) accounts only for the specific moisture capacity C_m which is defined by the **Retention Model** in the **Porous Matrix** subnode:

$$\frac{C_m \partial p}{g \partial t}.$$

For the option **Linearized storage** the storage coefficient S_p in [Equation 4-12](#) is defined as

$$S_p = \epsilon_p \chi_f + (1 - \epsilon_p) \chi_p.$$

From liquid content enforces the first term of [Equation 4-12](#) to be defined in terms of liquid volume fraction as

$$\frac{\partial(\rho \theta_l)}{\partial t}$$

and the **User defined** option allows to define the storage coefficient S_p in Equation 4-12 directly.



Note that some options are only available in some modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Unsaturated Porous Medium

Richards' Equation > Unsaturated Porous Medium

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > interface > Unsaturated Porous Medium

Dual Porosity Medium

Use the Dual Porosity Medium if the flow through macropores is governed by Darcy's law and the micropores do not significantly contribute to the overall flow in the porous medium.

The feature has three default subfeatures—the **Fluid** subfeature where the fluid density ρ (SI unit: kg/m^3) and dynamic viscosity μ (SI unit: $\text{Pa}\cdot\text{s}$) are defined, the **Macropores** subfeature where the hydraulic properties of the macroporous matrix are defined, and the **Micropores** where the hydraulic properties of the microporous matrix are defined.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the **Definitions** node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** or **Hydraulic conductivity** (in the **Macropores** and **Micropores** subnodes) should be set according to the coordinate system selected in this section.

STORAGE MODEL

Choose from a list one of the following formulations:

- **From density and porosity** (default), uses the formulation equivalent to [Equation 4-9](#)
- **Linearized storage**, where the following linear equation is used to define the storage in the macropores as $S_p = \varepsilon_p \chi_f + (1 - \varepsilon_p) \chi_p$ and in the micropores as $S_{p,m} = \varepsilon_{p,m} \chi_f + (1 - \varepsilon_{p,m}) \chi_p$.
- **Poroelastic storage**, where $S_p = \varepsilon_p \chi_f$ in the macropores and $S_{p,m} = \varepsilon_{p,m} \chi_f$ in the micropores.
- **Quasistatic**, where the storage term is set to zero
- **User defined** to directly specify the storage coefficient S_p and $S_{p,m}$ for the macro- and micropores respectively.



• [Storage Models](#) in the *Subsurface Flow Module User's Guide*

INTERPOROSITY FLOW

Specify the **Fluid Transfer Function** α_w which is used to calculate the interporosity flow, the mass exchange between macro- and microscale system:

$$Q_{ip} = \alpha_w (p_M - p_m) \quad (4-13)$$

with p_M being the pressure within the macropores and p_m the pressure within the micropores.



Find more information in the *Subsurface Flow Module User's Guide*

- [Dual Porosity & Dual Permeability Modeling](#)
 - [Dual Porosity and Dual Permeability Theory](#)
-



Note that some options are only available in some modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Dual Porosity/Permeability > Dual Porosity Medium

Richards' Equation > Dual Porosity/Permeability > Dual Porosity Medium

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Domains > Dual Porosity/Permeability > Dual Porosity Medium

Dual Permeability Medium

Use the **Dual Permeability Medium** feature to model transport in a saturated dual permeability medium where Darcy's Law describes the transport in each system.

The feature has three default subfeatures—the **Fluid** subfeature where the fluid density ρ (SI unit: kg/m^3) and dynamic viscosity μ (SI unit: $\text{Pa}\cdot\text{s}$) are defined, the **Macropores** subfeature where the hydraulic properties of the macroporous matrix are defined, and the **Micropores** where the hydraulic properties of the microporous matrix are defined.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the **Definitions** node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** or **Hydraulic conductivity** (in the **Macropores** and **Micropores** subnodes) should be set according to the coordinate system selected in this section.

STORAGE MODEL

With this option under the **Porous Medium** section, [Equation 4-9](#) is formulated with respect to a storage term (analogous to [Equation 4-4](#)). Therefore, choose from a list one of the following formulations:

- **From density and porosity** (default), uses the formulation of [Equation 4-4](#)
- **Linearized storage**, where the following linear equation is used to define the storage:

$$S_{p,i} = \varepsilon_{p,i}\chi_f + (1 - \varepsilon_{p,i})\chi_{p,i}$$

with $i=M$ for the macropores and $i=m$ for the micropores.

- **Poroelastic storage**, where $S_{p,i} = \varepsilon_{p,i} \chi_f$
- **Quasistatic**, where the storage term is set to zero
- **User defined** to directly specify the storage coefficient S_p



• [Storage Models](#) in the *Subsurface Flow Module User's Guide*

INTERPOROSITY FLOW

Specify the **Fluid Transfer Function** α_w which is used to calculate the interporosity flow, which is the mass exchange between macro- and microscale system:

$$Q_{ip} = \alpha_w(p_M - p_m) \quad (4-14)$$

with p_M being the pressure in the macropores and p_m the pressure in the micropores.



Find more information in the *Subsurface Flow Module User's Guide*

- [Dual Porosity & Dual Permeability Modeling](#)
- [Dual Porosity and Dual Permeability Theory](#)



Note that some options are only available in some modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > **Dual Porosity/Permeability** > **Dual Permeability Medium**

Richards' Equation > **Dual Porosity/Permeability** > **Dual Permeability Medium**

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Domains > **Dual Porosity/Permeability** > **Dual Permeability Medium**

Unsaturated Dual Permeability Medium

Use the **Unsaturated Dual Permeability Medium** feature to model the transport in an unsaturated dual permeability medium where Richards' Equation ([Equation 4-12](#)) describes the transport in each system.

$$\begin{aligned}\theta_M \left(\rho \left(S_{e,M} S_{p,M} + \frac{C_{m,M}}{\rho g} \right) \frac{\partial p_M}{\partial t} + \nabla(\rho \mathbf{u}_M) \right) &= -Q_{ip} + \theta_M Q_m \\ (1 - \theta_M) \left(\rho \left(S_{e,m} S_{p,m} + \frac{C_{m,m}}{\rho g} \right) \frac{\partial p_m}{\partial t} + \nabla(\rho \mathbf{u}_m) \right) &= Q_{ip} + (1 - \theta_M) Q_m \\ p &= \theta_M p_M + (1 - \theta_M) p_m\end{aligned}\tag{4-15}$$

θ_M is the volume fraction of the macropores and Q_{ip} is the interporosity flow, which is the mass transfer from the macroscale to the microscale system. The indices **M** and **m** stand for macropores and micropores, respectively.

The feature has three default subfeatures—the [Fluid](#) subfeature where the fluid density ρ (SI unit: kg/m³) and dynamic viscosity μ (SI unit: Pa·s) are defined, the [Macropores](#) subfeature where the hydraulic properties of the macroporous matrix are defined, and the [Micropores](#) where the hydraulic properties of the microporous matrix are defined.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the **Definitions** node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** or **Hydraulic conductivity** (in the [Macropores](#) and [Micropores](#) subnodes) should be set according to the coordinate system selected in this section.

STORAGE MODEL

With this option, define the first term in [Equation 4-15](#). Therefore, choose from a list one of the following formulations:

- From retention model
- Linearized storage (default)
- From liquid content
- User defined

If **From retention model** is chosen, the first term of Equation 4-15 accounts only for the specific moisture capacity $C_{m,i}$ which is defined by the **Retention Model** in the **Macropores** (i=M) or **Micropores** (i=m) subnode:

$$\frac{C_{m,i} \partial p}{g \frac{\partial t}}.$$

For the option **Linearized storage** the storage coefficient $S_{p,i}$ in Equation 4-15 is defined as

$$S_{p,i} = \epsilon_{p,i} \chi_f + (1 - \epsilon_{p,i}) \chi_{p,i}.$$

From liquid content enforces the first term of Equation 4-15 to be defined in terms of liquid volume fraction as

$$\frac{\partial(\rho \theta_{l,i})}{\partial t}$$

- and the **User defined** option allows to define the storage coefficient $S_{p,i}$ in Equation 4-15 directly.



- [Storage Models](#) in the *Subsurface Flow Module User's Guide*
-

INTERPOROSITY FLOW

Specify the **Fluid Transfer Function** α_w which is used to calculate the interporosity flow, which is the mass exchange between macro- and microscale system:

$$Q_{ip} = \alpha_w (p_M - p_m) \quad (4-16)$$

with p_M being the pressure within the macroscale system and p_m the pressure within the microscale system.



- [Dual Porosity & Dual Permeability Modeling](#) in the *Subsurface Flow Module User's Guide*
-

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Dual Porosity/Permeability > Unsaturated Dual Permeability Medium

Richards' Equation > Dual Porosity/Permeability > Unsaturated Dual Permeability Medium

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Domains > Dual Porosity/Permeability > Unsaturated Dual Permeability Medium

Macropores

This node defines the porosity and material properties of the macroporous matrix of the [Dual Porosity Medium](#), the [Dual Permeability Medium](#), or the [Unsaturated Dual Permeability Medium](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

VOLUME FRACTION

Here enter the **Volume Fraction of Macropores**, θ_M .

MATRIX PROPERTIES

The default **Porosity** $\epsilon_{p,M}$ of the solid matrix of the macropore system is taken **From material**. Select **User defined** to enter a value or expression for $\epsilon_{p,M}$.

If required by a **Storage model** in the parent node, specify the **Effective compressibility** of the porous matrix of the macropore system. The default value is $\chi_{p,M} = 10^{-4}$ 1/Pa.

Select a **Permeability model** to specify the capacity of the porous material to transmit flow. For the macropore settings only the Darcian Flow models are available. Select between the following options for the **Permeability model**:

- Select **Permeability** (default) to define the permeability of the porous matrix. The default **Permeability** κ_M (SI unit: m^2) uses the value **From material**, as defined by the **Porous material**. For **User defined** select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value.
- Select **Hydraulic conductivity** or define a combination of fluid permeability and dynamic viscosity. For **Hydraulic conductivity** K_M (SI unit: m/s) select **Isotropic** to

define a scalar value or **Diagonal** or **Symmetric** to define a tensor value. The default is $2.94 \cdot 10^{-4}$ m/s.

- Select **Kozeny–Carman** to define the permeability from the porosity and mean particle diameter. Enter the mean **Particle diameter** $d_{p,M}$ (SI unit: m), the default value is 0.5 mm.

RETENTION MODEL

For an [Unsaturated Dual Permeability Medium](#) select between the following options for the retention model:

- **van Genuchten**, to specify the retention model using the van Genuchten relationship. Enter the constitutive relation constants α (the default value is 1, and the SI unit is 1/m), n (default value is 2), and l (default value is 0.5). The constitutive parameter m is equal to $1 - 1/n$.
- **Brooks and Corey** to use the retention model according to the Brooks and Corey relationships.
- **User defined** to use a different retention model. Enter user defined expressions for the **Effective saturation** $S_{e,M}$, the **Liquid volume fraction** $\theta_{l,M}$. The default is $\epsilon_{p,M}$ (the porosity variable). Specify the **Specific moisture capacity** $C_{m,M}$ (SI unit: 1/m). The default value is 0 (1/m). Enter an expression for the **Relative permeability** $\kappa_{r,M}$. The default value is 1.

If there is a residual volume of liquid that cannot move through the pore network, also enter a value between 0 and 1 for the **Residual liquid volume fraction** θ_r . The default value is 0.



[Retention and Permeability Relationships](#) in the *Subsurface Flow Module User's Guide*

LOCATION IN USER INTERFACE

A default **Macropores** node is automatically added to the [Dual Porosity Medium](#), the [Dual Permeability Medium](#), and the [Unsaturated Dual Permeability Medium](#) node.

Context Menus

Darcy's Law > *porous medium feature* > **Macropores**

Richards' Equation > *porous medium feature* > **Macropores**

Micropores

This node defines the porosity and material properties of the microporous matrix of the [Dual Porosity Medium](#), [Dual Permeability Medium](#), or the [Unsaturated Dual Permeability Medium](#) parent node.

MODEL INPUT

This section is only available for the [Unsaturated Dual Permeability Medium](#) parent node and contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

MATRIX PROPERTIES

The default **Porosity** $\epsilon_{p,m}$ of the solid matrix of the micropore system is taken **From material**. Select **User defined** to enter a value or expression for $\epsilon_{p,m}$.

If required by a **Storage model** in the parent node, specify the **Effective compressibility** of the microscale porous matrix. The default value is $\chi_{p,m} = 10^{-4}$ 1/Pa.

Select a **Permeability model** to specify the capacity to transmit flow for the porous material of the microscale system. Here, only the Darcian Flow models are available. Select between the following options for the **Permeability model**:

- Select **Permeability** (default) to define the permeability of the porous matrix. The default **Permeability** κ_m (SI unit: m^2) uses the value **From material**, as defined by the **Porous material**. For **User defined** select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value.
- Select **Hydraulic conductivity** or define a combination of fluid permeability and dynamic viscosity. For **Hydraulic conductivity** K_m (SI unit: m/s) select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value. The default is $2.94 \cdot 10^{-4}$ m/s.
- Select **Kozeny–Carman** to define the permeability from the porosity and mean particle diameter. Enter the mean **Particle diameter** $d_{p,m}$ (SI unit: m), the default value is 0.5 mm.

RETENTION MODEL

For an [Unsaturated Dual Permeability Medium](#) select between the following options for the retention model:

- **From macropores** to take over the retention model settings from the **Macropores** node.

- **van Genuchten**, to specify the retention model using the van Genuchten relationship. Enter the constitutive relation constants α (the default value is 1, and the SI unit is 1/m), n (default value is 2), and l (default value is 0.5). The constitutive parameter m is equal to $1 - 1/n$.
- **Brooks and Corey** to use the retention model according to the Brooks and Corey relationships.
- **User defined** to use a different retention model. Enter user defined expressions for the **Effective saturation** $S_{e,m}$, the **Liquid volume fraction** $\theta_{l,m}$. The default is $\epsilon_{p,m}$ (the porosity variable). Specify the **Specific moisture capacity** $C_{m,m}$ (SI unit: 1/m). The default value is 0 (1/m). Enter an expression for the **Relative permeability** $\kappa_{r,m}$. The default value is 1.

If there is a residual volume of liquid that cannot move through the pore network, also enter a value between 0 and 1 for the **Residual liquid volume fraction** θ_r . The default value is 0.



Retention and Permeability Relationships in the *Subsurface Flow Module User's Guide*

INITIAL VALUES

Define the initial pressure of the microscale system. The default is to use the initial values defined in the **Initial Values** feature node which means that the same initial values apply to the macroscale and the microscale system.

Select **User Defined** to specify an initial pressure for the microscale system $p_{m0,init}$.



Note that some options are only available in some modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/

LOCATION IN USER INTERFACE

A default **Micropores** node is automatically added to the **Dual Porosity Medium**, the **Dual Permeability Medium**, and the **Unsaturated Dual Permeability Medium** node.

Context Menus

Darcy's Law > *porous medium feature* > **Micropores**

Richards' Equation > *porous medium feature* > **Micropores**

Mass Source

The **Mass Source** node adds a mass source Q_m , which appears on the right-hand side of the Darcy's Law equation ([Equation 4-9](#)).

$$\frac{\partial}{\partial t}(\rho \epsilon) + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (4-17)$$

For the Fracture Flow interface, the mass source is applicable to the right-hand side of [Equation 4-40](#) in the *Subsurface Flow Module User's Guide* (where it is multiplied with the fracture thickness).

MASS SOURCE

Enter a value or expression for the **Mass source** Q_m (SI unit: $\text{kg}/(\text{m}^3 \cdot \text{s})$). The default is $0 \text{ kg}/(\text{m}^3 \cdot \text{s})$.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Mass Source

Richards' Equation > Mass Source

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > interface > Mass Source

Contributing Velocity

The **Contributing Velocity** node adds a velocity contribution $\mathbf{u}_{\text{ctrb}}$ to the velocity field $\mathbf{u}_d$ that is computed from the flow model selected in the parent [Porous Medium](#), [Unsaturated Porous Medium](#), or [Dual Porosity Medium](#) feature. The total velocity field used in the continuity equation is therefore

$$\mathbf{u} = \mathbf{u}_d + \mathbf{u}_{\text{ctrb}} \quad (4-18)$$

Such velocity contributions can arise in multiphysics models that involve, for example, transport of solutes. If several **Contributing Velocity** nodes are added under the same parent feature, the contributions are treated additively. The sum of all contributing velocity fields is available in the variables `name.Uctrb` (material frame) and `name.uctrb` (spatial frame), with `name` being the **Name** of the physics interface (for example, `d1`).

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list to specify the coordinate system in which the contributing velocity field is given. The default is the **Global coordinate system**. Note that the coordinate system used to specify the components of the contributing velocity field can be selected independently of the coordinate system of the parent feature.

CONTRIBUTING VELOCITY

The default contributing velocity field $\mathbf{u}_{\text{ctrb}}$ is **User defined**, which lets you enter the components of the contributing velocity field explicitly. If a matching velocity field is announced by another feature, it will be available in the selection list.

ADVANCED

Select the **Frame** in which the contributing velocity field is specified. The default is the **Reference configuration**, which means that the velocity field follows the material under, for example, a rotation of the domain. If instead your contributing velocity field follows a fixed direction in space, select the **Deformed configuration**. This section is only displayed if **Advanced Physics Options** is activated and the spatial and material frame of the component differ, such as in the presence of a **Moving Mesh** or **Solid Mechanics** interface.

LOCATION IN USER INTERFACE

Context Menus

If the selected *porous medium feature* is [Porous Medium](#), [Unsaturated Porous Medium](#), or [Dual Porosity Medium](#):

Darcy's Law > *porous medium feature* > **Contributing Velocity**

Richards' Equation > *porous medium feature* > **Contributing Velocity**

Ribbon

Physics tab with **Porous Medium**, **Unsaturated Porous Medium**, or **Dual Porosity Medium** selected:

Attributes > **Contributing Velocity**

Initial Values

The **Initial Values** node adds an initial value for the pressure that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver.

If you have the Subsurface Flow Module licensed, you can specify the pressure either directly or as an expression for the pressure head, H_p , or the hydraulic head, H ; the hydraulic head and the pressure head relate to the pressure p as

$$H_p = \frac{p}{\rho g}; \quad H = H_p + D$$

where ρ is the fluid density (SI unit: kg/m^3); g denotes the acceleration of gravity (SI unit: m/s^2); and D is the elevation (SI unit: m).

INITIAL VALUES

Enter a value or expression for the initial value of the **Pressure** p (SI unit: Pa). The default value is 0 Pa.

In case the Subsurface Flow Module is available, click the **Pressure head** button to enter a value or expression for H_p (SI unit: m). The default is 0 m. Click the **Hydraulic head** button to enter a value or expression for H (SI unit: m). The default is 0 m.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Initial Values

Richards' Equation > Initial Values

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > *interface* > Initial Values

Gravity

The **Gravity** node is automatically added when **Include gravity** is selected at interface level in the **Darcy's Law** settings and it is active in all domains in which the Darcy's Law interface is applied.

COORDINATE SYSTEM SELECTION

The **Global coordinate system** is selected by default. The **Coordinate system** list contains any additional coordinate systems that the model includes. It can be used when prescribing the direction of the gravitational forces.

GRAVITY

Specify either the acceleration vector or — in case you have a Subsurface Flow Module license — the elevation.

When **Acceleration** is selected from the list, enter the components of the **Gravity vector** $\mathbf{g}$. The default value is $\mathbf{g\_const}$ which is the physical constant having the value 9.8066 m/s^2 .

- For 3D and 2D axisymmetric models, the default value is $-\mathbf{g\_const}$ in the z direction.
- For 2D models, the default value is $-\mathbf{g\_const}$ in the y direction.

When **Elevation** is selected from the list, specify the **Elevation** D (SI unit: m). The **Acceleration of gravity** is taken **From physics interface** by default but could be overridden when **User defined** is selected.

When the checkbox **Include gravity** is not selected in the Darcy's Law interface Settings, the elevation D is set equal to zero.

Select the checkbox **Specify reference position** to define a reference position for the gravity calculation.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Gravity

Richards' Equation > Gravity

Cross Section

Use this node with 1D components to model domains with another cross-sectional area than the global one that is used in the interface **Physical Model** section. In 1D geometries, the pressure is assumed to be constant in the radial direction, and Darcy's Law accounts for that.

CROSS SECTION

Enter values for the **Cross-sectional area** A_c to set the cross section of the domain in the plane perpendicular to the 1D geometry.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Cross Section

Richards' Equation > Cross Section

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > *interface* > Cross Section

Thickness

Use this node with 2D and 1D axisymmetric components to model domains with another thickness than the overall thickness that is specified in the interface **Physical Model** section. In 2D geometries, the pressure is assumed to be constant in the out-of-plane direction (the z direction with default spatial coordinate names). In 1D axisymmetric geometries the thickness represents the z direction.

THICKNESS

Specify a value for the **Thickness** d_z of the domain in the out-of-plane direction. This value replaces the overall thickness in the domains that are selected in the **Domain Selection** section, and is used to multiply some terms into the heat equation.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Thickness

Richards' Equation > Thickness

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > *interface* > Thickness

Porous Electrode Coupling

Use the **Porous Electrode Coupling** node to define a mass source based on the volumetric current densities of one or multiple **Porous Electrode Reaction** nodes in an Electrochemistry interface.

The source (or sink) is proportional to the **Molar mass (kg/mol)** of the reacting species, the current densities and the stoichiometric coefficients according to Faraday's law as defined by summation over the Reaction Coefficient subnodes.

SPECIES

Based on the number of species required for the model, use the **Add** () and **Delete** () buttons under the table to add or remove **Species**. Then enter a value for the **Molar mass (kg/mol)** in the applicable rows for each species.

The setting in the table will determine the number of available fields for entering the stoichiometric coefficients of the Reaction Coefficient subnodes.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > **Porous Electrode Coupling**

Richards' Equation > **Porous Electrode Coupling**

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Domains > *interface* > **Porous Electrode Coupling**

Electrode Surface Coupling

Use the **Electrode Surface Coupling** node to define a combined wall and inflow/outflow boundary condition based on current densities of one or multiple Electrode Reaction nodes in an Electrochemistry interface.

The flow is proportional to the **Molar mass (kg/mol)** of the reacting species, the current densities and the stoichiometric coefficients according to Faraday's law as defined by summation over the Reaction Coefficient subnodes.

SPECIES

Based on the number of species required for the model, use the **Add** () and **Delete** () buttons under the table to add or remove **Species**. Then enter a value for the **Molar mass (kg/mol)** in the applicable rows for each species.

The setting in the table will determine the number of available field for entering the stoichiometric coefficients of the Reaction Coefficient subnodes.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > **Electrode Surface Coupling**

Richards' Equation > **Electrode Surface Coupling**

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Boundaries > *interface* > **Electrode Surface Coupling**

Pressure

Use the **Pressure** node to specify the pressure on a boundary. In many cases the distribution of pressure is known, giving a Dirichlet condition $p = p_0$ where p_0 is a known pressure given as a number, a distribution, or an expression involving time, t , for example.

PAIR SELECTION

If this node is selected from the **Pairs** menu, choose the pair to apply this condition to. A pair must be created first. See [Identity](#), [Contact](#), and [Sector Symmetry Pairs](#) in the *COMSOL Multiphysics Reference Manual* for more details.

PRESSURE

Enter a value or expression for the **Pressure** p_0 (SI unit: Pa). Enter a relative pressure value in p_0 (SI unit: Pa).

For the Subsurface Flow Module, the **Pressure** node provides the pressure P_0 as a condition on edges in 3D models. Then select the edges under **Edge Selection**.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog.

As an attribute to the [Fracture](#) boundary condition or as boundary condition to [The Fracture Flow Interface](#) (which are available in the Subsurface Flow Module and the Porous Media Flow Module), the pressure can also be specified on edges (in 3D) or

points (in 2D) surrounding the fracture domain. In other cases, pressure constraints on edges or points are not implemented.



If the boundary condition is adjacent to an [Unsaturated Dual Permeability Medium](#) feature it constrains the pressures in the [Macropores](#) and [Micropores](#) to p_0 .

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Pressure

Richards' Equation > Pressure

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Boundaries > interface > Pressure

Pairs > interface > Pressure

Mass Flux

Use the **Mass Flux** node to specify the mass flux into or out of the model domain through some of its boundaries. It is often possible to determine the mass flux from the pumping rate or from measurements. With this boundary condition, positive values correspond to flow into the model domain:

$$\mathbf{n} \cdot \rho \frac{\mathbf{K}}{\mu} (\nabla p + \rho g \nabla D) = N_0$$

where N_0 is a value or expression for the specified inward (or outward) Darcy flux. D is the elevation head which is set to zero for other than Subsurface Flow Module applications.

MASS FLUX

Enter a value or expression for the **Inward mass flux** N_0 . A positive value of N_0 represents an inward mass flux whereas a negative value represents an outward mass flux. The units are based on the geometric entity: **Boundaries**: (SI unit: $\text{kg}/(\text{m}^2 \cdot \text{s})$).

LOCATION IN USER INTERFACE

Context Menus

Darcy’s Law > Mass Flux

Richards’ Equation > Mass Flux

Ribbon

Physics tab with *interface* as Darcy’s Law or Richards’ Equation selected:

Boundaries > *interface* > Mass Flux

Line Mass Source

The **Line Mass Source** node adds mass flow originating from a tube of infinitely small radius.

SELECTION

The **Line Mass Source** feature is available for all dimensions, but the applicable selection differs between the dimensions.

MODEL DIMENSION	APPLICABLE GEOMETRICAL ENTITY
2D	Points
2D Axisymmetry	Points not on the symmetry axis
3D	Edges

LINE MASS SOURCE

Enter a value or expression for the source strength, N_0 (SI unit: kg/(m·s)). A positive value results in mass injection from the line into the computational domain, and a negative value means that the mass is removed from the computational domain.

Line sources located on a boundary affect the adjacent computational domains. This effect makes the physical strength of a line source located in a symmetry plane twice the given strength.

Fracture Flow

The equivalent feature available for fracture flow is the **Mass Source** attribute of the [Fracture](#) boundary condition and the **Mass Source** point feature of the Fracture Flow Interface, both available in 3D only.

LOCATION IN USER INTERFACE

According to the model dimension as mentioned in [Selection](#), this feature is available for the *geometrical entities* **Edges** or **Points**.

Context Menus

Darcy's Law > *geometrical entity* > **Line Mass Source**

Richards' Equation > *geometrical entity* > **Line Mass Source**

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

geometrical entity > **Line Mass Source**

Point Mass Source

The **Point Mass Source** node models mass flow originating from an infinitely small sphere centered around a point. It is available for points in 3D geometries.

POINT MASS SOURCE

Enter a value or expression for the source strength, N_0 (SI unit: kg/(s)). A positive value results in mass injection from the point into the computational domain, and a negative value means that the mass is removed from the computational domain.

Point sources located on a boundary or on an edge affect the adjacent computational domains. This has the effect, for example, that the physical strength of a point source located in a symmetry plane is twice the given strength.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > **Points** > **Point Mass Source**

Richards' Equation > **Points** > **Point Mass Source**

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Points > **Point Mass Source**

Inlet

The **Inlet** node adds a boundary condition for the inflow (or outflow) perpendicular (normal) to the boundary. It has three options that can be used to specify inlet condition on a boundary, as follows:

VELOCITY

Enter a value or expression for the **Normal inflow velocity** U_0 (SI unit: m/s). A positive value of U_0 represents an inflow velocity. A negative value represents an outflow velocity. The inlet velocity boundary condition is implemented as;

$$\mathbf{n} \cdot \rho \frac{\kappa}{\mu} (\nabla p + \rho g \nabla D) = \rho U_0$$

where U_0 is a value or expression for the specified inward (or outward) Darcy velocity. A positive value of the velocity U_0 corresponds to flow into the model domain whereas a negative value represents an outflow. D is the elevation head which is set to zero for other than Subsurface Flow Module applications.

PRESSURE

Use the **Pressure** option to specify the inlet pressure on a boundary. In many cases the distribution of pressure is known, giving a Dirichlet condition $p = p_0$ where p_0 is a known pressure given as a number, a distribution, or an expression involving time, t , for example. Enter a value or expression for the **Pressure** p_0 (SI unit: Pa).

MASS FLOW

If you select **Mass flow** as the inlet condition, there are four options to specify the **Mass flow type**: the total **Mass flow rate** M_0 (SI unit: kg/s), the **Pointwise mass flux** N_0 (kg/(m²·s)), the **Standard flow rate** Q_{sv} (m³/s), and the **Standard flow rate** Q_{scm} (in number of SCCM units).

With the options **Mass flow rate**, **Standard flow rate**, and **Standard flow rate SCCM**, positive values correspond to flow into the model domain:

$$-\int_{\partial\Omega} \rho(\mathbf{u} \cdot \mathbf{n}) dS = M_0$$

where M_0 is a value or expression for the specified inward (or outward) Darcy flux.

For the **Standard flow rate** Q_{sv} , which is a volumetric flow rate of a gas at standard density according to the SEMI standard E12-0303, the equation above is divided by the standard density term:

$$-\int_{\partial\Omega} \frac{\rho}{\rho_{std}} (\mathbf{u} \cdot \mathbf{n}) dS = Q_{sv}$$

ρ_{std} can be specified as mean molar mass m_m divided by a standard molar volume V_m , or as a function of standard pressure p_{st} and standard temperature T_{st} :

$$\rho_{std} = \frac{p_{st} m_m}{RT_{st}}$$

The **Standard flow rate SCCM** option is equivalent to the Standard flow rate option, except that the flow rate is entered directly in SCCMs (standard cubic centimeters per minute) without the requirement to specify units. Here, the dimensionless number of SCCM units Q_{sccm} should be specified.

In the **Pointwise mass flux** boundary condition, positive values correspond to flow into the model domain:

$$\mathbf{n} \cdot \rho \frac{\kappa}{\mu} (\nabla p + \rho g \nabla D) = N_0$$

where N_0 is a value or expression for the specified inward (or outward) Darcy flux.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Inlet

Richards' Equation > Inlet

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Boundaries > interface > Inlet

Symmetry

The **Symmetry** node describes a symmetry boundary. The following condition implements the symmetry condition on an axis or a flow divide:

$$\mathbf{n} \cdot \frac{\kappa}{\mu} \nabla p = 0$$

$$\mathbf{n} \cdot \frac{\kappa}{\mu} (\nabla p + \rho g \nabla D) = 0$$

For axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** node that is valid on the axial symmetry boundaries only.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Symmetry

Richards' Equation > Symmetry

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Boundaries > interface > Symmetry

No Flow

The **No Flow** node is the default boundary condition stating that there is no flow across impermeable boundaries. The mathematical formulation is:

$$\mathbf{n} \cdot \rho \frac{\kappa}{\mu} (\nabla p + \rho g \nabla D) = 0$$

where $\mathbf{n}$ is the vector normal to the boundary. D is the elevation head which is set to zero for any other than Subsurface Flow Module applications.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > No Flow

Richards' Equation > No Flow

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Boundaries > interface > No Flow

Flux Discontinuity

Use the **Flux Discontinuity** node to specify a mass flux discontinuity through an interior boundary. The condition is represented by the following equation:

$$-\mathbf{n} \cdot (\rho \mathbf{u}_1 - \rho \mathbf{u}_2) = N_0$$

In this equation, $\mathbf{n}$ is the vector normal (perpendicular) to the interior boundary, ρ is the fluid density, $\mathbf{u}_1$ and $\mathbf{u}_2$ are the Darcy velocities in the adjacent domains (as defined in Equation 4-19) and N_0 is a specified value or expression for the flux discontinuity.

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p \quad (4-19)$$

For this boundary condition, a positive value of N_0 corresponds to a flow discontinuity in the opposite direction to the normal vector of the interior boundary.

MASS FLUX

Enter a value or expression for the **Inward mass flux** N_0 (SI unit: $\text{kg}/(\text{m}^2 \cdot \text{s})$). A positive value of N_0 represents a mass flux discontinuity in the opposite direction to the normal vector of the interior boundary.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Thin Structures > Flux Discontinuity

Richards' Equation > Thin Structures > Flux Discontinuity

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Thin Structures > Flux Discontinuity

Pairs > Flux Discontinuity

Outlet

The **Outlet** node adds a boundary condition for the outflow (or inflow) perpendicular (normal) to the boundary. It has different options that can be used to specify inlet condition on a boundary, as follows:

VELOCITY

Enter a value or expression for the **Normal outflow velocity** U_0 (SI unit: m/s). A positive value of U_0 represents an outflow velocity whereas a negative value represents an inflow velocity.

$$-\mathbf{n} \cdot \rho \frac{\kappa}{\mu} \nabla p = \rho U_0$$

where U_0 is a specified value or expression for the outward (or inward) Darcy velocity.

PRESSURE

Similar to the inlet **Pressure** option, the outlet pressure on a boundary can be specified. Enter a value or expression for the **Pressure** p_0 (SI unit: Pa).

DISCHARGE

Enter a value or expression for the discharge Q_D to specify the volumetric flow rate through the boundary. The mass flux $\rho \mathbf{u}$ is related to the discharge as follows:

$$\int_{\partial\Omega} \rho \mathbf{u} \cdot \mathbf{ds} = \rho Q_D. \quad (4-20)$$

The mean normal outflow velocity at the boundary can then be calculated as $U_{\text{out}} = Q_D/A$, where A is the cross-sectional area of the boundary.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Outlet

Richards' Equation > Outlet

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equation** selected:

Boundaries > interface > Outlet

Precipitation

The **Precipitation** node adds a boundary condition for the inflow (of rain) perpendicular (normal) to the boundary.

Enter a value or expression for the **Precipitation rate** which then contributes to the mass flux at the selected boundaries.

Select the **Slope correction** checkbox to apply a correction that depends on the normal to the boundary and the vertical axis. This is needed when the boundary is inclined.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Hydraulics > Precipitation

Richards' Equation > Hydraulics > Precipitation

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Hydraulics > Precipitation

Periodic Condition

Use this node to add a periodic condition to at least two boundaries (source and destination boundaries). It applies a pressure condition as follows:

$$p_{\text{src}} = p_{\text{dst}} + \Delta p$$

FLOW CONDITION

Specify either a **Pressure difference** Δp or a **Mass flow** $\dot{m}$. If the mass flow option is selected, the pressure difference is computed such that

$$\dot{m} = \int_{\text{dst}} \rho \mathbf{u} \cdot \mathbf{n}$$

BOUNDARY SELECTION AND ORIENTATION



The software usually automatically identifies the boundaries as either source or destination boundaries, as indicated in the selection list. This works fine for cases like opposing parallel boundaries. In other cases, right-click **Periodic Flow Condition** and select **Manual Destination Selection** to control the destination.

More details on how to handle curved boundaries and mapping between source and destination are located in the [Periodic Boundary Conditions](#) section in the *Comsol Multiphysics Reference Manual*.

Interior Wall

The **Interior Wall** boundary condition can only be applied on interior boundaries. It is similar to the **No Flux** boundary available on exterior boundaries except that it applies on both sides of an interior boundary. It allows discontinuities of velocity and pressure across the boundary. The **Interior Wall** boundary condition can be used to avoid meshing thin structures by applying no-flux condition on interior curves and surfaces instead.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Thin Structures> Interior Wall

Richards' Equation > Thin Structures> Interior Wall

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Thin Structures> Interior Wall

Barrier

The **Barrier** boundary condition models interior permeable walls, membranes, geotextiles, or perforated plates as thin permeable barriers. The **Barrier** boundary condition can only be applied on interior boundaries.

WALL

Enter a value or expression for the **Thickness** d_b (SI unit: m, the default is 0.1 m) and for the **Permeability** κ_b (SI unit: m^2). The default **Permeability** κ_b uses the value **From material**. For **User defined** select **Isotropic** to define a scalar value or **Diagonal**, **Symmetric**, or **Full** to define a tensor value and enter another value or expression in the field or matrix.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Thin Structures > Barrier

Richards' Equation > Thin Structures > Barrier

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Thin Structures > Barrier

Pressure Head

Use the **Pressure Head** node to specify the pressure head (instead of the pressure) on a boundary. It adds this boundary condition for the pressure head $H_p = H_{p0}$, where H_{p0} is a known pressure head given as a number, a distribution, or an expression involving time, t , for example. The dimension of the pressure head is length (SI unit: m).

PRESSURE HEAD

Enter a value or expression for the **Pressure head** H_{p0} (SI unit: m). The default is 0 m.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Hydraulics > Pressure Head

Richards' Equation > Hydraulics > Pressure Head

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Hydraulics > Pressure Head

Pairs > Pressure Head

Hydraulic Head

Use the **Hydraulic Head** node to specify the hydraulic head (instead of the pressure) on a boundary. This adds the Dirichlet condition for the hydraulic head $H = H_0$ where H_0 is a known hydraulic head given as a number, a distribution, or an expression involving time, t , for example.

HYDRAULIC HEAD

Enter a value or expression for the **Hydraulic head** H_0 (SI unit: m). The default is 0 m.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Hydraulics > Hydraulic Head

Richards' Equation > Hydraulics > Hydraulic Head

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Hydraulics > Hydraulic Head

Pairs > Hydraulic Head

Atmosphere/Gauge

The **Atmosphere/Gauge** node specifies an atmospheric pressure or gauges the pressure to the atmospheric value. This means that the total hydraulic potential reduces to the gravitational pressure at the free surface. At a free surface, such as a spring or a seepage face, the pressure is atmospheric. If the pressures in the model is gauged to the atmospheric value ($p = 0$), the total hydraulic potential reduces to the gravitational potential at the free surface — for example, the height of the free surface multiplied by the fluid weight, or $\rho_f g D$. This boundary condition sets the pressure at the boundary to zero and $p = 0$.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog.

LOCATION IN USER INTERFACE

Context Menus

Darcy's Law > Hydraulics > Atmosphere/Gauge

Richards' Equation > Hydraulics > Atmosphere/Gauge

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

Boundaries > Hydraulics > Atmosphere/Gauge

Pervious Layer

The **Pervious Layer** node provides a boundary condition that describes a mass flux through a semi-pervious layer connected to an external fluid source at different pressure, pressure head, or hydraulic head. The model domain might connect to a larger body of water through the semi-pervious layer. This condition is represented with the following boundary condition:

$$\mathbf{n} \cdot \rho \frac{\kappa}{\mu} (\nabla p + \rho g \nabla D) = \rho R_b \left[\frac{(p_b - p)}{\rho g} + (D_b - D) \right] \quad (4-21)$$

In this equation, p_b (SI unit: Pa) and D_b (SI unit: m) are the pressure and the elevation of the distant fluid source, respectively, and R_b (SI unit: 1/s) is the conductance of materials between the source and the model domain (conductance to flow in the semi-pervious layer adjacent to the boundary). Typically $R_b = K'/B'$, where K' is the hydraulic conductivity (SI unit: m/s) of the layer and B' (SI unit: m) is its thickness. Using logical relationships, it is possible to activate these expressions at different times or under various flow conditions.

When the pressure head H_p is specified instead of the pressure, the boundary condition is the following:

$$\mathbf{n} \cdot \rho K \nabla (H_p + D) = \rho R_b [(H_{pb} - H_p) + (D_b - D)] \quad (4-22)$$

H_{pb} is the pressure head (SI unit: m) at the edge of the layer.

When the hydraulic head H is specified instead of the pressure head, the boundary condition becomes:

$$\mathbf{n} \cdot \rho K \nabla H = \rho R_b [H_b - H] \tag{4-23}$$

H_b is the hydraulic head (SI unit: m) at the edge of the layer.

K in Equation 4-22 and Equation 4-23 is the hydraulic conductivity.

At a free surface, such as a water table or seepage face, the pressure is atmospheric (here taken to be zero), so the total hydraulic potential equals gravitational potential, which is defined on D .



Gravity effects are not active by default. Select the checkbox **Include gravity** to activate the acceleration of gravity. Setting the elevation D to zero also turns off gravity effects.

PERVIOUS LAYER

Specify the material properties whether to specify an external pressure, pressure head, or hydraulic head.

Select an **External variable to specify** — **Pressure and elevation** to specify the external pressure, **Pressure head and elevation** to specify the external pressure head, or **Hydraulic head**. For all selections, enter a value for the **Conductance R_b** (SI unit: 1/s).

- For **Pressure and elevation** enter an **External pressure p_b** (SI unit: Pa) and **External elevation D_b** (SI unit: m).
- For **Pressure head and elevation** enter an **External pressure head H_{pb}** (SI unit: m), H_{pb} (SI unit: m) and **External elevation D_b** (SI unit: m).
- For **Hydraulic head** enter the **Hydraulic head H_b** (SI unit: m).

LOCATION IN USER INTERFACE

Context Menus

Darcy’s Law > Hydraulics > Pervious Layer

Richards’ Equation > Hydraulics > Pervious Layer

Ribbon

Physics tab with **Darcy’s Law** or **Richards’ Equation** selected:

Boundaries > Hydraulics > Pervious Layer

Well

The **Well** feature is intended to model injection or production wells and is available with the Subsurface Flow Module.

WELL

Enter a value or expression for the **Well diameter** d_w (SI unit: m, the default is 0.1 m). Select the **Well type** from the list: **Production** or **Injection**.

Specify either the **Pressure** (SI unit: Pa), **Pressure head** (SI unit: m), **Hydraulic head** (SI unit: m), or the **Mass flow**. If you select **Mass flow**, specify the total **Mass flow rate** (SI unit: kg/s), the **Mass flow rate per unit length** (SI unit: kg/(m·s)) or the **Mass flux** (SI unit: kg/(m²·s)).

LOCATION IN USER INTERFACE

According to the model dimension this feature is available for the *geometrical entities* **Edges** (3D) or **Points** (2D).

Context Menus

Darcy's Law > *geometrical entity* > **Well**

Richards' Equation > *geometrical entity* > **Well**

Ribbon

Physics tab with **Darcy's Law** or **Richards' Equation** selected:

geometrical entity > **Well**

Fracture

Use the **Fracture** node to model flow along a boundaries representing fractures within a porous medium. By default, it adds the tangential form of the continuity equation and of Darcy's Law.

$$d_f \frac{\partial}{\partial t} (\epsilon_p \rho) + \nabla_T \cdot (d_f \rho \mathbf{u}) = d_f Q_m \quad (4-24)$$

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla_T p \quad (4-25)$$

The **Fracture** feature has two default subfeatures — the **Fluid (Fracture)** subfeature where the fluid density ρ (SI unit: kg/m³) and dynamic viscosity μ (SI unit: Pa·s) are

defined and the [Fracture Material](#) subfeature where the permeability κ (SI unit: m^2) and porosity ϵ_p (dimensionless) are specified.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the **Definitions** node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** or **Hydraulic conductivity** (in the [Fracture Material](#) subnode) should be set according to the coordinate system selected in this section.

APERTURE

Enter a value for the **Fracture thickness** d_f .

FLOW MODEL

Define if the flow follows a linear or nonlinear pressure-velocity relationship.

- **Darcian flow** (default) defines a linear relationship using [Equation 4-24](#) and [Equation 4-25](#).
- **Non-Darcian flow** defines a nonlinear relationship using [Equation 4-24](#) and the pressure-velocity relationship

$$\nabla_T p = -\frac{\mu}{\kappa} \mathbf{u} - \beta \rho |\mathbf{u}| \mathbf{u}$$

where the nonlinear parameter β (SI unit: $1/\text{m}$) is further specified in the [Fracture Material](#) subfeature.

STORAGE MODEL

[Equation 4-24](#) can also be formulated with respect to a storage term.

$$\rho S \frac{\partial p}{\partial t} + \nabla \cdot \rho \left[-\frac{\kappa}{\mu} (\nabla p) \right] = Q_m$$

The storage coefficient S can be interpreted as the weighted compressibility of the porous material and the fluid. Define the time dependent storage term by selecting one of the following options from the list:

- **From density and porosity** (default), uses the formulation of [Equation 4-24](#)
- **Linearized storage**, where the following linear equation is used to define the storage:

$$S = \varepsilon_p \chi_f + (1 - \varepsilon_p) \chi_p$$

- **User defined** to directly specify the storage coefficient S



- [About Darcian and Non-Darcian Flow](#) in the *Porous Media Flow Module User's Guide*
- [Storage Models](#) in the *Subsurface Flow Module User's Guide*

FRACTURE MODEL

Choose between two options to model fractures that have different characteristics.

- **Highly conductive fracture** represents a fracture filled with a highly permeable material.
- **Thin conducting barrier** represents a fracture that is filled with a low permeable material and therefore acts as a barrier for the flow which adds a jump in the pressure across the boundary:

$$-\mathbf{n}_{u,d} \cdot (\rho \mathbf{u})_{u,d} = \rho \frac{(p_{d,u} - p_{u,d}) \kappa_b}{d_b \mu}$$

The barrier thickness d_b and permeability κ_b are defined in the [Fracture Material](#) subnode.

In addition, either on the upside, downside, or on both sides a highly conductive fracture material is present and [Equation 4-24](#) and [Equation 4-25](#) are solved for the highly conductive side. Further specifications are done in the [Fracture Material](#) subnode.



Note that the option to specify the **Fracture Model** is only available for the Fracture feature within the Darcy's Law interface.

For the [Fracture](#) boundary node several subnodes are available from the context menu (right-click the parent node) or from the **Physics** toolbar, **Attributes** menu. As they are almost identical to the boundary conditions of the Fracture Flow interface, see [Domain, Boundary, Edge, Point, and Pair Nodes for the Fracture Flow Interface](#) in

the *Subsurface Flow User's Guide* for further description. Note that **Precipitation** is only available for the Fracture Flow interface.

	This feature requires a specific license. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/ .
	<i>Flow in a Fractured Reservoir</i> : Application Library path Subsurface_Flow_Module/Fluid_Flow/fractured_reservoir_flow

LOCATION IN USER INTERFACE

Context Menus

- Darcy's Law > Fracture**
- Richards' Equation > Fracture**
- Fracture Flow > Fracture**

Ribbon

Physics tab with *interface* as **Darcy's Law** or **Richards' Equations** selected:

Boundaries > interface > Fracture

With **Fracture Flow** selected:

Boundaries > Fracture

Fluid (Fracture)

This node defines the material properties of the fluid of the **Fracture** parent node. The fluid can be specified as general gas or liquid or as an ideal gas.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

FLUID PROPERTIES

The available options for the **Fluid type** are **Gas/Liquid** (default) or **Ideal gas**. The selection determines how the density and compressibility of the fluid are set. The latter

is only required if a linearized storage model has been selected in the parent [Fracture](#) node. For all fluid types specify the dynamic viscosity μ (SI unit: Pa·s).

Gas/Liquid

This option specifies the **Density** and the **Compressibility** for a general gas or liquid.

Ideal Gas

For **Ideal gas** it uses the ideal gas law to describe the fluid. In this case, specify the thermodynamics properties. Select a **Gas constant type** — **Specific gas constant** R_s (the default) or **Mean molar mass** M_n (SI unit: J/(mol·K)). For **Mean molar mass** the universal gas constant $R = 8.314$ J/(mol·K) is used as the built-in physical constant. For both properties, the defaults use values **From material**. For **User defined** enter another value or expression.

If required by a **Storage Model**, the **Compressibility** of the ideal gas is calculated as $\chi_f = 1/p_A$ with the absolute pressure p_A (Pa).



If **Hydraulic conductivity** is selected as the **Permeability model** (see the [Fracture Material](#) section), dynamic viscosity is not available. The hydraulic conductivity is defined using a combination of fluid and fracture material properties and replaces the need of defining the dynamic viscosity.

LOCATION IN USER INTERFACE

A default **Fluid** node is automatically added to the [Fracture](#) node.

Context Menus

Darcy's Law > **Fracture** > **Fluid**

Richards' Equation > **Fracture** > **Fluid**

Fracture Flow > **Fracture** > **Fluid**

Fracture Material

This node defines the porosity and material properties of the solid matrix of the [Fracture](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

BARRIER PROPERTIES

This section appears if the **Thin conducting barrier** option for the **Fracture model** is chosen in the parent **Fracture** feature.

Enter the values for the barrier thickness d_b and permeability κ_b .

Note that the **Thin Conductive barrier** option is only available for the Fracture feature within the Darcy’s Law interface.

FRACTURE MATERIAL PROPERTIES

If the **Thin conducting barrier** option for the **Fracture model** is chosen in the parent **Fracture** feature you can specify the **Conductive layer position** here. Choose between **Both sides**, **Upside**, and **Downside**. A sketch above shows the position of conductive layers and barrier within the fracture.

The **Porosity** ϵ_p of the fracture material is taken **From material** by default. Select **User defined** to enter a value or expression for ϵ_p .

If required by a **Storage model** in the parent node, specify the **Effective compressibility** of the porous matrix. The default value is $\chi_p = 10^{-4}$ 1/Pa.

Select a **Permeability model** to specify the capacity of the fracture material to transmit flow. Which options are available depends on the **Flow model** setting in the parent node.

- Select **Permeability** (default) to define the permeability of the fracture material.
- Select **Hydraulic conductivity** or define a combination of fluid permeability and dynamic viscosity. For **Hydraulic conductivity** K (SI unit: m/s) select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value. The default is $2.94 \cdot 10^{-4}$ m/s.
- Select **Cubic law** to define the permeability from the fracture thickness specified in the parent node and the roughness factor f_f .

If the Non-Darcian Flow Model is selected in the parent **Fracture** feature enter a value for the **Inertial resistance coefficient** β .



- [About Darcian and Non-Darcian Flow](#) in the *Porous Media Flow Module User’s Guide*
- [Permeability Models](#) in the *Porous Media Flow Module User’s Guide*

LOCATION IN USER INTERFACE

A default **Fracture Material** node is automatically added to the **Fracture** node.

Context Menus

Darcy's Law > Fracture > Fracture Material

Richards' Equation > Fracture > Fracture Material

Fracture Flow > Fracture > Fracture Material

Theory for the Richards' Equation Interface

The [Richards' Equation Interface](#) theory is described in this section:

- [About Richards' Equation](#)
- [Retention and Permeability Relationships](#)
- [References for the Richards' Equation Interface](#)

About Richards' Equation

Richards' equation models flow in variably saturated porous media. With variably saturated flow, hydraulic properties change as fluids move through the medium, filling some pores and draining others.

This discussion of the Richards' Equation interface begins with the propagation of a single liquid (oil or water). The pore space not filled with liquid contains an immobile fluid (air) at atmospheric pressure.

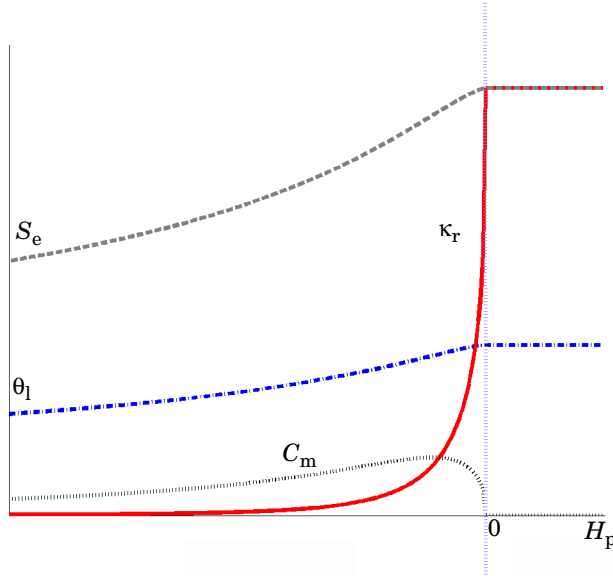


Figure 4-1: Fluid retention and permeability functions that vary with pressure head, as given by Van Genuchten formulas available in the Richards' Equation interface.

Many efforts to simplify and improve the modeling of flow in variably saturated media have produced a number of variants of Richards' equation since its appearance. The form that COMSOL Multiphysics solves is very general and allows for time-dependent changes in both saturated and unsaturated conditions (see [Ref. 1](#) and [Ref. 2](#)):

$$\rho \left(\frac{C_m}{\rho g} + S_e S_p \right) \frac{\partial p}{\partial t} + \nabla \cdot \rho \left(-\frac{\kappa_r \kappa_s}{\mu} (\nabla p + \rho g \nabla D) \right) = Q_m \quad (4-26)$$

where the pressure, p , is the dependent variable. In this equation, C_m represents the specific moisture capacity, S_e denotes the effective saturation, S_p is the storage coefficient, κ_s gives the hydraulic permeability at saturation, μ is the fluid dynamic viscosity, κ_r denotes the relative permeability, ρ is the fluid density, g is acceleration of gravity, D represents the elevation, and Q_m is the fluid source (positive) or sink (negative).

The fluid velocity across the faces of an infinitesimally small surface is

$$\mathbf{u} = -\frac{\kappa_r \kappa_s}{\mu} (\nabla p + \rho g \nabla D)$$

where $\mathbf{u}$ is the velocity vector. The porous medium consists of pore space, fluids, and solids, but only the liquids move. The equation above describes the flux as distributed across a representative surface. To characterize the fluid velocity in the pores, COMSOL Multiphysics also divides $\mathbf{u}$ by the liquid volume fraction, θ_l . This interstitial, pore or average linear velocity is $\mathbf{u}_a = \mathbf{u}/\theta_l$.

Retention and Permeability Relationships

Richards' equation appears deceptively similar to the saturated flow equation set out in the Darcy's Law interface, but it is notoriously nonlinear (Ref. 3). Nonlinearities arise because the material and hydraulic properties θ_l , S_e , C_m , and κ_r vary for unsaturated conditions (for example, negative pressure) and reach a constant value at saturation (for example, pressure of zero or above). The volume of liquid per porous medium volume, θ_l , ranges from a small residual value θ_r to the total porosity ϵ_p . Its value is given in a constitutive relation in the model commonly defined as:

$$\theta_l = \theta_r + S_e(\theta_s - \theta_r) \quad (4-27)$$

The variable θ_s is the saturated liquid volume fraction and therefore $\theta_s = \epsilon_p$. The effective saturation, S_e has a maximum value of 1 at saturation and usually follows a nonlinear relationship if the porous medium is unsaturated. The specific moisture capacity, C_m , describes the change in θ_l as the solution progresses, the slope on a plot of θ_l versus pressure (or pressure head):

$$C_m = \frac{\partial \theta_l}{\partial H_p}$$

The relative permeability, κ_r , increases with moisture content and varies from a nominal value to 1 at saturation, which reveals that the fluid moves more readily when the porous medium is fully wet.

The Richards' Equation interface in COMSOL Multiphysics includes three retention models to define θ_l , S_e , C_m , and κ_r :

- The analytic formulas of van Genuchten.
- The analytic formulas of Brooks and Corey.
- User-defined expression.

The analytic formulas of van Genuchten (Ref. 4) and Brooks and Corey (Ref. 5) are so frequently used that they are synonymous with this variably saturated flow modeling. Posed in terms of pressure head $H_p = p/(\rho g)$, the analytic expressions require data for

the saturated θ_s and residual θ_r liquid volume fractions as well as constants α , n , m , and l , which specify a particular medium type.

VAN GENUCHTEN

The van Genuchten equations define saturation when the fluid pressure is atmospheric (that is, $H_p = 0$). For unsaturated conditions ($H_p < 0$) the effective saturation is defined as:

$$S_e = \begin{cases} \frac{1}{(1 + |\alpha H_p|^n)^m} & H_p < 0 \\ 1 & H_p \geq 0 \end{cases}$$

For the liquid volume fraction [Equation 4-27](#) is used. The relative permeability is defined as

$$\kappa_r = S_e^l \left[1 - \left(1 - S_e^{\frac{1}{m}} \right)^m \right]^2$$

which results in $\kappa_r = 1$ at saturation ($S_e = 1$). The specific moisture capacity is defined as:

$$C_m = \frac{\alpha m}{1 - m} (\theta_s - \theta_r) S_e^{\frac{1}{m}} \left(1 - S_e^{\frac{1}{m}} \right)^m$$

here, the constitutive parameter m is equal to $1 - 1/n$. The functions are highly nonlinear which is also depicted in [Figure 4-2](#)

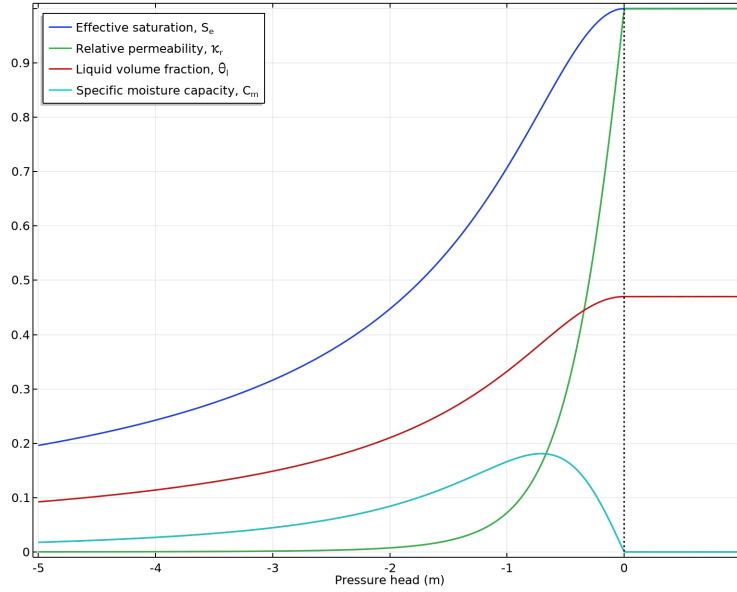


Figure 4-2: Van Genuchten retention functions for $\alpha = 1 \text{ m}^{-1}$, $n = 2$, and $l = 0.5$.

BROOKS AND COREY

With the Brooks and Corey approach, an air-entry pressure distinguishes saturated ($H_p > -1/\alpha$) and unsaturated ($H_p < -1/\alpha$) flow so that

$$S_e = \begin{cases} \frac{1}{|\alpha H_p|^n} & H_p < -\frac{1}{\alpha} \\ 1 & -\frac{1}{\alpha} \leq H_p \end{cases}$$

Again [Equation 4-27](#) describes the liquid volume fraction. The relative permeability is defined as

$$\kappa_r = S_e^{\frac{2}{n} + l + 2}$$

which results in $\kappa_r = 1$ at saturation ($S_e = 1$). The specific moisture capacity is defined as:

$$C_m = \begin{cases} \frac{-n}{H_p}(\theta_s - \theta_r)S_e & H_p < -\frac{1}{\alpha} \\ 0 & -\frac{1}{\alpha} \leq H_p \end{cases}$$

Here, the constitutive parameter m is equal to $1 - 1/n$. The functions are highly nonlinear which is also depicted in [Figure 4-3](#)

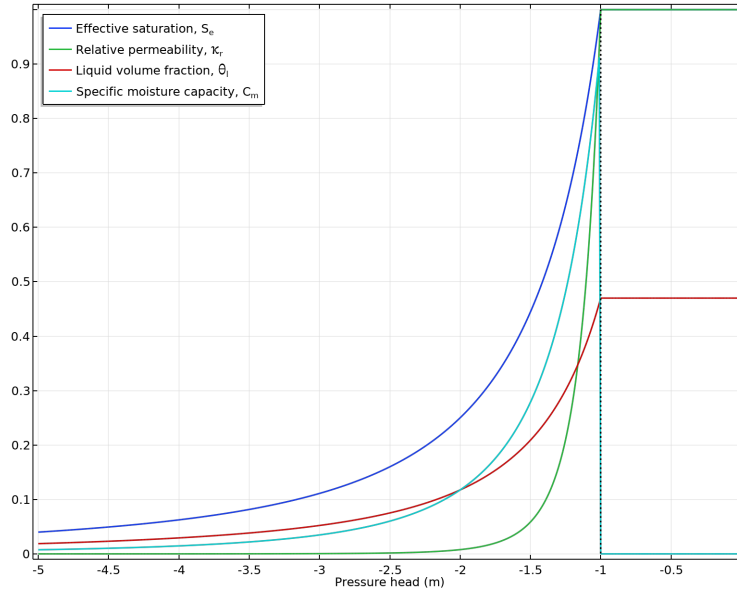


Figure 4-3: The Brooks and Corey retention functions for $\alpha = 1 \text{ m}^{-1}$, $n = 2$, and $l = 0.5$.

USER DEFINED

COMSOL Multiphysics also provides user-defined options for those who want to incorporate experimental data or arbitrary expressions to define these relationships. Enter expressions for θ_l , S_e , C_m , and κ_r directly.

References for the Richards' Equation Interface

1. J. Bear, *Dynamics of Fluids in Porous Media*, Elsevier Scientific Publishing, 1972.
2. J. Bear, *Hydraulics of Groundwater*, McGraw Hill, 1979.

3. R.A. Freeze, "Three-dimensional, Transient, Saturated-unsaturated Flow in a Groundwater Basin," *Water Resour. Res.*, vol. 7, no. 2, 1971.
4. M.Th. van Genuchten, "A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils," *Soil Sci. Soc. Am. J.*, vol. 44, 1980.
5. R.H. Brooks and A.T. Corey, "Properties of Porous Media Affecting Fluid Flow," *J. Irrig. Drainage Div., ASCE Proc.*, vol. 72 (IR2), 1966.

The Richards' Equation Interface

The **Richards' Equation (dl)** interface (), found under the **Porous Media and Subsurface Flow** branch () when adding a physics interface, is used to analyze flow in variably saturated porous media. The analytic formulas of van Genuchten, Brooks, and Corey are available for modeling variably saturated flow, where hydraulic properties change as fluids move through the porous medium, filling some pores and draining others. The physics interface can be used for stationary and time-dependent analysis.

When this physics interface is added, these default nodes are also added to the **Model Builder** — **Richards' Equation: Unsaturated Porous Medium** (which adds the equation for the pressure and provides an interface for defining the fluid material and its properties, including the retention model), **No Flow** (the default boundary condition), and **Initial Values**. Then, from the **Physics** toolbar, add other nodes that implement, for example, boundary conditions and mass sources. You can also right-click **Richards' Equation** to select physics features from the context menu.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `dl`.

PHYSICAL MODEL

Enter a **Reference pressure level** p_{ref} (SI unit: Pa). The default value is 1 [atm].

GRAVITY EFFECTS

Gravity effects are active by default. Clear the checkbox **Include gravity** to deactivate the acceleration of gravity.

Under **Acceleration of gravity**, enter the value for the acceleration of gravity (SI unit m/s^2). The default value is taken from the predefined physical constant `g_const`, which is the standard acceleration of gravity on earth.

DEPENDENT VARIABLES

The dependent variable (field variable) is for the **Pressure**. The name can be changed but the names of fields and dependent variables must be unique within a model.

DISCRETIZATION

You can choose the order of the shape functions used for the pressure variable solved by the Richards' Equation interface. The default shape functions are **Quadratic** Lagrange.

To display all settings available in this section, click the **Show** button () and select **Advanced Physics Options**.

	• Theory for the Richards' Equation Interface • Domain, Boundary, Edge, Point, and Pair Nodes for the Richards' Equation Interface • Physical Constants in the <i>COMSOL Multiphysics Reference Manual</i>
	• <i>Variably Saturated Flow</i>: Application Library path Subsurface_Flow_Module/Fluid_Flow/variably_saturated_flow • <i>Pesticide Transport and Reaction in Soil</i>: Application Library path Subsurface_Flow_Module/Solute_Transport/pesticide_transport

Domain, Boundary, Edge, Point, and Pair Nodes for the Richards' Equation Interface

[The Richards' Equation Interface](#) has the same domain, boundary, edge, point, and pair nodes described for [The Darcy's Law Interface](#).

	Domain, Boundary, Edge, Point, and Pair Nodes for the Darcy's Law Interface
	In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.



For axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** node that is valid on the axial symmetry boundaries only.



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Flownet Plot

The Flownet plot is available for 2D and 2D axisymmetric models. This plot is created automatically when solving a model that includes a Richards' equation interface, and it is created by adding a Streamline plot for the velocity field and a contour plot for the pressure.

Theory for the Two-Phase Darcy's Law Interface

About the Two-Phase Darcy's Law Interface

The Two-Phase Darcy's Law interface combines Darcy's law ([Equation 4-1](#)) with the continuity equation ([Equation 4-2](#)) for the average density ρ . In the presence of two miscible fluid phases, the average density ρ and average viscosity μ depend on the composition of the mixture ([Ref. 1](#)). In the Two-Phase Darcy's Law interface these dependencies are given by

$$1 = s_1 + s_2 \quad (4-28)$$

$$\rho = s_1 \rho_1 + s_2 \rho_2 \quad (4-29)$$

$$\frac{1}{\mu} = s_1 \frac{\kappa_{r1}}{\mu_1} + s_2 \frac{\kappa_{r2}}{\mu_2} \quad (4-30)$$

Here, s_1 and s_2 represent the saturation of each phase, ρ_1 and ρ_2 the densities, and κ_{r1} and κ_{r2} the relative permeabilities. When either of the fluids is compressible, its density can be related to the pressure (for instance using the ideal gas law).

Beside the continuity equation for the mixture, the Two-Phase Darcy's Law interface also solves the transport equation for the fluid content of one of the phases, $c_1 = s_1 \rho_1$:

$$\frac{\partial}{\partial t}(\epsilon_p c_1) + \nabla \cdot (c_1 \mathbf{u}) = \nabla \cdot D_c \nabla c_1 \quad (4-31)$$

Here, D_c (SI unit: m^2/s) is the capillary diffusion coefficient, which can be directly specified or derived from a capillary pressure expression. Normally, the fluid content c_1 will be the concentration of the wetting phase.

When *capillary pressure* is selected as capillary model, the capillary diffusion coefficient is computed from the saturation of one of the phases and the capillary pressure:

$$D_c = \frac{\kappa_{r1}}{\mu_1} \kappa(s_1 - 1) \frac{\partial p_c}{\partial s_1} \quad (4-32)$$

The capillary pressure is defined as the pressure difference between the phases, and it can be defined as a function of saturation, $p_c(s_1)$.

Capillary Pressure Models

The capillary pressure can be a user defined expression, or it can be derived from van Genuchten (Ref. 2) or Brooks and Corey (Ref. 3) models.

For van Genuchten model, the expression for the capillary pressure as a function of saturation follows the curve

$$p_c = p_{ec} \left(\frac{1}{s_1^{1/m}} - 1 \right)^{1-m} \quad (4-33)$$

where p_{ec} is the entry capillary pressure and m is a constitutive exponent.

For the Brooks and Corey model, the capillary pressure curve depends on saturation as

$$p_c = p_{ec} \frac{1}{s_1^{1/\lambda}} \quad (4-34)$$

where λ is the pore distribution index.

When capillary pressure is selected as capillary model, the relative permeabilities are also determined by the van Genuchten or Brooks and Corey models. For the van Genuchten model, the relative permeabilities are given by

$$\begin{aligned} \kappa_{r1} &= s_1^{l_{vG}} (1 - (1 - s_1^{1/m_{vG}})^{m_{vG}})^2 \\ \kappa_{r2} &= (1 - s_1)^{l_{vG}} (1 - s_1^{1/m_{vG}})^{2m_{vG}} \end{aligned}$$

where m_{vG} and l_{vG} are constitutive constants.

For the Brooks and Corey model, the relative permeabilities are given by

$$\begin{aligned} \kappa_{r1} &= s_1^{(3+2/\lambda)} \\ \kappa_{r2} &= (1 - s_1)^2 (1 - s_1^{(1+2/\lambda)}) \end{aligned}$$

where λ is again the pore distribution index.

References for the Two-Phase Darcy's Law Interface

1. Z. Chen, G. Huan, and Y. Ma, *Computational Methods for Multiphase Flows in Porous Media*. Philadelphia: Society for Industrial and Applied Mathematics, 2006.
2. M.Th. van Genuchten, "A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils," *Soil Sci. Soc. Am. J.*, vol. 44, 1980.
3. R.H. Brooks and A.T. Corey, "Properties of Porous Media Affecting Fluid Flow," *J. Irrig. Drainage Div., ASCE Proc.*, vol. 72 (IR2), 1966.

The Two-Phase Darcy's Law Interface

The **Two-Phase Darcy's Law (tpdl)** interface (), found under the **Porous Media and Subsurface Flow** branch () when adding a physics interface, is used to simulate fluid flow through interstices in a porous medium. It solves Darcy's law for the total pressure and the transport of the fluid content for one fluid phase. The physics interface can be used to model low velocity flows or media where the permeability and porosity are very small, for which the pressure gradient is the major driving force and the flow is mostly influenced by the frictional resistance within the pores. The physics interface can be used for stationary and time-dependent analyses.

The main feature is the Fluids and Matrix Properties node, which provides an interface for defining the two immiscible fluids properties along with the porous medium properties. The physics interface is available in 2D, 2D axisymmetric, and 3D.

When this physics interface is added, the following default nodes are also added in the **Model Builder** — **Fluids and Matrix Properties**, **No Flow** (the default boundary condition), and **Initial Values**. Then, from the **Physics** toolbar, add other nodes that implement, for example, boundary conditions and mass sources. You can also right-click **Two-Phase Darcy's Law** to select physics features from the context menu.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `tpdl`.

PHYSICAL MODEL

Enter a **Reference pressure level** p_{ref} (SI unit: Pa). The default value is `1 [atm]`.

DEPENDENT VARIABLES

The dependent variables (field variables) are the **Pressure** and **Fluid content I**. The name can be changed but the names of fields and dependent variables must be unique within a component.

CONSISTENT STABILIZATION

To display this section, click the **Show More Options** button () and select **Stabilization** from the **Show More Options** dialog.

There are two consistent stabilization methods available — **Streamline diffusion** and **Crosswind diffusion**. **Streamline diffusion** is active by default. The **Residual** setting applies to both the consistent stabilization methods. **Approximate residual** is the default setting and it means that derivatives of the capillary diffusion tensor components are neglected. This setting is usually accurate enough and computationally faster. If required, select **Full residual** instead.

DISCRETIZATION

You can choose the order of the shape functions used for the pressure and fluid content variables solved by the Two-Phase Darcy’s Law interface. The default shape functions are **P2+P1** Lagrange.

To display all settings available in this section, click the **Show More Options** button () and select **Advanced Physics Options** from the **Show More Options** dialog.

	• Domain, Boundary, and Pair Nodes for the Two-Phase Darcy’s Law Interface • Theory for the Two-Phase Darcy’s Law Interface

Domain, Boundary, and Pair Nodes for the Two-Phase Darcy’s Law Interface

The Two-Phase Darcy’s Law Interface has these domain, boundary, and pair nodes, listed in alphabetical order, available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

	In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.

- Fluid and Matrix Properties
- Inlet
- Initial Values
- Symmetry
- Interior Wall
- Mass Flux
- No Flux
- Outlet
- Pressure and Saturation



For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** node that is valid on the axial symmetry boundaries only.

Fluid and Matrix Properties

The **Fluids and Matrix Properties** node adds Equation 4-35 and Equation 4-36 and defines properties including density, dynamic viscosity, relative permeability, and porosity.

$$\frac{\partial}{\partial t}(\rho \varepsilon_p) + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (4-35)$$

$$\frac{\partial}{\partial t}(c_1 \varepsilon_p) + \nabla \cdot (c_1 \mathbf{u}) = \nabla \cdot D_c \nabla c_1 \quad (4-36)$$

CAPILLARY MODEL

The list defaults to **Capillary diffusion** or choose **Capillary pressure**.

For **Capillary diffusion**, choose **Isotropic** to define a scalar or **Diagonal, Symmetric**, or **Full** to enter a tensor, and then enter values or expressions in the D_c (SI unit: m^2/s) field or fields.

For **Capillary pressure**, select a **Pressure model** — **van Genuchten** (the default), **Brooks and Corey**, or **User defined**.

- For **van Genuchten**, enter an **Entry capillary pressure** p_{ec} (SI unit: Pa), the **Constitutive relations constant** m_{vG} (dimensionless), and **Constitutive relations constant** I_{vG} (dimensionless).
- For **Brooks and Corey**, enter an **Entry capillary pressure** p_{ec} (SI unit: Pa) and the **Pore size distribution index** λ_p (dimensionless).
- For **User defined**, enter a **Capillary pressure** p_c (SI unit: Pa).

FLUID 1 PROPERTIES

Select the material to use for **Fluid 1**. Select **Domain material** (the default value) to use the material defined for the domain. Select another material to use that material's properties.

Define the properties for **Fluid 1**:

The default **Density** ρ_1 (SI unit: kg/m^3) uses values **From material** based on the **Fluid 1** selection. For **User defined** enter another value or expression. The default is $0 \text{ kg}/\text{m}^3$.

Select a **Dynamic viscosity** μ_1 (SI unit: $\text{Pa}\cdot\text{s}$). The default uses values **From material** as defined by the **Fluid 1** selection. For **User defined** enter another value or expression. The default is $0 \text{ Pa}\cdot\text{s}$.

Select the **Relative permeability** κ_{r1} (a dimensionless number between 0 and 1) for fluid 1. The default is 1.

FLUID 2 PROPERTIES

The settings to define the properties for **Fluid 2** are similar to those for **Fluid 1**.

MATRIX PROPERTIES

Select the material to use as porous matrix. Select **Domain material** (the default) from the **Porous material** list to use the material defined for the porous domain. Select another material to use that material's properties.

The default **Porosity** ϵ_p (a dimensionless number between 0 and 1) uses the value **From material**, defined by the **Porous material** selection. For **User defined** enter another value or expression. The default is 0.

The default **Permeability** κ (SI unit: m^2) uses the value **From material**, as defined by the **Porous material** selection. For **User defined** select **Isotropic** to define a scalar or **Diagonal**, **Symmetric**, or **Full** to enter a tensor and enter other values or expressions in the field or matrix.

Initial Values

The **Initial Values** node adds initial values for the pressure and the saturation of fluid 1 (that is, the fraction of that fluid inside the pore space) that can serve as initial conditions for a transient simulation or as an initial guess for a nonlinear solver.

INITIAL VALUES

Enter a value or expression for the initial value of the **Pressure** p (SI unit: Pa). The default value is 0 Pa. Enter a value or expression for the initial value of the **Saturation fluid 1** s_1 (a dimensionless number between 0 and 1). The default value is 0. The saturation of **Fluid 2** is then calculated as $s_2 = 1 - s_1$.

No Flux

The **No Flux** node is the default boundary condition stating that there is no flow across impervious boundaries. The mathematical formulation is:

$$\mathbf{n} \cdot \rho \mathbf{u} = 0$$

where $\mathbf{n}$ is the vector normal to the boundary.

Also, the No Flux boundary enforces a zero gradient condition for the fluid content across the selected boundary

$$\mathbf{n} \cdot \nabla c_1 = 0$$

Pressure and Saturation

Use the **Pressure and Saturation** node to specify the pressure and fluid volume fractions on a boundary. In many cases the distribution of pressure and saturation are known as numbers, distributions, or expressions involving time, t , for example.

PRESSURE AND SATURATION

Enter a value or expression for the **Pressure** p (SI unit: Pa), and for the **Saturation fluid 1** s_1 (a dimensionless number between 0 and 1). The default values are 0. The saturation of **Fluid 2** is then calculated as $s_2 = 1 - s_1$.

Mass Flux

Use the **Mass Flux** node to specify the mass flux into or out of the model domain through some of its boundaries. It is often possible to determine the mass flux from the pumping rate or from measurements. With this boundary condition, positive values correspond to flow into the model domain

$$-\mathbf{n} \cdot \rho \mathbf{u} = N_0$$

where N_0 is a value or expression for the inward (or outward) Darcy's flux that is specified.

MASS FLUX AND SATURATION

Enter a value or expression for the **Inward mass flux** N_0 (SI unit: $\text{kg}/(\text{m}^2 \cdot \text{s})$). A positive value of N_0 represents an inward mass flux, whereas a negative value represents an outward mass flux.

Enter a value or expression for the **Saturation fluid 1** s_1 (a dimensionless number between 0 and 1) in the mass flux. The default value is 0.

Inlet

The **Inlet** node adds a boundary condition for the inflow (or outflow) perpendicular (normal) to the boundary:

$$-\mathbf{n} \cdot \rho \mathbf{u} = (s_1 \rho_1 + s_2 \rho_1) U_0$$

where U_0 is a specified value or expression for the inward (or outward) Darcy's velocity. A positive value of the velocity U_0 corresponds to flow into the model domain whereas a negative value represents an outflow.

INLET

Enter a value or expression for the **Normal inflow velocity** U_0 (SI unit: m/s). A positive value of U_0 represents an inflow velocity, whereas a negative value represents an outward velocity.

Enter a value or expression for the **Saturation fluid 1** s_1 (a dimensionless number between 0 and 1). The default value is 0.

Outlet

The **Outlet** node adds a boundary condition for the outflow perpendicular (normal) to the boundary:

$$-\mathbf{n} \cdot D_c \nabla c_1 = 0$$

where D_c is the capillary diffusion (SI unit: m^2/s) and $c_1 = s_1 \rho_1$ is the fluid 1 content (SI unit: kg/m^3). This means that the normal gradient of fluid saturation does not change through this boundary.

PRESSURE

Enter a value or expression for the **Pressure** p . The default value is 0. If the reference pressure p_{ref} , defined at the physics interface level is 0, enter the absolute pressure. Otherwise, enter the relative pressure.

Symmetry

The **Symmetry** node describes a symmetry boundary where there is no tangential flow. The following condition implements the symmetry condition on an axis or a flow divide:

$$-\mathbf{n} \cdot \rho \mathbf{u} = 0$$

$$-\mathbf{n} \cdot D_c \nabla c_1 = 0$$

For axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** node that is valid on the axial symmetry boundaries only.

Interior Wall

The **Interior Wall** boundary condition can only be applied on interior boundaries.

It is similar to the **No Flux** boundary available on exterior boundaries except that it applies on both sides of an interior boundary. It allows discontinuities of velocity and pressure across the boundary. The **Interior Wall** boundary condition can be used to avoid meshing thin structures by applying no-flux condition on interior curves and surfaces instead.

Thin Barrier

The **Thin Barrier** boundary condition can only be applied on interior boundaries.

It is similar to the **Interior Wall** boundary available on interior boundaries except that it allows pressure and concentration gradients across the boundary. The **Thin Barrier** boundary condition can be used to avoid meshing thin permeable structures by applying flux condition on interior curves and surfaces instead.

Theory for the Fracture Flow Interface

The [Fracture Flow Interface](#) theory is described in this section:

- [About Fracture Flow](#)
- [Equations and Inputs](#)

About Fracture Flow

The Fracture Flow interface uses tangential derivatives to define the flow along the interior boundaries representing fractures within a porous (or solid) block. Use this physics interface on boundaries in 3D, 2D, and axisymmetric geometries.



Fracture Flow is also available as a boundary condition in the Darcy's Law and Richards' Equation interfaces.

The physics interface uses the tangential version of Darcy's law:

$$\mathbf{q}_f = -\frac{\kappa_f}{\mu} d_f (\nabla_T p + \rho g \nabla_T D) \quad (4-37)$$

Here, $\mathbf{q}_f$ is the volume flow rate per unit length in the fracture, κ_f is the fracture's permeability, μ is the fluid dynamic viscosity, d_f is the *aperture* or fracture thickness, ∇_T denotes the gradient operator restricted to the fracture's tangential plane, p is the pressure, ρ is the fluid density, g is the acceleration of gravity, and D represents the vertical coordinate.



Gravity effects are not active by default. Select the checkbox **Include gravity** to activate the acceleration of gravity. Setting the elevation D to zero also turns off gravity effects.

The variable $\mathbf{q}_f$ gives the volume flow rate per unit length of the fracture. The mean fluid velocity within the fracture is $\mathbf{u}_f = \mathbf{q}_f / d_f$.

It is also possible to use the hydraulic conductivity of the fracture, K_f (SI unit: m/s), to define the capacity to transmit flow instead of using the fracture's permeability κ_f and fluid viscosity μ . These quantities are related by

$$\frac{\kappa_f}{\mu} = \frac{K_f}{\rho g}$$

The hydraulic conductivity represents properties of both fluid and porous matrix. If the model is defined using the hydraulic conductivity, Equation 4-37 changes to

$$\mathbf{q}_f = -\frac{K_f}{\rho g} d_f (\nabla_T p + \rho g \nabla_T D) \quad (4-38)$$

The Cubic law equation describes the permeability of the fracture from the aperture or fracture's thickness d_f and the roughness factor f_f

$$\kappa_f = \frac{d_f^2}{12 f_f}$$

If the model is defined using the Cubic law equation, the expression for flow rate per unit length in the fracture in Equation 4-37 changes to

$$\mathbf{q}_f = -\frac{d_f^3}{12 \mu f_f} (\nabla_T p + \rho g \nabla_T D) \quad (4-39)$$

Together with the material properties, Equation 4-37 above, in combination with the continuity equation integrated over the fracture cross section, produces a single equation for the pressure.

$$d_f \frac{\partial}{\partial t} (\epsilon_f \rho) + \nabla_T \cdot (\rho \mathbf{q}_f) = d_f Q_m \quad (4-40)$$

where ϵ_f is the fracture porosity, and Q_m is the mass source term (SI unit: kg/(m³·s)). The aperture or fracture thickness d_f can vary along the fracture and therefore appears on both sides of the equation.

The physics interface solves for the same dependent variable as for the equation in the porous medium, the pressure p .

Equations and Inputs

Two material models are supported: Fluid and Fracture Properties and the Storage Model.

FLUID AND FRACTURE PROPERTIES

This is the default setting of the Fracture Flow interface, and implements the fracture flow as described in [Equation 4-40](#). The values for the fluid properties (density and dynamic viscosity) can be taken from the material node or be defined by an arbitrary expression. The same can be done for the fracture's properties, such as porosity and permeability. Enter the fracture's thickness in the corresponding field.

STORAGE MODEL

The governing equation takes the slightly different form, compared to [Equation 4-40](#):

$$\rho S_f d_f \frac{\partial p}{\partial t} - \nabla_T \cdot (\rho \mathbf{q}_f) = d_f Q_m$$

where S_f is the fracture storage coefficient (SI unit: 1/Pa).

In contrast to the Darcy's Law and Richards' Equation interfaces, the storage coefficient is always a user-defined expression in the Fracture Flow interface.

The Fracture Flow Interface

The **Fracture Flow (ff)** interface () , found under the **Porous Media and Subsurface Flow** branch () when adding a physics interface, is used to simulate flow along faults and fractures represented as boundaries. The physics interface uses a variant of Darcy's law and can be used for stationary and time-dependent analysis.

When this physics interface is added, these default nodes are also added to the **Model Builder** — **Fracture, No Flow** (the default boundary condition), and **Initial Values**. Then, from the **Physics** toolbar, add other nodes that implement, for example, mass sources and conditions on edges or points. You can also right-click **Fracture Flow** to select physics features from the context menu.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `ff`.

PHYSICAL MODEL

Enter a **Reference pressure level** p_{ref} (SI unit: Pa). The default value is 1 [atm].

GRAVITY EFFECTS

Gravity effects are not active by default. Select the checkbox **Include gravity** to activate the acceleration of gravity.

Under **Acceleration of gravity** enter the value for the acceleration of gravity g (SI unit: m/s^2). The default value is the predefined constant `g_const`.

DISCRETIZATION

You can choose the order of the shape functions used for the pressure variable solved by the Fracture Flow interface. The default shape functions are **Quadratic** Lagrange.

DEPENDENT VARIABLES

The dependent variable (field variable) is for the **Pressure**. The name can be changed but the names of fields and dependent variables must be unique within a model.

To display all settings available in this section, click the **Show** button () and select **Advanced Physics Options**.

	• Domain, Boundary, Edge, Point, and Pair Nodes for the Fracture Flow Interface • Theory for the Fracture Flow Interface • Physical Constants in the <i>COMSOL Multiphysics Reference Manual</i>

Domain, Boundary, Edge, Point, and Pair Nodes for the Fracture Flow Interface

The [Fracture Flow Interface](#) has the domain, boundary, edge, point, and pair nodes available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

	In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.

DOMAIN

- [Fracture](#)
- [Initial Values](#)
- [Unsaturated Porous Medium](#)
- [Mass Source](#)
- [Thickness](#)

	Storage Model and Mass Source are described for the Darcy’s Law interface and are applied to boundaries instead of domains.

BOUNDARY CONDITIONS

The boundary conditions for this physics interface are similar to the boundary conditions for [The Darcy’s Law Interface](#) but are applied to edges in 3D and points in 2D. Edges and points form the “boundaries” of the geometries where the **Fracture Flow** interface is defined, which are faces in 3D and boundaries in 2D.



In the descriptions for these nodes, replace the **Boundary Selection** section with the **Edge Selection** (3D) or **Point Selection** (2D) section when using the Fracture Flow interface.

The following nodes are available on exterior boundaries (edges and points) and described for [The Darcy’s Law Interface](#):

- [Pressure](#)
- [Pressure Head](#)
- [Hydraulic Head](#)
- [Inlet](#)
- [Outlet](#)
- [Mass Flux](#)
- [Symmetry](#)
- [No Flow](#) (the default)
- [Atmosphere/Gauge](#)
- [Pervious Layer](#)
- [Precipitation](#)

Note that some of the boundary conditions mentioned above are only available for certain modules. For detailed information, see www.comsol.com/products/specifications/.

The relevant physics interface condition at interior edges is continuity. In addition, the following edge conditions are available on interior edges:

- [Pressure](#)
- [Pervious Layer](#)



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Fracture

The **Fracture** feature for the Fracture Flow interface uses the same settings as the [Fracture](#) boundary condition for Darcy's Law except that the fracture model is not available. With the Fracture Flow interface, only highly conductive fractures can be modeled.

As the Fracture boundary condition, the Fracture feature has two subfeatures: [Fluid \(Fracture\)](#) and [Fracture Material](#).

Initial Values

The **Initial Values** node adds an initial value for the pressure, pressure head, or hydraulic head pressure that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver.

INITIAL VALUES

- Select the **Pressure** (the default) button to enter a value or expression for the initial value of the pressure p (SI unit: Pa). The default is 0 Pa.
- Select **Pressure head** to enter a value or expression for H_p (SI unit: m).
- Select **Hydraulic head** to enter a value or expression for H (SI unit: m).

Theory for the Brinkman Equations Interface

The [Brinkman Equations Interface](#) theory is described in this section:

- [About the Brinkman Equations](#)
- [Brinkman Equations Theory](#)
- [References for the Brinkman Equations Interface](#)

About the Brinkman Equations

The *Brinkman equations* describe fast-moving fluids in porous media with the kinetic potential from fluid velocity, pressure, and gravity to drive the flow. These equations extend Darcy's law to describe the dissipation of the kinetic energy by viscous shear as with the Navier–Stokes equations. Consequently, this physics interface well suits transitions between slow flow in porous media governed by Darcy's law and fast flow in channels described by the Navier–Stokes equations. Interesting uses of a Brinkman and Navier–Stokes coupling include modeling of the hyporheic zone near a river, the flow of oil through a reservoir to a well perforation, and non-Newtonian flows.

In porous domains, the flow variables and fluid properties are defined at any point inside the medium by means of averaging of the actual variables and properties over a certain volume surrounding the point. This control volume must be small compared to the typical macroscopic dimensions of the problem, but it must be large enough to contain many pores and solid matrix elements.

Porosity is defined as the fraction of the control volume that is occupied by pores. Thus, the porosity can vary from zero for pure solid regions to unity for domains of free flow.

The physical properties of the fluid, such as density and viscosity, are defined as *intrinsic volume averages* that correspond to a unit volume of the pores. Defined this way, they present the relevant physical parameters that can be measured experimentally, and they are assumed to be continuous with the corresponding parameters in the adjacent free flow.

The flow velocity is defined as a *superficial volume average*, and it corresponds to a unit volume of the medium including both the pores and the matrix. It is sometimes

called the Darcy velocity, defined as the volume flow rate per unit cross section of the medium. Such a definition makes the velocity field continuous across the boundaries between porous regions and regions of free flow.

Brinkman Equations Theory

The dependent variables in the Brinkman equations are the Darcy velocity and the pressure. The flow in porous media is governed by a combination of the continuity equation and the momentum equation, which together form the Brinkman equations:

$$\frac{\partial}{\partial t}(\epsilon_p \rho) + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (4-41)$$

$$\begin{aligned} \frac{\rho}{\epsilon_p} \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \frac{\mathbf{u}}{\epsilon_p} \right) = \\ -\nabla p + \nabla \cdot \left[\frac{1}{\epsilon_p} \left\{ \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right\} \right] - \left(\kappa^{-1} \mu + \frac{Q_m}{\epsilon_p^2} \right) \mathbf{u} + \mathbf{F} \end{aligned} \quad (4-42)$$

In these equations:

- μ (SI unit: kg/(m·s)) is the dynamic viscosity of the fluid
- $\mathbf{u}$ (SI unit: m/s) is the velocity vector
- ρ (SI unit: kg/m³) is the density of the fluid
- p (SI unit: Pa) is the pressure
- ϵ_p is the porosity
- κ (SI unit: m²) is the permeability of the porous medium, and
- Q_m (SI unit: kg/(m³·s)) is a mass source or sink

Influence of gravity and other volume forces can be accounted for via the force term $\mathbf{F}$ (SI unit: kg/(m²·s²)).

When the **Neglect inertial term (Stokes flow)** checkbox is selected, the term $(\mathbf{u} \cdot \nabla)(\mathbf{u}/\epsilon_p)$ on the left-hand side of Equation 4-42 is disabled.

The mass source, Q_m , accounts for mass deposit and mass creation within the domains. The mass exchange is assumed to occur at zero velocity.

The Forchheimer and Ergun drag options add a viscous force proportional to the square of the fluid velocity, $\mathbf{F} = -\rho\beta|\mathbf{u}|\mathbf{u}$, on the right-hand side of Equation 4-42; see [References for the Darcy's Law Interface](#) for details.

In case of a flow with variable density, Equation 4-41 and Equation 4-42 must be solved together with the equation of state that relates the density to the temperature and pressure (for instance the ideal gas law).

For incompressible flow, the density stays constant in any fluid particle, which can be expressed as

$$\frac{\partial}{\partial t}(\varepsilon_p \rho) + \mathbf{u} \cdot \nabla \rho = 0$$

and the continuity equation (Equation 4-41) reduces to

$$\rho \nabla \cdot \mathbf{u} = Q_m$$

References for the Brinkman Equations Interface

1. D. Nield and A. Bejan, *Convection in Porous Media*, 3rd ed., Springer, 2006.
2. M. Le Bars and M.G. Worster, “Interfacial Conditions Between a Pure Fluid and a Porous Medium: Implications for Binary Alloy Solidification,” *J. Fluid Mech.*, vol. 550, pp. 149–173, 2006.

The Brinkman Equations Interface

The **Brinkman Equations (br)** interface (), found under the **Porous Media and Subsurface Flow** branch () when adding a physics interface, is used to compute fluid velocity and pressure fields of single-phase flow in porous media in the laminar flow regime. The physics interface extends Darcy's law to describe the dissipation of the kinetic energy by viscous shear, similar to the Navier–Stokes equations. Fluids with varying density can be included at Mach numbers below 0.3. Also the viscosity of a fluid can vary, for example, to describe non-Newtonian fluids. To simplify the equations, select the Stokes–Brinkman flow feature to reduce the dependence on inertial effects when the Reynolds number is significantly less than 1. The physics interface can be used for stationary and time-dependent analyses.

The main node is the Porous Medium feature, which adds the Brinkman equations and provides interfaces for defining the fluid material and the porous matrix.

When this physics interface is added, the following default nodes are also added in the **Model Builder — Porous Medium, Wall** (the default boundary type, using **No slip** as the default boundary condition), and **Initial Values**. Then, from the **Physics** toolbar, add other nodes that implement, for example, boundary conditions and volume forces. You can also right-click **Brinkman Equations** to select physics features from the context menu.

The boundary conditions are essentially the same as for the Laminar Flow interface. Differences exist for the following boundary types: Outlet, Symmetry, Open Boundary, and Boundary Stress where the viscous part of the stress is divided by the porosity to appear as

$$\frac{1}{\varepsilon_p} \left\{ \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right\}$$

In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections such as **Discretization**, **Consistent Stabilization**, **Inconsistent Stabilization**, and **Advanced Settings**, all accessed by clicking the **Show** button () and choosing the applicable option. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `br`.

PHYSICAL MODEL

This node specifies the properties of the Brinkman Equations interface, which describe the overall type of fluid flow model.

Compressibility

By default the physics interface uses the **Incompressible flow** formulation of the Brinkman equations to model constant density flow. Alternatively, from the **Compressibility** list select **Weakly compressible flow**, which should be used when the pressure dependency of the density can be neglected, or **Compressible flow (Ma<0.3)** to solve for the full compressible flow of which, however, the Mach number must be below 0.3.

Swirl Flow

For 2D axisymmetric models, select the **Swirl flow** checkbox to include the swirl velocity component, that is the velocity component u_ϕ in the azimuthal direction. While u_ϕ can be nonzero, there can be no gradients in the ϕ direction.

Note that this feature is only available for specific modules. See www.comsol.com/products/specifications/ for a detailed overview.

Neglect Inertial Term (Stokes–Brinkman Flow)

The **Neglect inertial term (Stokes–Brinkman)** checkbox is selected by default to model flow at low Reynolds numbers for which the inertial term can be neglected. This results in the linear Stokes–Brinkman equations.

Include Gravity

Gravity is not included by default. Select the **Include gravity** checkbox to activate the acceleration of gravity. This automatically adds a global **Gravity** feature node to the interface model tree, and the buoyancy force is included in the equations.

Note that this feature is only available for specific modules. See www.comsol.com/products/specifications/ for a detailed overview.

Also, when the **Include gravity** checkbox is selected, the **Use reduced pressure** option changes the pressure formulation from using the total pressure (default) to using the reduced pressure. This option is suitable for configurations where the density changes are very small; otherwise, the default formulation can be used. For more information, see [Gravity](#).

Rotating Frame

When the **Rotating Frame** checkbox is selected, a global **Rotating Frame** feature is shown in the interface model tree, and the fictitious Coriolis, centrifugal and Euler forces, stemming from the change to a non-inertial frame of reference, are included in the Navier–Stokes equations.

For 2D components, the **Gravity** feature and the **Rotating Frame** feature are mutually exclusive, and thus can not be selected simultaneously. For 2D axisymmetric components, the **Rotating Frame** checkbox and feature are only available when also the **Swirl flow** checkbox is selected.

When the **Rotating Frame** checkbox is selected, the **Use reduced pressure** option changes the pressure formulation from using the total pressure (default) to using the reduced pressure. This option is suitable for configurations where the density changes are very small; otherwise, the default formulation can be used. For more information, see [Rotating Frame](#)

Include Buoyancy-Induced Turbulence

When either the **Gravity** or the **Rotating Frame** checkbox is selected, and a turbulence model that solves for the turbulent kinetic energy, k , is used, the **Include buoyancy-turbulence** checkbox is available. When selected, the **Buoyancy contribution** is by default set to **Automatic from multiphysics**. Contributions are only obtained from multiphysics couplings that support buoyancy-induced turbulence, such as Nonisothermal Flow. If the **Buoyancy contribution** is switched to **Used defined**, a text field for the **Turbulent Schmidt number** appears.

When the **Incompressible flow** or the **Weakly compressible flow** option is selected, the **Hydrostatic pressure approximation** checkbox is available (selected by default). See also [Theory for Buoyancy-Induced Turbulence](#) in the *CFD Module User's Guide*.

Porous Treatment of No Slip Condition

Choose how the **No Slip** condition on **Wall** boundaries and **Interior Wall** boundaries adjacent to **Porous Medium** should be treated. The options are **Standard no slip formulation** and **Porous slip** (the default). The latter option provides a unified treatment when the porous matrix is fully resolved as well as when it is under resolved ensuring

a smooth transition between regions with different resolutions; see No Slip under [Wall](#) in [The Laminar Flow and Creeping Flow Interfaces](#) section.

Reference Pressure Level

Enter a **Reference pressure level** p_{ref} (SI unit: Pa). The default value is 1 [atm].

Reference Temperature Level

Enter a **Reference temperature level** T_{ref} (SI unit: K). The default value is 293.15 [K].

Reference Position

If **Include gravity** or **Rotating frame** is selected, a **Reference position** r_{ref} (SI unit: m) can be specified which is then used for the calculation of the hydrostatic pressure.

TURBULENCE

Turbulent flow can be simulated by changing the **Turbulence model type** to **RANS-EVM** (Reynolds-Averaged Navier–Stokes, eddy-viscosity model). If **RANS-EVM** is activated, you can set the **Turbulence model** either to one of the algebraic turbulence models, **Algebraic yPlus** (the default) or **L-VEL**, or to one of the following two-equation turbulence models: k - ϵ , Low Reynolds Number k - ϵ . The **Wall treatment** is set to **Automatic** (default) or **Low Re** while **Wall functions** is the only option for k - ϵ .

For more information about turbulence modeling, see [Theory for the Turbulent Flow Interfaces](#) in the *CFD Module User's Guide*.

DEPENDENT VARIABLES

The following dependent variables (fields) are defined for this physics interface — the **Velocity field** $\mathbf{u}$ (SI unit: m/s) and its components, and the **Pressure** p (SI unit: Pa).

ADVANCED SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. Normally these settings do not need to be changed.

The **Use pseudo time stepping for stationary equation form** option adds pseudo time derivatives to the equation when the **Stationary equation** form is used in order to speed up convergence. When selected, a **CFL number expression** should also be defined. For

the default **Automatic** option, the local CFL number (from the Courant–Friedrichs–Lewy condition) is determined by a PID regulator.

	• Domain, Boundary, Point, and Pair Nodes for the Brinkman Equations Interface • Theory for the Brinkman Equations Interface • Pseudo Time Stepping for Laminar Flow Models and Pseudo Time Stepping in the COMSOL Multiphysics Reference Manual • Numerical Stability — Stabilization Techniques for Fluid Flow • Nitsche Constraints
	<i>Free Convection in a Porous Medium:</i> Application Library path Subsurface_Flow_Module/Heat_Transfer/convection_porous_medium

Domain, Boundary, Point, and Pair Nodes for the Brinkman Equations Interface

The [Brinkman Equations Interface](#) has the following domain, boundary, point, and pair nodes, listed in alphabetical order, available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

	In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.

These nodes are described in this section:

- [Porous Medium](#)
- [Initial Values](#)
- [Mass Source](#)
- [Gravity](#)
- [Volume Force](#)
- [Fluid Properties](#)
- [Rotating Frame](#)

The following nodes (listed in alphabetical order) are described for the Laminar Flow interface :

- [Flow Continuity](#)
- [Inlet](#)
- [Line Mass Source](#)
- [Boundary Stress](#)
- [Outlet](#)
- [Open Boundary](#)
- [Periodic Flow Condition](#)
- [Point Mass Source](#)
- [Pressure Point Constraint](#)
- [Symmetry](#)
- [Wall](#)



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Porous Medium

The **Porous Medium** node adds the Brinkman equations: [Equation 4-34](#) and [Equation 4-35](#) (excluding any mass sources). It has two default subfeatures — the [Fluid](#) subfeature where the fluid density ρ (SI unit: kg/m^3) and dynamic viscosity μ (SI unit: $\text{Pa}\cdot\text{s}$) are defined and the [Porous Matrix](#) subfeature where the permeability κ (SI unit: m^2) and porosity ε_p (dimensionless) are specified.

COORDINATE SYSTEM SELECTION

Select a coordinate system from the **Coordinate system** list for the interpretation of directions in anisotropic material properties. The default is the **Global coordinate system**, and the list contains any additional orthonormal coordinate system (except boundary coordinate systems) added under the Definitions node.

The subnodes inherit these coordinate system settings. In particular, the **Permeability** (in the **Porous Matrix** subnode) should be set according to the coordinate system selected in this section.

FLOW MODEL

Define if the flow follows a linear or nonlinear pressure-velocity relationship.

- **Darcian flow** (default) defines a linear relationship characteristic for low Reynolds number laminar flows.
- **Non-Darcian flow** defines a nonlinear relationship due to inertial or turbulence effects.

POROUS MEDIUM TURBULENCE MODEL

If in the parent feature node a turbulence model other than **Algebraic yPlus** or **L-VEL** is selected, define a model which complements the basic turbulence model and accounts for the influence of the porous matrix on turbulent flow in porous media.

- For **Default** and **Nakayama–Kuwahara** specify the **Coefficient of subgrid turbulence generation by porous matrix** C_{stpm} which by default is set to 0.212.
- For **Pedras–de Lemos**, specify the **Choice of the Pedras–de Lemos model coefficient**. When **User defined** is chosen, specify the **Coefficient of subgrid turbulence generation by porous matrix** C_{stpm} with default value 0.212. Choosing **Original** results in $C_{\text{stpm}} = 0.28 \cdot \sqrt{\epsilon_p}$, while choosing **Recalibrated** results in $C_{\text{stpm}} = 0.18$.
- If **Turbulence model type** is **RANS-RSM**, specify the **Production of Reynolds stresses by porous matrix**. The default is **Isotropic** and the other option is **Streamwise**.

DISTANCE EQUATION

This section is available for the turbulence models **Algebraic yPlus** or **L-VEL**. Select how the **Reference length scale** l_{ref} (SI unit: m) is defined — **Automatic** (default) or **Manual**:

- For **Automatic** the wall distance is automatically evaluated to one tenth of the shortest side of the geometry bounding box. This is usually quite accurate but it can sometimes give a too high value if the geometry consists of several slim entities. In such cases, define the reference length scale manually.
- For **Manual** it defines a different value or expression for the length scale. The default is 1 m.

l_{ref} controls the result of the distance equation. Objects that are much smaller than l_{ref} are effectively be diminished while the distance to objects much larger than l_{ref} are accurately represented.

LOCATION IN USER INTERFACE

Context Menus

Brinkman Equations > Porous Medium

Ribbon

Physics tab with **Brinkman Equations** selected:

Domains > Porous Medium

Fluid

This node defines the material properties of the fluid of the [Porous Medium](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

FLUID PROPERTIES

Specify the **Density** ρ and the **Dynamic viscosity** μ (SI unit: Pa·s) of the fluid. The defaults use values **From material**, for **User defined** enter a value or expression.

The dynamic viscosity describes the relationship between the shear stresses and the shear rate in a fluid. Intuitively, water and air have a low viscosity, and substances often described as thick, such as oil, have a higher viscosity.

If the Polymer Flow Module license is available and for Darcian flow, also non-Newtonian models are available and the **Constitutive Relation** can be switched between **Specify dynamic viscosity** (default) or **Inelastic non-Newtonian**.

Constitutive Relation

Use the default option to either get the dynamic viscosity from a material, or to add a user defined expression. The default option lets you either use the dynamic viscosity from a material or define it with a custom expression. The **Inelastic non-Newtonian** option provides several models and defines the **Apparent shear rate**.

Inelastic Non-Newtonian

The following inelastic non-Newtonian Models are available for the **Fluid** feature under the **Porous Medium** feature:

- Power Law

- Carreau
- Carreau–Yasuda
- Cross
- Cross–Williamson
- Sisko

The setup of these models in porous media is similar to their definition under the [Fluid Properties](#) feature.

Apparent Shear Rate

To define the apparent shear rate $\dot{\gamma}_{app}$, either enter a user-defined expression for the correction factor (α) or choose the **Capillary bundle approach** from the list and provide a value for the Tortuosity factor (C).



- Read more about the theory of [Non-Newtonian Fluids in Porous Media](#) in the *Polymer Flow Module User's Guide*

LOCATION IN USER INTERFACE

A default **Fluid** node is automatically added to the [Porous Medium](#) node.

Context Menus

Brinkman Equations > Porous Medium > Fluid

Porous Matrix

This node defines the porosity and material properties of the solid matrix of the [Porous Medium](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

MATRIX PROPERTIES

The **Porosity** ε_p (a dimensionless number between 0 and 1) uses by default the value **From material**. For **User defined** the default value is 1.

Select a **Permeability model** to specify the capacity of the porous material to transmit flow. Depending on the **Flow model** selection in the parent **Porous Medium** feature node you can choose between the following options:

- **Permeability** (the default) to directly enter the permeability of the porous matrix or **Kozeny–Carman** to define the permeability from the porosity and mean **Particle diameter** for **Darcian** flow or
- **Forchheimer**, **Ergun**, or **User defined** for **Non-Darcian** flow.

Note that some of these options are only available for certain modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/.

The default **Permeability** κ (SI unit: m^2) uses the value **From material**, as defined by the **Porous material** list. For **User defined** select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value.

For **Kozeny–Carman**, enter the mean **Particle diameter** d_p (SI unit: m), the default value is 0.5 mm.

In case a **Non-Darcian** flow model is chosen in the parent node, **Forchheimer** is the default selection for the **Permeability model**. For **Forchheimer**, the default **Permeability** κ (SI unit: m^2) uses the value **From material**. Furthermore, the dimensionless **Forchheimer parameter** c_F can be defined. The default value is 0.55.

If **Ergun** is selected, enter the mean **Particle diameter** d_p (default value: 0.5 mm). The permeability κ is then calculated using Equation 2-4 in the *Porous Media Flow Module User's Guide*.

For the option **User defined** the default **Permeability** uses the option **From material**. Furthermore, the **Inertial resistance coefficient** β (SI unit: $1/\text{m}$) can be defined. Its default value is 0.0.



- [About Darcian and Non-Darcian Flow](#) in the *Porous Media Flow Module User's Guide*
- [Permeability Models](#) in the *Porous Media Flow Module User's Guide*

LOCATION IN USER INTERFACE

A default **Porous Matrix** node is automatically added to the **Porous Medium** node.

Mass Source

The **Mass Source** node adds a mass source (or mass sink) Q_m to the right-hand side of the continuity equation: [Equation 4-41](#). This term accounts for mass deposit and/or mass creation in porous domains. The physics interface assumes that the mass exchange occurs at zero velocity.

$$\frac{\partial}{\partial t}(\varepsilon_p \rho) + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (4-43)$$

DOMAIN SELECTION

Only Porous Matrix domains are available.

MASS SOURCE

Enter a value or expression for the **Source term** Q_{br} (SI unit: kg/(m³·s)). The default is 0 kg/(m³·s).

LOCATION IN USER INTERFACE*Ribbon*

Physics tab with **Brinkman Equations** selected:

Domains > Mass Source

Volume Force

Use the **Volume Force** node to specify the force $\mathbf{F}$ on the right-hand side of [Equation 4-42](#). It then acts on each fluid element in the specified domains.

$$\frac{\rho}{\varepsilon_p} \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \frac{\mathbf{u}}{\varepsilon_p} \right) = -\nabla p + \nabla \cdot \left[\frac{1}{\varepsilon_p} \left\{ \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} \right\} \right] - \left(\kappa^{-1} \mu + \frac{Q_m}{\varepsilon_p^2} \right) \mathbf{u} + \mathbf{F} \quad (4-44)$$

VOLUME FORCE

Enter the components of **Volume force $\mathbf{F}$** (SI unit: N/m^3).

LOCATION IN USER INTERFACE

Context Menus

Brinkman Equations > Volume Source

Ribbon

Physics tab with **Brinkman Equations** selected:

Domains > Volume Source

Initial Values

The **Initial Values** node adds initial values for the velocity field and the pressure that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver.

INITIAL VALUES

Enter initial values or expressions for the **Velocity field $\mathbf{u}$** (SI unit: m/s) and the **Pressure p** (SI unit: Pa). The default values are 0 m/s and 0 Pa , respectively.

LOCATION IN USER INTERFACE

Context Menus

Brinkman Equations > Initial Values

Ribbon

Physics tab with **Brinkman Equations** selected:

Domains > Initial Values

Fluid Properties

The **Fluid Properties** node adds the momentum and continuity equations to solve for free flow in nonporous domains. The node also provides an interface for defining the material properties of the fluid.

MODEL INPUTS

Fluid properties, such as density and viscosity, can be defined through user inputs, variables, or by selecting a material. For the latter option, additional inputs — for example, temperature or pressure — may be required to define these properties.

Temperature

By default, the single-phase flow interfaces are set to model isothermal flow. Hence, the **Temperature** is **User defined** and defaults to 293.15 K. If a Heat Transfer interface is included in the component, the temperature may alternatively be selected from this physics interface. All physics interfaces have their own tags (**Name**). For example, if a Heat Transfer in Fluids interface is included in the component, the **Temperature (ht)** option is available.

Absolute Pressure

This input appears when a material requires the absolute pressure as a model input. The absolute pressure is used to evaluate material properties, but it also relates to the value of the calculated pressure field. There are generally two ways to calculate the pressure when describing fluid flow: either to solve for the absolute pressure or for a pressure (often denoted gauge pressure) that relates to the absolute pressure through a reference pressure.

The choice of pressure variable depends on the system of equations being solved. For example, in a unidirectional incompressible flow problem, the pressure drop over the modeled domain is probably many orders of magnitude smaller than the atmospheric pressure, which, when included, may reduce the stability and convergence properties of the solver. In other cases, such as when the pressure is part of an expression for the gas volume or the diffusion coefficients, it may be more convenient to solve for the absolute pressure.

The default **Absolute pressure** p_A is $p + p_{\text{ref}}$ where p is the dependent pressure variable from the Navier–Stokes equations, and p_{ref} is from the user input defined at the physics interface level. When p_{ref} is nonzero, the physics interface solves for a gauge pressure. If the pressure field instead is an absolute pressure field, p_{ref} should be set to 0.

The **Absolute pressure** field can be edited by clicking **Make All Model Inputs Editable** () and entering the desired value in the input field.

FLUID PROPERTIES

If density variations with respect to pressure are to be included in the computations, the flow must be set to compressible.

The **Dynamic viscosity** μ describes the relationship between the shear rate and the shear stresses in a fluid. Intuitively, water and air have low viscosities, and substances often described as thick (such as oil) have higher viscosities.

LOCATION IN USER INTERFACE

Context Menus

Brinkman Equations > Fluid Properties

Ribbon

Physics tab with **Brinkman Equations** selected:

Domains > Fluid Properties

Gravity

The **Gravity** global feature is automatically added when **Include gravity** is selected at the interface level in the **Physical Model** settings. It defines the gravity forces from the **Acceleration of gravity** value.

This feature requires a specific license. For a detailed overview of the functionality available in each product visit www.comsol.com/products/specifications/.

ACCELERATION OF GRAVITY

The **Acceleration of gravity** (SI unit m/s, default value $-g_{\text{const}}\mathbf{e}_z$ in 2D axial symmetry and 3D and $-g_{\text{const}}\mathbf{e}_y$ in 2D) is used to define the gravity forces. It should be a global quantity.

When in addition to **Include gravity** also **Rotating frame** is selected (only possible in 3D and 2D axial symmetry), the **Project gravity vector on rotation axis** checkbox is available (by default selected).

LOCATION IN USER INTERFACE

Context Menus

Brinkman Equations > Gravity

Rotating Frame

The **Rotating Frame** global feature is automatically added when **Rotating frame** is selected at the interface level in the **Physical Model** settings. It defines the fictitious Coriolis, centrifugal, and Euler forces from the settings in the **Rotating Frame** section.

This feature requires a specific license. For a detailed overview of the functionality available in each product visit www.comsol.com/products/specifications/.

ROTATING FRAME

In 2D, the axis of rotation is an out-of-plane axis. In this case, select a **Rotation axis base point** $\mathbf{r}_{bp}$.

In 2D axial symmetry, the axis of rotation is the symmetry axis.

In 3D, select an **Axis of rotation** — **x-axis**, **y-axis**, **z-axis** (the default), or **User defined**. For **User defined** enter a **Rotation axis direction** $\mathbf{e}_{ax}$ and a **Rotation axis base point** $\mathbf{r}_{bp}$.

Select a **Rotation speed** — **Angular velocity**, **Revolutions per time**, or **User defined**.

- For **Angular velocity**, enter a value for the angular velocity magnitude Ω_f .
- For **Revolutions per time**, enter an RPM value.
- For **User defined**, enter the **Rotation angle** α in radians as function of time (the variable t).

In case **Angular velocity** or **Revolutions per time** is selected, also select a **Rotational direction** — **Counterclockwise** (the default) or **Clockwise**.

LOCATION IN USER INTERFACE

Context Menus

Brinkman Equations > Rotating Frame

Theory for the Free and Porous Media Flow, Brinkman Interface

The [Free and Porous Media Flow, Brinkman Interface](#) uses the Navier–Stokes equations to describe the flow in open regions, and the Brinkman equations to describe the flow in porous regions.

The same fields, $\mathbf{u}$ and p , are solved for in both the free flow domains and in the porous domains. This means that the pressure in the free fluid and the pressure in the pores is continuous over the interface between a free flow domain and a porous domain. It also means that continuity is enforced between the fluid velocity in the free flow and the Darcy velocity in the porous domain. This treatment is one of several possible models for the physics at the interface. Examples of other models can be found in [Ref. 1](#).

The continuity in $\mathbf{u}$ and p implies a stress discontinuity at the interface between a free-flow domain and a porous domain. The difference corresponds to the stress absorbed by the rigid porous matrix, which is a consequence implicit in the formulations of the Navier–Stokes and Brinkman equations.

Reference for the Free and Porous Media Flow, Brinkman Interface

1. M.L. Bars and M.G. Worster, “Interfacial Conditions Between a Pure Fluid and a Porous Medium: Implications for Binary Alloy Solidification,” *J. Fluid Mech.*, vol. 550, pp. 149–173, 2006.

The Free and Porous Media Flow, Brinkman Interface

The **Free and Porous Media Flow, Brinkman (fp)** interface () is found under the **Porous Media and Subsurface Flow** branch () when adding a physics interface. It is used to compute fluid velocity and pressure fields of single-phase flow where free flow is connected to porous media. The Free and Porous Media Flow, Brinkman interface is used over at least two different domains: a free channel and a porous medium. The physics interface is well suited for transitions between slow flow in porous media, governed by the Brinkman equations, and fast flow in channels described by the Navier–Stokes equations. Fluids with varying density can be included at Mach numbers below 0.3. Also the viscosity of a fluid can vary, for example, to describe non-Newtonian fluids. The physics interface can be used for stationary and time-dependent analyses.

When this physics interface is added, the following default nodes are also added in the **Model Builder** — **Fluid Properties**, **Wall**, and **Initial Values**. Then, from the **Physics** toolbar, add a **Porous Medium** node to be used on the domain selection corresponding to the porous media, or add other nodes that implement, for example, boundary conditions and volume forces. You can also right-click **Free and Porous Media Flow, Brinkman** to select physics features from the context menu.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `fp`.

PHYSICAL MODEL

Compressibility

By default the physics interface uses the **Incompressible flow** formulation of the Navier–Stokes and Brinkman equations to model constant density flow. If required, select

Weakly compressible flow from the **Compressibility** list, to account for small variations in the density, which are only dependent on the temperature (nonisothermal flow) or **Compressible flow (Ma<0.3)** for fully compressible flow. However, for the flow modeled with this physics interface, the Mach number must be below 0.3.

Swirl Flow

For 2D axisymmetric components, select the **Swirl flow** checkbox to include the swirl velocity component, that is the velocity component u_ϕ in the azimuthal direction. While u_ϕ can be nonzero, there can be no gradients in the ϕ direction. Note that this feature is only available for specific modules. Visit www.comsol.com/products/specifications/ for a detailed overview.

Neglect Inertial Term

Select the **Neglect inertial term (Stokes flow)** checkbox if the inertial forces are small compared to the viscous forces.

Enable porous media domains

The **Enable porous media domains** checkbox is selected by default to solve the Brinkman equations in porous domains. If it is cleared, the **Porous Medium** node is no longer available in this interface.

Include Gravity

Gravity is not included by default. Select the **Include gravity** checkbox to activate the acceleration of gravity. This automatically adds a global **Gravity** feature node to the interface model tree, and the buoyancy force is included in the Equations.

Also, when the **Include gravity** checkbox is selected, the **Use reduced pressure** option changes the pressure formulation from using the total pressure (default) to using the reduced pressure. This option is suitable for configurations where the density changes are very small; otherwise, the default formulation can be used. For more information, see [Gravity](#).

Rotating Frame

When the **Rotating Frame** checkbox is selected, a global **Rotating Frame** feature is shown in the interface model tree, and the fictitious Coriolis, centrifugal and Euler forces, stemming from the change to a non-inertial frame of reference, are included in the Navier–Stokes equations. Note that this feature is only available for specific modules. Visit www.comsol.com/products/specifications/ for a detailed overview.

For 2D components, the **Gravity** feature and the **Rotating Frame** feature are mutually exclusive, and thus can not be selected simultaneously. For 2D axisymmetric

components, the **Rotating Frame** checkbox and feature are only available when also the **Swirl flow** checkbox is selected.

When the **Rotating Frame** checkbox is selected, the **Use reduced pressure** option changes the pressure formulation from using the total pressure (default) to using the reduced pressure. This option is suitable for configurations where the density changes are very small; otherwise, the default formulation can be used. For more information, see [Rotating Frame](#)

Include Buoyancy-Induced Turbulence

When either the **Gravity** or the **Rotating Frame** checkbox is selected, and a turbulence model that solves for the turbulent kinetic energy, k , is used, the **Include buoyancy-turbulence** checkbox is available. When selected, the **Buoyancy contribution** is by default set to **Automatic from multiphysics**. Contributions are only obtained from multiphysics couplings that support buoyancy-induced turbulence, such as Nonisothermal Flow. If the **Buoyancy contribution** is switched to **Used defined**, a text field for the **Turbulent Schmidt number** appears.

When the **Incompressible flow** or the **Weakly compressible flow** option is selected, the **Hydrostatic pressure approximation** checkbox is available (selected by default). See also [Theory for Buoyancy-Induced Turbulence](#) in the *CFD Module User's Guide*.

Porous Treatment of No Slip Condition

Choose how the **No Slip** condition on interior boundaries should be treated. The options are **Standard no slip formulation** and **Porous slip** (default). The latter option provides a unified treatment when the porous matrix is fully resolved as well as when it is under resolved ensuring a smooth transition between regions with different resolutions; see No Slip under [Wall](#) in [The Laminar Flow and Creeping Flow Interfaces](#) section.

Reference Pressure Level

Enter a **Reference pressure level** p_{ref} (SI unit: Pa). The default value is 1[atm].

Reference Temperature

Enter a **Reference temperature** T_{ref} (SI unit: K). The default value is 293.15[K].

Reference Position

If **Include gravity** or **Rotating frame** is selected, a **Reference position** r_{ref} (SI unit: m) can be specified which is then used for the calculation of the hydrostatic pressure.

TURBULENCE

Turbulent flow can be simulated by changing the **Turbulence model type** to **RANS-EVM** (Reynolds-Averaged Navier–Stokes, eddy-viscosity model), **RANS-RSM** (Reynolds-Averaged Navier–Stokes, Reynolds stress model), or **Large Eddy Simulation** (which is only available in 3D). If turbulent flow is activated, you can choose the **Turbulence model** and between different options for **Wall treatment**. For a description of the turbulence models, wall treatment options, and turbulence model parameters, see [Theory for the Turbulent Flow Interfaces](#) in the *CFD Module User’s Guide*.

DEPENDENT VARIABLES

The following dependent variables (fields) are defined for this physics interface — the **Velocity field u** (SI unit: m/s) and its components, and the **Pressure p** (SI unit: Pa).

	• Domain, Boundary, Point, and Pair Nodes for the Free and Porous Media Flow, Brinkman Interface • Theory for the Free and Porous Media Flow, Brinkman Interface
	<i>Forchheimer Flow</i>: Application Library path Subsurface_Flow_Module/Fluid_Flow/forchheimer_flow

Domain, Boundary, Point, and Pair Nodes for the Free and Porous Media Flow, Brinkman Interface

[The Free and Porous Media Flow, Brinkman Interface](#) has the following domain, boundary, point, and pair nodes, listed in alphabetical order, available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

	In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.

- [Fluid Properties](#)
- [Gravity](#)
- [Initial Values](#)
- [Porous Medium](#)
- [Rotating Frame](#)
- [Mass Source](#)
- [Volume Force](#)
- [Wall](#)

Note that some features are only available with certain COMSOL products (see www.comsol.com/products/specifications/).

The following nodes (listed in alphabetical order) are described for the Laminar Flow interface:

- [No Viscous Stress](#)
- [Flow Continuity](#)
- [Inlet](#)
- [Line Mass Source](#)
- [Outlet](#)
- [Open Boundary](#)
- [Periodic Flow Condition](#)
- [Point Mass Source](#)
- [Pressure Point Constraint](#)
- [Symmetry](#)



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Fluid Properties

Use the **Fluid Properties** node to define the fluid material, density, and dynamic viscosity.

FLUID PROPERTIES

The default **Fluid material** uses the **Domain material** (the material defined for the domain). Select another material as needed.

The default **Density** ρ (SI unit: kg/m^3) uses values **From material** based on the **Fluid material** selection. For **User defined** enter another value or expression. The default is $0 \text{ kg}/\text{m}^3$.

The **Dynamic viscosity** μ (SI unit: Pa·s) uses values **From material** based on the **Fluid material** selection. For **User defined** enter another value or expression. The default is 0 Pa·s.

Porous Medium

The **Porous Medium** node adds the Brinkman equations (excluding any mass sources) to the interface. It has two default subfeatures — the **Fluid** subfeature where the fluid density ρ (SI unit: kg/m³) and dynamic viscosity μ (SI unit: Pa·s) are defined and the **Porous Matrix** subfeature where the permeability κ (SI unit: m²) and porosity ϵ_p (dimensionless) are specified.

FLOW MODEL

Define if the flow follows a linear or nonlinear pressure-velocity relationship.

- **Darcian flow** (default) defines a linear relationship characteristic for low Reynolds number laminar flows.
- **Non-Darcian flow** defines a nonlinear relationship due to inertial or turbulence effects.

POROUS MEDIUM TURBULENCE MODEL

Defines a model which complements the basic turbulence model and accounts for the influence of the porous matrix on turbulent flow in Porous Media.

- **Default** and **Nakayama–Kuwahara** specify **Coefficient of subgrid turbulence generation by porous matrix** C_{stpm} which by default is set to 0.212.
- **Pedras-de Lemos** asks to specify **Choice of the Pedras-de Lemos model coefficient**. When **User defined** is chosen, specify **Coefficient of subgrid turbulence generation by porous matrix** C_{stpm} with default value 0.212. Choosing **Original** results in $C_{\text{stpm}} = 0.28 \cdot \sqrt{\epsilon_p}$, while choosing **Recalibrated** results in $C_{\text{stpm}} = 0.18$.

Fluid

This node defines the material properties of the fluid of the **Porous Medium** parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

FLUID PROPERTIES

Specify the **Density** ρ and the **Dynamic viscosity** μ (SI unit: Pa·s) of the fluid. The defaults use values **From material**, for **User defined** enter a value or expression.

The dynamic viscosity describes the relationship between the shear stresses and the shear rate in a fluid. Intuitively, water and air have a low viscosity, and substances often described as thick, such as oil, have a higher viscosity.

If the Polymer Flow Module license is available and for Darcian flow, also non-Newtonian models are available and the **Constitutive Relation** can be switched between **Specify dynamic viscosity** (default) or **Inelastic non-Newtonian**.

Constitutive Relation

Use the default option to either get the dynamic viscosity from a material, or to add a user defined expression. The default option lets you either use the dynamic viscosity from a material or define it with a custom expression. The **Inelastic non-Newtonian** option provides several models and defines the **Apparent shear rate**.

Inelastic Non-Newtonian

The following inelastic non-Newtonian Models are available for the **Fluid** feature under the **Porous Medium** feature:

- Power Law
- Carreau
- Carreau–Yasuda
- Cross
- Cross–Williamson
- Sisko

The setup of these models in porous media is similar to their definition under the [Fluid Properties](#) feature.

Apparent Shear Rate

To define the apparent shear rate $\dot{\gamma}_{\text{app}}$, either enter a user-defined expression for the correction factor (α) or choose the **Capillary bundle approach** from the list and provide a value for the Tortuosity factor (C).



- Read more about the theory of [Non-Newtonian Fluids in Porous Media](#) in the *Polymer Flow Module User's Guide*

Porous Matrix

This node defines the porosity and material properties of the solid matrix of the [Porous Medium](#) parent node.

MODEL INPUT

This section contains fields and values that are inputs for expressions defining material properties. If such user-defined property groups are added, the model inputs appear here.

MATRIX PROPERTIES

The **Porosity** ε_p (a dimensionless number between 0 and 1) uses by default the value **From material**. For **User defined** the default value is 1 which is equivalent to free flow.

Select a **Permeability model** to specify the capacity of the porous material to transmit flow. Depending on the **Flow model** selection in the parent **Porous Medium** feature node you can choose between the following options:

- **Permeability** (the default) to directly enter the permeability of the porous matrix or **Kozeny–Carman** to define the permeability from the porosity and mean **Particle diameter** for **Darcian** flow or
- **Forchheimer**, **Ergun**, or **User defined** for **Non-Darcian** flow.

Note that some of these options are only available for certain modules. For a detailed overview of the functionality available in each product, visit www.comsol.com/products/specifications/.

The default **Permeability** κ (SI unit: m^2) uses the value **From material**, as defined by the **Porous material** list. For **User defined** select **Isotropic** to define a scalar value or **Diagonal** or **Symmetric** to define a tensor value.

For **Kozeny–Carman**, enter the mean **Particle diameter** d_p (SI unit: m), the default value is 0.5 mm.

In case a **Non-Darcian** flow model is chosen in the parent node, **Forchheimer** is the default selection for the **Permeability model**. For **Forchheimer**, the default **Permeability** κ (SI unit: m^2) uses the value **From material**. Furthermore, the dimensionless **Forchheimer parameter** c_F can be defined. The default value is 0.55.

If **Ergun** is selected, enter the mean **Particle diameter** d_p (default value: 0.5 mm). The permeability κ is then calculated using [Equation 2-4](#) in the *Porous Media Flow Module User's Guide*.

For the option **User defined** the default **Permeability** uses the option **From material**. Furthermore, the **Inertial resistance coefficient** β (SI unit: $1/\text{m}$) can be defined. Its default value is 0.0.



- [About Darcian and Non-Darcian Flow](#) in the *Porous Media Flow Module User's Guide*
- [Permeability Models](#) in the *Porous Media Flow Module User's Guide*

Mass Source

Enter a value or expression for an optional mass source (or sink) **Source term** Q_m (SI unit: $\text{kg}/(\text{m}^3 \cdot \text{s})$). This term accounts for mass deposit or mass creation within porous domains. The physics interface assumes that the mass exchange occurs at zero velocity.

Volume Force

The **Volume Force** node specifies the force $\mathbf{F}$ on the right-hand side of the Navier–Stokes or Brinkman equations, depending on whether a [Fluid Properties](#) or [Porous Medium](#) node is active on the domain. It then acts on each fluid element in the specified domains.

VOLUME FORCE

Enter the components of the **Volume force** $\mathbf{F}$ (SI unit: N/m^3).

Initial Values

The **Initial Values** node adds initial values for the velocity field and the pressure that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver.

INITIAL VALUES

Enter initial values or expressions for the **Velocity field $\mathbf{u}$** (SI unit: m/s) and for the **Pressure p** (SI unit: Pa). The default values are 0 m/s and 0 Pa, respectively.

Gravity

The **Gravity** global feature is automatically added when **Include gravity** is selected at the interface level in the **Physical Model** settings. It defines the gravity forces from the **Acceleration of gravity** value.

This feature requires a specific license. For a detailed overview of the functionality available in each product visit www.comsol.com/products/specifications/.

ACCELERATION OF GRAVITY

The **Acceleration of gravity** (SI unit m/s, default value $-g_{\text{const}}\mathbf{e}_z$ in 2D axial symmetry and 3D and $-g_{\text{const}}\mathbf{e}_y$ in 2D) is used to define the gravity forces. It should be a global quantity.

When in addition to **Include gravity** also **Rotating frame** is selected (only possible in 3D and 2D axial symmetry), the **Project gravity vector on rotation axis** checkbox is available (by default selected).

Rotating Frame

The **Rotating Frame** global feature is automatically added when **Rotating frame** is selected at the interface level in the **Physical Model** settings. It defines the fictitious Coriolis, centrifugal, and Euler forces from the settings in the **Rotating Frame** section.

This feature requires a specific license. For a detailed overview of the functionality available in each product visit www.comsol.com/products/specifications/.

ROTATING FRAME

In 2D, the axis of rotation is an out-of-plane axis. In this case, select a **Rotation axis base point $\mathbf{r}_{\text{bp}}$** .

In 2D axial symmetry, the axis of rotation is the symmetry axis.

In 3D, select an **Axis of rotation** — **x-axis**, **y-axis**, **z-axis** (the default), or **User defined**. For **User defined** enter a **Rotation axis direction** $\mathbf{e}_{\text{ax}}$ and a **Rotation axis base point** $\mathbf{r}_{\text{bp}}$.

Select a **Rotation speed** — **Angular velocity**, **Revolutions per time**, or **User defined**.

- For **Angular velocity**, enter a value for the angular velocity magnitude Ω_f .
- For **Revolutions per time**, enter an RPM value.
- For **User defined**, enter the **Rotation angle** α in radians as function of time (the variable t).

In case **Angular velocity** or **Revolutions per time** is selected, also select a **Rotational direction** — **Counterclockwise** (the default) or **Clockwise**.

Wall

The **Wall** node includes a set of boundary conditions describing fluid-flow conditions at stationary, moving, and leaking walls.

BOUNDARY CONDITION

Select a **Boundary condition** for the wall.

- [No Slip](#)¹
- [Slip](#)
- [Leaking Wall](#)

No Slip

No slip is the default boundary condition for a stationary solid wall for laminar flow (and SST, Low Re k- ϵ , Algebraic yPlus, L-VEL, and Spalart–Allmaras turbulence models). The condition prescribes $\mathbf{u} = 0$; that is, the fluid at the wall is not moving.

Slip

The **Slip** option prescribes a no-penetration condition, $\mathbf{u} \cdot \mathbf{n} = 0$. It is implicitly assumed that there are no viscous effects at the slip wall and hence, no boundary layer develops. From a modeling point of view, this can be a reasonable approximation if the main effect of the wall is to prevent fluid from leaving the domain.

Leaking Wall

This boundary condition can be used to simulate a wall where fluid is leaking into or leaving the domain with the velocity $\mathbf{u} = \mathbf{u}_1$ through a perforated wall. The components of the **Fluid velocity** $\mathbf{u}_1$ on the leaking wall should be specified.

CONSTRAINT SETTINGS

This section is displayed by clicking the **Show** button () and selecting **Advanced Physics Options**.



- [Theory for the Wall Boundary Condition](#)
- [The Moving Mesh Interface](#) in the *COMSOL Multiphysics Reference Manual*

Theory for the Free and Porous Media Flow, Darcy Interface

The **Free and Porous Media Flow, Darcy** interface uses the Navier–Stokes equations to describe the flow in the free-flow region, implemented by the coupled Laminar Flow interface, and Darcy’s law, implemented by the coupled Darcy’s Law interface, to describe the flow in the porous region.

In the free-flow region, the velocity $\mathbf{u}$ and pressure p are solved for, whereas in the porous region only the pressure p is solved for, and the velocity is calculated from the pressure by Darcy’s law.

The coupling between the free and porous medium flow, implemented by the Free and Porous Media Flow Coupling, ensures conservation of mass across the free-porous interface, which is enforced by the condition

$$\mathbf{n} \cdot (\rho_{\text{ns}} \mathbf{u}_{\text{ns}}) = \mathbf{n} \cdot (\rho_{\text{d}} \mathbf{u}_{\text{d}}) \quad (4-45)$$

Here ρ_{ns} and $\mathbf{u}_{\text{ns}}$ are the density and velocity on the free-flow side of the interface, ρ_{d} and $\mathbf{u}_{\text{d}}$ the density and velocity on the porous side, and $\mathbf{n}$ is a vector normal to the interface. In addition the coupling enforces the balance of the normal forces by:

$$p_{\text{ns}} - \mathbf{n} \cdot \mathbf{K} \mathbf{n} = p_{\text{d}} \quad (4-46)$$

where p_{ns} and p_{d} are the pressure on the free-flow side and porous side of the interface, respectively, and $\mathbf{K}$ is the viscous stress tensor (defined only on the free-flow side). The **Include pressure jump across free-porous boundary** option allows to specify an additional pressure jump p_{j} , in order to model, for example, a resistive layer or other pressure losses at the free-porous interface, such that the balance of the normal forces reads:

$$p_{\text{ns}} - \mathbf{n} \cdot \mathbf{K} \mathbf{n} - p_{\text{d}} = p_{\text{j}} \quad (4-47)$$

For the coupling condition for the tangential velocity component, there are several options. One of the most widely used conditions is the **Beavers–Joseph** interface condition (Ref. 1):

$$(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})(\mathbf{u}_{\text{ns}} - \mathbf{u}_{\text{d}}) = \frac{\sqrt{\text{tr}(\boldsymbol{\kappa})/n}}{\mu_{\text{ns}} \alpha_{BJ}} (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}) \mathbf{K} \mathbf{n} \quad (4-48)$$

where a_{BJ} is the Beavers and Joseph parameter, κ is the permeability tensor of the porous medium (defined on the porous side of the interface), μ_{ns} is the fluid's dynamic viscosity on the free-flow side of the interface, and n is the number of space dimensions.

When the velocity in the porous domain is much smaller than the velocity in the free flow, one can use a modification of the Beavers–Joseph condition which is given by the **Beavers–Joseph–Saffman** interface condition (Ref. 2):

$$(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{u}_{ns} = \frac{\sqrt{\text{tr}(\kappa)/n}}{\mu_{ns}\alpha_{BJ}}(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{K}\mathbf{n} \quad (4-49)$$

For cases when the permeability tensor is not isotropic, the **Use projected permeability** option allows to take into account only the permeability in the direction of the tangential velocity component. This is accomplished by replacing the $\sqrt{\text{tr}(\kappa)/n}$ term on the rights hand side of Equation 4-48 and Equation 4-49 by

$$\sqrt{\mathbf{t} \cdot \kappa \mathbf{t}} \quad (4-50)$$

with $\mathbf{t}$ a unit vector tangent to the interface for 2D and 2D-axisymmetric cases, and for 3D cases by

$$\sqrt{\mathbf{v} \cdot \kappa \mathbf{v}} \quad (4-51)$$

with $\mathbf{v}$ given by

$$\mathbf{v} = \frac{(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{K}\mathbf{n}}{\|(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{K}\mathbf{n}\|} \quad (4-52)$$

If one considers the limit of the Beavers–Joseph condition for large values of the Beavers and Joseph parameter a_{BJ} , one obtains the **Continuous tangential velocity** condition:

$$(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})(\mathbf{u}_{ns} - \mathbf{u}_d) = 0 \quad (4-53)$$

The same limit for the Beavers–Joseph–Saffman condition results in the **No slip** condition:

$$(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{u}_{ns} = 0 \quad (4-54)$$

The **Viscous slip** condition allows for a more general coupling condition. This interface condition is given by

$$(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})(\mathbf{u}_{\text{ns}} - \mathbf{u}_b) = \frac{L_s}{\mu_{\text{ns}}}(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{K}\mathbf{n} \quad (4-55)$$

Here L_s is a slip length, and $\mathbf{u}_b$ a user-defined interfacial velocity.



Please find related information in the *CFD Module User's Guide*:

- [The Darcy's Law Interface](#)
- [The Thin-Film and Porous Media Flow Interface](#)
- [The Laminar Flow Interface](#)
- [Theory for the Single-Phase Flow Interfaces](#)

References

1. G.S. Beavers and D.D. Joseph, "Boundary Conditions at a Naturally Permeable Wall," *J. Fluid Mech.*, vol. 30, 1967.
2. P.G. Saffman, "On the Boundary Condition at the Surface of a Porous Medium," *Stud. Appl. Math.*, vol. 50, no. 2, pp. 93–101, 1971.

The Free and Porous Media Flow, Darcy Interface

The **Free and Porous Media Flow, Darcy** interface () combines the **Darcy's Law** and **Laminar Flow** interfaces to compute fluid velocity and pressure fields where free flow is connected to porous media. The physics interface is suited for situations when both the free flow and the flow in the porous media are laminar.

When a predefined **Free and Porous Media Flow, Darcy** interface is added from the **Porous Media and Subsurface Flow** branch () of the **Model Wizard** or **Add Physics** window, the **Darcy's Law** and **Laminar Flow** interfaces are added to the Model Builder. In addition, the **Multiphysics** node is added, which automatically includes the **Free and Porous Media Flow Coupling** multiphysics coupling.

On the Constituent Physics Interfaces

The **Darcy's Law** interface is used to simulate fluid flow through interstices in a porous medium. It can be used to model low-velocity flows in media where the permeability and porosity are very small, and for which the pressure gradient is the major driving force and the flow is mostly influenced by the frictional resistance within the pores.

The **Laminar Flow** interface is used to compute the velocity and pressure for a fluid flow in the laminar flow regime. A flow will remain laminar as long as the Reynolds number is below a certain critical value. At higher Reynolds numbers, disturbances have a tendency to grow and cause transition to turbulence.

SETTINGS FOR PHYSICS INTERFACES AND COUPLING FEATURES

When physics interfaces are added using the predefined couplings, for example **Free and Porous Media Flow, Darcy**, specific settings are included with the physics interfaces and the coupling features.

However, if physics interfaces are added one at a time, followed by the coupling features, these modified settings are not automatically included.

Free and Porous Media Flow Coupling (Multiphysics Coupling)

The **Free and Porous Media Flow Coupling** multiphysics coupling node () bidirectionally links the **Laminar Flow** and **Darcy's Law** interfaces.

SETTINGS

The **Label** is the default multiphysics coupling feature name.

The **Name** is used primarily as a scope prefix for variables defined by the coupling node. Refer to such variables in expressions using the pattern <name>.<variable_name>. In order to distinguish between variables belonging to different coupling nodes or physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first multiphysics coupling feature in the model) is nsd1.

DOMAIN SELECTION

When nodes are added from the context menu, you can select **Manual** (the default) from the **Selection** list to choose specific domains to define the multiphase flow coupling, or select **All domains** as needed.

When the **Free and Porous Media Flow Coupling** is added as an effect of adding a Free and Porous Media Flow, Darcy multiphysics interface, the selection is the intersection of the boundaries of the participating physics interfaces.

Only boundaries that are active in the participating physics interfaces (which are chosen from the **Coupled Interfaces** selection list) can be selected.

COUPLED INTERFACES

This section defines the physics involved in the multiphysics coupling. The **Free flow** and **Porous media flow** lists include all applicable physics interfaces.

The default values depend on how the coupling node is created.

- If it is added from the **Physics** ribbon (Windows users), **Physics** contextual toolbar (Mac and Linux users), or context menu (all users), then the first physics interface of each type in the component is selected as the default.
- If it is added automatically when a multiphysics interface is selected in the **Model Wizard** or **Add Physics** window, then the two participating physics interfaces are selected.

You can also select **None** from either list to uncouple the Free and Porous Media Flow Coupling node from a physics interface. If the physics interface is removed from the **Model Builder**, for example **Darcy's Law** is deleted, then the **Porous media flow** list defaults to **None** as there is nothing to couple to.

COUPLING SETTINGS

This section defines the settings for the **Free flow boundary condition**.

- When **No slip** is selected, the tangential component of the fluid velocity at the free-porous interface on the free-flow side (computed in the **Free flow** interface) is set to zero.
- When **Continuous tangential velocity** is selected, the tangential velocity component of the flow on both sides of the free-porous interface is continuous. In addition, continuity of normal stresses and conservation of mass across the interface are ensured.
- When **Beavers–Joseph** is selected, the difference of the tangential velocity components of the flow on both sides of the free-porous interface is proportional to the shear stress at the free-porous interface on the free-flow side. For this option, enter a value or expression for the **Beavers and Joseph parameter** a_{BJ} (dimensionless) that determines the proportionality. In addition, select the checkbox **Use projected permeability** to use only the component of the permeability tensor in the direction of the difference of the tangential velocities on both sides of the interface, instead of the trace of the permeability tensor, in determining the proportionality.
- When **Beavers–Joseph–Saffman** is selected, the tangential velocity component of the flow at the free-porous interface on the free-flow side is proportional to the shear stress at the interface on the free-flow side. For this option also, enter a value or expression for the **Beavers and Joseph parameter** a_{BJ} (dimensionless) that determines the proportionality. In addition, select the checkbox **Use projected permeability** to use only the component of the permeability tensor in the direction of the difference of the tangential velocities on both sides of the interface, instead of the trace of the permeability tensor, in determining the proportionality.
- When **Viscous slip** is selected, the difference of the tangential velocity components of the flow on the free-flow side and a user-supplied interfacial velocity is proportional to the shear stress at the free-porous interface on the free-flow side. Enter values for the **Slip length** L_s (SI unit: m, the default is $1\text{e-}7$ m), and an **Interfacial velocity** $\mathbf{u}_b$ (SI unit: m/s, the default is 0 m/s).

In all the above cases, additionally continuity of normal stresses and conservation of mass across the interface are ensured.

Select the **Include pressure jump across free-porous boundary** checkbox to specify an additional pressure jump p_j (SI unit: Pa, the default is 0 Pa), in order to model, for example, a resistive layer or other pressure losses at the free-porous interface.

	Please find related information in the <i>CFD Module User's Guide</i>: • The Darcy's Law Interface • Theory for the Darcy's Law Interface • The Laminar Flow Interface • Theory for the Single-Phase Flow Interfaces • Theory for the Free and Porous Media Flow, Darcy Interface

Shallow Water Equations Interface

The physics interface described in this chapter is found under the **Fluid Flow** > **Shallow Water Equations** branch ().

In this chapter:

- [The Shallow Water Equations, Time Explicit Interface](#)
- [Theory for the Shallow Water Equations Interface.](#)

The Shallow Water Equations, Time Explicit Interface

In this section:

- [The Shallow Water Equations, Time Explicit Interface](#)
- [Domain, Boundary, Edge, Point, and Pair Nodes for the Shallow Water Equations Interface](#)



Theory for the Shallow Water Equations Interface

The Shallow Water Equations, Time Explicit Interface

The **Shallow Water Equations, Time Explicit (swe)** interface (), found under the **Shallow Water Equations** branch () when adding a physics interface, is used to solve the Shallow Water equations in a 1D or 2D geometry. These equations model the flow of a free surface in a fluid under the assumption that the horizontal scale is much greater than the vertical length scale. They are frequently used for modeling both oceanographic and atmospheric fluid flow. Models of such systems can be used to predict areas affected by pollution, coastal erosion, and polar ice-cap melting, provided the fluid layer is shallow enough.

Comprehensive modeling of such phenomena using physical descriptions such as the Navier–Stokes equations can often be problematic, due to the scale of the modeling domains as well as the resolution of free surfaces. The shallow water equations, of which there are a number of representations, provide an easier description of such phenomena.

A typical configuration for the flow of fluid in a shallow layer is shown in [Figure 5-1](#). This physics interface approximates free surface problems where the thickness h of the fluid layer, or water depth, is small compared to the lateral dimensions of the geometry. The lower boundary of the fluid is treated as a nonpenetrable wall and has a height $h_b(x, y)$ over a reference xy -plane placed at $z = 0$. The topography of the bottom is assumed constant in time. Using the shallow water equations, the water depth h and water flux $\mathbf{q}$ are computed in a reduced dimension

of the problem. Free surface problems that would require a 3D geometry when modeled with the Navier–Stokes equations can be modeled in 2D instead. 2D problems can also be reduced to 1D.

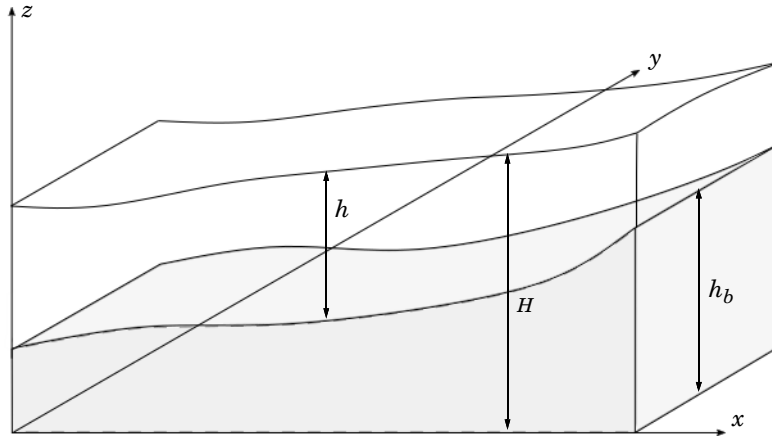


Figure 5-1: An example illustrating a typical configuration for shallow water equations. A water layer of thickness h flows over a nonflat bottom with topography represented by h_b . The total height of the free surface over the reference xy plane at $z = 0$ is represented by H . The gravity points downward ($-z$ direction) and gives rise to a hydrostatic pressure ranging from 0 atm at the free surface to a value of $g\rho h$ at the bottom.

Since the physics interface supports **Nodal discontinuous Lagrange** functions with **Constant Element order** for the discretization of the **Dependent variables**, the spatial derivatives are zero. When this physics interface is added, the following default nodes are also added in the **Model Builder** — **Domain Properties**, **Wall**, and **Initial Values**. Then, from the **Physics** toolbar, you can add other nodes that implement, for example, boundary conditions. You can also right-click **Shallow Water Equations, Time Explicit** to select physics features from the context menu.



The discontinuous Galerkin method, solved with the time-explicit method, is not compatible with the use of **Deformed Geometry** or **Moving Mesh**, except for problems where the deformed mesh stays constant during time stepping. See also [The Time-Explicit Algorithms](#) section.

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `swe`.

PHYSICAL MODEL

Select a value for **Acceleration of gravity** (SI unit: m/s^2). The default value is `gconst`. It should be a global quantity.

ADVANCED SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. Normally these settings do not need to be changed.

Select the **CFL number**. This CFL number will be used when defining the **Cell time scale expression** `swe.wtc` used in the **Time-Explicit Solver** if the **Time stepping** is set to **From expressions**. Note that the method will be unstable for CFL numbers larger than 1.



In the *COMSOL Multiphysics Reference Manual*:

- [References for the Solution Operation Nodes and Solvers](#)

DISCRETIZATION

The Shallow Water Equations, Time Explicit interface uses **Nodal discontinuous Lagrange** functions with **Constant Element order**.

DEPENDENT VARIABLES

The dependent variables (field variables) are the **Water depth** h (SI unit: m) and the **Water flux** $\mathbf{q}$ (SI unit: m^2/s). The names can be changed but the names of fields and dependent variables must be unique within a component.



- [Domain, Boundary, Edge, Point, and Pair Nodes for the Shallow Water Equations Interface](#)
- [Theory for the Shallow Water Equations Interface](#)



- *Dam Breaking on a Column, Shallow Water Equations*: Application Library path **Subsurface_Flow_Module/Fluid_Flow/dam_break_column_sw**
- *Tsunami Runup onto a Complex 3D Beach, Monai Valley*: Application Library path **Subsurface_Flow_Module/Fluid_Flow/monai_runup**

Domain, Boundary, Edge, Point, and Pair Nodes for the Shallow Water Equations Interface

For the Shallow Water Equations interface, the following domain, boundary, edge, point, and pair nodes (listed in alphabetical order) are described in this section:

- [Domain Properties](#)
- [Domain Source](#)
- [Initial Values](#)
- [Inlet](#)
- [Interior Wall](#)
- [Outlet](#)
- [Symmetry](#)
- [Wall](#)
- [Water Source](#)



In the *COMSOL Multiphysics Reference Manual* see [Table 2-4](#) for links to common sections and [Table 2-5](#) to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Domain Properties

Use the **Domain Properties** node to set the bottom topography and material properties.

BOTTOM TOPOGRAPHY

The **Bottom height** (SI unit: m) is defined as a user defined expression. It can depend on the geometry coordinates, but it is expected to be constant in time. The default is set to 0 m.

MATERIAL

The default **Density** ρ (SI unit: kg/m³) is taken **User defined**. Enter a different value or expression. The default is 1000 kg/m³. The density can also be set **From material**. In

that case, select the **Material** to use. By default, the material used is the **Domain material**. This corresponds to the material currently applied to the domain in question.

The value of the density is not used when solving but to compute the hydrostatic pressure and pressure force.

Domain Source

The **Domain Source** node specifies a momentum source on the right-hand side of the momentum equation

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot \left(\mathbf{u} \otimes \mathbf{q} + g \frac{h}{2} \mathbf{I} \right) = -gh \nabla h_B + \mathbf{F}$$

This source term can model different effects, such as bottom friction, the Coriolis force, or wind stresses.

Initial Values

The **Initial Values** node adds an initial value for the water depth and water flux that serve as an initial condition for a transient simulation. If more than one set of initial values is required, add additional **Initial Values** nodes from the **Physics** toolbar.

INITIAL VALUES

Select the set of variables to be defined in **Initial values** — **Primitive variables** (the default), or **Conservative variables**.

For the **Water depth**, select **Specify water depth** (the default) or **Specify total height**.

- For **Specify water depth** provide a value or an expression for the water depth. The default is 0 m.
- For **Specify total height** provide a value or an expression for the **Total height** of the water column H . The default is 0 m. When using this option, the initial water depth is calculated as

$$h = \max(0, H - h_b)$$

The variable used to define the water flux $\mathbf{q}$ will depend on the selection of the **Initial values** selection list:

- For **Primitive variables** provide a value or an expression for the components of the velocity $\mathbf{u}$. The default is 0 m/s. The initial water depth is then calculated as

$$\mathbf{q} = h\mathbf{u}$$

- For **Conservative variables** provide a value or an expression for the components of the water flux $\mathbf{q}$. The default is $0 \text{ m}^2/\text{s}$.

Inlet

Use the **Inlet** node to define boundaries where the fluid enters the domain. This condition uses the HLLC Riemann Solver ([Ref. 1](#)) to determine the incoming and outgoing characteristics and provide the appropriate flux.

FLOW PROPERTIES

Select the set of variables to be defined in **Input State** — **Primitive variables** (the default), or **Conservative variables**.

For the **Water depth**, select **User defined** (the default), **From domain values**, or **Specify total height**.

- For **User defined** provide a value or an expression for the water depth h_0 . The default is 0 m.
- When **From domain values** is selected, the value of the water depth used to define the external state is taken from the solution at the boundary. Note that at supercritical inlets both water depth and water flux should be specified.
- For **Specify total height** provide a value or an expression for the **Total height** of the water column H_0 . The default is 0 m. When using this option, the external water depth is calculated as

$$h_0 = \max(0, H_0 - h_b)$$

The variable used to define the water flux $\mathbf{q}_0$ will depend on the **Input state**:

- For **Primitive variables** the velocity is provided. If **User defined** is select, provide a value or an expression for the components of the velocity $\mathbf{u}_0$. The default is 0 m/s . The incoming water flux is then calculated as

$$\mathbf{q}_0 = h_0 \mathbf{u}_0$$

If **From domain values** is selected, the value of the velocity used to define the external state is taken from the solution at the boundary. Note that at supercritical inlets both water depth and water flux should be specified.

- For **Conservative variables** the water flux is required. If **User defined** is select, provide a value or an expression for the components of the water flux $\mathbf{q}_0$. The default value is $0 \text{ m}^2/\text{s}$. If **From domain values** is selected, the value of the water flux used to define the external state is taken from the solution at the boundary. Note that at supercritical inlets both water depth and water flux should be specified.

Outlet

The **Outlet** node defines boundaries where the fluid leaves the domain. This condition uses the HLLC Riemann Solver ([Ref. 1](#)) to determine the incoming and outgoing characteristics and provide the appropriate flux.

FLOW PROPERTIES

Select the set of variables to be defined in **Input State** — **Primitive variables** (the default), or **Conservative variables**.

For the **Water depth**, select **User defined** (the default), **From domain values**, or **Specify total height**.

- For **User defined** provide a value or an expression for the water depth h_0 . The default is 0 m.
- When **From domain values** is selected, the value of the water depth used to define the external state is taken from the solution at the boundary. Note that in supercritical outlets both water depth and water flux should be taken from domain values.
- For **Specify total height** provide a value or an expression for the **Total height** of the water column H_0 . The default is 0 m. When using this option, the external water depth is calculated as

$$h_0 = \max(0, H_0 - h_b)$$

The variable used to define the external water flux $\mathbf{q}_0$ will depend on the **Input state**:

- For **Primitive variables** the velocity is provided. If **User defined** is select, provide a value or an expression for the components of the velocity $\mathbf{u}_0$. The default is 0 m/s . The outgoing water flux is then calculated as

$$\mathbf{q}_0 = h_0 \mathbf{u}_0$$

If **From domain values** is selected, the value of the velocity used to define the external state is taken from the solution at the boundary. Note that in supercritical outlets both water depth and water flux should be taken from domain values.

- For **Conservative variables** the water flux is required. If **User defined** is select, provide a value or an expression for the components of the water flux $\mathbf{q}_0$. The default value is $0 \text{ m}^2/\text{s}$. If **From domain values** is selected, the value of the water flux used to define the external state is taken from the solution at the boundary. In supercritical outlets both water depth and water flux should be taken from domain values.

Wall

The **Wall** node includes a set of boundary conditions preventing fluid flow in the direction perpendicular to the boundary. A no-penetration condition, $\mathbf{u} \cdot \mathbf{n} = 0$, is enforced. Since viscous effects are neglected, no boundary layer develops.

The no-penetration condition is imposed weakly by setting the following flux:

$$\begin{aligned}\Gamma_h \cdot \mathbf{n} &= 0 \\ -\Gamma_q \cdot \mathbf{n} &= g \frac{h^2}{2}\end{aligned}$$



- [Theory for the Shallow Water Equations Interface](#)
-

Symmetry

The **Symmetry** boundary condition prescribes a no-penetration condition, $\mathbf{u} \cdot \mathbf{n} = 0$. This node sets the same flux condition as the **Wall** feature.

BOUNDARY SELECTION

For 2D axial symmetry, a boundary condition does not need to be defined for the symmetry axis at $r = 0$. The software automatically provides an **Axial Symmetry** node prescribing $\mathbf{u} \cdot \mathbf{n} = 0$ on the axial symmetry boundaries.

Interior Wall

The **Interior Wall** node prevents fluid flow in the direction perpendicular to the boundary. It can only be applied on interior boundaries.

It is similar to the [Wall](#) boundary condition available on exterior boundaries except that it applies on both sides of an interior boundary. It allows discontinuities of the dependent variables across the boundary. The Interior Wall condition can be used to avoid meshing thin structures by applying a no-penetration condition on interior curves and surfaces instead.

Water Source

The **Water Source** node specifies a height source in the continuity equation

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = Q$$

Theory for the Shallow Water Equations Interface

This section describes the theory for the Shallow Water Equations, Time Explicit interface:

- [The Shallow Water Equations](#)
- [The Numerical Method](#)
- [Wave Speed and Flow Regimes](#)
- [Forces on Boundaries](#)
- [References for the Shallow Water Equations Interfaces](#)

The Shallow Water Equations

Figure 5-2 shows a typical configuration for the flow of fluid in a shallow layer. The upper boundary is referred to as the free surface and the lower boundary of the water layer is referred to as the bottom. The bottom height, h_b , is measured with respect to a reference xy -plane placed at $z = 0$ and it is assumed to be constant in time. The total height of the free surface is denoted H . The height of the water layer is typically measured in relation to the bottom height and denoted h . In dry regions, $h = 0$ and $H = h_b$.

The equations of continuity and momentum of incompressible inviscid flow subjected to gravity in the vertical direction ($\mathbf{g} = [0, 0, -g]^T$) are

$$\rho \nabla \cdot \mathbf{u} = 0 \quad (5-1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (5-2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (5-3)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \quad (5-4)$$

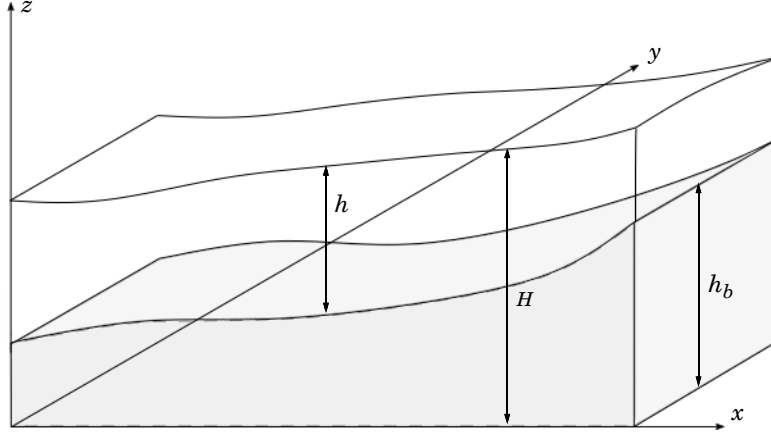


Figure 5-2: An example illustrating a typical configuration for shallow water equations. A water layer of thickness h flows over a nonflat bottom with topography represented by h_b . The total height of the free surface over the reference xy plane at $z = 0$ is represented by H . The gravity points downward ($-z$ direction) and gives rise to a hydrostatic pressure ranging from 0 atm at the free surface to a value of $p = \rho g h$ at the bottom.

Assuming that the water depth h is much smaller than the lateral dimensions of the geometry, the vertical accelerations can be neglected, $\frac{Dw}{Dt} = 0$, and Equation 5-4 reduces to the hydrostatic pressure relation

$$\frac{\partial p}{\partial z} = -\rho g \Rightarrow p = \rho g(h + h_b - z) \quad (5-5)$$

Both $h(x, y)$ and $h_b(x, y)$ are independent of z , implying that $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$. Equation 5-2 and Equation 5-3 then reduces to

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial}{\partial x}(h + h_b) \quad (5-6)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial}{\partial y}(h + h_b) \quad (5-7)$$

which is the momentum equation of the 2D shallow water equations in nonconservative form. Note that u and v represent the horizontal velocity components averaged over h .

The continuity equation is recovered from Equation 5-1 when assuming no penetration on both the free surface and bottom boundaries:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} u h + \frac{\partial}{\partial y} v h = 0 \quad (5-8)$$

See Chapter 2 in Ref. 1 for more details. Equation 5-6, Equation 5-7, and Equation 5-8 can be combined to express the shallow water equations in conservative form

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0$$

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x} \left(u q_x + g \frac{h^2}{2} \right) + \frac{\partial}{\partial y} (u q_y) = -g h \frac{\partial h_b}{\partial x} \quad (5-9)$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial x} (v q_x) + \frac{\partial}{\partial y} \left(v q_y + g \frac{h^2}{2} \right) = -g h \frac{\partial h_b}{\partial y} \quad (5-10)$$

q_x and q_y are the components of the water flux in the x and y directions, respectively. The vector $\mathbf{q} = [q_x, q_y, 0]^T$ can be obtained from the horizontally averaged velocity $\mathbf{u} = [u, v, 0]^T$ as

$$\mathbf{q} = \mathbf{u} h \quad (5-11)$$

Two different sets of variables are used in the context of the shallow water equations: the conservative variables h and $\mathbf{q}$, and the primitive variables h and $\mathbf{u}$.

The Numerical Method

The physics interface solves the shallow water equations using the Discontinuous Galerkin (DG) method in space together with explicit time stepping. The computations are restricted to transient problems. The basis functions are the nodal discontinuous Lagrange shape functions with constant order.

The numerical method used solves wave problems of the form

$$\mathbf{d}_a \frac{\partial \mathbf{w}}{\partial t} + \nabla \cdot \Gamma(\mathbf{w}) = \mathbf{f}$$

When solving the shallow water equations, $\mathbf{d}_a$ is an identity matrix, $\mathbf{w}$ is the vector of conservative variables, Γ represents the conservative fluxes, and $\mathbf{f}$ includes the source terms:

$$\mathbf{w} = \begin{bmatrix} h \\ \mathbf{q} \end{bmatrix}$$

$$\Gamma(\mathbf{w}) = \begin{bmatrix} \Gamma_h \\ \Gamma_q \end{bmatrix} = \begin{bmatrix} \mathbf{q}^T \\ \mathbf{q} \otimes \mathbf{u} + g \frac{h^2}{2} \mathbf{I} \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} 0 \\ -gh \nabla h_b \end{bmatrix}$$

Here, Γ_h and Γ_q represent the fluxes of the continuity and momentum equations, respectively.

Discontinuous Galerkin methods use shape functions that are continuous in the interior of each mesh element, but discontinuous across element boundaries. A numerical flux must be defined between element boundaries and the dependent variables are not required to be continuous on faces between adjacent mesh elements. The default numerical flux is the **Lax–Friedrichs** flux, which uses an average of the physical fluxes together with a penalty term.

After discretization in space, the system of equations is integrated in time. The standard procedure is to use an explicit **Runge–Kutta** method. The physics interface uses the third-order SSP (strong stability preserving) Runge–Kutta method per default.

The time step in explicit computations is limited by the CFL condition; see [Time Explicit Integrator](#) in the *COMSOL Multiphysics Reference Manual*. The maximum stable time step is proportional to the smallest mesh element size h and inversely proportional to the maximum wave speed in the domain. On unstructured meshes with local refinement regions only a few elements are small, yet these dictate the overall time step for the whole problem. In such cases, the use of [Local Time Stepping](#) is

advisable. COMSOL Multiphysics provides the **Adams–Bashforth 3 (local)** time stepping method.



Theory for the Wave Form PDE in the *COMSOL Multiphysics Reference Manual*.

The bottom topography h_b is entered as a user-defined expression. In order to ensure that its gradient can be evaluated and is constant on every element, it is first projected onto the variable `swe.hb_linear` using linear Lagrangian shape functions. For plotting purposes, the provided expression for h_b is also projected onto the variable `swe.hb_constant` using the same piecewise discontinuous shape functions as for h . These projections are done in a Stationary study step that must be computed before the Time Dependent study step is used to compute the shallow water equations. When starting a new model from the **Model Wizard**, the stationary study step is added per default when selecting a Time Dependent study.

Wave Speed and Flow Regimes

The fluxes of the momentum and continuity equations, $\Gamma(\mathbf{w})$, provide information about the speed of the waves. The eigenvalues of the Jacobian matrix of the fluxes evaluated at a direction provided by an arbitrary unit vector $\mathbf{n} = [n_x, n_y, 0]^T$ are

$$\lambda_1 = \mathbf{u} \cdot \mathbf{n} - c, \lambda_2 = \mathbf{u} \cdot \mathbf{n}, \lambda_3 = \mathbf{u} \cdot \mathbf{n} + c$$

where c is the wave celerity,

$$c = \sqrt{gh}$$

The eigenvalues of the Jacobian of the fluxes provide information about the velocity at which information is propagated. When $|\mathbf{u} \cdot \mathbf{n}| > c$, the eigenvalues have different sign and information is propagated both upward and downward. If $|\mathbf{u} \cdot \mathbf{n}| < c$, all eigenvalues have the same sign and all waves travel in the same direction.

The Froude number **Fr** is a dimensionless number typically used to define the flow regime by specifying the ratio of the inertial forces to gravity forces. In case of the Shallow Water Equations, it is defined as

$$\mathbf{Fr} = \frac{|\mathbf{u}|}{c}$$

The flow regime can be divided into subcritical flow, $\text{Fr} < 1$, and supercritical flow, $\text{Fr} > 1$. This classification is specially important when defining boundary conditions. In supercritical flows, all characteristics must be specified at inlets and extrapolated from the domain at outlets. In subcritical flows, the characteristics are partially specified from exterior values and partially extrapolated from interior values for both inlets and outlets.

Forces on Boundaries

The physics interface provides variables to compute the pressure force acting on boundaries. In the shallow water equations the pressure is assumed to be hydrostatic

$$p = \rho g(h + h_b - z)$$

The pressure force per unit length on a boundary can be obtained integrating along h

$$\mathbf{F}_p = \int_h^0 \rho g z \mathbf{n} dz = \rho g \frac{h^2}{2} \mathbf{n}$$

The components of the vector $\mathbf{F}_p$ are available as variables in the Shallow Water Equations, Time Explicit physics interface and can be accessed in 2D using `swe.Fpx` and `swe.Fpy`. In 1D only the `swe.Fpx` component is defined. Note that these variables represent a force per unit length; the total force on a boundary is obtained integrating over the whole boundary.

References for the Shallow Water Equations Interfaces

1. E.F. Toro, *Shock capturing methods for free-surface shallow flows*, John Wiley & Sons, 2001.

Chemical Species Transport Interfaces

This chapter describes the physics interfaces found under the **Chemical Species Transport** branch (). It summarizes the physics interfaces including the different fields, material properties, and boundary conditions associated with them, and suggests how and when to apply the conditions.

In this chapter:

- [The Transport of Diluted Species Interface](#)
- [The Transport of Diluted Species in Porous Media Interface](#)
- [The Transport of Diluted Species in Fractures Interface](#)
- [Theory for the Transport of Diluted Species Interface](#)

In addition, [The Reacting Flow in Porous Media Interface](#) is available under the **Chemical Species Transport** branch. It is described in the chapter [Multiphysics Interfaces and Couplings](#).

The Transport of Diluted Species Interface

The **Transport of Diluted Species (tds)** interface () , found under the **Chemical Species Transport** branch () , is used to calculate the concentration field of a dilute solute in a solvent. Transport and reactions of the species dissolved in a gas, liquid, or solid can be handled with this interface. The driving forces for transport can be diffusion by Fick's law, convection when coupled to a flow field, and migration, when coupled to an electric field.

The interface supports simulation of transport by convection and diffusion in 1D, 2D, and 3D as well as for axisymmetric components in 1D and 2D. The dependent variable is the molar concentration, c . Modeling multiple species transport is possible, whereby the physics interface solves for the molar concentration, c_i , of each species i .

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `tds`.

DOMAIN SELECTION

If any part of the model geometry should not partake in the mass transfer model, remove that part from the selection list.

OUT-OF-PLANE THICKNESS

For 2D components, the **Thickness** field (default value: 1 m) defines a parameter for the thickness of the geometry perpendicular to the two-dimensional cross-section. Both constant and varying thicknesses are supported. The value of this parameter is used, among other things, to automatically calculate molar flow rates from the total molar flux.

CROSS-SECTIONAL AREA

For 1D components, enter a **Cross-sectional area** A_c (SI unit: m^2) to define a parameter for the area of the geometry perpendicular to the 1D component. Both constant and varying areas are supported. The value of this parameter is used, among other things, to automatically calculate molar flow rates from the total molar flux. The default is 1 m^2 .

TRANSPORT MECHANISMS

Mass transport due to diffusion is always included. Use the checkboxes available under **Additional transport mechanisms** to control other transport mechanisms.

Note: Some of the additional transport mechanisms listed below are only available in certain products. For details see www.comsol.com/products/specifications/.

- By default, the **Convection** checkbox is selected. Clear the checkbox to disable convective transport.
- Select the **Migration in electric field** checkbox to activate transport of ionic species in an electric field. See further the theory section [Adding Transport Through Migration](#).



Transport of ionic species in an electric field is available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

Mass Transport in Porous Media

The **Mass transport in porous media** checkbox activates functionality specific to species transport in porous media:

- [Porous Medium](#)
- [Unsaturated Porous Medium](#)
- [Porous Electrode Coupling](#)
- [Volatilization](#)
- [Species Source](#)



Note: Mass transport in porous media is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

CONSISTENT STABILIZATION

To display this section, click the **Show** button () and select **Stabilization**.

- When the **Crosswind diffusion** checkbox is selected, a weak term that reduces spurious oscillations is added to the transport equation. The resulting equation system is always nonlinear. There are two options for the **Crosswind diffusion type**:
 - **Do Carmo and Galeão** — the default option. This type of crosswind diffusion reduces undershoots and overshoots to a minimum but can in rare cases give equation systems that are difficult to fully converge.
 - **Codina** — this option is less diffusive compared to the Do Carmo and Galeão option but can result in more undershoots and overshoots. It is also less effective for anisotropic meshes. The Codina option activates a text field for the **Lower gradient limit** g_{lim} . Its defaults to $0.1 [\text{mol}/\text{m}^3] / \text{tds.hElem}$, where tds.hElem is the local element size.
- For both consistent stabilization methods, select an **Equation residual**. **Approximate residual** is the default and means that derivatives of the diffusion tensor components are neglected. This setting is usually accurate enough and is computationally faster. If required, select **Full residual** instead.

INCONSISTENT STABILIZATION

To display this section, click the **Show** button () and select **Stabilization**. By default, the **Isotropic diffusion** checkbox is not selected, because this type of stabilization adds artificial diffusion and affects the accuracy of the original problem. However, this option can be used to get a good initial guess for under resolved problems.

ADVANCED SETTINGS

To display this section, click the **Show** button () and select **Advanced Physics Options**. Normally these settings do not need to be changed. Select a **Material Balance Form** — **Nonconservative** (the default) or **Conservative**. The conservative formulation should be used for compressible flow. See [Material Balance Formulation](#) for more information.

DISCRETIZATION

To display all settings available in this section, click the **Show** button () and select **Advanced Physics Options**.

The **Compute boundary fluxes** checkbox is activated by default so that COMSOL Multiphysics computes predefined accurate boundary flux variables. When this option is selected, the solver computes variables storing accurate boundary fluxes from each boundary into the adjacent domain.

If the checkbox is cleared, the COMSOL Multiphysics software instead computes the flux variables from the dependent variables using extrapolation, which is less accurate in postprocessing results but does not create extra dependent variables on the boundaries for the fluxes.

The flux variables affected in the interface are:

- ndflux_c (where c is the dependent variable for the concentration). This is the normal diffusive flux and corresponds to the boundary flux when diffusion is the only contribution to the flux term.
- ntflux_c (where c is the dependent variable for the concentration). This is the normal total flux and corresponds to the boundary flux plus additional transport terms, for example, the convective flux when you use the nonconservative form.

Also the **Apply smoothing to boundary fluxes** checkbox is available if the previous checkbox is selected. The smoothing can provide a more well-behaved flux value close to singularities.

For details about the boundary fluxes settings, see [Computing Accurate Fluxes](#) in the *COMSOL Multiphysics Reference Manual*.

The **Value type when using splitting of complex variables** setting should in most pure mass transfer problems be set to **Real**, which is the default. It makes sure that the dependent variable does not get affected by small imaginary contributions, which can occur, for example, when combining a Time Dependent or Stationary study with a frequency-domain study. For more information, see [Splitting Complex-Valued Variables](#) in the *COMSOL Multiphysics Reference Manual*.

DEPENDENT VARIABLES

The dependent variable name is the **Concentration c** by default. The names must be unique with respect to all other dependent variables in the component.

Add or remove species variables in the model and also change the names of the dependent variables that represent the species concentrations.

Enter the **Number of species**. Use the **Add concentration** () and **Remove concentration** () buttons as needed.

FURTHER READING

	• Theory for the Transport of Diluted Species Interface • Numerical Stabilization in the <i>COMSOL Multiphysics Reference Manual</i>. • In the <i>COMSOL Multiphysics Reference Manual</i>, see Table 2-4 for links to common sections and Table 2-5 for common feature nodes. You can also search for information: press F1 to open the Help window or Ctrl+F1 to open the Documentation window.
	• <i>Effective Diffusivity in Porous Materials</i>: Application Library path COMSOL_Multiphysics/Diffusion/effective_diffusivity • <i>Micromixer</i>: Application Library path COMSOL_Multiphysics/Fluid_Dynamics/micromixer

The Transport of Diluted Species in Porous Media Interface

This interface () , found under the **Chemical Species Transport** branch (), is used to calculate the species concentration and transport in free and porous media. The interface is the same as the *Transport of Diluted Species* interface but it uses other defaults: The [Mass Transport in Porous Media](#) property is selected, and a [Porous Medium](#) node is added by default. The interface includes reaction rate expressions and solute sources for modeling of species transport and reaction in porous media.

This interface is dedicated to modeling transport in porous media, including immobile and mobile phases, where the chemical species may be subjected to diffusion, convection, migration, dispersion, adsorption, and volatilization in porous media. It supports cases where either the solid phase substrate is exclusively immobile, or when a gas-filling medium is also assumed to be immobile.

It applies to one or more diluted species or solutes that move primarily within a fluid that fills (saturated) or partially fills (unsaturated) the voids in a solid porous medium. The pore space not filled with fluid contains an immobile gas phase. Models including a combination of porous media types can be studied.

The main feature nodes are the **Porous Medium** and **Unsaturated Porous Medium** nodes, which add the equations for the species concentrations and provide an interface for defining the properties of the porous media, as well as additional properties governing

adsorption, volatilization, dispersion and diffusion, migration, and the velocity field to model convection.

The physics interface can be used for stationary and time-dependent analysis.

When this physics interface is added, these default nodes are also added to the **Model Builder** — **Porous Medium, No Flux** (the default boundary condition), and **Initial Values**. Then, from the **Physics** toolbar, add other nodes that implement, for example, boundary conditions, reaction rate expressions, and species sources. You can also right-click **Transport of Diluted Species in Porous Media** to select physics features from the context menu.

SETTINGS

The rest of the settings are the same as [The Transport of Diluted Species Interface](#).

FURTHER READING

	• Mass Balance Equation for Transport of Diluted Species in Porous Media • Domain, Boundary, and Pair Nodes for the Transport of Diluted Species Interface
	• <i>Variably Saturated Flow and Transport — Sorbing Solute:</i> Application Library path Subsurface_Flow_Module/Solute_Transport/sorbing_solute Web link: www.comsol.com/model/variably-saturated-flow-and-transport-sorbing-solute-490

Domain, Boundary, and Pair Nodes for the Transport of Diluted Species Interface

[The Transport of Diluted Species Interface](#) has the following domain, boundary, point, and pair nodes, listed in alphabetical order, available from the **Physics** ribbon toolbar

(Windows users), **Physics** context menu (Mac or Linux users), or by right-clicking to access the context menu (all users).



- To add a node, go to the **Physics** toolbar, no matter what operating system you are using.
- Contextual subnodes (attributes) are available by clicking the parent node in the Model Builder, and then selecting the subnode from the **Attributes** menu.

• Adsorption	• Out-of-Plane Flux
• Concentration	• Outflow
• Continuity on Interior Boundaries	• Partition Condition
• Dispersion	• Periodic Condition
• Electrode Surface Coupling	• Point Mass Source
• Equilibrium Reaction	• Porous Matrix
• Fast Irreversible Surface Reaction	• Porous Medium
• Fluid	• Porous Electrode Coupling
• Fluid (Porous Medium)	• Reaction Coefficients
• Flux	• Reactions
• Flux Discontinuity	• Solid
• Fracture	• Species Properties
• Gas	• Species Source
• Inflow	• Surface Reactions
• Initial Values	• Surface Equilibrium Reaction
• Line Mass Source	• Symmetry
• Liquid	• Thin Diffusion Barrier
• Mass-Based Concentrations	• Thin Impermeable Barrier
• No Flux	• Unsaturated Porous Medium
• Open Boundary	• Volatilization

Some features require certain add-on modules. For details see www.comsol.com/products/specifications/



For axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** node that is valid on boundaries representing the symmetry axis.



In the *COMSOL Multiphysics Reference Manual*, see [Table 2-4](#) for links to common sections and [Table 2-5](#) for common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Prescribing Conditions on Fluid–Solid Interfaces

Defining conditions on interfaces between different phases, typically a wall in contact with a fluid, it is worthwhile to be careful with how variables in the conditions are evaluated. COMSOL Multiphysics supports that variables are defined differently on adjacent domains (or any other geometry dimension). Evaluating a variable with differing definitions, the mean value is returned on a boundary separating the different domains. The density, pressure, or temperature are examples of variables that may differ in a fluid and a solid. In order to specify on which side a variable is evaluated, the [up and down](#) operators (described in the *COMSOL Multiphysics Reference Manual*) can be used.

Species Properties

Use this node to define parameters specific to each of the species. It is available when the **Migration in electric field** checkbox is selected in the [Transport Mechanisms](#) section of the interface.

The node will change its label depending on what inputs are available in the node. If only **Charge** is available, it will be labeled “Species charges”.

CHARGE

Enter the charge number z_c (dimensionless, but requires a plus or minus sign) for each species.

ACTIVITY

This section is available when the **Species Activity** combo box is set to **Debye–Hückel** or **User defined**.

If **Debye–Hückel** is selected, enter the ion size number, a_0 , for each species. If **User defined** is selected, enter the activity coefficient, f , for each species.

Fluid

The settings in this node are dependent on the checkboxes selected under **Transport Mechanisms** in the **Settings** window for the Transport of Diluted Species interface. It includes only the sections required by the activated transport mechanisms. It has all the equations defining transport of diluted species as well as inputs for the material properties.

MODEL INPUTS

The temperature model input is always available. Select the source of the **Temperature**. For **User defined**, enter a value or expression for the temperature in the input field. This input option is always available.

You can also select the temperature solved for by a Heat Transfer interface added to the model component. These physics interfaces are available for selection in the **Temperature** list.

CONVECTION

If transport by convection is active, the velocity field of the solvent needs to be specified. Select the source of the **Velocity field**. For **User defined**, enter values or expressions for the velocity components in the input fields. This input option is always available.

You can also select the velocity field solved for by a Fluid Flow interface added to the model component. These physics interfaces are available for selection in the **Velocity field** list.

DIFFUSION

Use this section to specify diffusion coefficients describing the diffusion of each species in the solvent fluid.

Use the **Source** list to select diffusion coefficients defined in a material or a Chemistry interface.

Select **Material** to use a diffusion coefficient in a material available in the model.

User-defined property groups including a diffusion coefficient output property are available for selection in the **Fluid diffusion coefficient** list.

Select **Chemistry** to use a diffusion coefficient from a Chemistry interface added to model component. Diffusion coefficients are automatically defined when **Calculate transport properties** is selected on the interface level of a Chemistry interface. All defined diffusion coefficients are available for selection in the **Fluid diffusion coefficient** list.

For **User defined**, enter a value or expression for the **Fluid diffusion coefficient** D_c of each species in the corresponding input field. This can be a scalar value for isotropic diffusion or a tensor describing anisotropic diffusion. Select the appropriate tensor type — **Isotropic**, **Diagonal**, **Symmetric**, or **Full** that describes the diffusion transport, and then enter the values in the corresponding element (one value for each species).

Note that multiple species, as well as Migration in Electric fields (described below) is only available for certain COMSOL Multiphysics add-on products. See details: www.comsol.com/products/specifications/.

MIGRATION IN ELECTRIC FIELD

This section is available when the **Migration in electric field** checkbox is selected. From the **Electric potential** list, select the source of the electric field.

- Enter a value or expression for the **Electric potential** V , which is **User defined**; this input option is always available.
- Select the electric potential solved by an AC/DC-based interface that has also been added to the model.
- Select the electric potential defined or solved by Electrochemistry interface that has been added to the component.

By default the **Mobility** is set to be calculated based on the species diffusivity and the temperature using the **Nernst–Einstein relation**. For **User defined**, and under **Mobility**, select the appropriate scalar or tensor type — **Isotropic**, **Diagonal**, **Symmetric**, or **Full** — and type in the value of expression of the mobility $u_{m,c}$.

Enter the **Charge number** z_c (dimensionless, but requires a plus or minus sign) for each species.

The temperature (if you are using mobilities based on the Nernst–Einstein relation) is taken from **Model Inputs** section.

Note that the migration in electric fields feature is only available in some COMSOL products. See details: www.comsol.com/products/specifications/.

EXAMPLE MODELS

	• <i>Separation Through Dialysis</i>: Application Library path Chemical_Reaction_Engineering_Module/Mixing_and_Separation/dialysis Web link: www.comsol.com/model/separation-through-dialysis-258
	• <i>Transport in an Electrokinetic Valve</i>: Application Library path Microfluidics_Module/Fluid_Flow/electrokinetic_valve Web link: www.comsol.com/model/transport-in-an-electrokinetic-valve-603

Solid

Use this node to describe diffusion in a solid phase domain. In this feature, the activities of the species are assumed to be ideal.

DIFFUSION

Specify the diffusion coefficients describing the diffusion of each species in the solid.

Enter a value or expression for the **Diffusion coefficient** D_c of each species in the corresponding input field. This can be a scalar value for isotropic diffusion or a tensor describing anisotropic diffusion. Select the appropriate tensor type — **Isotropic**, **Diagonal**, or **Symmetric** that describes the diffusion transport, and then enter the values in the corresponding element (one value for each species).

ADVANCED

This section is only available with the **Advanced Physics Options** selected and when the model contains a split between the material frame and the spatial frame. A split of these frames occurs when using a **Moving Mesh** node or a Structural Mechanics interface like **Solid Mechanics**. In such a model, the reference geometry (the material frame) differs from the deformed geometry (the spatial frame). The diffusive transport of species is solved for on the material frame, but the equation is adjusted for geometry deformations so that species conservation is achieved on the spatial frame. Use the **Diffusion model** to define the frame of reference for the diffusion tensor.

Select **Reference geometry based (Lagrangian)** to specify the diffusion tensor in the undeformed geometry, implying that the diffusion is defined in the material frame (indicated by coordinates X, Y, Z). This is the preferred setting for an anisotropic

diffusion in combination with solid body translation or rotation. Also in general when the diffusion tensor components have been measured in an undeformed geometry.

Select **Deformed geometry based (Eulerian)** to specify the diffusion tensor in the deformed geometry. This means that the diffusion is defined in the spatial frame (indicated by coordinates x, y, z). Use this setting when the diffusion tensor components have been measured in a deformed geometry.

The **Advanced Physics Options** is found in the **Show More Options** dialog, accessed by clicking the **Show More Options** button ().

Note that multiple species are only available for certain COMSOL Multiphysics add-on products. See details: www.comsol.com/products/specifications/.

Initial Values

The **Initial Values** node specifies the initial values for the concentration of each species. These serve as an initial guess for a stationary solver or as initial conditions for a transient simulation.

DOMAIN SELECTION

If there are several types of domains with different initial values defined, it might be necessary to remove some domains from the selection. These are then defined in an additional **Initial Values** node.

INITIAL VALUES

Enter a value or expression for the initial value of the **Concentration** or concentrations, c_i . This also serves as a starting guess for stationary problems.

Mass-Based Concentrations

Use the **Mass-Based Concentrations** node to add postprocessing variables for mass-based concentrations (SI unit: kg/m^3) and mass fractions (dimensionless) for all species.

MIXTURE PROPERTIES

The default **Solvent density** ρ_{solvent} is taken **From material**. For **User defined**, enter a value or expression manually. Define the **Molar mass** of each species, which is needed to calculate the mass-based concentration.

Reactions

Use the **Reactions** node to account for the consumption or production of species through chemical reactions. Define the rate expressions as required.

DOMAIN SELECTION

From the **Selection** list, choose the domains on which to define rate expression or expressions that govern the source term in the transport equations.

Several reaction nodes can be used to account for different reactions in different parts for the modeling geometry.

REACTION RATES

The **Chemistry** list shows all the Chemistry interfaces that are coupled to the current mass transport interface. The coupling conditions for the Chemistry interfaces are:

- Active
- **Define variables for porous pellets** (under the **Pellet Chemistry** section) is cleared
- **Species solved for** (under the **Species Matching** section) set to the current mass transport interface

The reaction rates can be taken from a coupled Chemistry interface automatically or by specifying a rate expression R_i (SI unit: $\text{mol}/(\text{m}^3 \cdot \text{s})$) for species i :

- With a **Chemistry** interface selected, the species mass sources will be set to the corresponding reaction rates in the **Chemistry** according to the **Species Matching** table. In the case of partial coupling, the mass sources for the uncoupled species will be set to 0.
- **User defined.** Enter a value or expression in the field. Note that if you have the *Chemistry* interface available, provided with the *Chemical Reaction Engineering Module*, the reaction rate expressions can be generated automatically and picked up from the lists. For an example, see the application *Fine Chemical Production in a Plate Reactor* as linked below.

REACTING VOLUME

This section is only available when the [Mass Transport in Porous Media](https://www.comsol.com/products/specifications/) property is available and selected. See www.comsol.com/products/specifications/ for more details on availability.

When specifying reaction rates for a species in porous media, the specified reaction rate may have the basis of the total volume, the pore volume, or in the case of using the

volume of a particular phase (applicable when modeling an unsaturated porous medium).

- For **Total volume**, the reaction expressions, in $\text{mol}/(\text{m}^3 \cdot \text{s})$, are specified per unit volume of the model domain (multiplied by unity).
- For **Pore volume**, the reaction expressions, in $\text{mol}/(\text{m}^3 \cdot \text{s})$, are specified per unit volume of total pore space. The reaction expressions will be multiplied by the domain porosity, ϵ_p . (ϵ_p equals unity for nonporous domains).
- For **Liquid phase**, the reaction expressions, in $\text{mol}/(\text{m}^3 \cdot \text{s})$, are specified per unit volume of liquid in the pore space. The expressions will be multiplied by the liquid volume fraction θ_l . (θ_l equals the porosity (ϵ_p) on Porous Medium domains).
- For **Gas phase**, the reaction expressions, in $\text{mol}/(\text{m}^3 \cdot \text{s})$, are multiplied by the gas volume fraction $\theta_g = \epsilon_p - \theta_l$. θ_g equals 0 for Porous Medium domains.

HEAT SOURCE

The heat source will be defined as reaction heat from whole Chemistry if the **Chemistry** list is set to a coupled chemistry interface, otherwise it is defined as the specified value (SI unit: W/m^3).

FURTHER READING

See the theory chapter on chemical species transport, starting with the section *Mass Balance Equation*.



- *Fine Chemical Production in a Plate Reactor*: Application Library path
Chemical_Reaction_Engineering_Module/Reactors_with_Mass_and_Heat_Transfer/plate_reactor
Web link:
www.comsol.com/model/fine-chemical-production-in-a-plate-reactor-8589

No Flux

This node is the default boundary condition on exterior boundaries. It should be used on boundaries across which there is no mass flux, typically solid walls where no surface reactions occur. The condition applied for each species corresponds to

$$-\mathbf{n} \cdot (-D \nabla c) = 0$$

where $\mathbf{n}$ denotes the outward pointing normal of the boundary. When the mass transport includes migration of ionic species, the no flux condition is:

$$-\mathbf{n} \cdot (-D\nabla c - zu_{\text{m}}Fc\nabla\phi) = 0$$

CONVECTION

By default, the feature prescribes a vanishing flux due to diffusion and migration in an electric field. This is the appropriate no flux condition when the relative convective velocity at the boundary is zero. When the fluid velocity at the boundary is not equal to that of the boundary, it is often convenient to prescribe the total flux including the convection. To do this select **Include** in the **Convection** section.

When including the convection, the no flux condition prescribed is

$$-\mathbf{n} \cdot (-D\nabla c + \mathbf{u}c) = 0$$

or

$$-\mathbf{n} \cdot (-D\nabla c - zu_{\text{m}}Fc\nabla\phi + \mathbf{u}c) = 0$$

when migration of ionic species is included.

Inflow

Use this node to specify all species concentrations at an inlet boundary.

If you want to specify the concentration of a subset of the partaking species, this can be done by using the [Concentration](#) node instead.

For the **Electroanalysis** interface, this node is available when you select the **Convection** checkbox on the physics interface **Settings** window.

CONCENTRATION

For the concentration of each species $c_{0,e}$ (SI unit: mol/m^3), enter a value or expression.

BOUNDARY CONDITION TYPE

The option **Concentration constraint** constrains the concentration values on the boundary by the use of pointwise constraints.

The other option, **Flux (Danckwerts)** can be used when the concentration at the boundary is not known, or when it varies in a nontrivial manner. This may, for example, be useful when reactions with high reaction rates occur in the vicinity of the inlet. In this case the concentration far upstream of the boundary is instead prescribed. The **Flux (Danckwerts)** condition prescribes the total flux defined by the upstream concentration and the fluid velocity at the boundary.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options**. You can find details about the different constraint settings in the section [Constraint Reaction Terms](#) in the *COMSOL Multiphysics Reference Manual*.

FURTHER READING

See the theory chapter in the section [Danckwerts Inflow Boundary Condition](#).

Outflow

Apply this condition at outlets boundaries where species should be transported out of the model domain by fluid motion or by an electric field (in the case of ions). It is assumed that convection and migration (in an electric field) are the dominating transport mechanisms across the boundary, and therefore that the diffusive transport can be ignored, that is:

$$\mathbf{n} \cdot (-D\nabla c) = 0$$

Note that the **Convection** or the **Migration in electric field** transport mechanisms needs to be included for this node to be available.

Concentration

This condition node adds a boundary condition for the species concentration. For example, a $c = c_0$ condition specifies the concentration of species c .

CONCENTRATION

Individually specify the concentration for each species. Select the checkbox for the **Species** to specify the concentration, and then enter a value or expression in the

corresponding field. To use another boundary condition for a specific species, click to clear the checkbox for the concentration of that species.

When defining conditions on boundaries between different phases, see the section [Prescribing Conditions on Fluid–Solid Interfaces](#).

CONSTRAINT SETTINGS

To display this section, click the **Show** button () and select **Advanced Physics Options**. You can find details about the different constraint settings in the section [Constraint Reaction Terms](#) in the *COMSOL Multiphysics Reference Manual*.

Flux

This node can be used to specify the species molar flux across a boundary. The flux can for example occur due to chemical reactions or a phase change at the boundary. The flux can also represent the transport to or from a surrounding environment currently not included model.

The prescribed flux of a species c is by default defined as

$$-\mathbf{n} \cdot (-D\nabla c) = J_0$$

where $\mathbf{n}$ denotes the outward pointing normal of the boundary. When the mass transport includes migration of ionic species, the flux is defined as

$$-\mathbf{n} \cdot (-D\nabla c - zu_m Fc\nabla\phi) = J_0$$

The flux prescribed, J_0 , can include any arbitrary user-specified expression. It can be constant or a function of a dependent variable or independent variable. Common examples are a flux dependent of the concentration, temperature, pressure, or the electric potential ϕ .

When defining conditions on boundaries between different phases, see the section [Prescribing Conditions on Fluid–Solid Interfaces](#).

CONVECTION

By default, the flux due to diffusion and migration in an electric field is prescribed. This is the appropriate flux condition when the relative velocity at the boundary is zero. When the fluid velocity is not equal to that of the boundary, it is often convenient to prescribe the total flux, including the convection. To do this select **Include** in the **Convection** section.

When including the convection, the prescribed flux is defined as

$$-\mathbf{n} \cdot (-D\nabla c + \mathbf{u}c) = J_0$$

or

$$-\mathbf{n} \cdot (-D\nabla c - zu_m Fc\nabla\phi + \mathbf{u}c) = J_0$$

when migration of ionic species is included.

INWARD FLUX

Select the Species checkbox for the species for which to specify the flux, and enter a value or expression for the inward flux in the corresponding field. Use a minus sign when specifying a flux directed out of the system. To use another boundary condition for a specific species, click to clear the checkbox for that species.

External convection

Set **Flux type** to **External convection** to prescribe a flux to or from an exterior domain (not modeled) assumed to include convection. The exterior can for example include a forced convection to control the temperature or to increase the mass transport. In this case the prescribed mass flux corresponds to

$$J_0 = k_c(c_b - c)$$

where k_c is a mass transfer coefficient and c_b is the bulk concentration, the typical concentration far into the surrounding exterior domain.

Symmetry

The **Symmetry** node can be used to represent boundaries where the species concentration is symmetric, that is, where there is no mass flux across the boundary.

This boundary condition is identical to that of the [No Flux](#) node.

Flux Discontinuity

This node represents a discontinuity in the mass flux across an interior boundary:

$$-\mathbf{n} \cdot [(\mathbf{J} + \mathbf{u}c)_u - (\mathbf{J} + \mathbf{u}c)_d] = N_0 \quad \mathbf{J} = -D\nabla c$$

where the value N_0 (SI unit: $\text{mol}/(\text{m}^2\cdot\text{s})$) specifies the jump in total flux at the boundary. This can be used to model a boundary source, for example a surface reaction, adsorption, or desorption.

FLUX DISCONTINUITY

In this section the jump in species flux (or surface source) is specified.

Select the Species checkbox for the species to specify and enter a value or expression for the material flux jump in the corresponding field. To use a different boundary condition for a specific species, click to clear the checkbox for the flux discontinuity of that species.

Partition Condition

The **Partition Condition** node can be used to prescribe the relation between the concentration of a solute species in two adjoining immiscible phases. It can for example be used on interior boundaries separating two liquid phases, a gas-liquid interface, or on a boundary separating a liquid phase and a solid or porous media.

Select **Partition coefficient** from the **Relation** list to prescribe a linear relationship between the concentration on either side of a boundary. For a species concentration c_i , the ratio between the concentration on the upside and on the downside of the boundary ($c_{i,u}$ and $c_{i,d}$ respectively) is then defined in terms of a partition coefficient K_i in the manner of

$$K_i = \frac{c_{i,u}}{c_{i,d}}$$

Use the associated input field to prescribe the partition coefficient for each species.

Select **User defined** from the **Relation** list to set up a nonlinear relation between the up and downside concentrations. For such relations $f(c_i)_u = f(c_i)_d$, where $f(c_i)_u$ is the expression enforced on the upside. Use the associated input fields to enter the upside and downside expressions for each species.

The up and downside of the selected boundary is indicated with a red arrow in the Graphics window. The arrow points from the downside into the upside. Select the **Reverse direction** checkbox to reverse the direction of the arrow on the selected

boundary, and update the definition of the up and downside concentrations accordingly.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

FURTHER READING

For an example of using a partition condition, see this application example:



Separation Through Dialysis: Application Library path
Chemical_Reaction_Engineering_Module/Mixing_and_Separation/dialysis

Periodic Condition

The **Periodic Condition** node can be used to define periodicity for the mass transport between two sets of boundaries. The node prescribes continuity in the concentration and the mass flux between the “source” and the “destination” side, respectively. Note that these names are arbitrary and does not influence the direction in which mass is transported. It is dictated by mass transfer equations in the adjacent domains.

BOUNDARY SELECTION

The node can be activated on more than two boundaries, in which case the feature tries to identify two separate surfaces that each consist of one or several connected boundaries.

DESTINATION SELECTION

For more complex geometries, it might be necessary to specify the destination selection manually. To do so, right-click the **Periodic Condition** node and choose **Manual Destination Selection**. You can then specify the boundaries that constitute the destination surfaces in the **Destination Selection** section.

CONSTRAINT SETTINGS

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. This section contains settings for specifying the type of constraint and whether to use a pointwise or weak constraint. See

[Constraint Settings](#) in the *COMSOL Multiphysics Reference Manual* for more information.

ORIENTATION OF SOURCE

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. For information about the **Orientation of Source** section, see [Orientation of Source and Destination](#) in the *COMSOL Multiphysics Reference Manual*.

ORIENTATION OF DESTINATION

This section appears if the setting for **Transform to intermediate map** in the **Orientation of Source** section is changed from the default value, **Automatic**, and **Advanced Physics Options** is selected in the **Show More Options** dialog. For information about the **Orientation of Destination** section, see [Orientation of Source and Destination](#) in the *COMSOL Multiphysics Reference Manual*.

MAPPING BETWEEN SOURCE AND DESTINATION

To display this section, click the **Show More Options** button () and select **Advanced Physics Options** in the **Show More Options** dialog. For information about the **Mapping Between Source and Destination** section, see [Mapping Between Source and Destination](#) in the *COMSOL Multiphysics Reference Manual*.

FURTHER READING

For an example of using a periodic condition, see this application example:



The KdV Equation and Solitons: Application Library path
COMSOL_Multiphysics/Equation_Based/kdv_equation

Line Mass Source

The **Line Mass Source** feature models mass flow originating from a tube or line region with an infinitely small radius.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

SELECTION

The **Line Mass Source** feature is available for all dimensions, but the applicable selection differs between the dimensions.

MODEL DIMENSION	APPLICABLE GEOMETRICAL ENTITY
2D	Points
2D Axisymmetry	Points not on the symmetry axis and the symmetry axis
3D	Edges

SPECIES SOURCE

Enter the source strength, $\dot{q}_{l,c}$, for each species (SI unit: mol/(m·s)). A positive value results in species injection from the line into the computational domain, and a negative value means that the species is removed from the computational domain.

Line sources located on a boundary affect the adjacent computational domains. This effect makes the physical strength of a line source located in a symmetry plane twice the given strength.

FURTHER READING

See the section [Mass Sources for Species Transport](#).

Point Mass Source

The **Point Mass Source** feature models mass flow originating from an infinitely small domain around a point.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

SPECIES SOURCE

Enter the source strength, $\dot{q}_{p,c}$, for each species (SI unit: mol/s). A positive value results in species injection from the point into the computational domain, and a negative value means that the species is removed from the computational domain.

Point sources located on a boundary or on an edge affect the adjacent computational domains. This has the effect, for example, that the physical strength of a point source located in a symmetry plane is twice the given strength.

FURTHER READING

See the section [Mass Sources for Species Transport](#).

Open Boundary

Use this node to set up mass transport across boundaries where both convective inflow and outflow can occur. On the parts of the boundary where fluid flows into the domain, an exterior species concentration is prescribed. On the remaining parts, where fluid flows out of the domain, a condition equivalent to the **Outflow** node is instead prescribed.

The direction of the flow across the boundary is typically calculated by a fluid flow interface and is provided as a model input to the Transport of Diluted Species interface.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

EXTERIOR CONCENTRATION

Enter a value or expression for the **Exterior concentration**.

Thin Diffusion Barrier

Use this boundary condition to model a thin layer through which mass is transported by diffusion only. The node is applicable on interior boundaries and can be used to avoid meshing thin structures.

THIN DIFFUSION BARRIER

Specify the **Layer thickness**, d_s , and input a **Diffusion coefficient**, $D_{s,c}$, for each of the species included.

Thin Impermeable Barrier

This feature models a thin mass transfer barrier. It is available on interior boundaries and introduces a discontinuity in the concentration across the boundary. On each side, a no-flux condition is prescribed for the mass transport implying that it acts as a barrier. The feature can be used to avoid meshing thin structures.

Solving a model involving coupled fluid flow and mass transfer, the Thin Impermeable Barrier feature can be combined with an Interior Wall feature in order to model a thin solid wall.

Equilibrium Reaction

Use this node to model a reaction where the kinetics is so fast that the equilibrium condition is fulfilled at all times. The node solves for an additional degree of freedom (the reaction rate R_{eq}) to fulfill the equilibrium condition at all times in all space coordinates.

If the **Apply equilibrium condition on inflow boundaries** checkbox is selected, the specified inflow concentration values in all active **Inflow** boundary nodes for the physics interface are modified to comply with the equilibrium condition.



- A necessary requirement for this feature to be available is that two or more species are solved for by the interface.
- This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

EQUILIBRIUM CONDITION

The **Chemistry** list shows two ways to define an equilibrium reaction.

- A coupled chemistry — The **Equilibrium reaction formula** is available when the **Chemistry** list is set to a coupled chemistry interface. All the equilibrium reaction formulas in the coupled chemistry will appear in the formula list. Select one of them, then all coefficients (stoichiometric coefficients, equilibrium constant) will be taken from the coupled reaction automatically. There would be no any source term contributions if the **Equilibrium reaction formula** list is set to **None**.
- **None** — If **None** is selected from the **Chemistry** list, an equilibrium reaction will be defined by settings from fields of **Equilibrium definition**, **Equilibrium constant**, and **Unit activity concentration**.

The default value for **Equilibrium definition** list is **Equilibrium constant** or select **User defined**. For either option, the **Apply equilibrium condition on inflow boundaries** checkbox is selected by default.

For **Equilibrium constant**, enter an **Equilibrium constant** K_{eq} (dimensionless). Also enter a value or expression for the **Unit activity concentration** C_{a0} (SI unit: mol/m³).

Selecting **Equilibrium constant** defines an equilibrium condition based on the stoichiometric coefficients, the species activities, and the law of mass action.

For **User defined**, enter an **Equilibrium expression** E_{eq} (dimensionless).

STOICHIOMETRIC COEFFICIENTS

This section is available when the **Chemistry** list is set to **None**.

Enter a value for the stoichiometric coefficient ν_c (dimensionless). The default is 0. Use negative values for reactants and positive values for products in the modeled reaction.

Species with a stoichiometric coefficient value of 0 are not affected by the **Equilibrium Reaction** node.

HEAT SOURCE

The heat source will be defined from reaction heat from this equilibrium reaction.

- An equilibrium reaction defined from a **Chemistry** (from the **Equilibrium reaction formula** list). The heat source is defined as the same as that of equilibrium reaction in Chemistry.
- **None** selected in the **Chemistry** list. Enter a value or expression for the heat source (SI unit: W/m³).

Surface Reactions

The **Surface Reactions** node can be used to account for the species boundary flux due to chemical reactions occurring on a surface (heterogeneous reactions). For a domain species participating in a surface reaction, the boundary flux corresponds to the reaction rate at the surface.

SURFACE REACTION RATE

Specify the surface reaction rate J_0 of each species resulting from the reactions. Note that if you have the Chemistry interface available, provided with the Chemical Reaction Engineering Module, the reaction rate expressions can be automatically generated and picked up from the lists.

When defining conditions on boundaries between different phases, see the section [Prescribing Conditions on Fluid–Solid Interfaces](#).

FURTHER READING

For an example of using the Surface Reactions node, see this application example:



Chemical Vapor Deposition of GaAs: Application Library path
Chemical_Reaction_Engineering_Module/Reactors_with_Mass_and_Heat_Transfer/gaas_cvd

Surface Equilibrium Reaction

Use this node to model an equilibrium reaction on a boundary (surface). The settings for this node are similar to [Equilibrium Reaction](#). Note that a necessary requirement for this feature to be available is that two or more species are solved for by the interface.

When defining conditions on boundaries between different phases, see the section [Prescribing Conditions on Fluid–Solid Interfaces](#).



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

Fast Irreversible Surface Reaction

This boundary node defines an irreversible reaction where the kinetics is so fast that the only factor limiting the reaction rate is the transport of a species to the reacting surface.

The node will set the **Rate limiting species concentration to zero** at the boundary, and balance the fluxes of the species participating in the reaction and the current densities according to the Stoichiometric Coefficients settings.

When defining conditions on boundaries between different phases, see the section [Prescribing Conditions on Fluid–Solid Interfaces](#).



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

Porous Electrode Coupling

Use this node to add a molar source in a domain that is coupled to one or multiple **Porous Electrode Reaction** nodes of an **Electrochemistry Interface**.

The molar source is calculated from the number of electrons, stoichiometric coefficients, and volumetric current densities of the coupled porous electrode reactions specified in the **Reaction Coefficients** subnodes.

In the **Transport of Concentrated Species** interface, the molar sources (or sinks) are multiplied by the species molar masses to obtain the corresponding mass sources.

Additional **Reaction Coefficients** subnodes are available from the context menu (right-click the parent node) as well as from the **Physics** toolbar, **Attributes** menu.

Note that if you are also modeling the momentum transport and expect a nonnegligible total mass source or sink, which is often the case in gas diffusion electrodes, you need to also add a corresponding mass source/sink in the **Fluid Flow** interface. (A **Reacting Flow** multiphysics node adds the associated source/sink term in a **Fluid Flow** interface automatically.)



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

Reaction Coefficients

Add this node to the **Electrode Surface Coupling** and **Porous Electrode Coupling** features to define molar fluxes and sources based on electrode current densities in an **Electrochemistry** interface.

The molar flux or source is proportional to the stoichiometric coefficients and the current density according to Faraday's law.

Current densities from **Electrode Reaction** (i_{loc} , SI unit: A/m²) or **Porous Electrode Reaction** nodes (i_v , SI unit: A/m³) of any **Electrochemistry** interface in the model are available for selection as the **Coupled reaction**, and user-defined expressions are also supported.

Enter the **Number of participating electrons** n_m (dimensionless) and the **Stoichiometric coefficient** v_c (dimensionless) as explained in the theory section linked below.

Use multiple subnodes to couple to multiple reactions.

Electrode Surface Coupling

Use this node to define a flux boundary condition based on current densities of one or multiple [Electrode Reaction](#) nodes in an Electrochemistry interface.

The flux is proportional to the current densities and the stoichiometric coefficients according to Faraday's law as defined by summation over the [Reaction Coefficients](#) subnodes.

Note that if you are also modeling the momentum transport and expect a nonnegligible total mass flux over the boundary, which is often the case for gas diffusion electrodes, you need to also add a corresponding Electrode Surface Coupling node in the Fluid Flow interface.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

Porous Medium

Use this node to model the concentration of diluted species transported by a solvent (mobile fluid phase) through interstices in a solid porous medium. In addition to transport due to convection and diffusion, the node contains functionality to include species evolution through adsorption and dispersion.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

Species transport through a porous medium is affected both by properties of the fluid phase, and as well as properties of the solid matrix. These properties are defined in the [Fluid \(Porous Medium\)](#) and [Porous Matrix](#) subnodes respectively. The Porous Medium node supports material properties using a [Porous Material](#) node (described in the *COMSOL Multiphysics Reference Manual*), where properties are defined per phase in a similar manner.

Add an [Adsorption](#) or a [Dispersion](#) subfeature to the **Porous Medium** to account for the corresponding transport mechanism.

Fluid (Porous Medium)

Use this node to specify the mass transfer in the mobile fluid solvent filling the pores of a porous medium. It is used as a subnode to [Porous Medium](#).

MODEL INPUTS

The temperature model input is always available. By default, the **Temperature** model input is set to **Common model input**, and the temperature is controlled from [Default Model Inputs](#) under **Global Definitions** or by a locally defined [Model Input](#). If a Heat Transfer interface is included in the component, it controls the temperature **Common model input**. Alternatively, the temperature field can be selected from another physics interface. All physics interfaces have their own tags (**Name**). For example, if a Heat Transfer in Fluids interface is included in the component, the **Temperature (ht)** option is available for T .

You can also select **User defined** from the **Temperature** model input in order to manually prescribe T .

CONVECTION

If transport by convection is active, the velocity field of the solvent needs to be specified. For **User defined**, enter values or expressions for the velocity components in the input fields. This input option is always available. You can also select the velocity field solved for by a Fluid Flow interface added to the model component. These physics interfaces are available for selection in the **Velocity field** list.

DIFFUSION

Use this section to specify diffusion coefficients describing the diffusion of each species in the solvent fluid.

Use the **Source** list to select to pick up diffusion coefficients defined in a material or a Chemistry interface.

Select **Material**, and a **Fluid material** to use a diffusion coefficient in a material available in the model. User-defined property groups including a diffusion coefficient output property are available for selection in the **Fluid diffusion coefficient** list.

Select **Chemistry** to use a diffusion coefficient from a Chemistry interface added to model component. Diffusion coefficients are automatically defined when **Calculate transport properties** is selected on the interface level of a Chemistry interface. All defined diffusion coefficients are available for selection in the **Fluid diffusion coefficient** list.

For **User defined**, enter a value or expression for the **Fluid diffusion coefficient** $D_{F,i}$ of each species in the corresponding input field.

In a porous medium the diffusivity is reduced due to the fact that the solid grains impede Brownian motion. Select an **Effective diffusivity model** to account for the reduced diffusivity. The available models are **Millington and Quirk model** (the default), **Bruggeman model**, **Tortuosity model**, or **No correction**. For **Tortuosity model**, enter a value for the tortuosity $\tau_{F,i}$ (dimensionless).

Note that multiple species, as well as Migration in Electric fields (described below) is only available for certain COMSOL Multiphysics add-on products. For details, see www.comsol.com/products/specifications/.

MIGRATION IN ELECTRIC FIELD

This section is available when the **Migration in electric field** checkbox is selected. From the **Electric potential** list, select the source of the electric field.

- For **User defined**, enter a value or expression for the **Electric potential** V . This input option is always available.
- Select the electric potential solved by an AC/DC-based interface that has added to the component.
- Select the electric potential defined or solved for by an Electrochemistry interface added to the component.

By default the **Mobility** is set to be calculated based on the species effective diffusivity and the temperature using the **Nernst–Einstein relation**. For **User defined**, select the appropriate scalar or tensor type — **Isotropic**, **Diagonal**, **Symmetric**, or **Full** — and type in the value or expression of the effective mobility $u_{me,i}$.

Enter the **Charge number** z_c for each species.

Porous Matrix

This node sets the porosity when modeling transport of diluted species in a **Porous Medium** or an **Unsaturated Porous Medium**. The porosity corresponds to the volume fraction occupied by the species containing fluid.

- By default the porosity of the solid matrix, ϵ_p , is taken **From material**. The **Porous Matrix** node supports the use of a **Porous Material** added to the Materials branch. In this case the porosity is defined in the manner of

$$\varepsilon_p = 1 - \sum_i \theta_{s,i} - \sum_i \theta_{\text{imf},i}$$

where $\theta_{s,i}$ and $\theta_{\text{imf},i}$ are the volume fractions given in **Solid** and **Immobile Fluids** subnodes under the **Porous Material** node.

- Select **From pellet bed densities** to compute the porosity using the (dry bulk) **Bed density** ρ_b and the (single phase) **Pellet density** ρ_{pe} . The porosity is then defined from

$$\varepsilon_p = 1 - \frac{\rho_b}{\rho_{pe}}$$

- For **User defined**, enter a value or expression for the porosity.

Dispersion

Local variations in the velocity as the fluid flows around solid particles lead to mechanical mixing, referred to as dispersion. Use this feature to account for dispersion in a [Porous Medium](#) or a [Unsaturated Porous Medium](#)

This subfeature is available when both the **Mass transfer in porous media mass transfer** checkbox and the **Convection** checkbox are selected in the **Settings** window for the physics interface.

Select the **Specify dispersion for each species individually** checkbox to specify the dispersion tensor D_D (SI unit: m^2/s) for each species separately. When not selected the same dispersion tensor D_D is used for all species.

Select an option from the **Dispersion tensor** list — **Dispersivity** or **User defined**.

Select **Dispersivity** to specify the dispersion in terms of dispersivities (SI unit: m). Select an option from the **Dispersivity model** list: **Isotropic** (the default) or **Transverse isotropic** based on the properties of the porous media. For isotropic porous media, specify the longitudinal and transverse dispersivities. For transverse isotropic porous media, specify the longitudinal, horizontal transverse, and vertical transverse dispersivities.

For **User defined**, specify the dispersion components in terms of constants or expressions. Select **Isotropic**, **Diagonal**, **Symmetric**, or **Full** to enable the appropriate tensor components.

Unsaturated Porous Medium

Use this node to model the concentration of diluted species transported by a solvent (mobile fluid phase) through interstices in a solid porous medium. In addition to transport due to convection and diffusion, the node contains functionality to include species evolution through adsorption and dispersion.

Use this node to model the concentration of diluted species transported by a liquid (mobile fluid phase) in a partially filled solid porous medium. The interstices of the porous medium contains the liquid carrier phase and gas pockets. Apart from convection and diffusion, the node contains functionality to include species evolution through adsorption, dispersion, and volatilization.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

The properties of each phase present are defined using the [Liquid](#), the [Gas](#), and the [Porous Matrix](#) subnodes respectively. The Unsaturated Porous Medium node supports material properties using a [Porous Material](#) node (described in the *COMSOL Multiphysics Reference Manual*), where properties are defined per phase in a similar manner.

Add an [Adsorption](#) or a [Dispersion](#) subfeature to the **Unsaturated Porous Medium** to account for the corresponding transport mechanism.

Liquid

Use this node to specify the mass transfer in the mobile liquid solvent present in the pores of the [Unsaturated Porous Medium](#).

MODEL INPUTS

The temperature model input is always available. By default, the **Temperature** model input is set to **Common model input**, and the temperature is controlled from [Default Model Inputs](#) under **Global Definitions** or by a locally defined [Model Input](#). If a Heat Transfer interface is included in the component, it controls the temperature **Common model input**. Alternatively, the temperature field can be selected from another physics interface. All physics interfaces have their own tags (**Name**). For example, if a Heat Transfer in Fluids interface is included in the component, the **Temperature (ht)** option is available for T .

You can also select **User defined** from the **Temperature** model input in order to manually prescribe T .

SATURATION

Select **Saturation** or **Liquid volume fraction** from the list.

For **Saturation**, enter a value for s (dimensionless) between 0 and 1. The liquid volume fraction is then computed from the saturation and porosity as $\theta_l = \varepsilon_p s$.

For **Liquid volume fraction**, enter a value for θ_l (dimensionless) between 0 and the value of the porosity.

Select a **Liquid fraction time change**: **Liquid fraction constant in time** (the default), **Time change in liquid fraction**, or **Time change in pressure head**.

- For **Time change in fluid fraction**, enter $d\theta/dt$ (SI unit: 1/s).
- For **Time change in pressure head**, enter dH_p/dt (SI unit: m/s) and a **Specific moisture capacity** C_m (SI unit: 1/m). If a Darcy's Law interface is included in the component, the time change in pressure head solved for can be selected.

CONVECTION

If transport by convection is active, the velocity field of the solvent needs to be specified. For **User defined**, enter values or expressions for the velocity components in the input fields. This input option is always available. You can also select the velocity field solved for by a Fluid Flow interface added to the model component. These physics interfaces are available for selection in the **Velocity field** list.

DIFFUSION

Use this section to specify diffusion coefficients describing the diffusion of each species in the solvent liquid.

Use the **Source** list to select to pick up diffusion coefficients defined in a material or a Chemistry interface.

Select **Material**, and a **Liquid material** to use a diffusion coefficient in a material available in the model. User-defined property groups including a diffusion coefficient output property are available for selection in the **Liquid diffusion coefficient** list.

Select **Chemistry** to use a diffusion coefficient from a Chemistry interface added to model component. Diffusion coefficients are automatically defined when **Calculate transport properties** is selected on the interface level of a Chemistry interface. All

defined diffusion coefficients are available for selection in the **Liquid diffusion coefficient** list.

For **User defined**, enter a value or expression for the **Liquid diffusion coefficient** $D_{L,i}$ of each species in the corresponding input field.

In a porous medium the diffusivity is reduced due to the fact that the solid grains impede Brownian motion. Select an **Effective diffusivity model, liquid** to account for the reduced diffusivity in the liquid. The available models are **Millington and Quirk model** (the default), **Bruggeman model**, **Tortuosity model**, or **No correction**. For **Tortuosity model**, enter a value for the tortuosity $\tau_{L,i}$ (dimensionless).

Note that multiple species, as well as Migration in Electric fields (described below) is only available for certain COMSOL Multiphysics add-on products. For details see: www.comsol.com/products/specifications/.

MIGRATION IN ELECTRIC FIELD

This section is available when the **Migration in electric field** checkbox is selected. From the **Electric potential** list, select the source of the electric field.

- For **User defined**, enter a value or expression for the **Electric potential** V . This input option is always available.
- Select the electric potential solved by an AC/DC-based interface that has added to the component.
- Select the electric potential defined or solved for by an Electrochemistry interface added to the component.

By default the **Mobility** is set to be calculated based on the species effective diffusivity and the temperature using the **Nernst–Einstein relation**. For **User defined**, select the appropriate scalar or tensor type — **Isotropic**, **Diagonal**, **Symmetric**, or **Full** — and type in the value or expression of the effective mobility $u_{me,i}$.

Enter the **Charge number** z_c for each species.

Gas

Use this node to specify the mass transfer in the gas phase present in the pores of the **Unsaturated Porous Medium**.

DIFFUSION

The settings for the diffusion of each species in the gas phase are identical to those in the [Liquid](#) subnode.

VOLATILIZATION

Enter a value for the volatilization isotherm $k_{G,c}$ (dimensionless) for each species.

Adsorption

Use this node to model adsorption of the (fluid phase) solute species onto the surface of the porous matrix. It is available as a subnode to the [Porous Medium](#) and the [Unsaturated Porous Medium](#) nodes.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

MATRIX PROPERTIES

The density of the porous media is needed when modeling adsorption to the surface of the porous matrix. Choose to input either the **Dry bulk density** ρ , or the **Solid phase density** ρ_s . The former is the density of the porous matrix including empty pores, while the latter corresponds to the density of the pure solid phase. The density can be defined from the domain material by selecting **From material**, or from a user defined expression. When a **Porous Material** is used on the selection, the density will be requested from a **Solid** subfeature. Several **Solid** features can be used to model a homogeneous mixture of several solid components.

ADSORPTION

Select an **Adsorption isotherm** — **Langmuir**, **Freundlich**, **Toth**, **BET**, or **User defined** to specify how to compute c_p , the amount of species sorbed to the solid phase (moles per unit dry weight of the solid):

- For **Langmuir**:

$$c_P = c_{P\max} \frac{K_L c}{1 + K_L c} \quad , \quad K_P = \frac{\partial c_P}{\partial c} = \frac{K_L c_{P\max}}{(1 + K_L c)^2}$$

Enter a **Langmuir constant** $k_{L,c}$ (SI unit: m³/mol) and an **Adsorption maximum** $c_{p,\max,c}$ (SI unit: mol/kg).

- For **Freundlich**:

$$c_P = K_F \left(\frac{c}{c_{\text{ref}}} \right)^N \quad , \quad K_P = \frac{\partial c_P}{\partial c} = N \frac{c_P}{c}$$

Enter a **Freundlich constant** $k_{F,c}$ (SI unit: mol/kg), a **Freundlich exponent** $N_{F,c}$ (dimensionless), and a **Reference concentration** $c_{\text{ref},c}$ (SI unit: mol/m³).

- For **Toth**:

$$c_P = c_{P\max} \frac{b_T c}{(1 + (b_T c)^{N_T})^{1/N_T}}$$

and

$$K_P = \frac{\partial c_P}{\partial c} = c_{P\max} b_T (1 + (b_T c)^{N_T})^{-\left(1 + \frac{1}{N_T}\right)}$$

Enter a **Toth constant** $b_{T,c}$ (SI unit: m³/mol), a **Toth exponent** $N_{T,c}$ (dimensionless), and an **Adsorption maximum** $c_{p,\max,c}$ (SI unit: mol/kg).

- For **BET (Brunauer–Emmett–Teller)**:

$$c_P = \frac{K_B c_0 c}{(c_S - c) \left(1 + (K_B - 1) \frac{c}{c_S} \right)}$$

and

$$K_P = \frac{\partial c_P}{\partial c} = \frac{K_B c_0 c_S ((K_B - 1) c^2 + c_S^2)}{(c - c_S)^2 (c_S + (K_B - 1) c)^2}$$

Enter a **BET constant** $K_{B,c}$ (dimensionless), a **Monolayer adsorption capacity** $c_{0,c}$ (SI unit: mol/kg), and an **Saturation concentration** $c_{S,c}$ (SI unit: mol/m³).

- For **User defined** enter an **Adsorption isotherm** $c_{P,c}$ (SI unit: mol/kg):

$$c_p = f(c)$$

For more information, see [Adsorption](#) in the theory section.

FURTHER READING

See the theory chapter in the section [Mass Balance Equation for Transport of Diluted Species in Porous Media](#).

Volatilization

This feature is available when the **Mass transfer in porous media** checkbox is selected in the **Settings** window for the physics interface.

Use this feature to model mass transfer at the boundary due to volatilization. The species dissolved in the liquid are assumed to be vaporized at the boundary, and transported into the surrounding bulk region due to convection and diffusion. The mass transfer at the boundary is defined as

$$-\mathbf{n} \cdot \mathbf{J}_c = -h_c(k_{G,c}c - c_{Gatm,c})$$

where h_c is the mass transfer coefficient, $k_{G,c}$ the volatilization coefficient, and $c_{Gatm,c}$ the concentration in the surrounding atmosphere.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

VOLATILIZATION

Enter a **Mass transfer coefficient** h_c defining the transfer into the surrounding media. This can be given by boundary layer theory. When assuming that no convective flow is present in the surrounding, the mass transfer coefficient can be defined from the gas diffusion coefficient D_{Gc} and the thickness of the diffusion layer d_s in the manner of

$$h_c = \frac{D_{Gc}}{d_s}$$

Also give the atmospheric concentration for each species, $c_{Gatm,c}$. The **Volatilization** coefficient $k_{G,c}$ for each species are taken from the adjacent [Unsaturated Porous Medium](#) domain.

Species Source

In order to account for consumption or production of species in porous domains, the **Species Source** node adds source terms expressions S_i to the right-hand side of the species transport equations.

DOMAIN SELECTION

From the **Selection** list, choose the domains on which to define rate expression or expressions that govern the source term in the transport equations.

If there are several types of domains, with subsequent and different reactions occurring within them, it might be necessary to remove some domains from the selection. These are then defined in an additional **Species Source** node.

SPECIES SOURCE

Add a source term S_i (SI unit: $\text{mol}/(\text{m}^3 \cdot \text{s})$) for each of the species solved for. Enter a value or expression in the field of the corresponding species.

Hygroscopic Swelling

The **Hygroscopic Swelling** multiphysics coupling node () is used for moisture concentration coupling between the Solid Mechanics interface and either the Transport of Diluted Species or Transport of Diluted Species in Porous Media interfaces.

Hygroscopic swelling is an effect of internal strain caused by changes in moisture content. This volumetric strain can be written as

$$\epsilon_{\text{hs}} = \beta_{\text{h}} M_{\text{m}} (c_{\text{mo}} - c_{\text{mo,ref}})$$

where β_{h} is the coefficient of hygroscopic swelling, M_{m} is the molar mass, c_{mo} is the moisture concentration, and $c_{\text{mo,ref}}$ is the strain-free reference concentration.

This feature requires a license of either the MEMS Module or the Structural Mechanics Module. The multiphysics feature will appear automatically if both the Transport of Diluted Species and the Solid Mechanics interfaces are added to the same component. For the most current information about licensing, please see www.comsol.com/products/specifications/.

FURTHER READING

More information about how to use hygroscopic swelling can be found in the [Hygroscopic Swelling](#) section in the *Structural Mechanics Module User's Guide*.

More information about multiphysics coupling nodes can be found in the section [The Multiphysics Branch](#) in the *COMSOL Multiphysics Reference Manual*.

Out-of-Plane Flux

The out-of-plane flux node is used to prescribe a flux in an out-of-plane domain, and it can be used to reduce a model geometry to 2D, or even 1D, when the concentration variation is small in one or more directions. This could be the case for example when the object to model is thin or slender. [Figure 6-1](#) shows examples of possible situations in which this type of geometry reduction can be applied.

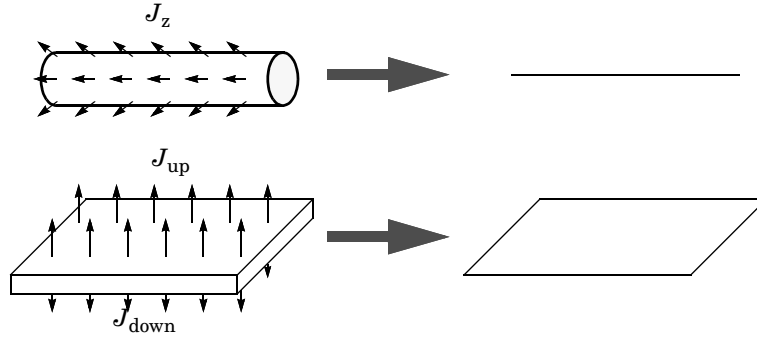


Figure 6-1: Geometry reduction from 3D to 1D (top) and from 3D to 2D (bottom).

For a 1D component this node adds a single out-of-plane molar flux $J_{0,z,i}$ for species i . For a 2D component two fluxes can be prescribed for each species; $J_{0,u,i}$ for the upside of the domain, and $J_{0,d,i}$ for the downside of the domain.

The fluxes are added as a source contribution to the right-hand side of the transport equation. For example, in 2D components with diffusion, convection, and out-of-plane flux present, the mass transfer of species i is defined by the mass conservation equation

$$\frac{\partial c_i}{\partial t} + \nabla \cdot \mathbf{J}_i + \mathbf{u} \cdot \nabla c_i = R_i + S_{\text{opf},i}, \quad (6-1)$$

where $S_{\text{opf},i}$ is the out-of-plane source for species i

$$j_{\text{opf},i} = \frac{J_{0,i}}{d_z} = \frac{J_{0,\text{u},i} + J_{0,\text{d},i}}{d_z}.$$

For external convection on the upside and the downside of the domain, the out-of-plane flux is

$$J_{0,i} = k_{\text{c},\text{u},i}(c_{\text{b},\text{u},i} - c_i) + k_{\text{c},\text{d},i}(c_{\text{b},\text{d},i} - c_i)$$

where k_c is a mass transfer coefficient and c_b is the bulk concentration, the typical concentration far into the surrounding exterior domain. The prescribed flux, J_0 , can include any arbitrary user-specified expressions. It can be a constant or a function of a dependent variable or independent variables.

UPSIDE INWARD FLUX

For 1D components, enter the **cross-sectional perimeter** P_c to get the out-of-plane flux

$$J_{0,i} = P_c J_{0,z,i}$$

The default value of P_c is the circumference. Either keep the default value, for a circular cross-section shape, or edit the value to get a user-defined shape of the out-of-plane cross-section.

The available flux type options are **General inward flux** and **External convection**. Select the **Species** checkbox for the species for which to specify the flux, and enter a value or expression for the inward flux in the corresponding field. Use a minus sign when specifying a flux directed out of the system. To use another boundary condition for a specific species, click to clear the checkbox for that species.

Set **Flux type** to **External convection** to prescribe a flux to or from an exterior domain (not modeled) assumed to include convection. The exterior can for example include a forced convection to control the temperature or to increase the mass transport. In this case the prescribed mass flux corresponds to

$$J_0 = k_c(c_b - c)$$

where k_c is a mass transfer coefficient and c_b is the bulk concentration, the typical concentration far into the surrounding exterior domain.

DOWNSIDE INWARD FLUX

The available options are **General inward flux** and **External convection**. The settings are the same as for the [Upside Inward Flux](#) section.

FURTHER READING

For an example of using the Out-of-Plane Flux node, see this application example:



Cross-Flow Mass Transfer in a Thin Domain: Application Library path
Chemical_Reaction_Engineering_Module/Tutorials/thin_domain

Fracture

Use this node to model mass transport along thin fractures in porous media. The node assumes that the transport in the tangential direction along the fracture is dominant as a result of lower flow resistance.

The Fracture feature has two default subfeatures — the **Fluid (Fracture)** subfeature where convection and diffusion are specified and the **Fracture Material** subfeature where the porosity ϵ_p is defined.



This feature is only available in a limited set of add-on products. See www.comsol.com/products/specifications/ for more details on availability.

FRACTURE PROPERTIES

Specify a value for the **Fracture thickness** d_f .

For the **Fracture** boundary condition, there are a number of **Attributes** available which can be accessed either via the **Physics** Ribbon toolbar (for Windows) or by right-clicking on the **Fracture** node. These attributes are identical to the **Boundary, Edge, Point, and Pair Nodes for the Transport of Diluted Species in Fractures Interface**.

Fluid (Fracture)

Use this node to specify the mass transfer in the mobile fluid solvent filling the pores of a fracture.

It uses the same input options as the **Fluid (Porous Medium)** subfeature under the **Porous Medium** feature. There are further details about the settings.

MODEL INPUTS

The temperature model input is always available. By default, the **Temperature** model input is set to **Common model input**, and the temperature is controlled from **Default Model Inputs** under **Global Definitions** or by a locally defined **Model Input**. If a Heat Transfer interface is included in the component, it controls the temperature **Common model input**. Alternatively, the temperature field can be selected from another physics interface. All physics interfaces have their own tags (**Name**). For example, if a Heat Transfer in Fluids interface is included in the component, the **Temperature (ht)** option is available for T.

CONVECTION

If transport by convection is active, the velocity field of the solvent needs to be specified. Select an option from the **Velocity field** list to specify the convective velocity along the fracture. For a consistent model, use a Fracture feature in a Darcy's Law interface to compute the fluid flow velocity in the fracture.

For **User defined**, enter values or expressions for the velocity components in the table shown

DIFFUSION

Use this section to specify diffusion coefficients describing the diffusion of each species in the solvent fluid.



The **Diffusion** settings are similar to those for the diffusion coefficients in the [Fluid \(Porous Medium\)](#) node under [Porous Medium](#).

Fracture Material

Specify the **Porosity** ϵ_p (dimensionless) of the porous matrix. This is by default taken **From material**. Select **From pellet bed densities** to compute the porosity from the (dry bulk) **Bed density** ρ_b and the (single phase) **Pellet density** ρ_{pe} through the relation

$$\epsilon_p = 1 - \frac{\rho_b}{\rho_{pe}}$$

For **User defined**, enter a value or expression for the porosity.

The Transport of Diluted Species in Fractures Interface

The **Transport of Diluted Species in Fractures (dsf)** interface () , found under the **Chemical Species Transport** branch () , is used to model the transport of a solute species along thin porous fractures, taking into account diffusion, dispersion, convection, and chemical reactions. The fractures are defined by boundaries and the solute species is assumed to be present in a solvent.

The interface supports simulation of species transport along boundaries in 2D and 3D, and axisymmetric components in 2D. The dependent variable is the molar concentration, c . Modeling multiple species transport is possible, whereby the physics interface solves for the molar concentration, c_i , of each species i .



This interface is only available in a limited set of add-on products. For a detailed overview of which interfaces are available in each product, visit www.comsol.com/products/specifications/

SETTINGS

The **Label** is the default physics interface name.

The **Name** is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the `name` string must be unique. Only letters, numbers, and underscores (`_`) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first physics interface in the model) is `dsf`.

BOUNDARY SELECTION

If the model geometry includes boundaries that should not be included in the mass transfer simulation, remove those from the selection list.

OUT-OF-PLANE THICKNESS

For 2D components, the **Thickness** field (default value: 1 m) defines a parameter for the thickness of the geometry perpendicular to the two-dimensional cross-section. Both constant and varying thicknesses are supported. The value of this parameter is used,

among other things, to automatically calculate molar flow rates from the total molar flux.

TRANSPORT MECHANISMS

Use the **Convection** checkbox, available under **Additional transport mechanisms**, to control whether to also include convective transport.

CONSISTENT STABILIZATION

To display this section, click the **Show** button () and select **Stabilization**. Use this section to control the application of the available consistent stabilization methods; **Streamline diffusion** and **Crosswind diffusion**.

- When the **Crosswind diffusion** checkbox is selected, a weak term that reduces spurious oscillations is added to the transport equation. The resulting equation system is always nonlinear. There are two options for the **Crosswind diffusion type**:
 - **Do Carmo and Galeão** — the default option. This type of crosswind diffusion reduces undershoots and overshoots to a minimum but can in rare cases give equation systems that are difficult to fully converge.
 - **Codina**. This option is less diffusive compared to the Do Carmo and Galeão option but can result in more undershoots and overshoots. It is also less effective for anisotropic meshes. The Codina option activates a text field for the **Lower gradient limit** g_{lim} . Its default value is $0.1 [\text{mol}/\text{m}^3) / \text{tds}.\text{helem}$, where $\text{tds}.\text{helem}$ is the local element size.
- For both consistent stabilization methods select an **Equation residual**. **Approximate residual** is the default and means that derivatives of the diffusion tensor components are neglected. This setting is usually accurate enough and is computationally faster. If required, select **Full residual** instead.

INCONSISTENT STABILIZATION

To display this section, click the **Show** button () and select **Stabilization**. By default, the **Isotropic diffusion** checkbox is not selected, because this type of stabilization adds artificial diffusion and affects the accuracy of the original problem. However, this option can be used to get a good initial guess for under resolved problems.

DISCRETIZATION

To display all settings available in this section, click the **Show** button () and select **Advanced Physics Options**.

The **Value type when using splitting of complex variables** setting should in most pure mass transfer problems be set to **Real**, which is the default. It makes sure that the dependent variable does not get affected by small imaginary contributions, which can occur, for example, when combining a Time Dependent or Stationary study with a frequency-domain study. For more information, see [Splitting Complex-Valued Variables](#) in the *COMSOL Multiphysics Reference Manual*.

DEPENDENT VARIABLES

The dependent variable name is **Concentration c** by default. A dependent variable name must be unique with respect to all other dependent variables in the component.

Add or remove species variables in the model and also change the names of the dependent variables that represent the species concentrations.

Enter the **Number of species**. Use the **Add concentration** () and **Remove concentration** () buttons as needed.

FURTHER READING



- [Mass Transport in Fractures](#) in the theory section.
- [Numerical Stabilization](#) in the *COMSOL Multiphysics Reference Manual*.
- [Domain, Boundary, and Pair Nodes for the Transport of Diluted Species Interface](#)
- In the *COMSOL Multiphysics Reference Manual*, see [Table 2-4](#) for links to common sections and [Table 2-5](#) for common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

Boundary, Edge, Point, and Pair Nodes for the Transport of Diluted Species in Fractures Interface

The [Transport of Diluted Species in Fractures Interface](#) has the following boundary, edge, point, and pair nodes, listed in alphabetical order, available from the **Physics**

ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or by right-clicking to access the context menu (all users).

- [Concentration](#)
- [Flux](#)
- [Fracture](#)
- [Inflow](#)
- [Initial Values](#)
- [No Flux](#)
- [Outflow](#)
- [Reactions](#)
- [Species Source](#)

Furthermore, there are the following attributes to the **Fracture** node

- [Adsorption](#)
- [Dispersion](#)

Adsorption

Use this node to model adsorption of the fluid phase species onto the porous media surface of the fracture.

MATRIX PROPERTIES

Use the **Porous material** list to define a material specifying the matrix properties on the current selection. By default the **Boundary material** is used as [The Transport of Diluted Species in Fractures Interface](#) is active on boundaries only. The **Density** ρ of the porous media is needed when modeling adsorption to the surface of the porous matrix. By default the option **From material** is selected.

ADSORPTION

Select an **Adsorption isotherm** — **Langmuir** (the default), **Freundlich**, **Toth**, **BET**, or **User defined** to specify how to compute c_p , the amount of species sorbed to the solid phase (moles per unit dry weight of the solid). Also activate the checkbox for the species c_i for which adsorption takes place.

- For **Langmuir**:

$$c_P = c_{P\max} \frac{K_L c}{1 + K_L c} \quad \frac{\partial c_P}{\partial c} = \frac{K_L c_{P\max}}{(1 + K_L c)^2}$$

Enter a **Langmuir constant** $k_{L,c}$ (SI unit: m^3/mol) and an **Adsorption maximum** $c_{P,\max,c}$ (SI unit: mol/kg):

- For **Freundlich**:

$$c_P = K_F \left(\frac{c}{c_{\text{ref}}} \right)^N \quad \frac{\partial c_P}{\partial c} = N \frac{c_P}{c} \quad \text{Freundlich}$$

Enter a **Freundlich constant** $k_{F,c}$ (SI unit: mol/kg), a **Freundlich exponent** $N_{F,c}$ (dimensionless), and a **Reference concentration** $c_{\text{ref},c}$ (SI unit: mol/m^3).

- For **Toth**:

$$c_P = c_{P\max} \frac{b_T c}{(1 + (b_T c)^{N_T})^{1/N_T}}$$

and

$$\frac{\partial c_P}{\partial c} = c_{P\max} b_T (1 + (b_T c)^{N_T})^{-\left(1 + \frac{1}{N_T}\right)}$$

Enter a **Toth constant** $b_{T,c}$ (SI unit: m^3/mol), a **Toth exponent** $N_{T,c}$ (dimensionless), and an **Adsorption maximum** $c_{P,\max,c}$ (SI unit: mol/kg).

- For **BET (Brunauer–Emmett–Teller)**:

$$c_P = \frac{K_B c_0 c}{(c_S - c) \left(1 + (K_B - 1) \frac{c}{c_S} \right)}$$

and

$$\frac{K_B c_0 c_S ((K_B - 1) c^2 + c_S^2)}{(c - c_S)^2 (c_S + (K_B - 1) c)^2}$$

Enter a **BET constant** $K_{B,c}$ (dimensionless), a **Monolayer adsorption capacity** $c_{0,c}$ (SI unit: mol/kg), and a **Saturation concentration** $c_{S,c}$ (SI unit: mol/m^3).

- For **User defined**:

$$c_p = f(c)$$

Enter an adsorption isotherm $c_{p,c}$ (SI unit: mol/kg).

For more information, see [Adsorption](#) in the theory section.

FURTHER READING

See the theory chapter in the section [Mass Balance Equation for Transport of Diluted Species in Porous Media](#).

Concentration

Use this node to specify the species concentration on a fracture boundary (applied in points in 2D and along edges in 3D). For example, a $c = c_0$ condition specifies the concentration of species c .

CONCENTRATION

Individually specify the concentration for each species. Select the checkbox for the **Species** to specify the concentration, and then enter a value or expression in the corresponding field. To use another boundary condition for a specific species, click to clear the checkbox for the concentration of that species.

CONSTRAINT SETTINGS

To display this section, click the **Show** button () and select **Advanced Physics Options**. You can find details about the different constraint settings in the section [Constraint Reaction Terms](#) in the *COMSOL Multiphysics Reference Manual*.

Dispersion

Local variations in the velocity as the fluid flows around solid particles lead to mechanical mixing, referred to as dispersion. Use this feature to account for dispersion in a Fracture.

This subfeature is available when the **Convection** checkbox is selected in the **Settings** window for the physics interface.

Select the **Specify dispersion for each species individually** checkbox to specify the dispersion tensor D_D (SI unit: m^2/s) for each species separately. When not selected the same dispersion tensor D_D is used for all species.

Select an option from the **Dispersion tensor** list — **Dispersivity** or **User defined**.

Select **Dispersivity** to specify the dispersion in terms of dispersivities (SI unit: m). Select an option from the **Dispersivity model** list: **Isotropic** (the default) or **Transverse isotropic** based on the properties of the porous media. For isotropic porous media, specify the longitudinal and transverse dispersivities. For transverse isotropic porous media, specify the longitudinal, horizontal transverse, and vertical transverse dispersivities.

For **User defined**, specify the dispersion components in terms of constants or expressions. Select **Isotropic**, **Diagonal**, **Symmetric**, or **Full** to enable the appropriate tensor components.

Flux

This node can be used to specify the species flux across a boundary of a porous fracture (applied in points in 2D and along edges in 3D). The flux of species c is defined as

$$\mathbf{n} \cdot d_{\text{fr}}(D_c \nabla c) = d_{\text{fr}} N_0$$

where N_0 is an arbitrary user-specified flux expression. For example, N_0 can represent a flux due to chemical reactions, or a phase change. A positive N_0 implies that the concentration inside the fracture increases.

INWARD FLUX

The available options are **General inward flux** and **External convection**. If the latter is chosen, define a **Mass transfer coefficient** and a **Bulk concentration**. Specify the flux of each species individually. To use another boundary condition for a specific species, make sure that the checkbox for the mass fraction of that species is cleared.

Inflow

Use this node to specify all species concentrations at a fracture inlet. The condition is applied in points in 2D and along edges in 3D.

If you want to specify the concentration of a subset of the partaking species, this can be done by using the [Concentration](#) node instead.

CONCENTRATION

For the concentration of each species $c_{0,c}$ (SI unit: mol/m³), enter a value or expression.

BOUNDARY CONDITION TYPE

This section in the settings is only available for some products. Search for “Inflow” on the page: www.comsol.com/products/specifications/ for more details on availability.

The option **Concentration constraint** constrains the concentration values on the boundary by the use of pointwise constraints. The other option, **Flux (Danckwerts)** can be more stable and fast to solve when high reaction rates are anticipated in the vicinity of the inlet. Oscillations on the solutions can also be avoided in such cases. The latter condition uses a flux boundary condition based on the velocity across the boundary and the concentration values. See further details in the theory section.

CONSTRAINT SETTINGS

To display this section, click the **Show** button () and select **Advanced Physics Options**. You can find details about the different constraint settings in the section [Constraint Reaction Terms](#) in the *COMSOL Multiphysics Reference Manual*.

FURTHER READING

See the theory chapter in the section [Danckwerts Inflow Boundary Condition](#).

No Flux

This node can be used to specify that the species flux across a boundary of a porous fracture is zero. The condition is applied in points in 2D and along edges in 3D.

Outflow

Set this condition at fracture outlets where species are transported out of the model domain by fluid motion. The condition is applied in points in 2D and along edges in 3D. It is assumed that convection is the dominating transport mechanism across outflow boundaries, and therefore that diffusive transport can be ignored, that is:

$$\mathbf{n} \cdot (-D_c \nabla c) = 0$$

Reactions

Use the **Reactions** node to account for the consumption or production of species through chemical reactions in the fracture. Define the rate expressions as required.

BOUNDARY SELECTION

From the **Selection** list, choose the boundaries on which to define rate expression or expressions that govern the source term in the transport equations.

Several reaction nodes can be used to account for different reactions in different parts of the fracture.

REACTION RATES

Add a rate expression R_i for species i . Enter a value or expression in the field. Note that if you have the Chemistry interface available, included with the Chemical Reaction Engineering Module, the reaction rate expressions can be automatically generated and picked up from the lists.

REACTING VOLUME

When specifying reaction rates for a species in a fracture, the specified reaction rate may have the basis of the pore volume of the fracture, or the total volume.

- For **Total volume**, the reaction expressions in are specified per unit volume of the fracture. The reaction expressions will be multiplied by the fracture thickness d_{fr} .
- For **Pore volume**, the reaction expressions in $\text{mol}/(\text{m}^3 \cdot \text{s})$ are specified per unit volume of total pore space in the fracture. The reaction expressions will be multiplied by the fracture thickness d_{fr} and the fracture porosity, ϵ_{p} .

Species Source

In order to account for consumption or production of species in a fracture, the **Species Source** node adds source terms expressions S_i to the right-hand side of the species transport equations.

BOUNDARY SELECTION

From the **Selection** list, choose the boundaries on which to define expressions that govern the source term in the transport equations.

If there are several different parts of the fracture, with subsequent and different sources occurring within them, it might be necessary to remove some boundaries from the selection. The sources in these can then be defined using an additional **Species Source** node.

SPECIES SOURCE

Add a source term S_i for each of the species solved for. Enter a value or expression in the field of the corresponding species.

Theory for the Transport of Diluted Species Interface

The [Transport of Diluted Species Interface](#) provides a predefined modeling environment for studying the evolution of chemical species transported by diffusion and convection. The physics interface assumes that all species present are dilute; that is, that their concentration is low compared to a solvent fluid or solid. As a rule of thumb, a mixture containing several species can be considered dilute when the concentration of the solvent is more than 90 mol%. Due to the dilution, mixture properties such as density and viscosity can be assumed to correspond to those of the solvent.

When studying mixtures that are not dilute, the mixture and transport properties depend on the composition, and a different physics interface is recommended. See [The Transport of Concentrated Species Interface](#) in the *Chemical Reaction Engineering Module User's Guide* for more information.

Fick's law governs the diffusion of the solutes, dilute mixtures, or solutions, while the phenomenon of ionic migration is sometimes referred to as *electrokinetic flow*. The Transport of Diluted Species interface supports the simulations of chemical species transport by convection, migration, and diffusion in 1D, 2D, and 3D as well as for axisymmetric components in 1D and 2D.

In this section:

- [Adding Transport Through Migration](#)
- [Material Balance Formulation](#)
- [Crosswind Diffusion](#)
- [Danckwerts Inflow Boundary Condition](#)
- [Equilibrium Reaction Theory](#)
- [Mass Balance Equation](#)
- [Mass Sources for Species Transport](#)
- [Solving a Diffusion Equation Only](#)
- [Supporting Electrolytes](#)
- [References](#)

Note: Some features explained in this section require certain add-on modules. For details see www.comsol.com/products/specifications/

The section also includes the theory for [The Transport of Diluted Species in Porous Media Interface](#):

- [Adsorption](#)
- [Convection in Porous Media](#)
- [Diffusion in Porous Media](#)
- [Dispersion](#)
- [Mass Balance Equation for Transport of Diluted Species in Porous Media](#)
- [Mass Transport in Fractures](#)
- [Reactions](#)

Mass Balance Equation

The default node attributed to the Transport of Diluted Species interface models chemical species transport through diffusion and convection and solves the mass conservation equation for one or more chemical species i :

$$\frac{\partial c_i}{\partial t} + \nabla \cdot \mathbf{J}_i + \mathbf{u} \cdot \nabla c_i = R_i \quad (6-2)$$

[Equation 6-2](#) in its form above includes the transport mechanisms diffusion and convection. If *Migration in Electric Field* is activated (only available in some add-on products), the migration mechanism will be added to the equation as well. See more details in the section [Adding Transport Through Migration](#).

- c_i is the concentration of the species (SI unit: mol/m³)
- D_i denotes the diffusion coefficient (SI unit: m²/s)
- R_i is a reaction rate expression for the species (SI unit: mol/(m³·s))
- $\mathbf{u}$ is the mass averaged velocity vector (SI unit: m/s)
- $\mathbf{J}_i$ is the mass flux diffusive flux vector (SI unit: mol/(m²·s))

The mass flux relative to the mass averaged velocity, $\mathbf{J}_i$ (SI unit: mol/(m²·s)), is associated with the mass balance equation above and used in boundary conditions and flux computations. The Transport of Diluted Species interface always includes mass transport due to molecular diffusion. In this case the mass flux $\mathbf{J}_i$ defines the diffusive flux vector

$$\mathbf{J}_i = -D\nabla c_i \quad (6-3)$$

An input field for the diffusion coefficient is available.

When *Migration in Electric Fields* is activated, the migration term is also added to the diffusive flux vector as shown in the section [Adding Transport Through Migration](#).

The third term on the left side of [Equation 6-2](#) describes the convective transport due to a velocity field $\mathbf{u}$. This field can be expressed analytically or obtained from coupling the physics interface to one that solves for fluid flow, such as *Laminar Flow*. Note that all fluid flow interfaces solve for the mass averaged velocity.

On the right-hand side of the mass balance equation ([Equation 6-2](#)), R_i represents a source or sink term, typically due to a chemical reaction or desorption on a porous matrix. To specify R_i , another node must be added to the Transport of Diluted Species interface — the **Reaction** node for example, which includes an input field for specifying a reaction expression using the variable names of all participating species.

Equilibrium Reaction Theory

The feature Equilibrium Reaction is described in this section. A chemical equilibrium reaction system is defined by the stoichiometry of the reaction and the relation between the chemical activities of the chemical species participating in the reaction (the equilibrium condition).

The kinetics of the reaction is so fast that the equilibrium condition is fulfilled at all times in all space coordinates.

The equilibrium condition is commonly based on the stoichiometric coefficients, ν_i (dimensionless), of the reaction; the species activities of the reacting species a_i (dimensionless); and an equilibrium constant, K_{eq} (1) according to

$$K_{\text{eq}} = \frac{\prod_{i \in \text{products}} a_i^{\nu_i}}{\prod_{i \in \text{reactants}} a_i^{-\nu_i}}$$

where the species activities are defined as

$$a_i = \gamma_{c,i} \frac{c_i}{c_{a0}}$$

where c_{a0} (SI unit: mol/m³) is the standard molarity, and $\gamma_{c,i}$ (dimensionless) an activity coefficient.

Defining the stoichiometric coefficients positive for products and negative for reactants, the above equilibrium condition can also be written

$$K_{\text{eq}} = \prod_i a_i^{v_i}$$

The **Equilibrium Reaction** node solves for a reaction rate so that the equilibrium condition is always fulfilled in the domain. It is available for the Chemical Reaction Engineering Module; Battery Design Module; Corrosion Module; Electrochemistry Module; Electrodeposition Module; and Fuel Cell & Electrolyzer Module.



$\gamma_{e,i}$ is set to unity when the Equilibrium constant is selected in the Settings window. For nonunity activity coefficients, a user-defined equilibrium condition can be used.

EQUILIBRIUM REACTIONS AND INFLOW BOUNDARY CONDITIONS

Contradictory constraints arise if the boundary conditions for concentrations or activities are set so that the domain equilibrium condition is not fulfilled. Special treatment is therefore needed at Inflow boundaries, where the concentrations are set for all species in the mass transport interfaces.

One way of avoiding competing constraints on an inflow boundary is to add an additional reaction coordinate degree of freedom, solved for to create a set of modified inflow concentrations that fulfill the domain equilibrium condition. The reaction coordinate gives rise to a concentration shift, which is the offset to the inflow concentrations provided by the user. The shift for each species obeys the stoichiometry of the reaction and the equilibrium expression. The modified inflow concentrations are then used in the boundary conditions for the domain mass transport equations. The resulting modified inflow concentrations can be seen as the stationary solution for a batch reactor with the user inflow concentrations as initial concentrations. In addition, the domain reaction rate degree of freedom of the equilibrium reaction is constrained to zero on all Inflow boundaries.

EQUILIBRIUM REACTIONS AND CONCENTRATION BOUNDARY CONDITIONS

No special treatment is made with regards to input concentration values of the Concentration boundary node. Using this feature, you can explicitly set one or a set of concentrations, and the equilibrium condition acts on the rest of the concentrations.

However, there is no solution to the problem if more concentrations than the number of species minus the number of equilibrium reactions are set using this feature.

EQUILIBRIUM REACTIONS AND TIME-DEPENDENT SIMULATIONS

Spurious oscillations may occur in a time-dependent problem if the initial conditions do not fulfill the equilibrium condition. Since equilibrium reactions are assumed to be infinitely fast, the solution is to initialize the problem using an additional study step, solving for a stationary problem with all nonequilibrium reaction rates set to zero. Manual scaling of the reaction rate dependent variables is needed in this study step.

Material Balance Formulation

The default node attributed to [The Transport of Diluted Species Interface](#) assumes chemical species transport through diffusion and convection (depending on the modules licensed, a checkbox to activate migration is available) and implements the mass balance equation in [Equation 6-2](#).

Two formulations of the mass balance are available, these are referred to as the nonconservative and conservative form:

$$\text{nonconservative: } \frac{\partial c}{\partial t} + \nabla \cdot \mathbf{J}_i + \mathbf{u} \cdot \nabla c = R \quad (6-4)$$

$$\text{conservative: } \frac{\partial c}{\partial t} + \nabla \cdot \mathbf{J}_i + \nabla \cdot (c\mathbf{u}) = R \quad (6-5)$$

In these equations $\mathbf{J}_i$ is the diffusive flux vector, $\mathbf{u}$ is the solvent velocity field, and R is a production or consumption rate expression. The nonconservative is often more numerically robust, and therefore chosen by default for the interface. This formulation however disregards density changes in the fluid. This can be seen by expanding the conservative convection term using the chain rule. One of the resulting terms, $c\nabla \cdot \mathbf{u}$, equals zero for an incompressible fluid which results in the nonconservative form. To switch between the two formulations, click the **Show** button () and select **Advanced Physics Options**.

Solving a Diffusion Equation Only

Remove the convection term from [Equation 6-4](#) and [Equation 6-5](#) by clearing the Convection checkbox in the Transport Mechanisms section for [The Transport of Diluted Species Interface](#). The equation then becomes

$$\frac{\partial c}{\partial t} = \nabla \cdot \mathbf{J}_i + R$$

Mass Sources for Species Transport

Note: The features below are only available in a limited set of add-on products. For a detailed overview of which features are available in each product, visit www.comsol.com/products/specifications/

There are two types of mass sources in the Transport of Diluted Species interface: point sources and line sources.

POINT SOURCE

A point source is theoretically formed by assuming a mass injection/ejection, Q_c (SI unit: $\text{mol}/(\text{m}^3 \cdot \text{s})$), in a small volume δV and then letting the size of the volume tend to zero while keeping the total mass flux constant. Given a point source strength, $\dot{q}_{p,c}$ (SI unit: mol/s), this can be expressed as

$$\lim_{\delta V \rightarrow 0} \int_{\delta V} Q_c = \dot{q}_{p,c} \quad (6-6)$$

An alternative way to form a point source is to assume that mass is injected/extracted through the surface of a small object. Letting the object surface area tend to zero while keeping the mass flux constant results in the same point source. For this alternative approach, effects resulting from the physical object's volume need to be neglected.

The weak contribution

$$\dot{q}_{p,c} \text{test}(c)$$

is added at a point in the geometry. As can be seen from Equation 6-6, Q_c must tend to plus or minus infinity as δV tends to zero. This means that in theory the concentration also tends to plus or minus infinity.

Observe that “point” refers to the physical representation of the source. A point source can therefore only be added to points in 3D components and to points on the symmetry axis in 2D axisymmetry components. Other geometrical points in 2D components represent physical lines.

The finite element representation of [Equation 6-6](#) corresponds to a finite concentration at a point with the effect of the point source spread out over a region around the point. The size of the region depends on the mesh and on the strength of the source. A finer mesh gives a smaller affected region but also a more extreme concentration value. It is important not to mesh too finely around a point source since this can result in unphysical concentration values. It can also have a negative effect on the condition number for the equation system.

LINE SOURCE

A line source can theoretically be formed by assuming a source of strength $Q_{l,c}$ (SI unit: $\text{mol}/(\text{m}^3 \cdot \text{s})$), located within a tube with cross section δS and then letting δS tend to zero while keeping the total mass flux per unit length constant. Given a line source strength, $\dot{q}_{l,c}$ (SI unit: $\text{mol}/(\text{m} \cdot \text{s})$), this can be expressed as

$$\lim_{\delta S \rightarrow 0} \int_{\delta S} Q_{l,c} = \dot{q}_{l,c} \quad (6-7)$$

As in the point source case, an alternative approach is to assume that mass is injected/extracted through the surface of a small object. This results in the same mass source, but requires that effects resulting from the physical object's volume are neglected.

The weak contribution

$$q_{l,c} \text{test}(c)$$

is added on lines in 3D or at points in 2D (which represent cut-through views of lines). Line sources can also be added on the axisymmetry line in 2D axisymmetry components. It cannot, however, be added on geometrical lines in 2D since those represent physical planes.

As with a point source, it is important not to mesh too finely around the line source.



For feature node information, see [Line Mass Source](#) and [Point Mass Source](#).

Note: Migration is only available in a limited set of add-on products. For a detailed overview of which features are available in each product, visit www.comsol.com/products/specifications/

In addition to transport due to convection and diffusion, the Transport of Diluted Species interface supports ionic species transport by *migration*. This is done by selecting the Migration in Electric Field checkbox under the Transport Mechanisms section for the physics interface. The mass balance then becomes:

$$\frac{\partial c_i}{\partial t} + \nabla \cdot (-D_i \nabla c_i - z_i u_{m,i} F c_i \nabla V + c_i \mathbf{u}) = R_i \quad (6-8)$$

where

- c_i (SI unit: mol/m³) denotes the concentration of species i
- D_i (SI unit: m²/s) is the diffusion coefficient of species i
- $\mathbf{u}$ (SI unit: m/s) is the fluid velocity
- F (SI unit: A·s/mol) refers to Faraday's constant
- V (SI unit: V) denotes the electric potential
- z_i (dimensionless) is the charge number of the ionic species, and
- $u_{m,i}$ (SI unit: mol·s/kg) is its ionic mobility

In this case the diffusive flux vector is

$$\mathbf{J}_i = -D_i \nabla c_i - z_i u_{m,i} F c_i \nabla V$$

The velocity, $\mathbf{u}$, can be a computed fluid velocity field from a Fluid Flow interface or a specified function of the spatial variables x , y , and z . The potential can be provided by an expression or by coupling the system of equations to a current balance, such as the Electrostatics interface. Sometimes it is assumed to be a supporting electrolyte present, which simplifies the transport equations. In that case, the modeled charged species concentration is very low compared to other ions dissolved in the solution. Thus, the species concentration does not influence the solution's conductivity and the net charge within the fluid.

The Nernst–Einstein relation can in many cases be used for relating the species mobility to the species diffusivity according to

$$u_{m,i} = \frac{D_i}{RT}$$

where R (SI unit: J/(mol·K)) is the molar gas constant and T (SI unit: K) is the temperature.

Note: In the *Nernst–Planck Equations* interface, the ionic species contribute to the charge transfer in the solution. It includes an *electroneutrality condition* and also computes the electric potential field in the electrolyte. For more information, see [Theory for the Nernst–Planck Equations Interface](#). This interface is included in the Chemical Reaction Engineering Module.

Supporting Electrolytes

In *electrolyte* solutions, a salt can be added to provide a high electrolyte conductivity and decrease the ohmic losses in a cell. These solutions are often called *supporting electrolytes*, buffer solutions, or carrier electrolytes. The added species, a negative and a positive ion pair, predominates over all other species. Therefore, the supporting electrolyte species can be assumed to dominate the current transport in the solution. In addition, the predominant supporting ions are usually selected so that they do not react at the electrode surfaces since the high conductivity should be kept through the process, that is, they should not be electroactive species. This also means that the concentration gradients of the predominant species in a supporting electrolyte are usually negligible.

Modeling and solving for a supporting electrolyte in the *Electrostatics* or *Secondary Current Distribution* interfaces will give a potential distribution that drives the migration in the Transport of Diluted Species Interface.

The current density vector is proportional to the sum of all species fluxes as expressed by Faraday’s law:

$$\mathbf{i} = F \sum_i z_i \mathbf{N}_i$$

The electroneutrality condition ensures that there is always a zero net charge at any position in a dilute solution. Intuitively, this means that it is impossible to create a current by manually pumping positive ions in one direction and negative ions in the other. Therefore, the convective term is canceled out to yield the following expression for the electrolyte current density, where j denotes the supporting species:

$$\mathbf{i} = F \sum_j -z_j^2 u_{m,j} F c_j \nabla \phi \quad (6-9)$$

Equation 6-9 is simply Ohm's law for ionic current transport and can be simplified to

$$\mathbf{i} = -\kappa \nabla \phi \quad (6-10)$$

where κ is the conductivity of the supporting electrolyte. A current balance gives the current and potential density in the cell

$$\nabla \cdot \mathbf{i} = 0$$

which, in combination with Equation 6-10, yields:

$$\nabla \cdot (-\kappa \nabla \phi) = 0 \quad (6-11)$$

Equation 6-11 can be easily solved using the Electrostatics or Secondary Current Distribution interface and, when coupled to the Transport in Diluted Species interface, the potential distribution shows up in the migration term.

Crosswind Diffusion

Transport of diluted species applications can often result in models with a very high cell Péclet number — that is, systems where convection or migration dominates over diffusion. Streamline diffusion and crosswind diffusion are of paramount importance to obtain physically reasonable results. The Transport of Diluted Species interface provides two crosswind diffusion options using different formulations. Observe that crosswind diffusion makes the equation system nonlinear even if the transport equation is linear.

DO CARMO AND GALEÃO

This is the formulation described in [Numerical Stabilization](#) in the *COMSOL Multiphysics Reference Manual*. The method reduces over- and undershoots to a minimum, even for anisotropic meshes.

In some cases, the resulting nonlinear equation system can be difficult to converge. This can happen when the cell Péclet number is very high and the model contains many thin layers, such as contact discontinuities. You then have three options:

- Refine the mesh, especially in regions with thin layers.
- Use a nonlinear solver with a constant damping factor less than one.
- Switch to the Codina crosswind formulation.

CODINA

The Codina formulation is described in [Ref. 1](#). It adds diffusion strictly in the direction orthogonal to the streamline direction. Compared to the do Carmo and Galeão formulation, the Codina formulation adds less diffusion but is not as efficient at reducing over- and undershoots. It also does not work as well for anisotropic meshes. The advantage is that the resulting nonlinear system is easier to converge and that underresolved gradients are less smeared out.

Danckwerts Inflow Boundary Condition

Constraining the composition to fixed values at an inlet to a reactor may sometimes result in issues with unreasonably high reaction rates or singularities at the inlet boundary. These problems may many times be mitigated by using a flux boundary condition instead, based on the upstream concentrations and the fluid velocity at the boundary. In chemical engineering, this type of flux boundary condition is also known as a Danckwerts condition.

Use the Danckwerts condition to specify inlet conditions for domains where high reaction rates are anticipated in the vicinity to the inlet ([Ref. 2](#)).

Given an upstream concentration $c_{i,0}$, the Danckwerts inflow boundary condition prescribed the total flux as

$$\mathbf{n} \cdot (\mathbf{J}_i + \mathbf{u}c_i) = \mathbf{n} \cdot (\mathbf{u}c_{i,0}) \quad (6-12)$$

VARIABLY SATURATED POROUS MEDIA

The following equations for the molar concentrations, c_i , describe the transport of solutes in a variably saturated porous medium for the most general case, when the pore space is primarily filled with liquid but also contain pockets or immobile gas:

$$\begin{aligned} \frac{\partial}{\partial t}(\theta_l c_i) + \frac{\partial}{\partial t}(\rho c_{p,i}) + \frac{\partial}{\partial t}(\theta_g c_{g,i}) + \mathbf{u} \cdot \nabla c_i = \\ \nabla \cdot [(D_{D,i} + D_{e,i}) \nabla c_i] + R_i + S_i \end{aligned} \quad (6-13)$$

On the left-hand side of Equation 6-13, the first three terms correspond to the accumulation of species within the liquid, solid, and gas phases, while the last term describes the convection due to the velocity field $\mathbf{u}$ (SI unit: m/s).

In Equation 6-13 c_i denotes the concentration of species i in the liquid (SI unit: mol/m³), $c_{p,i}$ the amount adsorbed to solid particles (moles per unit dry weight of the solid), and $c_{g,i}$ the concentration of species i in the gas phase.

The equation balances the mass transport throughout the porous medium using the porosity ϵ_p , the liquid volume fraction θ_l ; the dry bulk density, $\rho = (1 - \epsilon_p)\rho_s$, and the solid phase density ρ_s .

For saturated porous media, the liquid volume fraction θ_l is equal to the porosity ϵ_p , but for unsaturated porous media, they are related by the saturation s as $\theta_l = \epsilon_p s$. The resulting gas volume fraction in the case of an unsaturated porous medium is

$$\theta_g = \epsilon_p - \theta_l = (1 - s)\epsilon_p$$

On the right-hand side of Equation 6-13, the first term introduces the spreading of species due to mechanical mixing resulting from the porous media (dispersion), as well as from diffusion and volatilization to the gas phase. The dispersion tensor is denoted D_D (SI unit: m²/s) and the effective diffusion by D_e (SI unit: m²/s).

The last two terms on the right-hand side of Equation 6-13 describe production or consumption of the species; R_i is a reaction rate expression which can account for reactions in the liquid, solid, or gas phase, and S_i is an arbitrary source term, for example due to a fluid flow source or sink.

Adsorption

The time evolution of the adsorption, the solute transport to or from the solid phase, is defined by assuming that the amount of solute adsorbed to the solid, $c_{P,i}$, is a function of the concentration in the fluid c_i . This implies that the solute concentration in the liquid and solid phase are in instant equilibrium. The adsorption term can be expanded to give

$$\frac{\partial}{\partial t}(\rho c_{P,i}) = \rho \frac{\partial c_{P,i}}{\partial c_i} \frac{\partial c_i}{\partial t} - c_{P,i} \rho_s \frac{\partial \epsilon_P}{\partial t} = \rho K_{P,i} \frac{\partial c_i}{\partial t} - c_{P,i} \rho_s \frac{\partial \epsilon_P}{\partial t} \quad (6-14)$$

where $K_{P,i} = \partial c_{P,i} / \partial c_i$ is the adsorption isotherm.

Volatilization

Volatilization is the process where a solute species in the liquid is transported to the gas phase due to vaporization. Assuming that the amount of solute in the gas phase, $c_{G,i}$, is a linear function of the liquid phase concentration, the volatilization term is defined as

$$\frac{\partial}{\partial t} \theta_g c_{G,i} = \theta_g \frac{\partial c_{G,i}}{\partial c_i} \frac{\partial c_i}{\partial t} + k_{G,i} c_i \frac{\partial \theta_g}{\partial t} = \theta_g k_{G,i} \frac{\partial c_i}{\partial t} + k_{G,i} c_i \frac{\partial \theta_g}{\partial t} \quad (6-15)$$

where $k_{G,i} = \partial c_{G,i} / \partial c_i$ is the linear volatilization.

SATURATED POROUS MEDIA

In the case of transport in a saturated porous medium, $\theta_l = \epsilon_p$, and the governing equations are

$$\frac{\partial}{\partial t}(\epsilon_p c_i) + \frac{\partial}{\partial t}(\rho c_{P,i}) + \mathbf{u} \cdot \nabla c_i = \nabla \cdot [(D_{D,i} + D_{e,i}) \nabla c_i] + R_i + S_i \quad (6-16)$$

Convection in Porous Media

Convection (also called advection) describes the movement of a species, such as a pollutant, with the bulk fluid velocity. The velocity field $\mathbf{u}$ corresponds to a superficial volume average over a unit volume of the porous medium, including both pores and matrix. This velocity is sometimes called *Darcy velocity*, and defined as volume flow

rates per unit cross section of the medium. This definition makes the velocity field continuous across the boundaries between porous regions and regions with free flow.



The velocity field to be used in the Model Inputs section on the physics interface can, for example, be prescribed using the velocity field from a Darcy's Law or a Brinkman Equations interface.

The average linear fluid velocities $\mathbf{u}_a$, provides an estimate of the fluid velocity within the pores:

$$\mathbf{u}_a = \frac{\mathbf{u}}{\varepsilon_p} \quad \text{Saturated}$$

$$\mathbf{u}_a = \frac{\mathbf{u}}{\theta_l} \quad \text{Unsaturated}$$

where ε_p is the porosity and $\theta_l = s\varepsilon_p$ the liquid volume fraction, and s the saturation, a dimensionless number between 0 and 1.

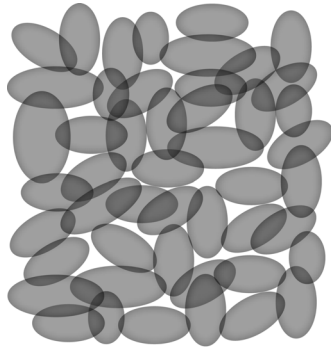


Figure 6-2: A block of a porous medium consisting of solids and the pore space between the solid grains. The average linear velocity describes how fast the fluid moves within the pores. The Darcy velocity attributes this flow over the entire fluid-solid face.

MATERIAL BALANCE FORMULATION

The Transport of Diluted Species interface in porous media includes two formulations of the material balance. The conservative formulation of the species equations in [Equation 6-13](#) is written as:

$$\frac{\partial}{\partial t}(\theta_l c_i) + \frac{\partial}{\partial t}(\rho c_{P,i}) + \frac{\partial}{\partial t}(\theta_g c_{G,i}) + \nabla \cdot \mathbf{u} c_i = \nabla \cdot [(D_{D,i} + D_{e,i}) \nabla c_i] + R_i + S_i \quad (6-17)$$

If the conservative formulation is expanded using the chain rule, then one of the terms from the convection part, $c_i \nabla \cdot \mathbf{u}$, would equal zero for an incompressible fluid and would result in the nonconservative formulation described in [Equation 6-13](#).

When using the nonconservative formulation, which is the default, the fluid is assumed incompressible and divergence free: $\nabla \cdot \mathbf{u} = 0$. The nonconservative formulation improves the stability of systems coupled to a momentum equation (fluid flow equation).



To switch between the two formulations, click the **Show** button () and select **Advanced Physics Options**. In the section **Advanced Settings** select either **Nonconservative** (the default) or **Conservative**. The conservative formulation should be used for compressible flow.

Diffusion in Porous Media

The effective diffusion in porous media, D_e , depends on the structure of the porous material and the phases involved. Dependent on whether the transport of diluted species occurs in free flow or in saturated or unsaturated porous media, the effective diffusivity is defined as:

$$\begin{aligned} D_e &= D_F && \text{Free Flow} \\ D_e &= \frac{\epsilon_p}{\tau_L} D_L && \text{Saturated Porous Media} \\ D_e &= \frac{\theta_l}{\tau_L} D_L && \text{Unsaturated Porous Media} \\ D_e &= \frac{\theta_l}{\tau_L} D_L + \frac{\theta_g}{\tau_G} k_G D_G && \text{Unsaturated with Volatilization} \end{aligned}$$

Here D_F , D_L , and D_G are the single-phase diffusion coefficients for the species diluted in a fluid, a pure liquid, and a gas phase, respectively (SI unit: m^2/s), and τ_F , τ_L , and τ_G are the corresponding tortuosity factors (dimensionless).

The tortuosity factor accounts for the reduced diffusivity due to the fact that the solid grains impede Brownian motion. The interface provides predefined expressions to

compute the tortuosity factors in partially saturated porous media according to the Millington and Quirk model (Ref. 3):

$$\tau_L = \theta_l^{-7/3} \epsilon_p^2, \tau_G = \theta_g^{-7/3} \epsilon_p^2$$

and Bruggeman model

$$\tau_L = \theta_l^{-5/2} \epsilon_p^2, \tau_G = \theta_g^{-5/2} \epsilon_p^2$$

For saturated porous media $\theta_l = \epsilon_p$. The fluid tortuosity for the Millington and Quirk model is

$$\tau_L = \epsilon_p^{-1/3}$$

and for the Bruggeman model the tortuosity is defined as

$$\tau_L = \epsilon_p^{-1/2}$$

User defined expressions for the tortuosity factor can also be applied.

Dispersion

The contribution of dispersion to the mixing of species typically overshadows the contribution from molecular diffusion, except when the fluid velocity is very low.

The spreading of mass, as a fluid travel through a porous medium is caused by several contributing effects. Local variations in fluid velocity lead to mechanical mixing referred to as dispersion occurs because the fluid in the pore space flows around solid particles, so the velocity field varies within pore channels. The spreading in the direction parallel to the flow, or *longitudinal dispersivity*, typically exceeds the *transverse dispersivity* from up to an order of magnitude. Being driven by the concentration gradient alone, molecular diffusion rate is low relative to the mechanical dispersion, except at very low fluid velocities.

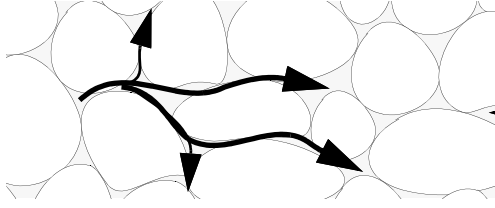


Figure 6-3: Spreading of fluid around solid particles in a porous medium.

The dispersion is controlled through the dispersion tensor D_D . The tensor components can either be given by user-defined values or expressions or derived from the directional dispersivities.

Using the longitudinal and transverse dispersivities in 2D, the dispersivity tensor components are (Ref. 5):

$$D_{Dii} = \alpha_L \frac{u_i^2}{|\mathbf{u}|} + \alpha_T \frac{u_j^2}{|\mathbf{u}|}$$

$$D_{Dij} = D_{Dji} = (\alpha_L - \alpha_T) \frac{u_i u_j}{|\mathbf{u}|}$$

In these equations, D_{Dii} (SI unit: m^2/s) are the principal components of the dispersivity tensor, and D_{Dji} and D_{Dij} are the cross terms. The parameters α_L and α_T (SI unit: m) specify the longitudinal and transverse dispersivities; and u_i (SI unit: m/s) stands for the velocity field components.

In order to facilitate modeling of stratified porous media in 3D, the tensor formulation by Burnett and Frind (Ref. 4) can be used. Consider a transverse isotropic media, where the strata are piled up in the z direction, the dispersivity tensor components are:

$$\begin{aligned}
D_{Lxx} &= \alpha_1 \frac{u^2}{|\mathbf{u}|} + \alpha_2 \frac{v^2}{|\mathbf{u}|} + \alpha_3 \frac{w^2}{|\mathbf{u}|} \\
D_{Lyy} &= \alpha_1 \frac{v^2}{|\mathbf{u}|} + \alpha_2 \frac{u^2}{|\mathbf{u}|} + \alpha_3 \frac{w^2}{|\mathbf{u}|} \\
D_{Lzz} &= \alpha_1 \frac{w^2}{|\mathbf{u}|} + \alpha_2 \frac{u^2}{|\mathbf{u}|} + \alpha_3 \frac{v^2}{|\mathbf{u}|} \\
D_{Lxy} &= D_{Lyx} = (\alpha_1 - \alpha_2) \frac{uv}{|\mathbf{u}|} \\
D_{Lxz} &= D_{Lzx} = (\alpha_1 - \alpha_3) \frac{uw}{|\mathbf{u}|} \\
D_{Lyz} &= D_{Lzy} = (\alpha_1 - \alpha_3) \frac{vw}{|\mathbf{u}|}
\end{aligned} \tag{6-18}$$

In Equation 6-18 the fluid velocities u , v , and w correspond to the components of the velocity field $\mathbf{u}$ in the x , y , and z directions, respectively, and α_1 (SI unit: m) is the longitudinal dispersivity. If z is the vertical axis, α_2 and α_3 are the dispersivities in the transverse horizontal and transverse vertical directions, respectively (SI unit: m). Setting $\alpha_2 = \alpha_3$ gives the expressions for isotropic media shown in Bear (Ref. 5 and Ref. 6).

Adsorption

As species travel through a porous medium they typically attach to (adsorb), and detach (desorb) from the solid phase, which slows chemical transport through the porous medium. Adsorption and desorption respectively reduces or increases species concentrations in the fluid. The adsorption properties vary between chemicals, so a plume containing multiple species can separate into components (Ref. 7). The Adsorption feature includes four predefined and one user-defined relationships to predict the solid concentrations, c_{pi} from the concentration in the liquid phase, c_i :

$$\begin{aligned}
c_P &= c_{P\max} \frac{K_L c}{1 + K_L c} && \text{Langmuir} \\
c_P &= K_F \left(\frac{c}{c_{\text{ref}}} \right)^N && \text{Freundlich} \\
c_P &= c_{P\max} \frac{b_T c}{(1 + (b_T c)^{N_T})^{1/N_T}} && \text{Toth} \\
c_P &= \frac{K_B c_0 c}{(c_S - c) \left(1 + (K_B - 1) \frac{c}{c_S} \right)} && \text{BET}
\end{aligned} \tag{6-19}$$

The above equations contains the following parameters:

- Freundlich ([Ref. 8](#))
 - Freundlich constant K_F (SI unit: mol/kg), Freundlich exponent N_F (dimensionless), and reference concentration c_{ref} (SI unit: mol/m³).
- Langmuir ([Ref. 9](#))
 - Langmuir constant K_L (SI unit: m³/mol), and adsorption maximum $c_{P\max}$ (SI unit: mol/kg).
- Toth ([Ref. 10](#))
 - Toth constant b_T (SI unit: m³/mol), Toth exponent N_T (dimensionless), and adsorption maximum $c_{P\max}$ (SI unit: mol/kg).
- BET (Brunauer–Emmett–Teller, [Ref. 11](#))
 - BET constant K_B (dimensionless), the monolayer adsorption capacity c_0 (SI unit: mol/kg) and the saturation concentration, c_S (SI unit: mol/m³).

These predefined expressions are adsorption isotherms that describe the amount of species sorbed to the solid. Applying the isotherms, as defined in the [Adsorption](#) section above, assumes that adsorption is at equilibrium locally. This implies that the species phase transfer, between the fluid and the solid phase (within the porous medium), is much faster than the transport within the fluid phase.

Using a [Species Source](#) feature, arbitrary expressions can be entered to define, for example, nonequilibrium and temperature-dependent adsorption laws, in the manner of Fetter ([Ref. 12](#)), or Bear and Verruijt ([Ref. 13](#)).

The retardation factor, **RF**, describes how adsorption slows the solute velocity, $\mathbf{u}_c$, relative to the average linear velocity of the fluid, $\mathbf{u}_a$, as in

$$\text{RF} = 1 + \frac{\rho_b}{\theta} \frac{\partial c_p}{\partial c} = \frac{\mathbf{u}_a}{\mathbf{u}_c}$$

If the contaminant moves at the average linear velocity of the fluid for $\text{RF} = 1$. For $\text{RF} > 1$, the contaminant velocity is lower than the fluid velocity owing to residence time on solids.

Reactions

Chemical reactions of all types influence species transport in porous media. Examples include biodegradation, radioactive decay, transformation to tracked products, temperature- and pressure-dependent functions, exothermic reactions, and endothermic reactions. The reactions represent change in species concentration per unit volume porous medium per time. Reaction terms are used on the right-hand side of the governing equation to represent these processes. For reactions in a fluid phase, multiply the expression by the fluid volume fraction θ . Similarly, solid phase reaction expressions include the bulk density, ρ_b , and gas phase reactions include the gas volume fraction, α_v .

The following expressions define some common types of reactions:

$$\begin{aligned} R_{Li} &= -\theta \frac{\ln 2}{\lambda_{Li}} c_i && \text{Radioactive decay — liquid} \\ R_{Pi} &= -\rho_b \frac{\ln 2}{\lambda_{Pi}} \left(\frac{\partial c_{Pi}}{\partial c_i} \right) c_i && \text{Radioactive decay — solid} \\ R_{Gi} &= -\frac{\ln 2}{\lambda_{Gi}} \alpha_v \left(\frac{\partial c_{Gi}}{\partial c} \right) c_i && \text{Radioactive decay — gas} \\ R_{Lk} &= \theta \zeta_{Li} c_i && \text{Creation from parent } c_{Li} \text{ — liquid} \\ R_{Pk} &= \rho_b \zeta_{Pi} \left(\frac{\partial c_{Pi}}{\partial c_i} \right) c_i && \text{Creation from sorbed parent } c_{Pi} \text{ — solid} \\ R_{Gk} &= -\zeta \alpha_v \left(\frac{\partial c_{Gi}}{\partial c} \right) c_i && \text{Reaction — gas} \end{aligned}$$

where λ is the chemical half life, ζ is a reaction rate, and the subscripts L , P , and G denote liquid, solid, and gas phases, respectively. In the equations, the reactions either depend on liquid concentration c_i or solid phase concentrations c_{Pi} obtained using the sorption derivative with c_i or gas phase concentration c_{Gi} depending on the gas volume fraction, the volatilization, and the liquid concentration.

Reaction rates can vary with results from other equations in your model, such as temperature. For example, enter the Arrhenius rate law given in [Ref. 14](#):

$$\zeta_T = \zeta_R \exp \left[\frac{E_a(T - T_R)}{R_u T T_R} \right] \quad (6-20)$$

In [Equation 6-20](#), T denotes the current absolute temperature, T_R denotes the reference absolute temperature, E_a is the activation energy, and R_u is the universal gas constant.

Mass Transport in Fractures

When thin fractures occur in porous media, fluid flow tends to move faster along the fracture than in the surrounding media. The transport of chemical species therefore also occur also faster in the direction of the fractures.

The fluid flow in a fracture can be modeled using Darcy's law formulated in a thin sheet of porous medium (a fracture):

$$\mathbf{u} = \frac{\kappa}{\mu} \nabla_t p$$

Here $\mathbf{u}$ is the tangential Darcy velocity, κ is the fracture permeability, μ the fluid's dynamic viscosity, and $\nabla_t p$ is the tangential gradient of the fluid pressure.

The equation to solve for mass transport of species c_i in a thin fracture, embedded in a porous media, is derived from [Equation 6-13](#). The resulting equation is:

$$d_{fr} \left(\frac{\partial \rho_b c_{P,i}}{\partial t} + \frac{\partial \varepsilon_p c_i}{\partial t} + \nabla_t \cdot (D_{e,i} \nabla_t c_i) + \mathbf{u} \cdot \nabla_t c_i \right) = d_{fr} R_i + d_{fr} S_i + n_0 \quad (6-21)$$

Here d_{fr} is the fracture thickness, $c_{P,i}$ the amount of species adsorbed to (or desorbed from) the porous matrix (moles per unit dry weight of the solid), ε_p is the fracture porosity, and D_e is the effective diffusivity. The first two terms on the right hand side represent source terms from reactions, and n_0 corresponds to out-of plane flux from the adjacent porous domain.

In order to arrive at the tangential differential equation, the gradient is split into the contributions normal and tangential to the fracture:

$$\nabla c_i = \nabla_n c_i + \nabla_t c_i$$

The normal gradient is defined in the direction normal to the boundary representing the fracture and the tangential gradient is defined along the boundary. Assuming that the variations in the normal (thin) direction of the fracture are negligible compared to those in the tangential direction, the gradient is simplified as:

$$\nabla c_i = \nabla_t c_i$$

Using [The Transport of Diluted Species in Fractures Interface](#), the transport along fracture boundaries alone is solved for. In this case the transport in the surrounding porous media neglected and the out-of plane flux n_0 vanishes.



See [Fracture](#) for more information about the boundary feature solving [Equation 6-21](#). See [The Transport of Diluted Species in Fractures Interface](#) for more information about the physics interface solving the equation on boundaries only.

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Heat Transfer Interfaces

This chapter has information about the heat transfer interfaces used for modeling, particularly in porous media. As with all other physical descriptions simulated by COMSOL Multiphysics®, any description of heat transfer can be directly coupled to any other physical process. This is particularly relevant for systems based on chemical reactions and mass transfer along with fluid flow. These physics interfaces are found under the **Heat Transfer** branch () when adding a physics interface.

In this chapter:

- [The Mechanisms for Heat Transfer](#)

The documentation of all features in the Heat Transfer and Heat Transfer in Porous Media interfaces are available in the *Heat Transfer Module User's Guide*, specifically in these sections:

- [The Heat Transfer Module Interfaces](#)
- [The Heat Transfer in Porous Media Interface](#)
- [The Heat Transfer in Fractures Interface](#)
- [Theory for the Heat Transfer Module](#)
- [Theory for Heat Transfer in Porous Media](#)
- [Theory for Heat Transfer in Thin Structures](#)

The Mechanisms for Heat Transfer

This section includes these topics:

- [Selecting the Right Heat Transfer Interface](#)
- [Heat Conduction](#)
- [Heat Convection and Conduction](#)



- [The Heat Transfer Module Interfaces](#)
- [Theory for the Heat Transfer Module](#)
- [The Heat Transfer in Porous Media Interface](#)
- [Theory for Heat Transfer in Porous Media](#)
- [The Heat Transfer in Fractures Interface](#)
- [Theory for Heat Transfer in Thin Structures](#)

Selecting the Right Heat Transfer Interface

The Heat Transfer branch included with the Subsurface Flow Module license includes a number of subbranches to describe energy transport.

Heat transfer through conduction and convection (both nonisothermal flow and conjugate heat transfer) in solid and free media is supported by physics interfaces shipped with the basic COMSOL Multiphysics license.

If you are using this module with enhanced Heat Transfer interface features, see also the Heat Transfer Module documentation for additional information. See [Where Do I Access the Documentation and Application Libraries?](#)

This module also includes [The Heat Transfer in Porous Media Interface](#). The physics interface itself supports heat transfer in porous media where the thermal conductivity and heat capacity are affected by the extent of porosity. As a porous media is made up of a solid matrix filled with a fluid, heat is transported at different rates through the two types of media. The Heat Transfer in Porous Media interface provides tools and support for defining an appropriate matrix model.

The Heat Transfer in Porous Media interface characterizes temperature distributions for geological phenomena and freely couple to other features in a model. The physics

interface applies to systems consisting of porous media, solids, and fluids. Included are physics interfaces to calculate effective properties for porous media consisting of fluids and solid components, or a rock formation with different mineral proportions. The physics interface also features predefined expressions to represent the geotherm as a radiogenic heat source.

The **Heat Transfer in Fractures (htlsh)** interface () , found in the **Thin Structures** physics area under the **Heat Transfer** branch () , is used to model heat transfer by conduction and convection in layered materials represented by boundaries. The interface is active on all boundaries where a layered material is defined, with a **Porous Medium** model active by default. All functionalities for including other boundary contributions, such as surface-to-ambient radiation, are also available.

Heat Conduction

The Heat Transfer interface in this module provides tools for analyzing heat transfer that is proportional to a temperature gradient, or conduction. It accounts explicitly for the geotherm as a heat source. This physics interface provides options to calculate thermal properties for multicomponent media. It incorporates boundary and source options to represent transversal fluxes such as convection and radiation at adjacent domains that you do not explicitly model.

The physics interface describes heat flow with negligible impacts of moving fluids. Just a few targets for conduction modeling include resistive heating in cores, estimating a surface heat flux, describing a temperature profile with depth, phase changes, exothermic reactions, and cooling earth analyses. The physics interface provides tools to consider radiative and convection heat transfers at boundaries so that you can focus on the physics in the domain of interest. With the COMSOL Multiphysics “just-type-it-in” modeling flexibility, it is straightforward to create nonlinear expressions where, for example, thermal conductivity varies with temperature as well as other physics.

Heat conduction figures into radiogenic decay, pressure-temperature phase changes, cooling-earth models, radiation, exothermic and endothermic reactions of solutes, microbial processes, diurnal heating, and many other earth processes. Conductive heat transfer can be long-term steady or exquisitely sensitive over tiny time increments. It can involve molten materials, mineral grains, fluids trapped within interstices, human-made structures, and molten rock. The heat conduction can operate within closed systems, but the geometry of interest often interacts at edges and surfaces with adjacent domains through moving fluids, conduction across a semi-insulating layer,

and radiation. The models can cover such large distances that the tiny amount of heat given off by the spontaneous decay of the radiogenic particles present in most rocks produces the discernible temperature gradient with depth known to many as the geotherm.

Heat Convection and Conduction

The [Heat Transfer in Porous Media Interface](#) can also set up analyses for heat transferred by convection and conduction for subsurface flow applications. Use this physics interface to describe heat carried by moving oil, water, or magma that you describe with a velocity field. Presumably the fluid velocity is nonzero but not necessarily so. You can couple the Heat Transfer interface to a Fluid Flow interface if you want to model, for example, rising magma, hot springs, liquid-steam transfers, conduction in a solid rock, buoyancy flow in streams, magma convection, and hot oil moving through a pipe. For models involving large depth changes, this physics interface provides a number of options to characterize the geothermal gradient.

The physics interface assumes that the model domain includes a single fluid moving through a domain with a number of immobile constituents, including several different solids and trapped fluids. Such is the case in many porous media.

The physics interface can also describe heat transferred with a moving fluid, which can be a surface flow or one confined to interstices in a porous medium. Analyses can cover thermal pollution migrating in a stream, transfer of hot oil into and up a well, steam injection, and diurnal heating in variably saturated soil.

In this context it is worth mentioning that it is possible to model inflow of heat through a virtual domain at inlet boundaries of fluid domains using the so-called [Inflow](#) boundary condition. Accordingly, an [Outflow](#) Boundary condition is available at the outlet. Furthermore, an [Open Boundary](#) condition is available to define a boundary as the limit between a nonsolid domain of the geometry and the rest of the same fluid domain that is not represented in the geometry. At the open boundary both inflow and outflow conditions are supported.

This physics interface easily combines with others for an unlimited number of interesting analyses. Consider, for example, that it is straightforward to include the impacts of temperature change on hydraulic conductivity, biodegradation and chemical reaction rates, electric and magnetic potential fields, and rock strength. Likewise, you can feed back results of other physics, including reactions that consume heat, to a heat transfer model.

Phase Change

When modeling heat transfer in solids, fluids, and porous media, the properties of a phase changing material are specified according to the apparent heat capacity formulation. This formulation gets its name from the fact that the latent heat is included as an additional term in the heat capacity.

The subnode **Phase Change Material** is available under the Solid, Fluid, or Porous Medium node.

With phase transitions in porous media, there is often a remaining phase that does not participate in the phase transition. For example, water in the soil, which does not freeze even at values far lower than 0 degrees, or water in food, which does not evaporate completely during cooking. To consider such residual phases [Immobile Fluids \(Porous Medium\)](#) can be used.



Frozen Inclusion: Application Library path
Subsurface_Flow_Module/Heat_Transfer/frozen_inclusion

A more detailed description of the functionalities mentioned above and the discussion of a variety of modeling techniques can be found in the following sections of the *Heat Transfer Module User's Guide*:

- [Heat Transfer Variables](#)
- [Using the Boundary Conditions for the Heat Transfer Interfaces](#)
- [Handling Frames in Heat Transfer](#)
- [Heat Transfer Consistent and Inconsistent Stabilization Methods](#)
- [Heat Transfer and Fluid Flow Coupling](#)
- [Solver Settings](#)
- [Theory for the Inflow Boundary Condition](#)
- [Theory for Heat Transfer with Phase Change](#)

See also [The Heat Transfer Module Interfaces](#) and [Theory for the Heat Transfer Module](#).

Multiphysics Interfaces and Couplings

The Subsurface Flow Module contains predefined multiphysics interfaces for poroelasticity and multiphase flow in porous media. When a predefined multiphysics interface is added from the **Model Wizard** or **Add Physics** window, it adds the constituent interfaces and the **Multiphysics** node which automatically includes one or more multiphysics couplings.

In this chapter:

- [Theory for the Poroelasticity Interfaces](#)
- [The Poroelasticity, Solid Interface](#)
- [Theory for the Multiphase Flow in Porous Media Interface](#)
- [The Multiphase Flow in Porous Media Interface](#)
- [The Reacting Flow in Porous Media Interface](#)
- [The Nonisothermal Flow, Brinkman Equations Interface](#)

Theory for the Poroelasticity Interfaces

Small Strain Poroelasticity

Poroelasticity theory describes the interaction between fluid flow and deformation in elastic porous media. This bidirectional multiphysics coupling relates fluid flow and solid deformation with linear constitutive relations. Isothermal conditions are assumed.

Of the two constitutive relations governing poroelastic behavior, one relates the stress, strain, and pore pressure:

$$\boldsymbol{\sigma} = \mathbf{C}:\boldsymbol{\varepsilon} - \alpha_B p_f \mathbf{I} \quad (8-1)$$

Here, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{\varepsilon}$ is the strain tensor, α_B is the *Biot-Willis coefficient*, and p_f is the fluid pore pressure. For linear elasticity, the coefficients in the elasticity matrix $\mathbf{C}$ must be measured under “drained” conditions by measuring the strain induced by a change in stress (or by measuring the induced stress by changes in strain) under constant pore pressure.

By splitting the above relation into volumetric and deviatoric parts, it can be seen that the deviatoric part (shear stress) is independent of the pore pressure. For an isotropic linear elastic material, this means that

$$\text{dev}(\boldsymbol{\sigma}) = 2G_d \text{dev}(\boldsymbol{\varepsilon})$$

where G_d is the shear modulus of the drained porous matrix.

The volumetric coupling is derived by taking the trace of the stress tensor $\boldsymbol{\sigma}$, as written in [Equation 8-1](#)

$$-p_m = \text{trace}(\boldsymbol{\sigma})/3 = K_d \varepsilon_{\text{vol}} - \alpha_B p_f$$

Here, the mean pressure p_m is positive in compression, K_d is the bulk modulus of the drained porous matrix, and the volumetric strain ε_{vol} is a measurement of the porous matrix dilation or contraction (negative in compression).

Note that the **Poroelasticity** multiphysics coupling adds the contribution from the pore pressure, $\alpha_B p_f$, as a load. In this setting, the variables for the stress tensor σ and mean pressure p_m are *effective stress* measures.



The mean pressure and the volumetric strain are available from the Solid Mechanics interface in the variables `solid.pm` and `solid.evol`, respectively. The fluid pore pressure is available from the Darcy's Law interface in the dependent variable `p`.

The other constitutive relation in Biot's theory of poroelasticity ([Ref. 1](#)) relates the increment in fluid content ζ to changes in volumetric strain and pore pressure. The pore pressure is proportional to the dilation of the porous matrix and the variation in fluid content:

$$p_f = M(\zeta - \alpha_B \epsilon_{vol}) \quad (8-2)$$

Biot and Willis ([Ref. 1](#)) measured the coefficients α_B and M with the unjacketed compressibility test and derived expressions for these coefficients in terms of solid and fluid bulk moduli (or compressibilities).

The variable M , sometimes called the Biot modulus, is the inverse of the storage coefficient S_p in the Darcy's law interface. It is defined through [Equation 8-2](#) as the change in fluid content due to changes in pore pressure under constant deformation (constant volumetric strain):

$$S_p = \frac{1}{M} = \left. \frac{\partial \zeta}{\partial p_f} \right|_{\epsilon_{ii}}$$

Using this definition, it is possible to measure the storage coefficient S_p directly in the lab, and, in the case of an ideal isotropic porous material, it can be calculated from basic material properties as

$$S_p = \frac{\epsilon_p}{K_f} + \frac{\alpha_B - \epsilon_p}{K_s} = \epsilon_p \chi_f + (\alpha_B - \epsilon_p) \chi_s \quad (8-3)$$

where ϵ_p is the initial porosity, K_f the fluid bulk modulus (the inverse of the fluid compressibility χ_f), and K_s the solid bulk modulus (the inverse of the solid compressibility χ_s); that is, the theoretical bulk modulus of a homogeneous block of the solid material making up the porous matrix.

The Biot–Willis coefficient, α_B , relates the volume of fluid expelled (or sucked into) a porous material element due to the volumetric change of the same element. This coefficient can be measured experimentally as the change in mean pressure due to changes in the pore pressure under constant deformation (constant volumetric strain). In the case of an ideal isotropic porous material it can be defined in terms of the drained and solid bulk moduli as

$$\alpha_B = \left. \frac{\partial p_m}{\partial p_f} \right|_{\epsilon_{ii}} = 1 - \frac{K_d}{K_s}$$

The drained bulk modulus K_d is always smaller than the solid bulk modulus K_s (a solid block is stiffer than a porous block made of the same material), and therefore the Biot–Willis coefficient is bounded to $\epsilon_p \leq \alpha_B \leq 1$.

The Biot–Willis coefficient depends on the properties of the porous matrix but not on the properties of the fluid. A soft porous matrix is represented by a Biot–Willis coefficient close to unit (since $K_d \ll K_s$), while for a stiff matrix, α_B is close to the porosity ϵ_p , since $K_d \approx (1 - \epsilon_p)K_s$.

By replacing $K_s = K_d/(1 - \alpha_B)$, the storage coefficient S_p in Equation 8-3 is calculated in terms of the porosity ϵ_p , the Biot–Willis coefficient α_B , and the bulk moduli of the fluid K_f and the drained porous matrix K_d as

$$S_p = \frac{\epsilon_p}{K_f} + (\alpha_B - \epsilon_p) \frac{(1 - \alpha_B)}{K_d} \quad (8-4)$$

Both for a soft porous matrix, $\alpha_B \approx 1$, and for a stiff porous matrix, $\alpha_B \approx \epsilon_p$, the lower bound for the poroelastic storage is $S_p \approx \epsilon_p/K_f$.

A special case of Biot’s theory of poroelasticity arises if both the solid and the fluid phases are assumed incompressible. In this case, it follows that $\chi_f = \chi_s = 0$ with the Biot–Willis coefficient $\alpha_B = 1$ and the storage coefficient $S_p = 0$. The corresponding governing equations are then identical to those derived for a biphasic porous medium with incompressible solid and fluid constituents (Ref. 3). This version of the poroelasticity coupling is available by selecting the **Biphasic** poroelasticity model, see [Poroelastic Coupling Properties](#).

The linear momentum balance for a fully saturated porous solid in equilibrium under gravitational load reads

$$-\nabla \cdot \boldsymbol{\sigma} = -\nabla(\alpha_B p_f) + \rho_{av} \mathbf{g} \quad (8-5)$$

where σ is the stress tensor computed from linear elasticity, and $\mathbf{g}$ is the gravitational acceleration. The average density ρ_{av} is computed from

$$\rho_{av} = \rho_f \epsilon_p + \rho_d$$

where ρ_f represents the fluid density, $\rho_d = (1 - \epsilon_p)\rho_s$ denotes the density of the dry porous matrix, and ρ_s is the density of the solid material.



The density of the dry porous matrix, ρ_d , and the drained bulk modulus, K_d , correspond to the user inputs for the density and bulk modulus in the Solid Mechanics interface.

The fluid-to-structure coupling enters as an additional volumetric force in the momentum equation as described in [Equation 8-5](#), which can be identified as an external load due to the fluid pressure, see [External Stress](#) in the *Structural Mechanics Module User's Guide* for details.

[Equation 8-5](#), which describes an equilibrium state (inertial effects are neglected), also applies to the case of a time-dependent flow model. This is a valid assumption in geotechnical and biological applications since the time scale of the inertial response is generally many orders of magnitude faster than the time scale of the flow. When you study the coupled process on the time scale of the flow, you can therefore assume that the solid reaches a new equilibrium immediately in response to a change in the flow conditions. This means that the stresses and strains change in time — even if [Equation 8-5](#) appears to be stationary — and that the structure-to-fluid coupling term, involving the rate of strain, is nonzero.

When **Include inertial terms** is selected, the average density is used in the inertial term for the momentum equation. In the presence of a volume force $\mathbf{f}$ (for example, gravity), the momentum balance then reads

$$\rho_{av} \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla \cdot \sigma = -\nabla(\alpha_B p_f) + \mathbf{f} \quad (8-6)$$

where $\mathbf{u}$ represents the solid displacement field.

Inertial effects are also included when modeling waves in saturated porous media, see the [Theory for the Poroelastic Waves Interfaces](#) section in the *Acoustics Module User's Guide* for details.

Darcy's Law

Darcy's law describes the flow and pore pressure fields in the poroelastic medium. The fluid equation comes from the mass conservation

$$\frac{\partial}{\partial t}(\rho_f \epsilon_p) + \nabla \cdot (\rho_f \mathbf{v}_d) = Q_m \quad (8-7)$$

where ρ_f is the fluid density, ϵ_p is the porosity, and Q_m is an external mass sink or source. The so-called Darcy velocity $\mathbf{v}_d$ is computed from Darcy's law

$$\mathbf{v}_d = -\frac{\kappa}{\mu} \nabla p_f$$

where κ is the permeability of the porous matrix, μ is the fluid viscosity, and p_f is the pore pressure. If gravity is included, Darcy's velocity is augmented by

$$\mathbf{v}_d = -\frac{\kappa}{\mu} (\nabla p_f - \rho_f \mathbf{g})$$

where $\mathbf{g}$ is the gravitational acceleration. Note that the Darcy velocity $\mathbf{v}_d$ is actually the relative velocity of the fluid with respect to the pore walls weighted by the porosity,

$$\mathbf{v}_d = \epsilon_p (\mathbf{v}_f - \mathbf{v}_s)$$

With the help of Equation 8-2, the first term in Equation 8-7 can be written

$$\frac{\partial}{\partial t}(\rho_f \epsilon_p) = \rho_f S_p \frac{\partial p_f}{\partial t} + \rho_f \alpha_B \frac{\partial \epsilon_{vol}}{\partial t}$$

where the poroelastic storage term S_p is calculated from Equation 8-4 for Biot poroelasticity and is equal to zero for biphasic poroelasticity. The second term is a mass source or sink

$$Q_{vol} = -\rho_f \alpha_B \frac{\partial}{\partial t} \epsilon_{vol}$$

where $\partial \epsilon_{vol} / \partial t$ is the rate of change in volumetric strain (of the porous matrix), ρ_f is the fluid density, and α_B is the Biot–Willis coefficient. You can interpret the right-hand-side term as the rate of expansion of the pore space. As ϵ_{vol} increases with time, the volume fraction available for the fluid increases and thereby gives rise to a fluid sink; as indicated by the negative sign in the source term Q_{vol} .

The poroelastic storage together with the expression for the mass source translate the mass conservation equation, Equation 8-7, into

$$\rho_f \mathbf{S}_p \frac{\partial p_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{v}_d) = -\rho_f \alpha_B \frac{\partial}{\partial t} \varepsilon_{\text{vol}} + Q_m \quad (8-8)$$

Large Strain Poroelasticity

When a porous body experiences large deformations (large strains or large rotations), the infinitesimal theory cannot be used. In a large deformation context, the momentum equation, [Equation 8-6](#), reads

$$\rho_{\text{av}} \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla \cdot \mathbf{P}^T = \nabla \cdot \mathbf{P}_p^T + \mathbf{f} \quad (8-9)$$

where the gradient $\nabla = \partial/\partial \mathbf{X}$ corresponds to the material gradient with respect to the undeformed configuration, and $\mathbf{P}$ is the (effective) first Piola–Kirchhoff stress tensor. See [Equation of Motion](#) in the *Structural Mechanics Theory* chapter for details.

The pore pressure contribution enters as a load in [Equation 8-9](#). The first Piola–Kirchhoff stress generated from the pore pressure reads

$$\mathbf{P}_p = -\alpha_B p_f \mathbf{J} \mathbf{F}^{-T} \quad (8-10)$$

where $\mathbf{F}$ is the deformation gradient and $\mathbf{J}$ is the volume ratio. In biphasic poroelasticity, the total stress in the mixture is often of interest. The corresponding Cauchy stress is computed as

$$\boldsymbol{\sigma}_{\text{mix}} = \boldsymbol{\sigma} - p_f \mathbf{I} \quad (8-11)$$

where $\boldsymbol{\sigma}$ is the (effective) Cauchy stress computed in the Solid Mechanics interface.

Darcy’s law is also written with respect to the undeformed configuration. The material velocity field reads

$$\mathbf{v}_d = -\frac{\kappa}{\mu} \nabla p_f$$

where κ is the material permeability tensor of the porous matrix, μ is the fluid viscosity, and p_f is the pore pressure. The symbol $\nabla = \partial/\partial \mathbf{X}$ corresponds to the gradient with respect to the undeformed configuration.

Darcy’s velocity in the current (deformed) configuration is then computed with the so-called *Piola transformation* ([Ref. 2](#))

$$\mathbf{v}_d = \frac{1}{J} \mathbf{F} \cdot \mathbf{V}_d$$

For numerical reasons, and also since the solid mechanics equations are solved in a *total Lagrangian formulation* (see [Lagrangian Formulation](#) for details), it is more efficient to write the mass conservation equation for the fluid in the undeformed configuration. It is then given by ([Ref. 2](#))

$$\frac{\partial}{\partial t}(\phi \rho_f) + \nabla \cdot (\rho_f \mathbf{V}_d) = Q_m \quad (8-12)$$

where $\phi = J \varepsilon_p$ is the Lagrangian porosity and Q_m is an external mass source or sink per unit reference volume. The material time derivative can be expanded as

$$\frac{\partial(J \varepsilon_p \rho_f)}{\partial t} = \rho_f \varepsilon_p \dot{J} + \rho_f J \dot{\varepsilon}_p + J \varepsilon_p \dot{\rho}_f \quad (8-13)$$

The current porosity is derived from the mass conservation equation of the solid. For Biot poroelasticity, it is expressed in terms of the volume change J and the pore pressure p_f as

$$\frac{\dot{\varepsilon}_p}{(\alpha_B - \varepsilon_p)} = \frac{\dot{J}}{J} + \frac{1}{K_s} \dot{p}_f$$

For biphasic poroelasticity, the current porosity has the simple closed-form expression

$$\varepsilon_p = 1 - \frac{1 - \varepsilon_0}{J}$$

where ε_0 stands for the initial (undeformed) porosity.

[Equation 8-13](#) can now be expressed as

$$\frac{\partial(J \varepsilon_p \rho_f)}{\partial t} = \rho_f \alpha_B \dot{J} + \rho_f J S_p \dot{p}_f$$

The poroelastic storage ([Equation 8-3](#)) in a large strain context still reads

$$S_p = \frac{\varepsilon_p}{K_f} + \frac{\alpha_B - \varepsilon_p}{K_s} \quad (8-14)$$

with the important difference that the current porosity in the deformed matrix, ε_p , is used. The fluid mass conservation [Equation 8-12](#) is then formulated as

$$\rho_f J S_p \frac{\partial p_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{V}_d) = -\rho_f \alpha_B \frac{\partial J}{\partial t} + Q_m \quad (8-15)$$

Similarly, the biphasic poroelasticity formulation reads

$$\nabla \cdot (\rho_f \mathbf{V}_d) = -\rho_f \frac{\partial J}{\partial t} + Q_m \quad (8-16)$$

since the first term on the left hand side vanishes ($S_p = 0$) and the Biot–Willis coefficient equals 1.



The choice between the large strain and small strain formulation is normally governed by the **Include geometric nonlinearity** checkbox in the study step. If some feature in the model forces geometric nonlinearity, such as **Contact**, but the strains and rotations are expected to be small, it is still possible to let the **Poroelasticity** node use the small strain formulation. For this, select **Geometrically linear** in the **Formulation** list under the **Linear Elastic Material** node within the Solid Mechanics interface to enforce a small strain formulation.

Then, Darcy's law is implemented according to the small strain formulation described by [Equation 8-8](#), the poroelastic storage coefficient S_p follows the small strain formulation described in [Equation 8-3](#), and the initial porosity ε_0 is used instead of the current porosity ε_p .

Unsaturated Poroelasticity

For unsaturated poroelasticity, the pore pressure is computed from the volume average of the liquid water and moist air pressures, p_l and p_m

$$p_f = s_l p_l + s_m p_m$$

here, s_l is the liquid saturation, and $s_m = 1 - s_l$ is the moist air saturation.

The liquid water and moist air pressures are computed by solving the continuity equations for dry air and moisture content

$$s_m \rho_a S_p^a \frac{\partial p_a}{\partial t} + \nabla \cdot \Gamma_a = \varepsilon_p \rho_a \frac{\partial s_l}{\partial t} - s_m \rho_a \alpha_B \frac{\partial \varepsilon_{vol}}{\partial t} - s_m \rho_a S_p^s \frac{\partial p_f}{\partial t}$$

$$s_m \rho_v S_p^v p_{v, \text{sat}} \frac{\partial \phi_w}{\partial t} + \nabla \cdot \Gamma_w = \varepsilon_p (\rho_v - \rho_l) \frac{\partial s_l}{\partial t} - \rho_w \alpha_B \frac{\partial \varepsilon_{\text{vol}}}{\partial t} - \rho_w S_p^s \frac{\partial p_f}{\partial t}$$

here, ρ_a stands for the dry air density, ρ_v is the vapor density, ρ_l is the liquid water density, ρ_w is the volume average water density (vapor and liquid), and ϕ_w is the relative humidity.

The storage coefficients, S_p^i , for vapor, dry air, and solid are defined as

$$S_p^v = \varepsilon_p \chi_v$$

$$S_p^a = \varepsilon_p \chi_a$$

$$S_p^s = (\alpha_B - \varepsilon_p) \chi_s$$

where χ_i stands for the i -component of the compressibility.

For a formulation equivalent to [Large Strain Poroelasticity](#), the continuity equations read

$$J s_m \rho_v S_p^v p_{v, \text{sat}} \frac{\partial \phi_w}{\partial t} + \nabla \cdot \Gamma_w = J \varepsilon_p (\rho_v - \rho_l) \frac{\partial s_l}{\partial t} - \rho_w \alpha_B \frac{\partial J}{\partial t} - J \rho_w S_p^s \frac{\partial p_f}{\partial t}$$

$$J s_m \rho_a S_p^a \frac{\partial p_a}{\partial t} + \nabla \cdot \Gamma_a = J \varepsilon_p \rho_a \frac{\partial s_l}{\partial t} - s_m \rho_a \alpha_B \frac{\partial J}{\partial t} - J s_m \rho_a S_p^s \frac{\partial p_f}{\partial t}$$

The moisture flux Γ_w and the dry air flux the Γ_a are described in details in the [Theory for Moisture Transport in Solids](#) chapter.

References for the Poroelasticity Interfaces

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The Poroelasticity, Solid Interface

The Poroelasticity, Solid Interface

The **Poroelasticity, Solid** interface () combines a transient formulation of Darcy's law with a quasistatic formulation of Solid Mechanics under isothermal conditions. The coupling occurs on the domain level, where the pore pressure from the Darcy's Law interface acts as a load for the Solid Mechanics interface, causing swelling or shrinkage. Changes in volumetric strain affect the porosity, acting as a mass source or sink in Darcy's law.

When a predefined **Poroelasticity, Solid** interface is added from the **Structural Mechanics** branch () of the **Model Wizard** or **Add Physics** windows, the **Solid Mechanics** and **Darcy's Law** interfaces are added to the Model Builder.

In addition, the **Multiphysics** node is added, which automatically includes the **Poroelasticity** multiphysics coupling.

On the Constituent Physics Interfaces

- The Solid Mechanics interface is described in [The Solid Mechanics Interface](#).
- The Darcy's Law interface is described in [The Darcy's Law Interface](#).



To model orthotropic or anisotropic porous materials requires the Structural Mechanics Module. For information about the constitutive equations and theory background, see [Structural Mechanics Theory](#).



The **Poroelasticity** multiphysics coupling handles both geometrically linear and nonlinear problems automatically according to the settings in material model used in the Solid Mechanics interface. In the default **Linear Elastic Material** node, geometric nonlinearity is controlled by the **Formulation** list. By default, it is set to **From study step**. Changing this setting to **Total Lagrangian** forces the checkbox **Include geometric nonlinearity** in the study step to be selected and grayed out.

Conversely, select **Geometrically linear** in the **Formulation** list to enforce a small strain formulation, even if the checkbox **Include geometric nonlinearity** is selected in the study. See [Geometric Nonlinearity for the Solid Mechanics Interface](#) in the *Structural Mechanics Module User's Guide*.

SETTINGS FOR PHYSICS INTERFACES AND COUPLING FEATURES

When physics interfaces are added using the predefined couplings, for example **Poroelasticity**, specific settings are included with the physics interfaces and the coupling features.

However, if the physics interfaces are added one at a time, followed by the coupling features, these modified settings are not automatically included.

For example, if single **Solid Mechanics** and **Darcy's Law** interfaces are added, an empty **Multiphysics** node appears in the model tree. You can choose from the available coupling features, but the settings in the constituent interfaces are not modified.

TABLE 8-1: MODIFIED SETTINGS FOR A POROELASTICITY INTERFACE.

PHYSICS INTERFACE OR COUPLING FEATURE	MODIFIED SETTINGS
Solid Mechanics	For the Solid Mechanics interface, under Structural Transient Behavior , the Structural transient behavior is set to Quasistatic .
Darcy's Law	For the Darcy's Law interface, a Poroelastic storage model is added by default.
Poroelasticity	The Domain Selection is the same as that of the participating physics interfaces. The corresponding Solid Mechanics and Darcy's Law interfaces are preselected in the Coupled Interfaces section.

	<i>Failure of a Multilateral Well</i>: Application Library path Subsurface_Flow_Module/Poromechanics/multilateral_well <i>Large Swelling in Polymer Hydrogels</i>: Application Library path Porous_Media_Flow_Module/Poromechanics/hydrogel_swelling

Poroelasticity (Multiphysics Coupling)

The **Poroelasticity** multiphysics coupling node () links the Solid Mechanics and Darcy's Law interfaces bidirectionally to account for poroelastic deformation.

SETTINGS

The **Label** is the default multiphysics coupling feature name.

The **Name** is used primarily as a scope prefix for variables defined by the coupling node. Refer to such variables in expressions using the pattern <name>.<variable_name>. In order to distinguish between variables belonging to different coupling nodes or physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first multiphysics coupling feature in the model) is poro1.

DOMAIN SELECTION

When nodes are added from the context menu, you can select **Manual** (the default) from the **Selection** list to choose specific domains to define the poroelasticity coupling, or select **All domains** as needed.

When **Poroelasticity** is added as an effect of adding a Poroelasticity interface, the selection is the same as for the participating physics interfaces.

Only domains that are active in the physics interfaces selected in the [Coupled Interfaces](#) section can be selected.

	The Poroelasticity coupling handles geometric nonlinearity automatically in order to apply a strain formulation that is consistent with the geometric nonlinearity settings in the solid material model for each domain selected, see Geometric Nonlinearity for the Solid Mechanics Interface in the Structural Mechanics Modeling chapter for details.

COUPLED INTERFACES

This section defines the physics involved in the multiphysics coupling. The **Solid mechanics** and **Flow in porous media** lists include all applicable physics interfaces.

The default values depend on how the coupling node is created.

- If it is added from the **Physics** ribbon (Windows users), **Physics** contextual toolbar (macOS and Linux users), or context menu (all users), then the first physics interface of each type in the component is selected as the default.
- If it is added automatically when a multiphysics interface is selected in the **Model Wizard** or **Add Physics** window, then the two participating physics interfaces are selected.

You can also select **None** from either list to uncouple the Poroelasticity node from a physics interface. If the physics interface is removed from the **Model Builder**, for example if **Darcy's Law** is deleted, then the **Flow in porous media** list defaults to **None** as there is nothing to couple to.



If a physics interface is deleted and then added to the model again, then in order to reestablish the coupling, you need to choose the physics interface again from the **Solid mechanics** or **Flow in porous media** lists. This is applicable to all multiphysics coupling nodes that would normally default to the once present interface. See [Multiphysics Modeling Workflow](#) in the *COMSOL Multiphysics Reference Manual*.

POROELASTIC COUPLING PROPERTIES

In the **Poroelasticity model** list, choose between **Biot** and **Biphasic**. The Biot poroelasticity model (default) is commonly used in geotechnical applications where the compressibility of the solid and/or fluid phase is important. In contrast, the biphasic model assumes that both the solid and the fluid phases are incompressible, which is valid for soft materials like biological tissues and hydrogels. Note that even if both constituents are modeled as incompressible, the porous medium can still undergo large volume changes as a result of fluid inflow or outflow.

In the **Reference pressure level** list, choose **User defined** to enter a reference pressure p_{ref} (SI unit: Pa) at which the pore pressure does not affect the porous matrix deformation. The default value is 1 atm. If applicable, the list also contains options to inherit the value from another physics interface that announces the reference pressure level as a model input. For example, the reference pressure level can often be inherited from the

corresponding input in the Darcy's Law interface that is part of the Poroelasticity coupling.

For the **Biot–Willis coefficient** list, the default is to take its value **From material**. Choose **User defined** to enter a Biot–Willis coefficient α_B (dimensionless) in the text field. The user input is only required if the **Poroelasticity model** is set to **Biot**.

Theory for the Multiphase Flow in Porous Media Interface

The model equations that are solved in the **Multiphase Flow in Porous Media** interface are based on the mass conservation of each phase and on an extended Darcy's law. The mass conservation equation for each phase is given by

$$\frac{\partial}{\partial t}(\epsilon_p \rho_i s_i) + \nabla \cdot (\rho_i \mathbf{u}_i) = Q_i \quad (8-17)$$

where ϵ_p (dimensionless) is the porosity, and the vector $\mathbf{u}_i$ should be interpreted as the volumetric flux of phase i (SI unit $\text{m}^3/(\text{m}^2 \cdot \text{s})$ or m/s). The volumetric fluxes are determined using the extended Darcy's law ([Ref. 2](#))

$$\mathbf{u}_i = -\frac{\kappa_{ri}}{\mu_i} \kappa (\nabla p_i - \rho_i \mathbf{g}) \quad (8-18)$$

where κ denotes the permeability (SI unit: m^2) of the porous medium, $\mathbf{g}$ the gravitational acceleration vector (SI unit: m/s^2), and μ_i the dynamic viscosity (SI unit: $\text{kg}/(\text{m} \cdot \text{s})$), p_i the pressure field (SI unit: Pa), and κ_{ri} the relative permeability (dimensionless) of phase i , respectively. The phase pressures p_i are related through the capillary pressure functions $p_{c_{ij}}$:

$$p_i - p_j = p_{c_{ij}}(s_1, \dots, s_N) \quad i \neq j \quad (8-19)$$

One phase pressure can be chosen independently so that $N - 1$ capillary pressure relations are needed to define the other phase pressures. In addition it is assumed that all phases together fill the pore space completely, so that we have

$$\sum_{i=1}^N s_i = 1 \quad (8-20)$$

Pressure-Saturation Formulation

The algebraic relations in [Equation 8-19](#) and [Equation 8-20](#) allow for a reduction of the number of dependent variables. An often used way is to eliminate $N - 1$ phase pressures and one of the saturations. This results in a so-called pressure-saturation

formulation (Ref. 3). The procedure in the **Multiphase Flow in Porous Media** interface is to pick one of the phases, let us say phase i_c , then express the volume fraction, or saturation, s_{i_c} , of this phase in terms of the saturations of the other phases and in addition to use the pressure, p_{i_c} , of this phase to define the other phase pressures:

$$s_{i_c} = 1 - \left(\sum_{i=1, i \neq i_c}^N s_i \right) \quad (8-21)$$

$$p_i = p_{i_c} + p_{c_i}(s_1, \dots, s_N) \quad \text{for } i \neq i_c \quad (8-22)$$

The equations for the saturations s_i , ($i \neq i_c$) are given by Equation 8-17 and Equation 8-18. To arrive at an equation for p_{i_c} , the conservation equations of all phases are summed. This results in

$$\frac{\partial}{\partial t} \left(\epsilon_p \sum_{i=1}^N \rho_i s_i \right) + \nabla \cdot \left(\sum_{i=1}^N \rho_i \mathbf{u}_i \right) = Q_{\text{tot}} \quad (8-23)$$

where the total mass source Q_{tot} is given by

$$Q_{\text{tot}} = \sum_{i=1}^N Q_i \quad (8-24)$$

The equations for the saturations s_i are solved in the **Phase Transport in Porous Media** interface (Equation 8-17 and Equation 8-18). The Equation 8-23 for the pressure field p_{i_c} , needed as an input to the **Phase Transport in Porous Media** interface, is solved for in the **Darcy's Law** interface: the **Multiphase Flow in Porous Media** multiphysics coupling interface replaces the equation

$$\frac{\partial}{\partial t} (\epsilon_p \rho_{\text{tot}}) + \nabla \cdot (\rho_{\text{tot}} \mathbf{u}_d) = Q_m \quad \text{where} \quad \mathbf{u}_d = -\frac{1}{\mu} \kappa (\nabla p - \rho_{\text{tot}} \mathbf{g}) \quad (8-25)$$

which is originally implemented in the **Darcy's Law** interface, with Equation 8-23 by adding the following terms to the left-hand side of Equation 8-25:

$$\nabla \cdot \left(\sum_{i=1}^N \rho_i \mathbf{u}_i - \rho_{\text{tot}} \mathbf{u}_d \right) \quad (8-26)$$

and by setting the right-hand side Q_m to be equal to Q_{tot} . The volume averaged density ρ_{tot} is given by

$$\rho_{\text{tot}} = \sum_{i=1}^N \rho_i s_i \quad (8-27)$$

Furthermore, the **Multiphase Flow in Porous Media** multiphysics coupling interface couples the user inputs for the porosity and permeability of the porous matrix in the **Darcy's Law** interface to the corresponding user input fields in the **Phase and Porous Media Transport Properties** feature, and couples the pressure field computed for in the **Darcy's Law** interface to the user input field for the pressure of the phase computed from the volume constraint.

Note that when the hydraulic conductivity of a domain is specified in the **Darcy's Law** interface (instead of the permeability), the coupled **Phase and Porous Media Transport Properties** feature is supplied with a permeability that is computed using a reference kinematic viscosity of $1.004 \cdot 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$ (kinematic viscosity of water at 293.15 K).

In the other direction, the multiphysics coupling node provides the averaged density (denoted by ρ_{tot} in Equation 8-26 above) and effective viscosity to the **Darcy's Law** interface.

Mass Source and Boundary Mass Source

When adding a Mass Source node in the coupled Phase Transport interface, it is by default assumed that the net mass source of all phases, Q_{tot} , equals zero. This implies

$$Q_{i_c} = - \sum_{i=1, i \neq i_c}^N Q_i \quad (8-28)$$

However, when mass is transferred from or to phases not accounted for in the phase transport interface — for instance, when mass is released or absorbed by the solid part of the porous matrix — select the **Mass transfer to other phases** checkbox. In this case the mass source for the phase calculated from the volume constraint should also be specified, and the net total mass source Q_{tot} is supplied as a right-hand side to the Darcy Equation 8-23 for p_{i_c} .

Similarly, when a Boundary Mass Source node is added in the coupled Phase Transport interface, the sum of the specified mass fluxes for all phases is supplied as a mass flux boundary condition for the Darcy Equation 8-23.

Boundary Conditions

When supplying boundary conditions for the coupled **Phase Transport in Porous Media** and **Darcy's Law** interfaces, bear in mind that the **Pressure** boundary condition in the **Darcy's Law** interface affects the pressure of the phase from the volume constraint, p_{i_e} . If the pressure of another phase needs to be supplied, then take into account the capillary pressure.

Since the coupled **Darcy's Law** interface uses the conservation equation for the total mass (Equation 8-23) to solve for p_{i_e} , the **Mass Flux** boundary condition in the **Darcy's Law** interface should be used to prescribe the mass flux of all phases together.

If the mass flux of the phase computed from the volume constraint needs to be prescribed at a boundary, and the total mass flux is not known a priori, it might be necessary to compute for the total mass flux in the form of a Lagrange multiplier by using the weak constraint formulation for the pressure boundary condition in the **Darcy's Law** interface.

Thin Barrier (Multiphysics Coupling)

The **Thin Barrier** multiphysics coupling uses the Extra Dimension tool to solve the equations for the pressure and saturations through the thin barrier structure's thickness. The thin structure has its domain represented by the product space between the lumped boundary and the additional dimension for the thickness. Inside this product space, it is convenient to split the gradient operator into a tangential part and a normal part: $\nabla = \nabla_{\mathbf{t}} + \nabla_{\mathbf{n}}$.

The following assumptions, used in deriving the **Thin Barrier** equations from the equations solved in the **Multiphase Flow in Porous Media** interface, justify the simplification $\nabla = \nabla_{\mathbf{n}}$:

- A thin barrier can be considered as a boundary of thickness significantly smaller than the typical lengths of the overall model.
- The permeability of the porous material inside the thin barrier is significantly smaller than the permeability of the porous material in the adjacent domains.
- There is no mass source or sink inside the thin barrier.

In addition, these assumptions imply that the effect of gravity on the phase transport can be neglected. This results in the following set of equations for the saturations s_i , ($i \neq i_c$) and the pressure p_{i_c} :

$$\frac{\partial}{\partial t}(\epsilon_p \rho_i s_i) + \nabla_{\mathbf{n}} \cdot (\rho_i \mathbf{u}_i) = 0 \quad \text{for } i \neq i_c \quad (8-29)$$

$$\mathbf{u}_i = -\frac{\kappa_{ri}}{\mu_i} \kappa \nabla_{\mathbf{n}} p_i \quad (8-30)$$

$$\frac{\partial}{\partial t} \left(\epsilon_p \sum_{i=1}^N \rho_i s_i \right) + \nabla_{\mathbf{n}} \cdot \left(\sum_{i=1}^N \rho_i \mathbf{u}_i \right) = 0 \quad (8-31)$$

Together with the algebraic relations

$$s_{i_c} = 1 - \left(\sum_{i=1, i \neq i_c}^N s_i \right) \quad (8-32)$$

$$p_i = p_{i_c} + p_{c_i}(s_1, \dots, s_N) \quad \text{for } i \neq i_c \quad (8-33)$$

Here ϵ_p (dimensionless) denotes the porosity and κ the permeability (SI unit: m^2) of the porous material in the thin barrier. Further more ρ_i denotes the density (SI unit: kg/m^3), μ_i the dynamic viscosity (SI unit: $\text{kg}/(\text{m}\cdot\text{s})$), p_i the pressure field (SI unit: Pa), $\mathbf{u}_i$ the (normal) volumetric flux (SI unit: m/s) and κ_{ri} the relative permeability (dimensionless) of phase i , respectively.

Since the interfaces between the thin barrier and the adjacent domains are boundaries between a more permeable and a less permeable porous domain, conditions analogous to the [Porous Medium Discontinuity Boundary Condition](#) are needed to couple the equation [Equation 8-29](#) for the saturations inside the thin barrier to the equations in the adjacent domains. These condition at the extremities of the extra dimension can be written as:

$$p_{c_i}^t(s_i^t)_{L=0} = \begin{cases} p_{ec,i}^t & \text{if } p_{c_i}^d(s_i^d) \leq p_{ec,i}^t \\ p_{c_i}^d(s_i^d) & \text{if } p_{c_i}^d(s_i^d) > p_{ec,i}^t \end{cases} \quad \text{for } i \neq i_c \quad (8-34)$$

where the superscript t is used to indicate the saturation and capillary pressure inside the thin barrier (the low permeable side of the interface), and the superscript d

indicates the domain adjacent to the downside of the thin barrier (the high permeable side), and

$$p_{c_i}^t(s_i^t)_{L=d_s} = \begin{cases} p_{ec,i}^t & \text{if } p_{c_i}^u(s_i^u) \leq p_{ec,i}^t \\ p_{c_i}^u(s_i^u) & \text{if } p_{c_i}^u(s_i^u) > p_{ec,i}^t \end{cases} \quad \text{for } i \neq i_c \quad (8-35)$$

where the superscript u indicates the domain adjacent to the upside of the thin barrier (again the high permeable side) and where d_s is the length of the extra dimension, or equivalently, the thickness of the thin structure.

The additional conditions are continuity of the fluxes for all phases and continuity of the pressure of the phase i_c computed from the volume constraint. Note that this last condition assumes that this phase is present both inside the thin barrier and in the adjacent domains. Furthermore, this boundary condition assumes that the phase i_c computed from the volume constraint is the wetting phase and it is necessary that the settings for the van Genuchten or Brooks and Corey capillary pressure model in the adjacent domains match this assumption.

	Please find related information under • Theory for the Phase Transport Interfaces in the <i>CFD Module User's Guide</i> and • Using Extra Dimensions in the <i>COMSOL Multiphysics Reference Manual</i>.

References

1. J. Bear, *Dynamics of Fluids in Porous Media*, Elsevier Scientific Publishing, 1972.
2. Z. Chen, G. Huan, and Y. Ma, *Computational Methods for Multiphase Flows in Porous Media*, Philadelphia: Society for Industrial and Applied Mathematics, 2006.
3. R. Helmig, *Multiphase Flow and Transport Processes in the Subsurface – A Contribution to the Modeling of Hydrosystems*, Springer–Verlag, 1997.

The Multiphase Flow in Porous Media Interface

The **Multiphase Flow in Porous Media** interface () combines the **Darcy's Law** interface with the **Phase Transport in Porous Media** interface to model the flow and transport of multiple immiscible phases in a porous medium.

When a predefined **Multiphase Flow in Porous Media** interface is added from the **Porous Media and Subsurface Flow** branch () of the **Model Wizard** or **Add Physics** window, **Phase Transport** and **Darcy's Law** interfaces are added to the Model Builder. In addition, the **Multiphysics** node is added, which automatically includes the **Multiphase Flow in Porous Media** multiphysics coupling.

On the Constituent Physics Interfaces

The **Phase Transport in Porous Media** interface is used to simulate the transport of immiscible multiple species in a porous medium and solves for their averaged volume fractions (also called saturations). The interfaces between the different phases are not tracked explicitly, although microscopic interfacial effects are taken into account in the macroscopic equations through the capillary pressure functions.

The **Darcy's Law** interface is used to simulate fluid flow through interstices in a porous medium. It can be used to model low-velocity flows in media where the permeability and porosity are very small, and for which the pressure gradient is the major driving force and the flow is mostly influenced by the frictional resistance within the pores.

SETTINGS FOR PHYSICS INTERFACES AND COUPLING FEATURES

When physics interfaces are added using the predefined couplings, for example **Multiphase Flow in Porous Media**, specific settings are included with the physics interfaces and the coupling features.

However, if physics interfaces are added one at a time, followed by the coupling features, these modified settings are not automatically included.

For example, if single **Phase Transport in Porous Media** and **Darcy's Law** interfaces are added, an empty **Multiphysics** node appears in the model tree. You can choose from the

available coupling features but the settings in the constituent interfaces are not modified.

TABLE 8-2: MODIFIED SETTINGS FOR A MULTIPHASE FLOW IN POROUS MEDIA INTERFACE.

PHYSICS INTERFACE OR COUPLING FEATURE	MODIFIED SETTINGS
Darcy's Law	In the Darcy's Law interface, linear shape functions are used for the discretization of the pressure.
Multiphase Flow in Porous Media	The Domain Selection is the same as that of the participating physics interfaces. The corresponding Phase Transport in Porous Media and Darcy's Law interfaces are preselected in the Coupled Interfaces section.

Multiphase Flow in Porous Media (Multiphysics Coupling)

The **Multiphase Flow in Porous Media** multiphysics coupling node () links bidirectionally the **Phase Transport in Porous Media** and **Darcy's Law** interfaces.

SETTINGS

The **Label** is the default multiphysics coupling feature name.

The **Name** is used primarily as a scope prefix for variables defined by the coupling node. Refer to such variables in expressions using the pattern <name>.<variable_name>. In order to distinguish between variables belonging to different coupling nodes or physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first multiphysics coupling feature in the model) is mfpm1.

DOMAIN SELECTION

When nodes are added from the context menu, you can select **Manual** (the default) from the **Selection** list to choose specific domains to define the multiphase flow coupling, or select **All domains** as needed.

When **Multiphase Flow in Porous Media** is added as an effect of adding a Multiphase Flow in Porous Media interface, the selection is the same as for the participating physics interfaces.

Only domains that are active in the participating physics interfaces (which are chosen from the **Coupled Interfaces** selection list) can be selected.

COUPLED INTERFACES

This section defines the physics involved in the multiphysics coupling. The **Phase transport** and **Flow in porous media** lists include all applicable physics interfaces.

The default values depend on how the coupling node is created.

- If it is added from the **Physics** ribbon (Windows users), **Physics** contextual toolbar (Mac and Linux users), or context menu (all users), then the first physics interface of each type in the component is selected as the default.
- If it is added automatically when a multiphysics interface is selected in the **Model Wizard** or **Add Physics** window, then the two participating physics interfaces are selected.

You can also select **None** from either list to uncouple the Multiphase Flow in Porous Media node from a physics interface. If the physics interface is removed from the **Model Builder**, for example **Darcy's Law** is deleted, then the **Flow in porous media** list defaults to **None** as there is nothing to couple to.



Please find related information under

- [The Phase Transport Interfaces](#) and
- [Theory for the Phase Transport Interfaces](#)

in the *CFD Module User's Guide*.

Well (Multiphysics Coupling)

The **Multiphase Flow in Porous Media** () interface contains an optional **Well** multiphysics coupling feature on points (in 2D) or edges (in 3D). The **Well** coupling node is intended to model injection or production wells and links bidirectionally the **Phase Transport in Porous Media** and **Darcy's Law** interfaces.

The Subsurface Flow Module license is required to use this coupling feature (see www.comsol.com/products/specifications/).

SETTINGS

The **Label** is the default multiphysics coupling feature name.

The default **Name** (for the first **Well** multiphysics coupling feature in the model) is wellmpp1.

POINT OR EDGE SELECTION

When nodes are added from the context menu, you can select **Manual** (the default) from the **Selection** list to choose specific points or edges, or select **All points** or **All edges** as needed.

COUPLED INTERFACES

This section controls which individual interfaces are coupled by the current coupling feature. If a physics interface is deleted and then added to the model again, then in order to reestablish the coupling, you need to choose the correct physics interface again from the **Phase transport** or **Flow in porous media** lists

WELL

Select whether the well is a production or injection well: for an injection well positive mass fluxes signify inflowing fluxes and for a production well positive mass fluxes are outflowing fluxes.

Either specify the total **Total mass flow rate** $\dot{M}_0$ (SI unit: kg/s), the **Total mass flow rate per unit length** $\dot{M}_1$ (SI unit: kg/(m·s)), the **Pressure** p_0 (SI unit: Pa), the **Pressure head** H_{p0} (SI unit: m), or the **Hydraulic head** H_0 (SI unit: m). The elevation D used to convert the hydraulic head to a pressure, is defined by the gravity effects in the coupled Darcy's Law interface.

Select the **Specify well diameter** checkbox to define the **Well diameter** d_w (SI unit: m). This averages the mass source on the boundary of a cylinder around the line. This option avoids obtaining an increasing pressure value at the line when meshing finer than this diameter. It makes use of the `circavg` operator.

If the well diameter is defined, the option to specify the **Total mass flux** N_0 (SI unit: kg/(m²·s)) is also available.

PHASES

The number of **Phase** sections depends on the number of phases defined in the coupled Phase Transport in Porous Media interface: the number of sections is equal to the number of phases not computed from the volume constraint, which is in turn equal to the number of phases defined in the coupled Phase Transport in Porous Media interface minus one.

Specify the boundary condition for each phase not computed from the volume constraint. You can choose to enter a value for the **Mass flow rate** $\dot{M}_{0,i}$ (SI unit: kg/s), the **Mass flow rate per unit length** $\dot{M}_{1,i}$ (SI unit: kg/(m·s)), or the **Volume fraction** $s_{0,i}$ (dimensionless). If the well diameter is specified, the option to specify the **Mass flux** $N_{0,i}$

(SI unit: $\text{kg}/(\text{m}^2 \cdot \text{s})$) is also available. For a production well, you can additionally choose the **Outflow** option. For an injection well you can additionally choose to specify the **Fraction of total volumetric flow rate** $s_{0,i}$ (dimensionless).

Fraction of total volumetric flow rate

When the **Fraction of total volumetric flow rate** $s_{0,i}$ is specified for a phase, a mass flow rate per unit length $M_{1,i}$ is supplied for that phase, which is given by

$$M_{1,i} = \rho_i s_{0,i} V_1 \quad (8-36)$$

where V_1 is the total volumetric flow rate per unit length, which is computed as follows: assume that there are N phases, and that phase 1 is the phase computed from the volume constraint, that for phases i , for $i = 2, \dots, p$, the fraction of total volumetric flow rate $s_{0,i}$ is supplied, and that for the phases i , for $i = p+1, \dots, N$, the mass flow rate per unit length $M_{1,i}$ is given. Then V_1 can be computed from the relation

$$V_1 = \frac{M_{1,1}}{\rho_1} + \sum_{i=2}^p V_1 s_{i,0} + \sum_{i=p+1}^N \frac{M_{1,i}}{\rho_i} \quad (8-37)$$

where the mass flow rate $M_{1,1}$ for the phase computed from the volume constraint is given by

$$M_{1,1} = M_1 - \sum_{i=2}^p V_1 \rho_i s_{i,0} - \sum_{i=p+1}^N M_{1,i} \quad (8-38)$$

Note that the total mass flow rate per unit length M_1 is readily available when any of the **Total mass flow rate**, **Total mass flow rate per unit length**, or the **Total mass flux** options is chosen. Furthermore, the total mass flow rate per unit length M_1 is also available as a Lagrange multiplier in case any of the **Pressure**, the **Pressure head**, or the **Hydraulic head** is given, as these conditions are implemented as weak constraints. For the mass flow rates per unit length $M_{1,i}$ of the individual phases, it holds similarly that they are readily available when any of the **Mass flow rate**, **Mass flow rate per unit length**, or the **Mass flux** options are chosen, and that for the **Volume fraction** option, implemented as a weak constraint, the mass flow rate per unit length $M_{1,i}$ is computed as a Lagrange multiplier.

Outflow

When the **Outflow** option is specified for a phase s_i , an outflowing mass flow rate per unit length $M_{1,i}$ is supplied for that phase, which is given by

$$M_{l,i} = \frac{\rho_i \kappa_{ri}}{\lambda_r \mu_i} M_l \quad \text{with} \quad \lambda_r = \sum_{i=1}^N \frac{\rho_i \kappa_{ri}}{\mu_i} \quad (8-39)$$

where κ_{ri} denotes the relative permeability of phase i .

Thin Barrier (Multiphysics Coupling)

The **Multiphase Flow in Porous Media** () interface contains an optional **Thin Barrier** multiphysics coupling feature on edges (in 2D) or surfaces (in 3D). The **Thin Barrier** coupling node can be used to avoid meshing interior permeable structures that are less permeable than the adjacent porous domains. It links bidirectionally the **Phase Transport in Porous Media** and **Darcy's Law** interfaces.

SETTINGS

The **Label** is the default multiphysics coupling feature name.

The default **Name** (for the first **Thin Barrier** multiphysics coupling feature in the model) is `tb1s1`.

BOUNDARY SELECTION

When nodes are added from the context menu, you can select **Manual** (the default) from the **Selection** list to choose specific boundaries, or select **All boundaries** as needed.

COUPLED INTERFACES

This section controls which individual interfaces are coupled by the current coupling feature. If a physics interface is deleted and then added to the model again, then in order to reestablish the coupling, you need to choose the correct physics interface again from the **Phase transport** or **Flow in porous media** lists

SHELL PROPERTIES

The **Thin Barrier** multiphysics coupling feature is designed to be only applicable for a **Single Layer Material**. Select the **Single Layer Material** to be used for the thin barrier from the **Layer** list. The **Thickness** of the thin barrier is set in the **Layer Definition** section of the **Shell Property Group** window of the **Single Layer Material**. Also use this section to set the number of **Mesh elements** that are used in the Extra Dimension to solve the equations through the thickness of the thin barrier layer.

MODEL INPUT

The material properties of the different phases, such as density and viscosity, can be defined through user inputs, variables, or by selecting a boundary material. For the latter option, additional inputs, for example temperature or pressure, may be required to define these properties. These model inputs appear in this section.

MATRIX PROPERTIES

Select the material to use as porous matrix inside the thin barrier. Select **Boundary material** (the default) from the **Porous material** list to use the material defined for the thin barrier boundary. Select another material to use that material's properties.

The default **Porosity** ε_p (a dimensionless number between 0 and 1) uses the value **From material**, defined by the **Porous material** selected. For **User defined** the default is 0.5.

The default **Permeability** κ (SI unit: m^2) uses the value **From material**, as defined by the **Porous material** selected. For **User defined** the default is $1\text{e-}9 \text{ m}^2$.

CAPILLARY PRESSURE

Select a **Capillary pressure model** — **User defined** (the default), **van Genuchten**, **Brooks and Corey**, or **Capillary diffusion**. The **van Genuchten** and **Brooks and Corey** options are only available when there are not more than two phases present in the model.

- For **User defined**, enter an expression for the **Capillary pressure** p_c (SI unit: Pa) for the phases that are not computed from the volume constraint. The pressure of the corresponding phase is equal to the pressure of the phase computed from the volume constraint plus the capillary pressure supplied for this phase.
- For **van Genuchten**, enter an **Entry capillary pressure** p_{ec} (SI unit: Pa), the **Constitutive relations constant** m_{vG} (dimensionless), and **Constitutive relations constant** l_{vG} (dimensionless).
- For **Brooks and Corey**, enter an **Entry capillary pressure** p_{ec} (SI unit: Pa) and the **Pore size distribution index** λ_p (dimensionless).
- For **Capillary diffusion**, enter a capillary diffusion coefficient D_c (SI unit: m^2/s) for the phases that are not computed from the volume constraint.

PHASE PROPERTIES

Select the material to use for each phase. Select **Boundary material** (the default) from the **Fluid** list to use the material defined for the domain. Select another material to use that material's properties.

Enter the **Density** ρ (SI unit: kg/m^3), **Dynamic viscosity** μ (SI unit: $\text{Pa}\cdot\text{s}$) and **Relative permeability** κ_r (a dimensionless number between 0 and 1) for each phase. The defaults for the **Density** and **Dynamic viscosity** are **From material**, and **User defined** for the **Relative permeability**. For **User defined**, the default values are 1000 kg/m^3 , $0.001 \text{ Pa}\cdot\text{s}$. and s_{ri}^2 , respectively.

The **Relative permeability** input fields are only available when the **Capillary pressure model** is set to **User defined** or **Capillary diffusion**. In the other cases (**van Genuchten** or **Brooks and Corey**) the relative permeabilities are defined by the **van Genuchten** or **Brooks and Corey** model. For the **van Genuchten** and **Brooks and Corey** model, specify the **Residual saturation** s_{ri} (dimensionless, default value 0) for each phase.

INITIAL VALUES

Enter a value or expression in the field for the volume fraction inside the thin barrier of each phase, except for the one computed from the volume constraint, and in the field for the pressure of the phase computed from the volume constraint.

	Please find related information under • Single Layer Materials in the <i>COMSOL Multiphysics Reference Manual</i> and • Theory for the Multiphase Flow in Porous Media Interface in the <i>Subsurface Flow Module User's Guide</i>.

The Reacting Flow in Porous Media Interface

The **Reacting Flow in Porous Media, Transport of Diluted Species** interface () is used to study the flow and chemical composition of a gas or liquid moving through the interstices of a porous medium.

It combines the **Brinkman Equations** and **Transport of Diluted Species in Porous Media** interfaces. The **Reacting Flow, Diluted Species** multiphysics coupling feature, which is added automatically, couples the fluid flow and mass transport.

The interface can be used for stationary and time-dependent analysis in 2D, 2Daxi, and 3D.

Detailed information is available in the section [The Reacting Flow in Porous Media Multiphysics Interfaces](#) of the *Chemical Reaction Engineering User's Guide*.

The Nonisothermal Flow, Brinkman Equations Interface

The **Nonisothermal Flow, Brinkman Equations** multiphysics coupling is used to model heat transfer and flow in porous media where both the temperature and flow fields have to be coupled. Here, the nonlocal couplings between porous media flow and heat transfer are set up automatically.

The interface can be used for stationary and time-dependent analysis in 2D, 2D axial symmetry, and 3D.

When the **Brinkman Equations** multiphysics interface is chosen from the **Fluid Flow > Nonisothermal Flow** branch of the **Model Wizard** or **Add Physics** windows, the **Heat Transfer in Porous Media** and the **Brinkman Equations** interfaces are added to the **Model Builder**. In addition, the **Multiphysics** node is added, which automatically includes the multiphysics coupling feature **Nonisothermal Flow**.

Furthermore, a **Porous Material** node with a **Fluid** and a **Solid** subnode is automatically generated under **Component > Materials**.

Settings for the Brinkman Equations Interface and the Coupling Feature

When physics interfaces are added using the predefined couplings, specific settings are included with the physics interfaces and the coupling features.

By default in the **Settings** window for **Brinkman Equations**, the **Compressibility** option is set to **Weakly compressible flow**. The **Density** ρ in the **Porous Medium > Fluid** subnode is automatically set to the variable from the **Nonisothermal Flow** multiphysics coupling feature. Other options for Compressibility are Incompressible flow or Compressible flow ($Ma < 0.3$).

For **Heat Transfer in Porous Media** in the **Porous Medium > Fluid** subnode, the **Absolute pressure**, p_A (**Model Input** section of the **Porous Medium > Fluid** node) and the **Velocity field**, $\mathbf{u}$ (**Heat Convection** section) are automatically set to the variables from the **Nonisothermal Flow** multiphysics coupling feature.

Note that if the physics interfaces are added one at a time, followed by the coupling feature, these modified settings may not be automatically included.

To get all the modified settings presented below automatically, you can right-click the component node, select **Add Multiphysics**, and add the **Nonisothermal Flow** coupling feature.



The settings for the interfaces and the coupling feature are described fully under the following links:

- [The Nonisothermal Flow, Brinkman Equations Interface](#) in the *CFD Module User's Guide*.
- [The Brinkman Equations Interface](#)
- [The Heat Transfer in Porous Media Interface](#) in the *Heat Transfer Module User's Guide*

For more general information on **Nonisothermal Flow** see

- [The Nonisothermal Flow and Conjugate Heat Transfer, Laminar Flow, Turbulent Flow and LES Interfaces](#) in the *CFD Module User's Guide*
- [Heat Transfer and Fluid Flow Coupling](#) in the *Heat Transfer Module User's Guide*
- [The Multiphysics Branch](#) in the *COMSOL Multiphysics Reference Manual*.

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