



Model created in COMSOL Multiphysics 6.4

Dispersion of Surface Plasmon Polariton in Thin Metal Embedded in Dielectric

Introduction

Surface plasmon polaritons (SPPs) are a special class of surface waves that propagate along the interface between materials with negative and positive dielectric constants, and decay in the direction normal to the interface. Most of the metals at optical frequencies possess a negative dielectric constant, and are widely used materials to excite such surface waves. An SPP propagates along the metal–dielectric interface until it completely loses its energy due to the metal absorption. The thickness of the metal layer plays an important role in this context as it controls the absorption of the structure.

In the case of a metal layer with a certain thickness, embedded in a dielectric, SPPs propagating on the top and bottom interfaces couple to each other and create a supermode. Such a structure behaves similarly to an optical slab waveguide with dispersion depending on the metal thickness. SPPs supported by thin metal layers could be useful in practical applications such as short-distance signal transmission and routing as well as to design power splitters and optical couplers.

Model Definition

The schematic of the model is shown in [Figure 1](#). A thin metal layer is covered by layers of dielectrics with certain thickness and is surrounded by air.

The expressions of the electric and magnetic fields of the TM surface waves or SPPs and the propagation constant are available in [Ref. 1](#).

As the SPPs propagate along the surface, they lose energy due to the metal absorption. The dispersion of the structure as well as the complex SPP wavenumber $k_{\text{spp}} = k_r + ik_i$ depends on the thickness of metal. Assume that the SPP wave propagates along the x direction. The electric field component of such a wave is represented by

$$E = e^{ik_r x} e^{-k_i x}$$

Therefore, at a distance x , the SPP intensity has decayed by a factor of $e^{-2k_i x}$. The propagation length, L of the SPP is defined as the distance at which the wave intensity decays by a factor $1/e$, and is defined as

$$L = \frac{1}{2k_i}$$

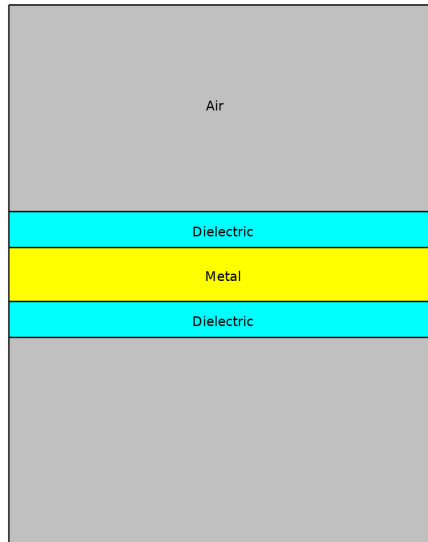


Figure 1: Schematic of the configuration. A thin metal layer is embedded in layers of dielectric with certain thickness. The setup is surrounded by air domains.

This model simulates SPPs propagating in a thin layer of aluminum embedded in dielectric layers, using material properties for aluminum from the built-in Optical Material Library. Numeric ports are used on the left and right boundaries of the model. The left port, with excitation turned on, launches the SPP, while the right port, with excitation turned off, absorbs the SPP without reflection. To get the mode field for both ports, two Boundary Mode Analysis study steps are added. Finally, another Boundary Mode Analysis study step with Auxiliary sweep of the wavelength is used to calculate the dispersion and the propagation length of SPP.

Results and Discussion

Figure 2 shows the electric field y -component of the SPP as it propagates from left to right. The intensity of the wave decays as it propagates due to metal absorption.

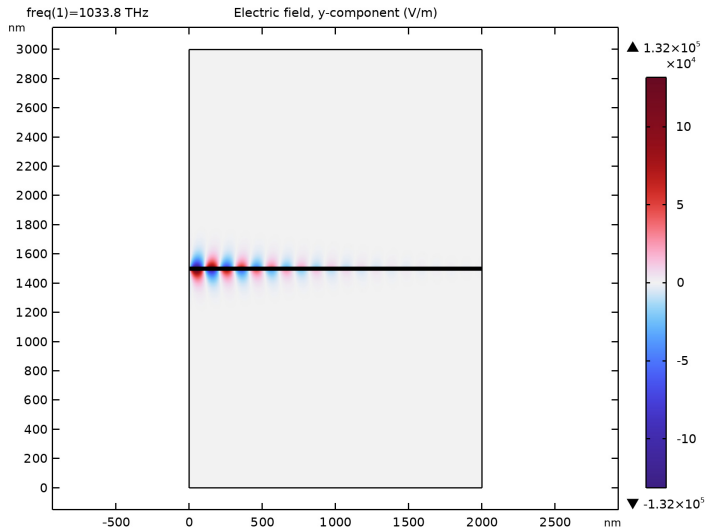


Figure 2: The y-component of the electric field for a wavelength of 290 nm (4.28 eV). The wave gets weaker, due to absorption, as it propagates from left to right.

Figure 3 shows the propagation constant of the SPP versus the photon energy and compares with the propagation constant of light in free space. As the photon energy reduces and moves away from the plasmon resonance frequency of aluminium, the SPP propagation constant converges with the propagation constant of free space light.

Figure 4 shows the propagation length versus the photon energy. The SPPs decay very fast when the photon energy is very high. This is due to the fact of strong metal absorption at high photon energy, and the loss gradually decreases as the photo energy reduces.

For more SPP simulation examples, see [Ref. 1](#).

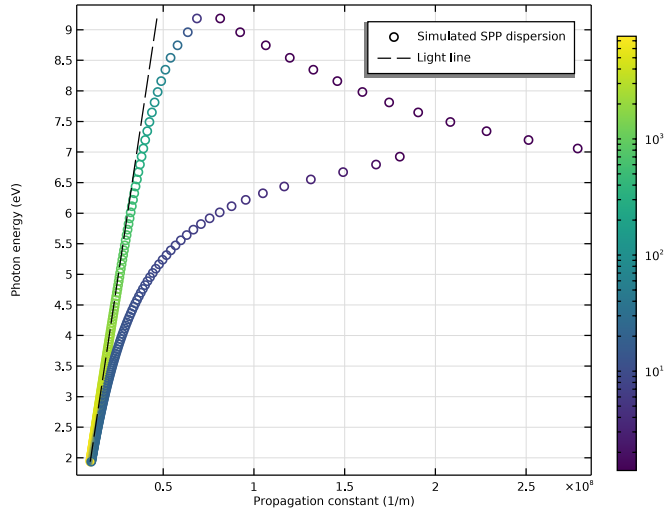


Figure 3: Photon energy versus the propagation constant.

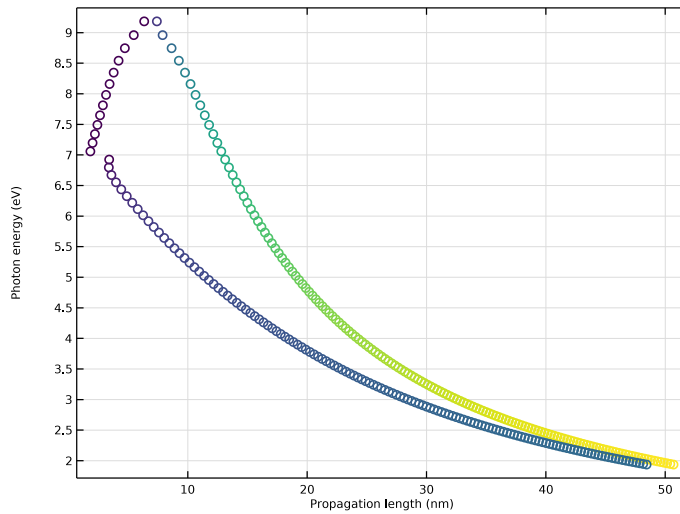


Figure 4: Photon energy versus the propagation length.

Reference


I. X. Chen, “Modeling Surface Plasmon Polaritons in COMSOL®,” COMSOL Blog, 12 Oct. 2022; www.comsol.com/blogs/modeling-surface-plasmon-polaritons-in-comsol/.

Application Library path: Wave_Optics_Module/Waveguides/
thin_metal_surface_plasmon_polariton




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Optics > Wave Optics > Electromagnetic Waves, Frequency Domain (ewfd)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies > Frequency Domain**.
- 6 Click  **Done**.

GEOMETRY I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **nm**.

GLOBAL DEFINITIONS

Parameters I

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
L	2[um]	2E-6 m	Width
H	3[um]	3E-6 m	Domain height
t_diel	4[nm]	4E-9 m	Dielectric layer thickness
t_metal	12[nm]	1.2E-8 m	Metal thickness
n_diel	2	2	Refractive index, dielectric
lda0	290[nm]	2.9E-7 m	Wavelength
f0	c_const/lda0	1.0338E15 1/s	Frequency

GEOMETRY I

The geometry consists of a thin metal layer embedded in a dielectric. The structure is surrounded by air domain.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click  **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type L.

4 In the **Height** text field, type H.

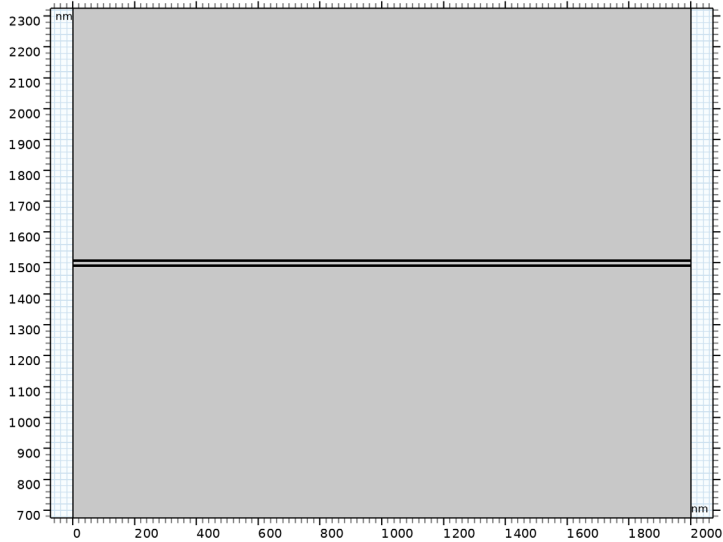
5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (nm)
Layer 1	$H/2 - t_{\text{diel}} - t_{\text{metal}}/2$
Layer 2	t_diel

6 Select the **Layers on top** checkbox.


7 Click  **Build Selected**.

In the **Graphics** window, click **Zoom In** twice to clearly visualize the metal and dielectric domains.



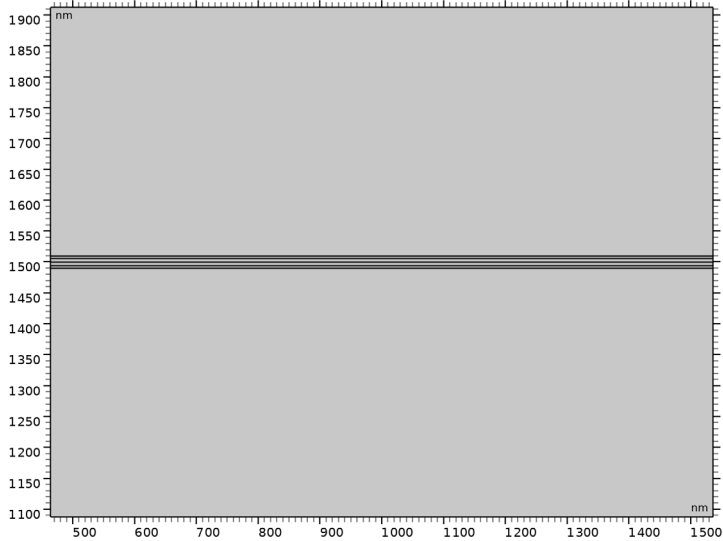
Rectangle 2 (r2)

Build another rectangle with similar width, and height equal to half of the total geometry. This will be useful to copy mesh from the top half of the geometry to the bottom half.

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type L.
- 4 In the **Height** text field, type H/2.

5 Click  **Build All Objects**.



In the **Graphics** window, click **Zoom In** twice to clearly visualize the metal and dielectric domains.



MATERIALS

Pick Aluminium as the metal from the Optical material library.

ADD MATERIAL

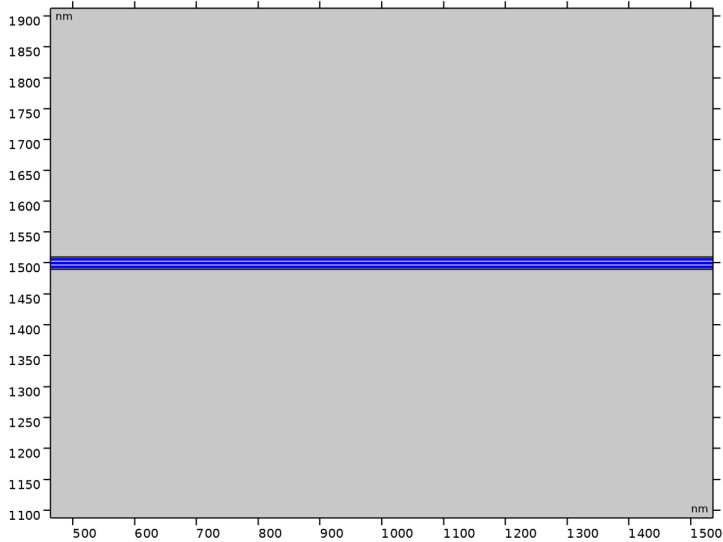
- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Built-in > Air**.
- 4 Right-click and choose **Add to Component 1 (comp1)**.
- 5 In the tree, select **Optical > Inorganic Materials > Al - Aluminium and aluminates > Models and simulations > Al (Aluminium) (Rakic et al. 1998: Lorentz-Drude model; n, k 0.0620-248 um)**.
- 6 Right-click and choose **Add to Component 1 (comp1)**.
- 7 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.

MATERIALS

Al (Aluminium) (Rakic et al. 1998: Lorentz-Drude model; n, k 0.0620-248 μm) (mat2)

1 Select Domains 3 and 4 only.

In the **Graphics** window, click **Zoom In** twice to clearly visualize the metal layer.



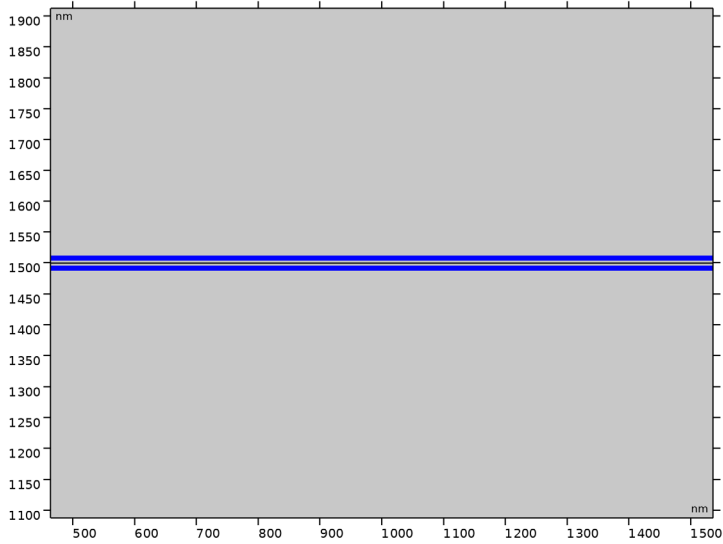
Dielectric

1 In the **Model Builder** window, right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type Dielectric in the **Label** text field.

3 Select Domains 2 and 5 only.

In the **Graphics** window, click **Zoom In** twice to clearly visualize the dielectric domains.



4 Locate the **Material Contents** section. In the table, enter the following settings:

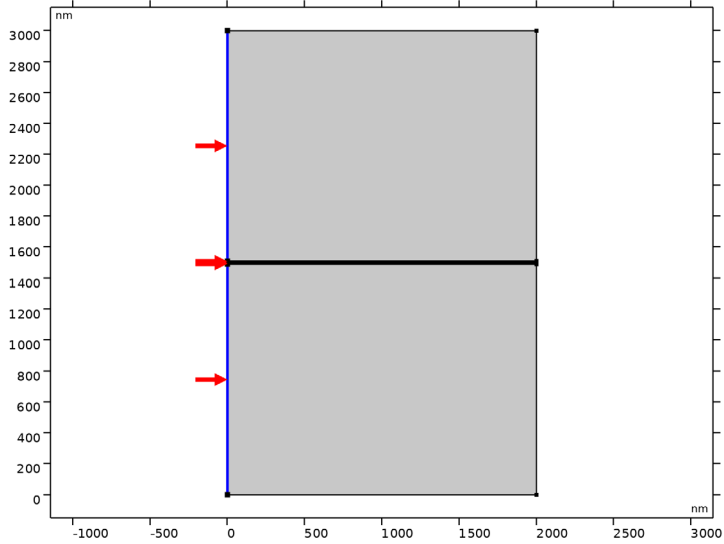
Property	Variable	Value	Unit	Property group
Refractive index, real part	n_{iso} ; $n_{ii} = n_{iso}$, $n_{ij} = 0$	n_{diel}	1	Refractive index

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

Port 1

I In the **Physics** toolbar, click  **Boundaries** and choose **Port**.

2 Select Boundaries 1, 3, 5, 7, 9, and 11 only.



3 In the **Settings** window for **Port**, locate the **Port Properties** section.

4 From the **Type of port** list, choose **Numeric**.

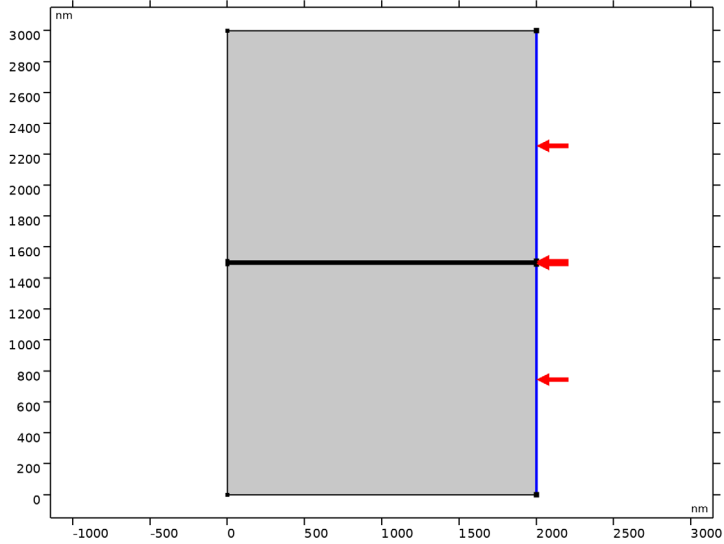
Port 2

1 Right-click **Port 1** and choose **Duplicate**.

2 In the **Settings** window for **Port**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.

4 Select Boundaries 14–19 only.



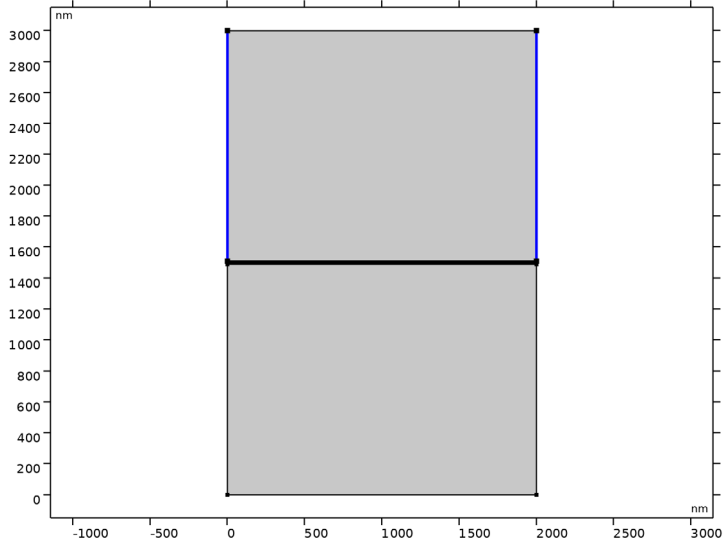
5 Locate the **Port Properties** section. From the **Wave excitation at this port** list, choose **Off**.

MESH 1

Distribution 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Distribution**.

2 Select Boundaries 11 and 19 only.



3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 From the **Distribution type** list, choose **Predefined**.

5 In the **Number of elements** text field, type 20.

6 In the **Element ratio** text field, type 100.

7 From the **Growth rate** list, choose **Exponential**.

Size

1 In the **Model Builder** window, click **Size**.

2 In the **Settings** window for **Size**, click to expand the **Element Size Parameters** section.

3 In the **Minimum element size** text field, type 0.6.

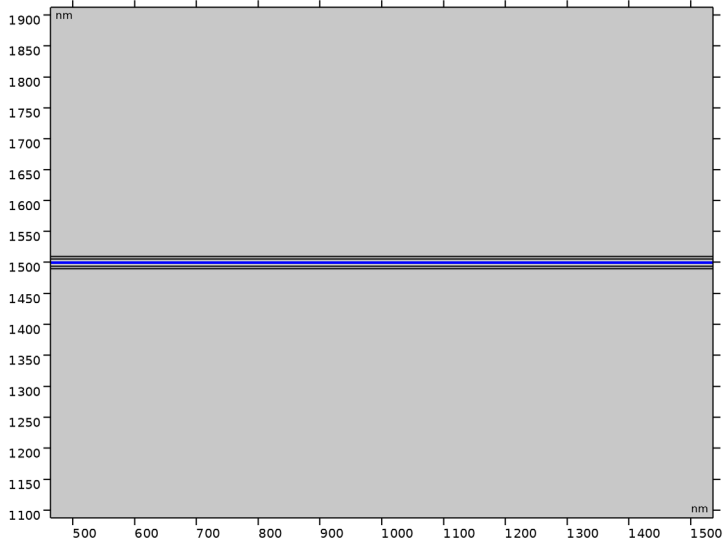
Distribution 2

1 In the **Model Builder** window, under **Component 1 (comp1) > Mesh 1** right-click **Distribution 1** and choose **Duplicate**.

2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.

4 Select Boundary 8 only.



5 Locate the **Distribution** section. From the **Distribution type** list, choose **Fixed number of elements**.

6 In the **Number of elements** text field, type $\text{round}(L/4[\text{nm}])$.

Set each element size to 4 nm to properly resolve the surface plasmon polariton.

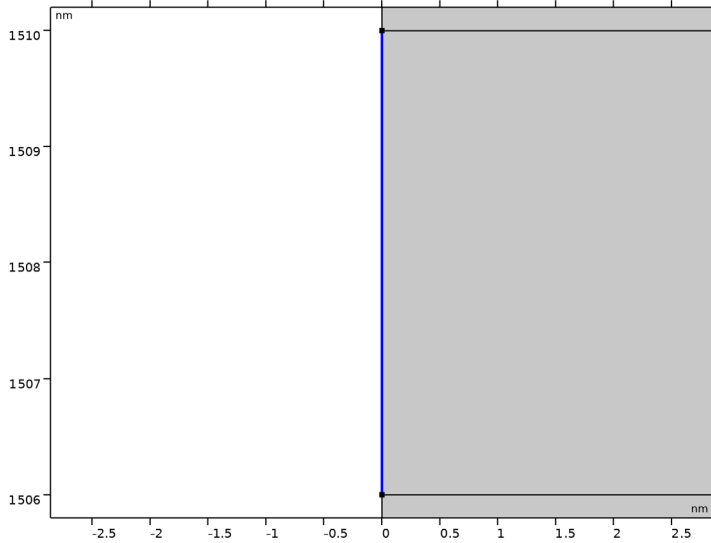
Distribution 3

1 Right-click **Distribution 2** and choose **Duplicate**.

2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.

4 Select Boundary 9 only.



In the **Graphics** window, click Zoom to Selection to clearly visualize the boundary layer.

5 Locate the **Distribution** section. In the **Number of elements** text field, type 10.

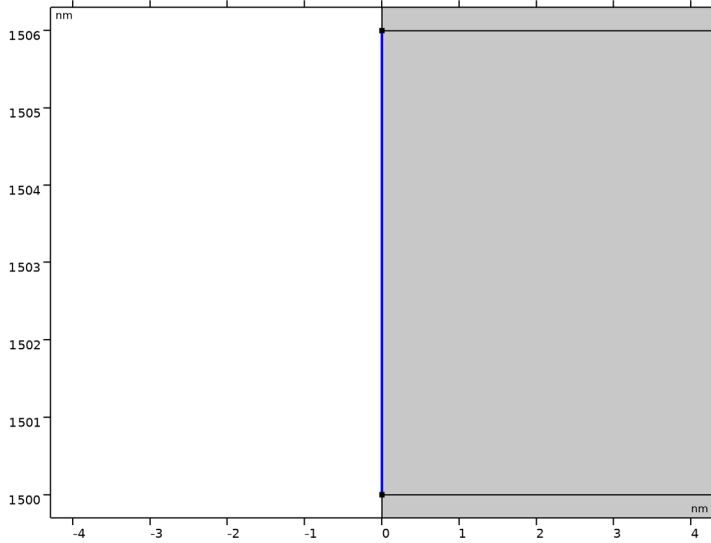
Distribution 4

1 Right-click **Distribution 3** and choose **Duplicate**.

2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.


4 Select Boundary 7 only.



In the **Graphics** window, click Zoom to Selection to clearly visualize the boundary layer.

5 Locate the **Distribution** section. In the **Number of elements** text field, type 25.

Mapped 1

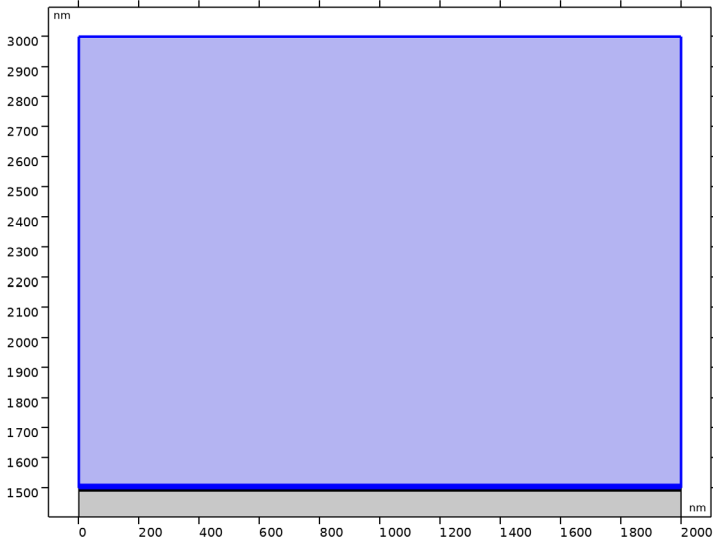
1 In the **Mesh** toolbar, click  **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domains 4–6 only.

In the **Graphics** window, click Zoom to Selection to properly visualize the top half of the geometry.



Copy Domain 1

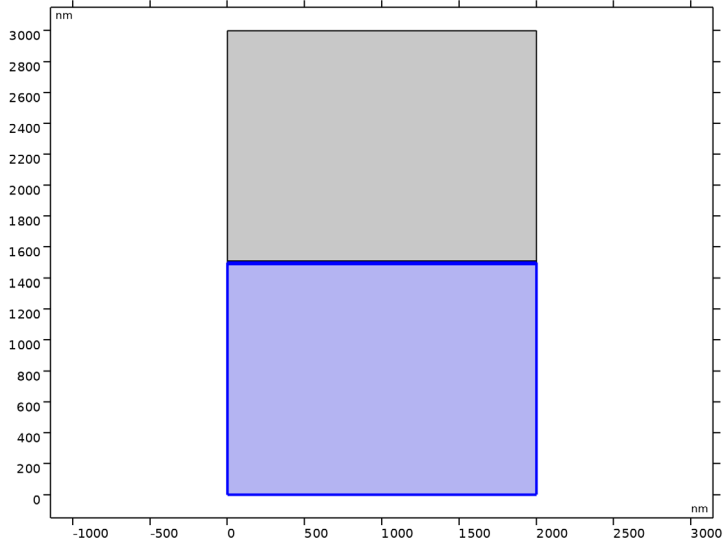
1 In the **Model Builder** window, right-click **Mesh 1** and choose **Copying Operations > Copy Domain**.

2 Select Domains 4–6 only.

3 In the **Settings** window for **Copy Domain**, locate the **Destination Domains** section.

4 Click to select the **Activate Selection** toggle button.

5 Select Domains 1–3 only.



6 Click  **Build All**.


STUDY 1

This study is used to simulate the field profile of surface plasmon polariton propagating in thin metal layer.

Step 1: Frequency Domain

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type f_0 .

Step 2: Boundary Mode Analysis

- 1 In the **Study** toolbar, click  **More Study Steps** and choose **Other > Boundary Mode Analysis**.
- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Mode analysis frequency** text field, type f_0 .
- 4 In the **Search for modes around shift** text field, type 1.4.
- 5 From the **Search method around shift** list, choose **Larger real part**.

Step 3: Boundary Mode Analysis 1

- 1 Right-click **Step 2: Boundary Mode Analysis** and choose **Duplicate**.

- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Port name** text field, type 2.

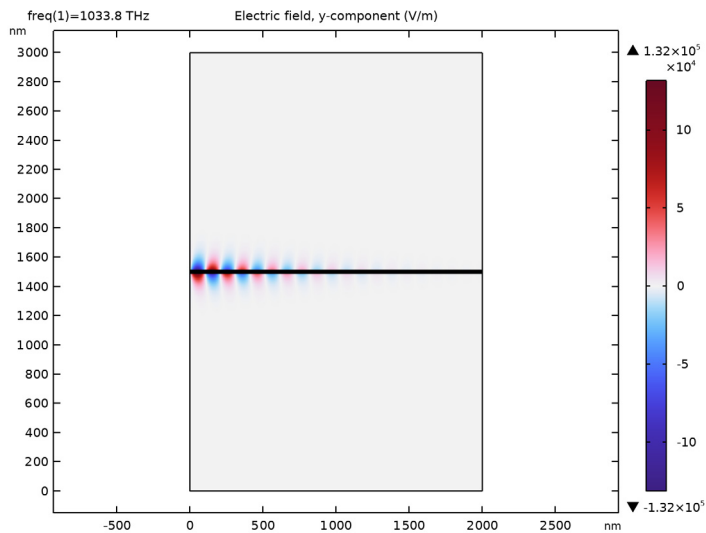
Step 1: Frequency Domain

- 1 In the **Model Builder** window, click **Step 1: Frequency Domain**.
- 2 Drag and drop below **Step 2: Boundary Mode Analysis I**.
- 3 In the **Study** toolbar, click **Compute**.

RESULTS

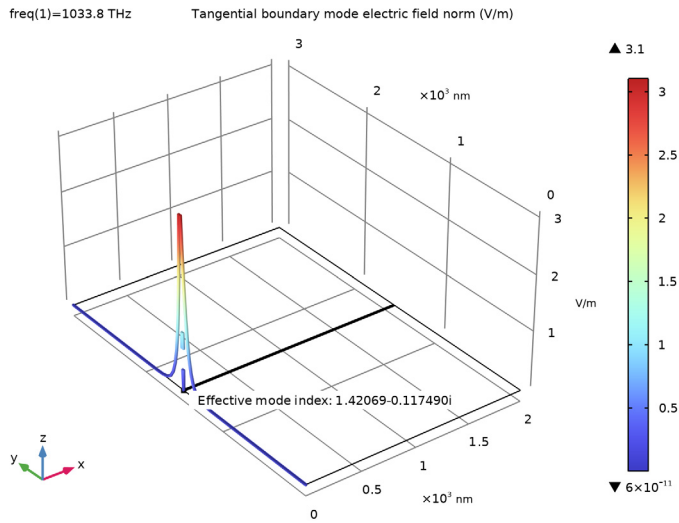
Surface 1

- 1 In the **Model Builder** window, expand the **Electric Field (ewfd)** node, then click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type $\text{ewfd} \cdot E_y$.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **Wave**.
- 5 From the **Scale** list, choose **Linear symmetric**.



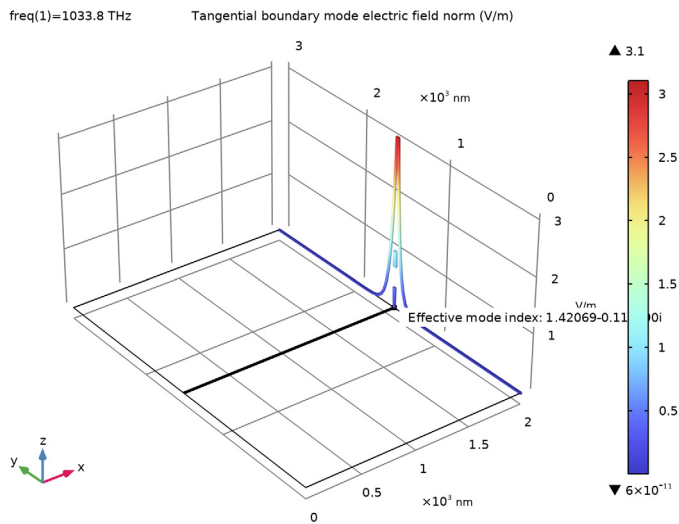
Electric Mode Field, Port 1 (ewfd)

In the **Model Builder** window, under **Results** click **Electric Mode Field, Port 1 (ewfd)**.




Electric Mode Field, Port 2 (ewfd)

In the **Model Builder** window, click **Electric Mode Field, Port 2 (ewfd)**.



ADD STUDY



- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Right-click and choose **Add Study**.

STUDY 2



Add another study to calculate the dispersion of thin metal embedded in dielectric versus frequency.

- 1 In the **Model Builder** window, click **Study 2**.
- 2 In the **Settings** window for **Study**, locate the **Study Settings** section.
- 3 Clear the **Generate default plots** checkbox.

Step 1: Boundary Mode Analysis


- 1 In the **Study** toolbar, click  **More Study Steps** and choose **Other > Boundary Mode Analysis**.
- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Mode analysis frequency** text field, type f_0 .
- 4 Select the **Desired number of modes** checkbox. In the associated text field, type 2.
- 5 From the **Search method around shift** list, choose **Larger real part**.
- 6 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 7 Click  **Add**.
- 8 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
lda0 (Wavelength)	range(135[nm], (640[nm] - (135[nm])) / 149, 640[nm])	m

- 9 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.
- 10 In the **Study** toolbar, click  **Compute**.

RESULTS

Propagation Constant

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Propagation Constant in the **Label** text field.

- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 2/Solution 4 (sol4)**.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the **Plot Settings** section.
- 6 Select the **x-axis label** checkbox. In the associated text field, type Propagation constant (1/m).
- 7 Select the **y-axis label** checkbox. In the associated text field, type Photon energy (eV).

First, plot the propagation constant of surface plasmon polariton in thin metal layer embedded in a dielectric.

Global I

- 1 Right-click **Propagation Constant** and choose **Global**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
<code>h_const*ewfd.freq</code>	eV	Photon energy (eV)

- 4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **All solutions**.
- 5 From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type `abs(ewfd.beta_1)`.
- 7 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
- 8 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.
- 9 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.
- 10 In the table, enter the following settings:

Legends
Simulated SPP dispersion

Color Expression I

- 1 Right-click **Global I** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, locate the **Expression** section.
- 3 In the **Expression** text field, type `-real(ewfd.beta_1)/imag(ewfd.beta_1)`.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **Viridis**.
- 5 From the **Scale** list, choose **Logarithmic**.

Global 2

1 In the **Model Builder** window, right-click **Propagation Constant** and choose **Global**.

Now, plot the propagation constant of light in free space.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
$h_const * \text{ewfd}.\text{freq}$	eV	Frequency (eV)

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **All solutions**.

5 From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type $\text{ewfd}.\text{k0}$.

7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

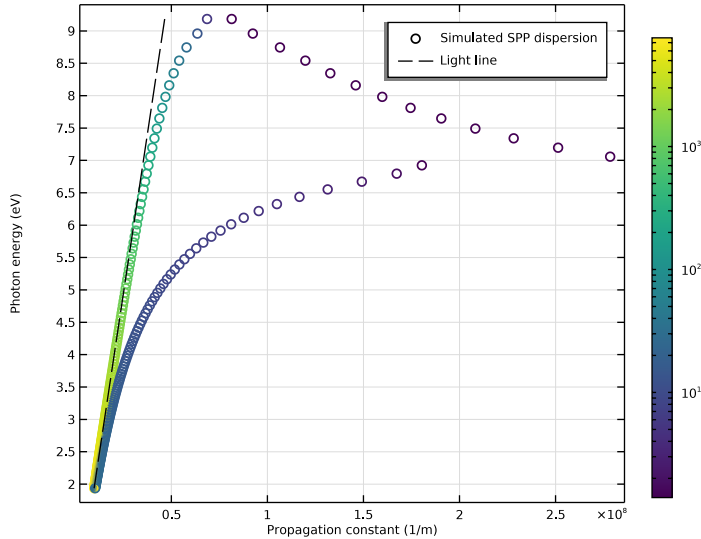
8 From the **Color** list, choose **Black**.

9 Locate the **Legends** section. From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
Light line

II In the **Propagation Constant** toolbar, click  **Plot**.



Propagation Length

1 Right-click **Propagation Constant** and choose **Duplicate**.

Plot the propagation length of SPP.

2 In the **Settings** window for **ID Plot Group**, type Propagation Length in the **Label** text field.

3 Locate the **Plot Settings** section. In the **x-axis label** text field, type Propagation length (nm).

4 Locate the **Legend** section. Clear the **Show legends** checkbox.

Global 1

1 In the **Model Builder** window, expand the **Propagation Length** node, then click **Global 1**.

2 In the **Settings** window for **Global**, locate the **x-Axis Data** section.

3 In the **Expression** text field, type $1/(2*\text{real}(\text{ewfd}.\text{beta}_1))$.

Global 2

In the **Model Builder** window, right-click **Global 2** and choose **Delete**.

Global 1

