



Model created in COMSOL Multiphysics 6.4

Surface Plasmon Polariton Excitation via Otto and Kretschmann Configurations

Introduction

Total internal reflection refers to the complete reflection of light, when it travels from a dense medium with a high refractive index to a medium with a lower refractive index. This phenomenon occurs at the interface between the two media, when the incident angle of light is larger than a limiting value, namely the critical angle. Even though the reflection is total, the tangential components of the electromagnetic field must be continuous across the interface and nonzero. However, the spatial distribution of the transmitted field should be limited in the sense that a continuous power flow within the medium of lower refractive index is not permitted. This demonstrates that the total internal reflection mechanism is accompanied by a nonradiative electromagnetic surface wave propagating along the interface with an evanescent wavefront in the normal direction within the transmitted domain.

In the case that another dense medium is brought close to the evanescent field, the total internal reflection mechanism may get disrupted, thereby leading to some power transmission into the new medium. This phenomenon, referred to as evanescent-wave coupling, is commonly used in nanophotonic devices such as waveguide sensors, couplers, and fiber tapping amongst many others.

The evanescent-wave coupling is the fundamental physics that enables excitation of surface plasmon polaritons (SPPs) via the Otto and Kretschmann configurations. For instance, when a metallic structure is brought into the vicinity of the evanescent field generated by the total internal reflection, it excites the free electrons in the metal and introduces SPPs propagating along the dielectric–metal interface. The optimal excitation of SPPs is achieved through the precise angle phase matching, which refers to the process of matching the tangential wave vector of the incident light to the propagation constant of the SPPs.

This model discusses the simulation setup for exciting SPPs using the Otto and Kretschmann configurations. It also analyzes the reflectance, transmittance, and SPP coupling of the systems. Both configurations involve the total internal reflection and evanescent-wave coupling phenomena. Angle phase matching is also performed to calculate the requirements to achieve optimal SPP excitation.

Model Definition

Figure 1 shows the two simulation setups: (a) Otto and (b) Kretschmann configurations. In both cases, SPPs are excited on a metal structure thanks to the evanescent-wave

coupling mechanism. The evanescent wave generated due to the total internal reflection at the prism layer, interacts with the plasma waves in metal, and excites SPPs.

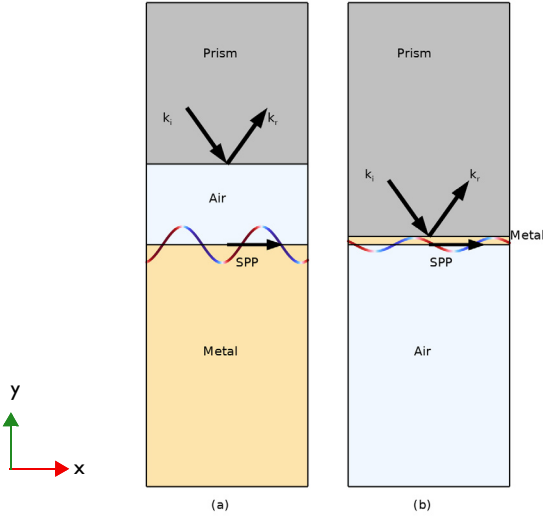


Figure 1: Excitation of surface plasmon polaritons using the (a) Otto and (b) Kretschmann configurations due to the total internal reflection mechanism. In both setups, the surface plasmon propagates in the x direction, along the metal-air interface, and decays in the y direction.

Imagine an electromagnetic wave propagating from a high index (refractive index n_i) to a low index medium (refractive index n_t), with the plane of incidence being the xy -plane at $z = 0$. Assuming that the interface between the two media is at $y = 0$, the transmitted wave vector can be expressed as

$$\mathbf{k}_t = \mathbf{x}k_x + \mathbf{y}k_y. \quad (1)$$

Here, $k_x = k_t \sin \theta_t$ and $k_y = k_t \cos \theta_t$, where k_t is the wavenumber of the transmitted wave band and θ_t is the angle of refraction.

Following Snell's law, $n_i \sin \theta_i = n_t \sin \theta_t$, where θ_i is the incident angle, the y -component of the transmitted wave vector can be written as

$$k_y = k_t \cos \theta_t = \pm k_t \sqrt{1 - \left(\frac{n_i}{n_t} \sin \theta_i\right)^2}. \quad (2)$$

When the condition of total internal reflection, $\sin \theta_i > n_t/n_i$, is satisfied, Equation 2 can be written as

$$k_y = \pm j k_t \sqrt{\left(\frac{n_i}{n_t} \sin \theta_i\right)^2 - 1} = \pm j \alpha. \quad (3)$$

Assume a transverse magnetic (TM) incident wave with magnetic field polarized in the z direction (perpendicular to the plane of incidence). The magnetic field component of such waves is given by

$$H_z = H_0 \exp[-j(\omega t - \mathbf{k} \cdot \mathbf{r})]. \quad (4)$$

Here, H_0 is the magnetic field amplitude. From Equation 4, the transmitted magnetic field component leads to

$$H_z = H_{t0} e^{-j(k_x x + k_y y)} = H_{t0} e^{-j k_x x - \alpha y}. \quad (5)$$

Here, H_{t0} is the transmitted magnetic field amplitude, α is the attenuation constant, and the '+' sign of k_y in Equation 2 is ignored as it corresponds to wave amplification along the y direction. Equation 5 shows that the transmitted wave is evanescent along the y direction, as the total internal reflection condition is satisfied.

In the Otto configuration, as is shown in panel (a) of Figure 1, the incident light is totally internally reflected back from the prism–air interface. A metal structure, that supports SPPs, is positioned near the interface within a few wavelengths of distance so that the evanescent wave can interact with the metal plasma wave and excites SPPs. By adequately tuning the angle of incidence, maximum SPP coupling can be achieved. In this situation, the reflectivity of the system almost vanishes as most of the waves are refracted into metal. Such a resonance condition appears over a very narrow range of incident angles.

The incident angle at which the optimum SPP excitation is achieved can be calculated using the angle phase matching. At this angle, the tangential wavenumber of the incident wave $k_x = n_t k_0 \sin \theta_{i0}$ is equal to the propagation constant β of the SPPs. The angle at which the optimum SPP excitation occurs is expressed as

$$\sin \theta_{i0} = \frac{\beta}{n_t k_0}. \quad (6)$$

In the Kretschmann configuration, as is shown in panel (b) of Figure 1, a metal thin film is deposited on a prism block. Here, the evanescent wave generated due to the total internal reflection penetrates through the metal layer, and SPPs are excited on the outer (metal–air) boundary. When the metal layer is thick, SPPs cannot be excited as the evanescent wave will be completely absorbed. The Kretschmann configuration is more

popular in practical applications as it resolves the challenging control of the precise prism-metal distance in the Otto configuration.

This model simulates excitation of SPPs using Otto and Kretschmann configurations, and perform angle phase matching analysis to calculate the required incident angle for the optimum SPP excitation. Silver is considered as the metal with material properties available from the built-in Optical Material Library. Periodic ports are used on the top and bottom boundaries of the model. The top port, with excitation turned on, launches the incident TM-light, and the bottom port, with excitation turned off, absorbs the transmitted wave. A Frequency Domain study step is used to solve for the domain field. An Auxiliary sweep of the incident angle is performed in the study step to evaluate the surface plasmon resonance condition by analyzing reflectance, transmittance, and SPP coupling of the systems.

Results and Discussion

Figure 2 shows the y -component of the electric field using the Otto configuration. The incident light is TM-polarized with an angle of incidence of 49° . The evanescent wave created due to the total internal reflection is strongly coupled to the metal in the form of

SPPs propagating along the x direction. The decay rate in the metal is much faster than in air thanks to the strong absorption.

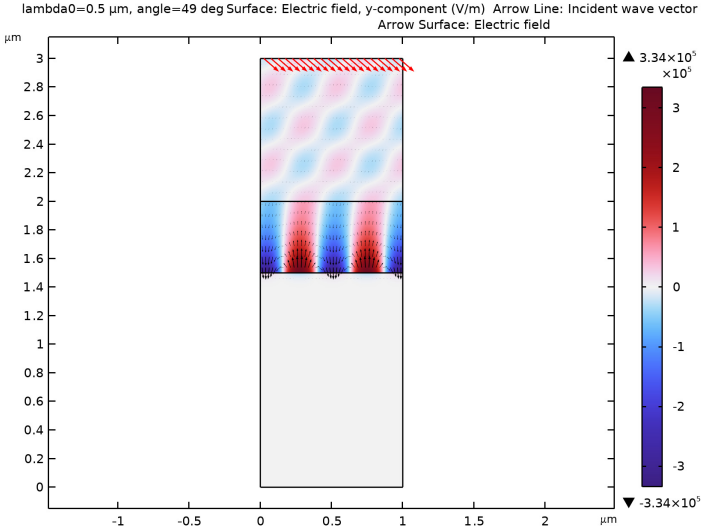


Figure 2: The y-component of the electric field for the Otto configuration. The wavelength is set to 500 nm and the angle of incidence is 49°. Red arrows indicate the incident light wave vector and black arrows indicate the electric field polarization.

Figure 3 shows the reflectance, transmittance, and SPP coupling curve versus the incident angle. The surface plasmon resonance condition is fulfilled at an angle of 49° where maximum SPP coupling is observed.

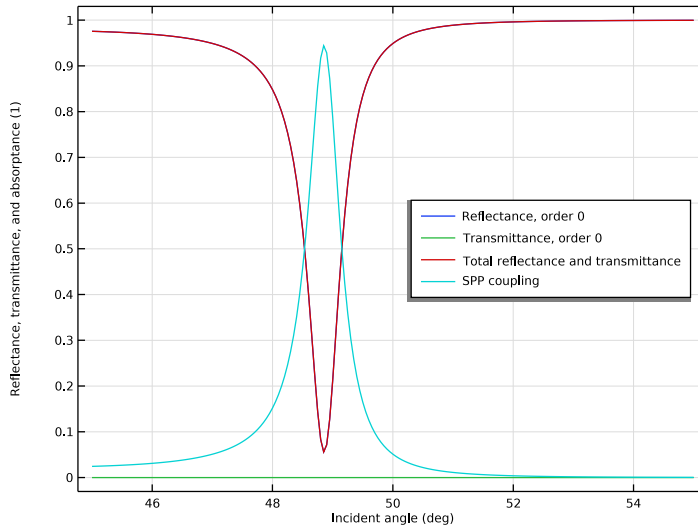


Figure 3: Reflectance, transmittance, and SPP coupling versus the incident angle.

Figure 4 shows a similar study, but with Kretschmann configuration. Here, SPPs are excited on the outer boundary of the thin metal layer (that is, the metal–air interface).

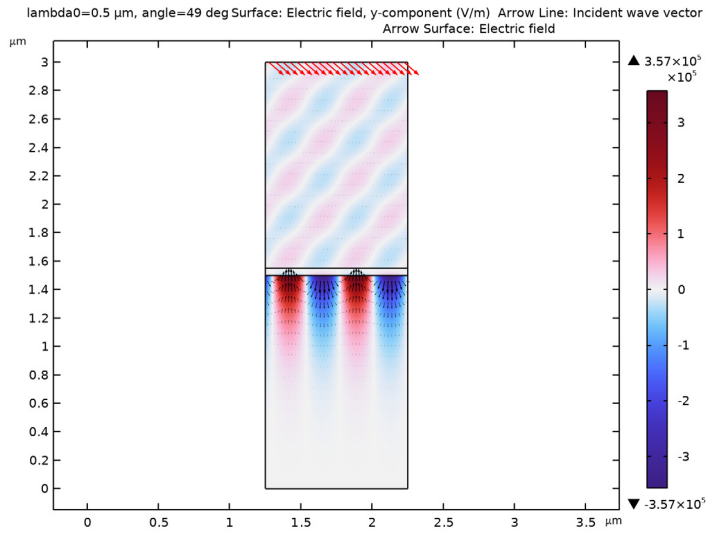


Figure 4: The y-component of the electric field for the Kretschmann configuration. The wavelength is set to 500 nm and the angle of incidence is 49° . Red arrows indicate the incident light wave vector and black arrows indicate the electric field polarization.

Figure 5 shows that the surface plasmon resonance condition is fulfilled at a 49° angle of incidence, similar to the case of the Otto configuration.

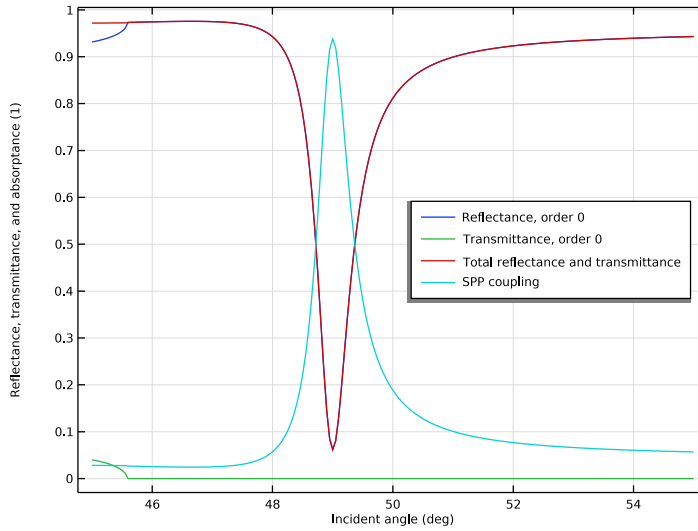


Figure 5: Reflectance, transmittance, and absorbance versus incident angle.

Figure 6 shows the tangential wavenumber of the incident wave and the propagation constant of the SPPs versus the angle of incidence. It shows that the angle phase matching is obtained when the incident angle is approximately 49° and optimum SPP is excited at this angle.

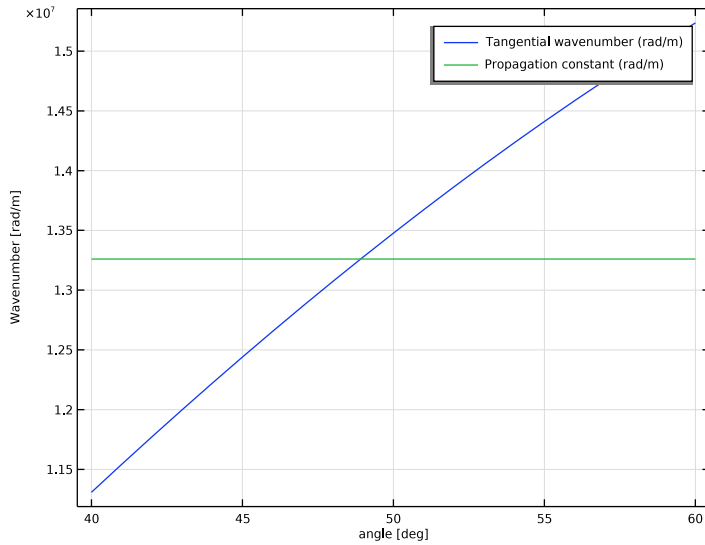


Figure 6: The tangential wavenumber of the incident wave and the propagation constant of the surface plasmon polariton wave versus the incident angle. The wavelength is set to 500 nm in free space.

For a modeling setup to use for calculating the dispersion relation and additional SPP simulation examples, see the blog post in [Ref. 1](#).

Reference


1. X. Chen, “Modeling Surface Plasmon Polaritons in COMSOL®,” COMSOL Blog, 12 Oct. 2022; www.comsol.com/blogs/modeling-surface-plasmon-polaritons-in-comsol/.

Application Library path: Wave_Optics_Module/Waveguides/
spp_otto_kretschmann




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, Use two physics interfaces to simulate the Otto and Kretschmann configurations independently, and another physics interface to calculate the angle phase matching.
- 2 click  **2D**.
- 3 In the **Select Physics** tree, select **Optics > Wave Optics > Electromagnetic Waves, Frequency Domain (ewfd)**.
- 4 Click **Add**.
- 5 Click **Add**, to add the second physics interface.
- 6 Click **Add**, to add the third physics interface.
- 7 Click  **Study**.
- 8 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Wavelength Domain**.
- 9 Click  **Done**.

GEOMETRY I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose μm .

GLOBAL DEFINITIONS

Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:


Name	Expression	Value	Description
L	1[μm]	1E-6 m	Simulation domain length
H	3[μm]	3E-6 m	Simulation domain height
lda0	500[nm]	5E-7 m	Wavelength
angle	0[deg]	0 rad	Incident angle
n_prism	1.4	1.4	Refractive index, prism

GEOMETRY 1

The geometry is simple, consisting of three rectangular geometries side by side, representing the (i) Otto and (ii) Kretschmann configurations, and (iii) to calculate the angle phase matching.

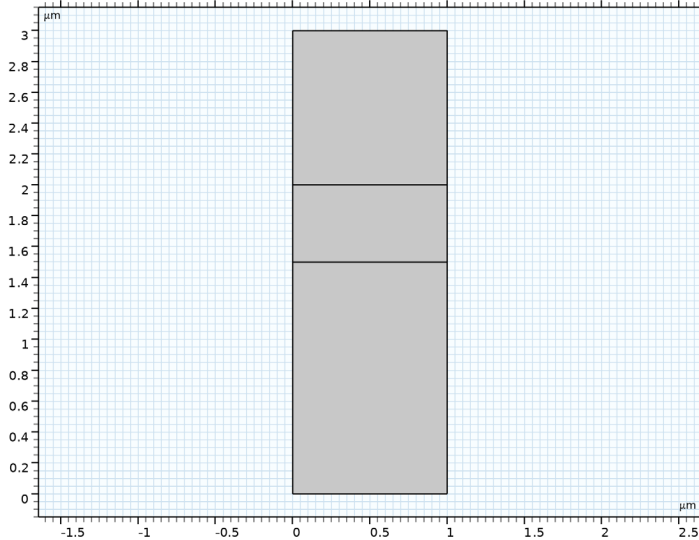
First, build the geometry of the Otto configuration.

Rectangle 1 (r1)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type L.
- 4 In the **Height** text field, type H.
- 5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (μm)
Layer 1	H/2
Layer 2	1da0

- 6 Click  **Build Selected**.





Rectangle 2 (r2)

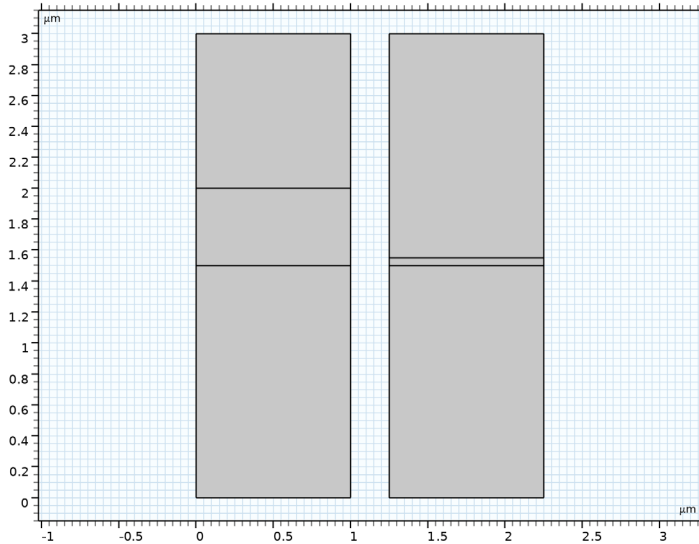
Next, build the geometry of the Kretschmann configuration.

- 1 Right-click **Rectangle 1 (r1)** and choose **Duplicate**.

- 2 In the **Settings** window for **Rectangle**, locate the **Position** section.
- 3 In the **x** text field, type $1.25*L$.
- 4 Locate the **Layers** section. In the table, enter the following settings:


Layer name	Thickness (μm)
Layer 2	$1d_0/10$

- 5 Click  **Build Selected**.
- 6 Click the  **Zoom Extents** button in the **Graphics** toolbar.




Rectangle 3 (r3)

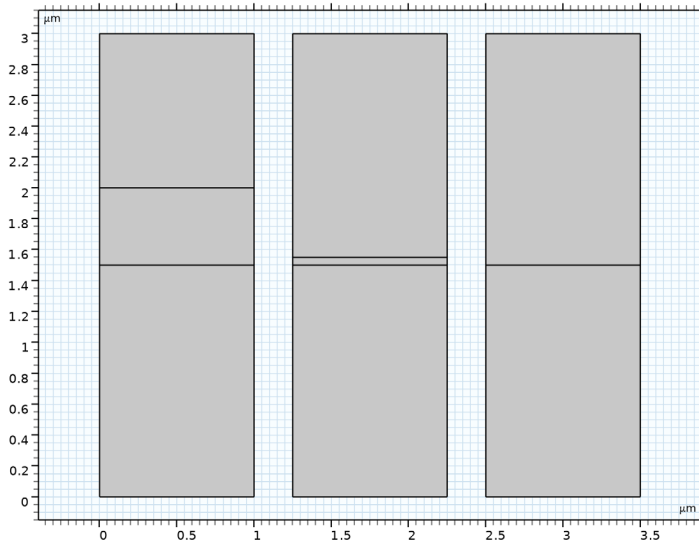
Now, build the geometry to calculate the angle phase matching.

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type L .
- 4 In the **Height** text field, type H .
- 5 Locate the **Position** section. In the **x** text field, type $2.5*L$.
- 6 Locate the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (μm)
Layer 1	$H/2$

7 Click  **Build All Objects**.


8 Click the  **Zoom Extents** button in the **Graphics** toolbar.



MATERIALS

Pick silver from the Optical material library.

ADD MATERIAL

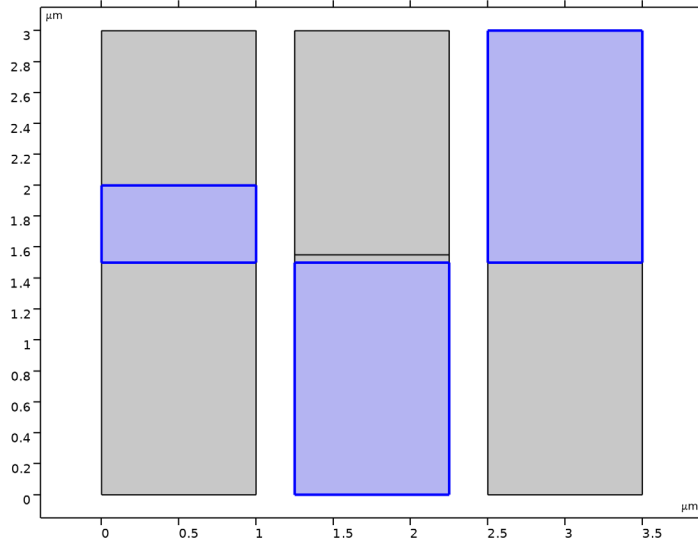
- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Optical** > **Inorganic Materials** > **Ag - Silver** > **Experimental data: bulk, thick film** > **Ag (Silver) (Johnson and Christy 1972: n,k 0.188-1.94 um)**.
- 4 Right-click and choose **Add to Component 1 (comp1)**.
- 5 In the tree, select **Built-in** > **Air**.
- 6 Right-click and choose **Add to Component 1 (comp1)**.

MATERIALS

Air (mat2)

- 1 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.
- 2 In the **Model Builder** window, under **Component 1 (comp1)** > **Materials** click **Air (mat2)**.

3 Select Domains 2, 4, and 8 only.

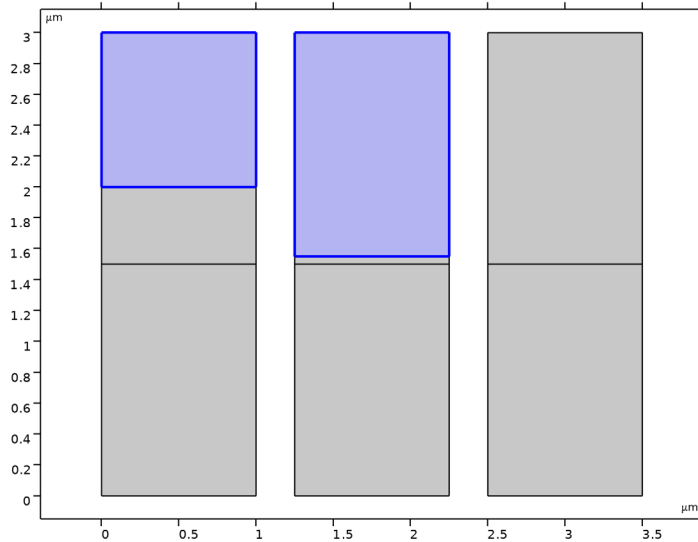


Prism

1 In the **Model Builder** window, right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type **Prism** in the **Label** text field.

3 Select Domains 3 and 6 only.




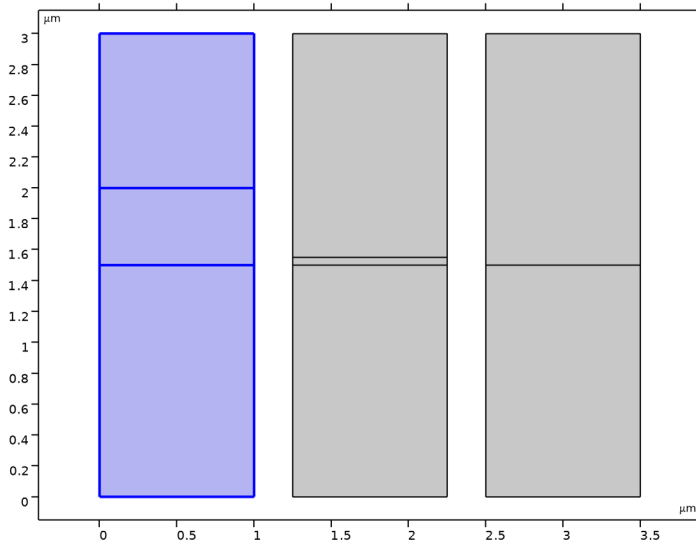
4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_{iso} ; $n_{ij} = n_{iso}$, $n_{ij} = 0$	n_{prism}	1	Refractive index

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

This physics interface is used to simulate the Otto configuration.

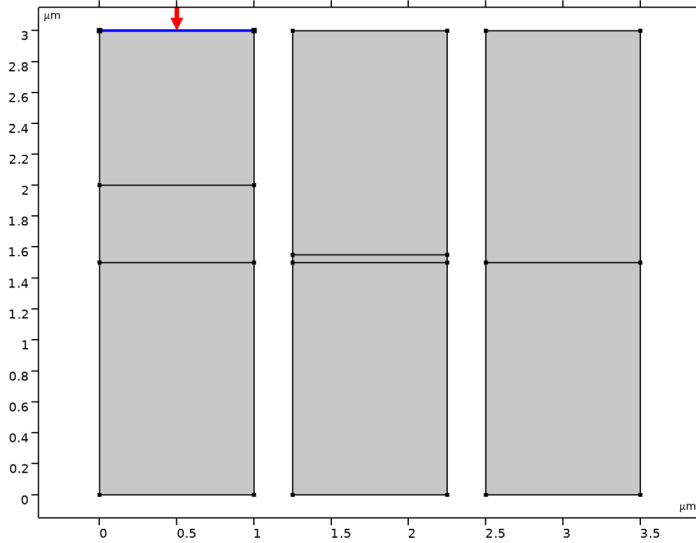
- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (ewfd)**.
- 2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Domain Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Domains 1–3 only.



Port 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Port**.

2 Select Boundary 7 only.



3 In the **Settings** window for **Port**, locate the **Port Properties** section.

4 From the **Type of port** list, choose **Periodic**.

5 Locate the **Port Mode Settings** section. From the **Input quantity** list, choose **Magnetic field**.

6 Specify the \mathbf{H}_0 vector as

0	x
0	y
1	z

7 In the α text field, type angle.

Port 2

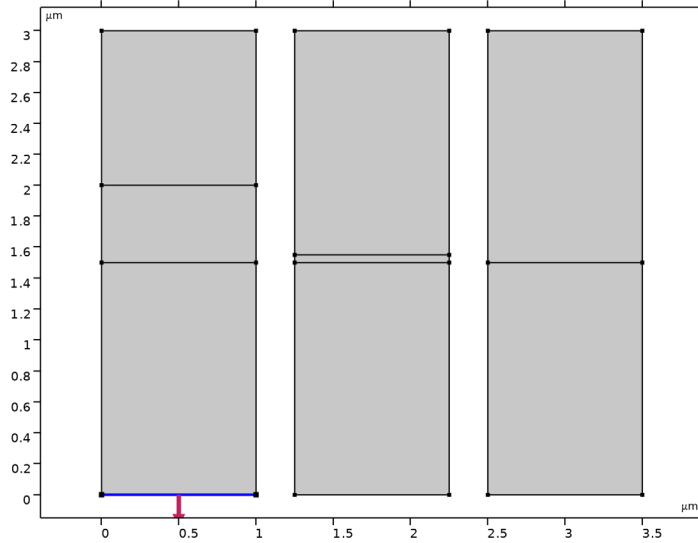
1 Right-click **Port 1** and choose **Duplicate**.

2 In the **Settings** window for **Port**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.

4 Select Boundary 2 only.

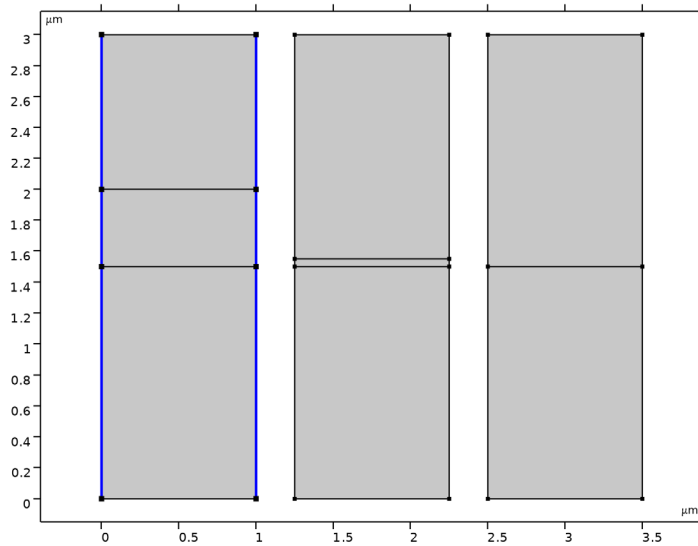
5 Locate the **Port Properties** section. From the **Wave excitation at this port** list, choose **Off**.



Periodic Condition 1

1 In the **Physics** toolbar, click  **Boundaries** and choose **Periodic Condition**.

2 Select Boundaries 1, 3, 5, and 8–10 only.




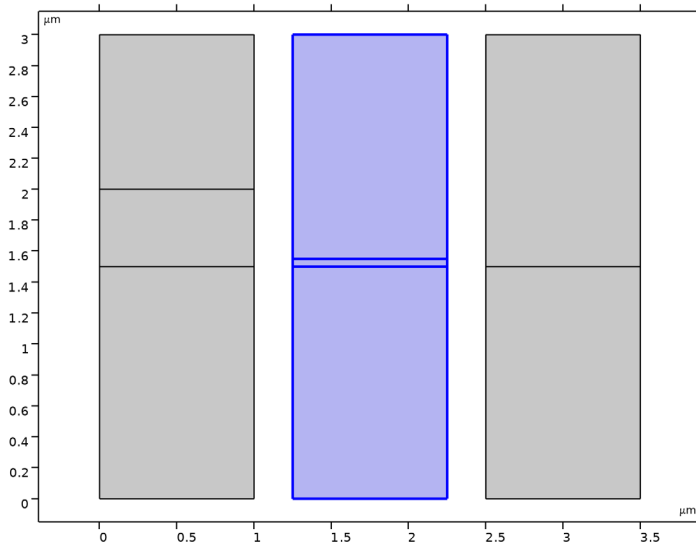
3 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.

- 4 From the **Type of periodicity** list, choose **Floquet periodicity**.
- 5 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN 2 (EWFD2)

This physics interface is used to simulate the Kretschmann configuration.

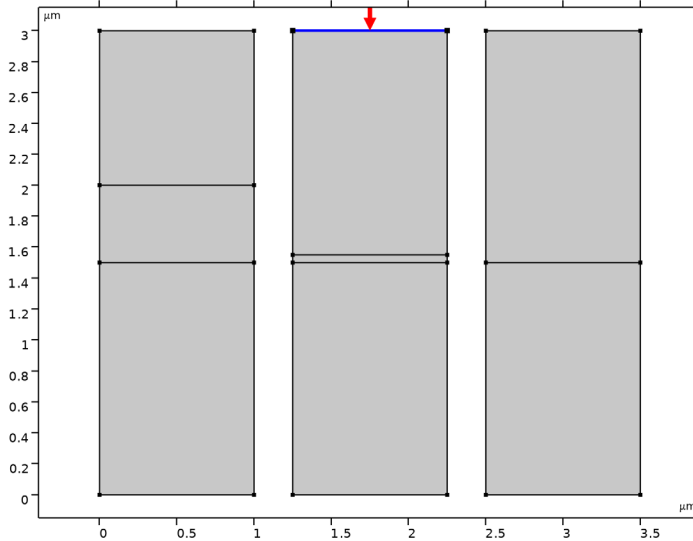
- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain 2 (ewfd2)**.
- 2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Domain Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Domains 4–6 only.



Port 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Port**.

2 Select Boundary 17 only.



3 In the **Settings** window for **Port**, locate the **Port Properties** section.

4 From the **Type of port** list, choose **Periodic**.

5 Locate the **Port Mode Settings** section. From the **Input quantity** list, choose **Magnetic field**.

6 Specify the \mathbf{H}_0 vector as

0	x
0	y
1	z

7 In the α text field, type angle.

Port 2

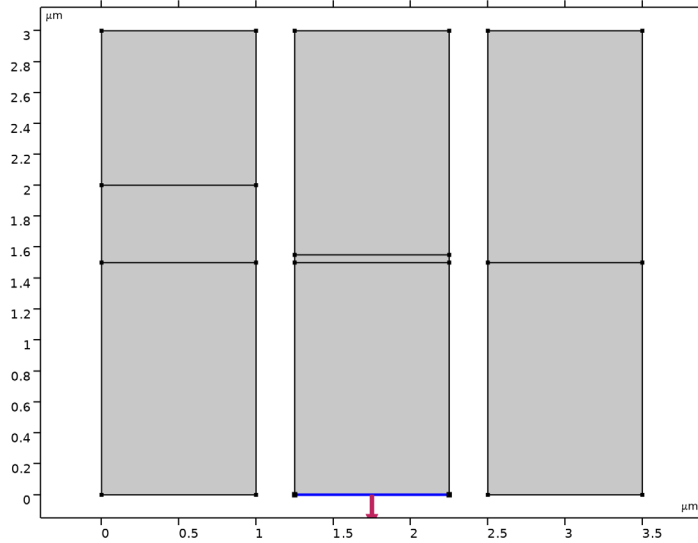
1 Right-click **Port 1** and choose **Duplicate**.

2 In the **Settings** window for **Port**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.

4 Select Boundary 12 only.

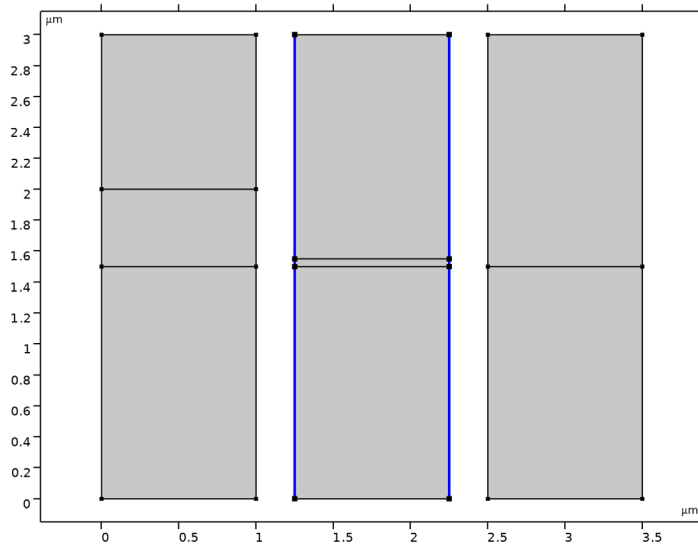
5 Locate the **Port Properties** section. From the **Wave excitation at this port** list, choose **Off**.



Periodic Condition 1

1 In the **Physics** toolbar, click  **Boundaries** and choose **Periodic Condition**.

2 Select Boundaries 11, 13, 15, and 18–20 only.




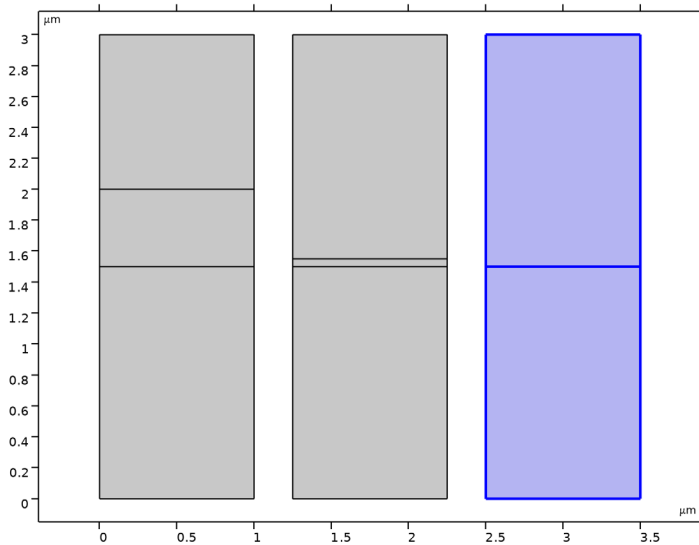
3 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.

- 4 From the **Type of periodicity** list, choose **Floquet periodicity**.
- 5 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN 3 (EWF3)

This physics interface is used to calculate the angle phase matching.

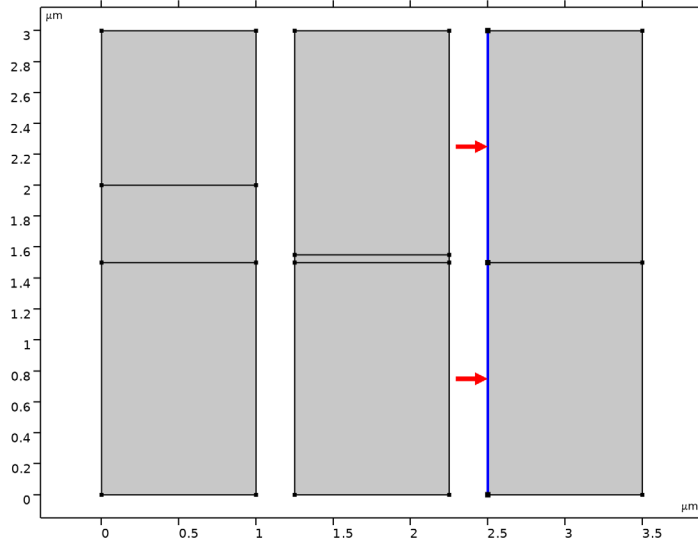
- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain 3 (ewfd3)**.
- 2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Domain Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Domains 7 and 8 only.



Port 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Port**.

2 Select Boundaries 21 and 23 only.



3 In the **Settings** window for **Port**, locate the **Port Properties** section.

4 From the **Type of port** list, choose **Numeric**.

Port 2

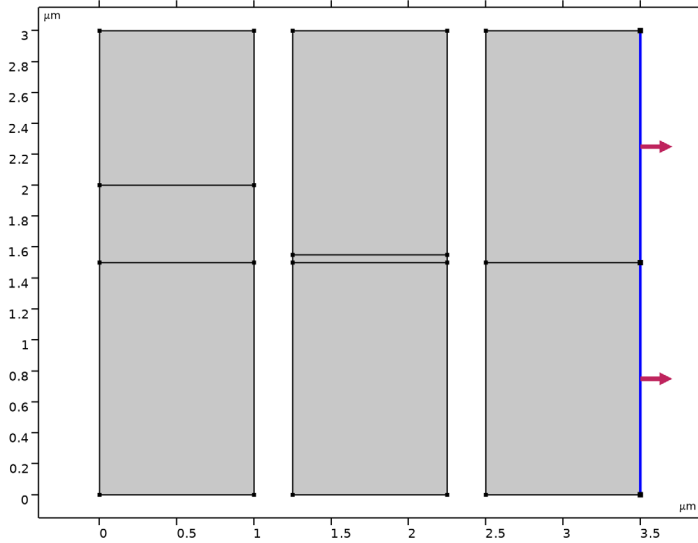
1 Right-click **Port 1** and choose **Duplicate**.

2 In the **Settings** window for **Port**, locate the **Boundary Selection** section.

3 Click  **Clear Selection**.

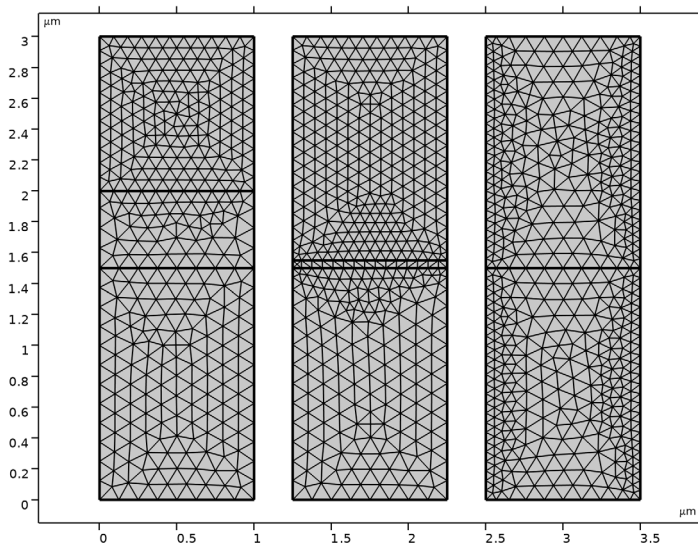
4 Select Boundaries 26 and 27 only.

5 Locate the **Port Properties** section. From the **Wave excitation at this port** list, choose **Off**.



MESH I

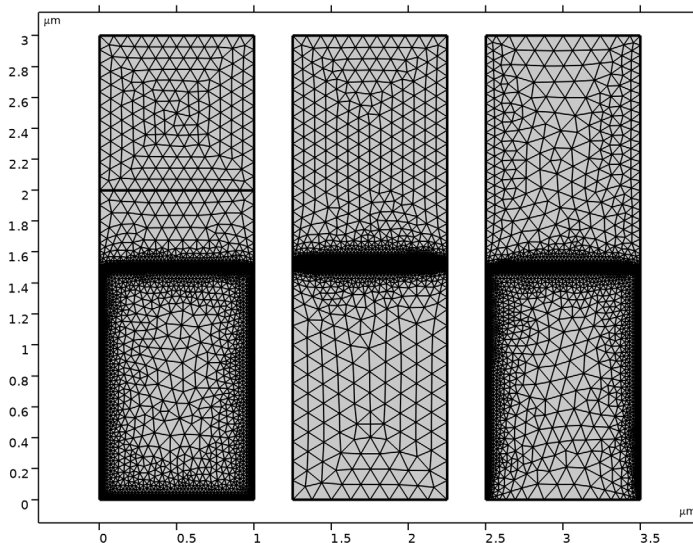
I In the **Model Builder** window, under **Component I (comp1)** right-click **Mesh I** and choose **Build All**.



- 2 In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Frequency Domain (ewfd)** section.
- 3 Select the **Resolve wave in lossy media** checkbox.
- 4 Locate the **Electromagnetic Waves, Frequency Domain 2 (ewfd2)** section. Select the **Resolve wave in lossy media** checkbox.
- 5 Locate the **Electromagnetic Waves, Frequency Domain 3 (ewfd3)** section. Select the **Resolve wave in lossy media** checkbox.

It applies a finer mesh to the lossy media and metal domains to properly resolve the electromagnetic waves.

- 6 Click  **Build All**.



STUDY, OTTO

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Study, Otto in the **Label** text field.

Step 1: Wavelength Domain

- 1 In the **Model Builder** window, under **Study, Otto** click **Step 1: Wavelength Domain**.
- 2 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.
- 3 In the **Wavelengths** text field, type 1da0.
- 4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkboxes for **Electromagnetic Waves**,

Frequency Domain 2 (ewfd2) and **Electromagnetic Waves, Frequency Domain 3 (ewfd3)**.
This study simulates the Otto configuration only.

5 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

6 Click  **Add**.

7 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Incident angle)		rad

8 In the table, click to select the cell at row number 1 and column number 2.

9 Click  **Range**.

10 In the **Range** dialog, type 45 in the **Start** text field.

11 In the **Step** text field, type 0.05.

12 In the **Stop** text field, type 55.

13 Click **Add**.

14 In the **Settings** window for **Wavelength Domain**, locate the **Study Extensions** section.

15 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Incident angle)	range (45,0.05,55)	deg

ADD STUDY

Add a study to simulate the Kretschmann configuration.

1 In the **Study** toolbar, click  **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Wavelength Domain**.

4 Right-click and choose **Add Study**.

5 In the **Study** toolbar, click  **Add Study** to close the **Add Study** window.

STUDY, KRETSCHMANN


In the **Settings** window for **Study**, type Study, Kretschmann in the **Label** text field.

1 In the **Model Builder** window, under **Study, Kretschmann** click **Step 1: Wavelength Domain**.

2 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.

- 3 In the **Wavelengths** text field, type 1da0.
- 4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkboxes for **Electromagnetic Waves, Frequency Domain (ewfd)** and **Electromagnetic Waves, Frequency Domain 3 (ewfd3)**.
- 5 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 6 Click **+ Add**.
- 7 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Incident angle)		rad

- 8 In the table, click to select the cell at row number 1 and column number 2.
- 9 Click  **Range**.
- 10 In the **Range** dialog, type 45 in the **Start** text field.
- 11 In the **Step** text field, type 0.05.
- 12 In the **Stop** text field, type 55.
- 13 Click **Add**.



14 In the **Settings** window for **Wavelength Domain**, locate the **Study Extensions** section.

15 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Incident angle)	range (45,0.05,55)	deg

ADD STUDY

Now, add another study to calculate the angle phase matching.


- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Wavelength Domain**.
- 4 Right-click and choose **Add Study**.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

STUDY, ANGLE PHASE MATCHING

In the **Settings** window for **Study**, type Study, Angle Phase Matching in the **Label** text field.

- 1 In the **Model Builder** window, under **Study, Angle Phase Matching** click **Step 1: Wavelength Domain**.
- 2 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.
- 3 In the **Wavelengths** text field, type $1da0$.
- 4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkboxes for **Electromagnetic Waves, Frequency Domain (ewfd)** and **Electromagnetic Waves, Frequency Domain 2 (ewfd2)**.

Step 2: Boundary Mode Analysis

- 1 In the **Study** toolbar, click  **More Study Steps** and choose **Other > Boundary Mode Analysis**.
- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Mode analysis frequency** text field, type $c_const/1da0$.
- 4 In the **Search for modes around shift** text field, type 5.
- 5 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkboxes for **Electromagnetic Waves, Frequency Domain (ewfd)** and **Electromagnetic Waves, Frequency Domain 2 (ewfd2)**.

Step 3: Boundary Mode Analysis I

- 1 Right-click **Step 2: Boundary Mode Analysis** and choose **Duplicate**.
- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Port name** text field, type 2.


Step 2: Boundary Mode Analysis, Step 3: Boundary Mode Analysis I

In the **Model Builder** window, under **Study, Angle Phase Matching**, Ctrl-click to select **Step 2: Boundary Mode Analysis** and **Step 3: Boundary Mode Analysis I**.

Step 1: Boundary Mode Analysis

Drag and drop above **Step 3: Wavelength Domain**.

Parametric Sweep


- 1 In the **Study** toolbar, click  **Parametric Sweep**.
- 2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
- 3 Click **+ Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
lda0 (Wavelength)	range (330, 15, 375) range (400, 25, 675)	nm

STUDY, OTTO

First, simulate the Otto configuration.


In the **Study** toolbar, click  **Compute**.

RESULTS

Study, Otto/Solution 1 (sol1)

In the **Model Builder** window, expand the **Results > Datasets** node, then click **Study, Otto/Solution 1 (sol1)**.

Selection

- 1 In the **Results** toolbar, click  **Attributes** and choose **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domains 1–3 only.

Electric Field (ewfd)

Create a plot of the electric field to visualize the surface plasmon polariton wave as is shown in [Figure 2](#).

- 1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd)**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (angle (deg))** list, choose **49**. At this angle maximum absorption occurs, as will be shown in the next figure.

Surface 1

- 1 In the **Model Builder** window, expand the **Electric Field (ewfd)** node, then click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type $ewfd.Ey$.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **Wave**.
- 5 From the **Scale** list, choose **Linear symmetric**.

Electric Field, Otto (ewfd)

- 1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd)**.
- 2 In the **Settings** window for **2D Plot Group**, type Electric Field, Otto (ewfd) in the **Label** text field.




Arrow Line 1

- 1 Right-click **Electric Field, Otto (ewfd)** and choose **Arrow Line**.
Plot the incident wave vector at the excitation port.
- 2 In the **Settings** window for **Arrow Line**, locate the **Expression** section.
- 3 In the **X-component** text field, type `ewfd.kIncx_1`.
- 4 In the **Y-component** text field, type `ewfd.kIncy_1`.
- 5 Locate the **Arrow Positioning** section. In the **Number of arrows** text field, type 20.
- 6 Locate the **Coloring and Style** section.
- 7 Select the **Scale factor** checkbox. In the associated text field, type 8E-9.

Selection 1

- 1 Right-click **Arrow Line 1** and choose **Selection**.
- 2 Select Boundary 7 only.

Arrow Surface 1

- 1 In the **Model Builder** window, right-click **Electric Field, Otto (ewfd)** and choose **Arrow Surface**.
Plot the electric field polarization.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.
- 3 In the **X-component** text field, type `ewfd.Ex`.
- 4 In the **Y-component** text field, type `ewfd.Ey`.
- 5 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type 40.
- 6 Find the **Y grid points** subsection. In the **Points** text field, type 40.
- 7 Locate the **Coloring and Style** section.
- 8 Select the **Scale factor** checkbox. In the associated text field, type 3E-7.
- 9 In the **Electric Field, Otto (ewfd)** toolbar, click  **Plot**.
- 10 From the **Color** list, choose **Black**.
- 11 In the **Electric Field, Otto (ewfd)** toolbar, click  **Plot**.
- 12 Click the  **Zoom Extents** button in the **Graphics** toolbar, to reproduce [Figure 2](#).

Reflectance, Transmittance, and SPP Coupling, Otto (ewfd)

Create a plot of the reflectance, transmittance, and SPP coupling, to reproduce [Figure 3](#).

- 1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorptance (ewfd)**.
- 2 In the **Settings** window for **ID Plot Group**, type Reflectance, Transmittance, and SPP Coupling, Otto (ewfd) in the **Label** text field.
- 3 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

Global I

- 1 In the **Model Builder** window, expand the **Reflectance, Transmittance, and SPP Coupling, Otto (ewfd)** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:


Expression	Unit	Description
ewfd.Atotal	1	SPP coupling

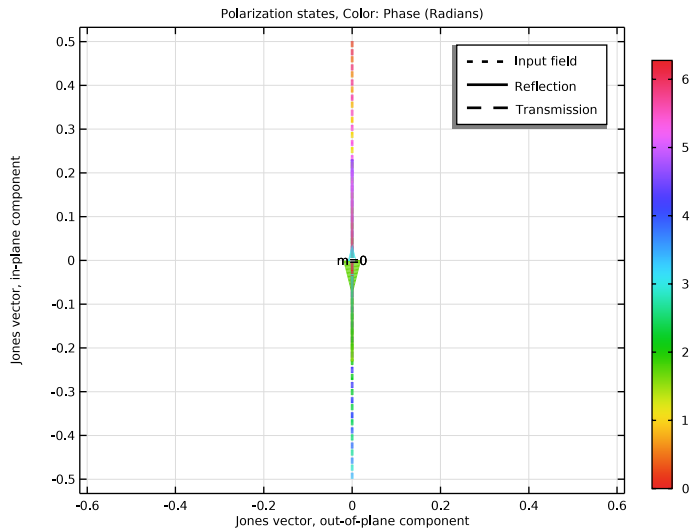
- 4 In the **Reflectance, Transmittance, and SPP Coupling, Otto (ewfd)** toolbar, click  **Plot**.

Polarization Plot, Otto (ewfd)

Visualize the polarization plot.


- 1 In the **Model Builder** window, under **Results** click **Polarization Plot (ewfd)**.
- 2 In the **Settings** window for **ID Plot Group**, type Polarization Plot, Otto (ewfd) in the **Label** text field.
- 3 Locate the **Data** section. In the **Parameter values (angle (deg))** list box, select **49**.

4 In the **Polarization Plot, Otto (ewfd)** toolbar, click  **Plot**.



STUDY, KRETSCHMANN

Now, simulate the Kretschmann configuration.


In the **Study** toolbar, click  **Compute**.

RESULTS

Study, Kretschmann/Solution 2 (sol2)

In the **Model Builder** window, under **Results > Datasets** click **Study, Kretschmann/Solution 2 (sol2)**.

Selection

- 1 In the **Results** toolbar, click  **Attributes** and choose **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domains 4–6 only.

Electric Field, Kretschmann (ewfd2)

Create a plot of the electric field to visualize the surface plasmon polariton wave as is shown in [Figure 4](#).

- 1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd2)**.

- 2 In the **Settings** window for **2D Plot Group**, type **Electric Field, Kretschmann (ewfd2)** in the **Label** text field.
- 3 Locate the **Data** section. From the **Parameter value (angle (deg))** list, choose **49**.

Surface 1

- 1 In the **Model Builder** window, expand the **Electric Field, Kretschmann (ewfd2)** node, then click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type **ewfd2.Ey**.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **Wave**.
- 5 From the **Scale** list, choose **Linear symmetric**.

Arrow Line 1




- 1 In the **Model Builder** window, right-click **Electric Field, Kretschmann (ewfd2)** and choose **Arrow Line**.
- 2 In the **Settings** window for **Arrow Line**, locate the **Expression** section.
- 3 In the **X-component** text field, type **ewfd2.kIncx_1**.
- 4 In the **Y-component** text field, type **ewfd2.kIncy_1**.
- 5 Locate the **Arrow Positioning** section. In the **Number of arrows** text field, type **20**.
- 6 Locate the **Coloring and Style** section.
- 7 Select the **Scale factor** checkbox. In the associated text field, type **8E-9**.

Selection 1

- 1 Right-click **Arrow Line 1** and choose **Selection**.
- 2 Select **Boundary 17** only.

Arrow Surface 1

- 1 In the **Model Builder** window, right-click **Electric Field, Kretschmann (ewfd2)** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.
- 3 In the **X-component** text field, type **ewfd2.Ex**.
- 4 In the **Y-component** text field, type **ewfd2.Ey**.
- 5 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type **40**.
- 6 Find the **Y grid points** subsection. In the **Points** text field, type **40**.
- 7 Locate the **Coloring and Style** section.

- 8 Select the **Scale factor** checkbox. In the associated text field, type 3E-7.
- 9 In the **Electric Field, Kretschmann (ewfd2)** toolbar, click  **Plot**.
- 10 From the **Color** list, choose **Black**.
- 11 In the **Electric Field, Kretschmann (ewfd2)** toolbar, click  **Plot**.
- 12 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Reflectance, Transmittance, and SPP Coupling, Kretschmann (ewfd2)

Create a plot of the reflectance, transmittance, and SPP coupling, to reproduce [Figure 5](#).

- 1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorptance (ewfd2)**.
- 2 In the **Settings** window for **ID Plot Group**, type Reflectance, Transmittance, and SPP Coupling, Kretschmann (ewfd2) in the **Label** text field.
- 3 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

Global I

- 1 In the **Model Builder** window, expand the **Reflectance, Transmittance, and SPP Coupling, Kretschmann (ewfd2)** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
ewfd2.Atotal	1	SPP coupling

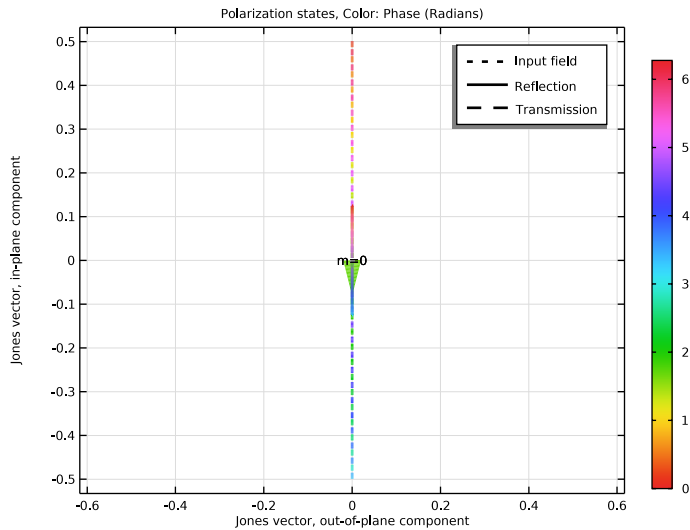
- 4 In the **Reflectance, Transmittance, and SPP Coupling, Kretschmann (ewfd2)** toolbar, click  **Plot**.

Polarization Plot, Kretschmann (ewfd2)

Visualize the polarization plot.


- 1 In the **Model Builder** window, under **Results** click **Polarization Plot (ewfd2)**.
- 2 In the **Settings** window for **ID Plot Group**, type Polarization Plot, Kretschmann (ewfd2) in the **Label** text field.
- 3 Locate the **Data** section. In the **Parameter values (angle (deg))** list box, select **49**.

- 4 In the **Polarization Plot, Kretschmann (ewfd2)** toolbar, click  **Plot**.



Instantaneous Electric Field Norm

Now, create a plot of the instantaneous electric field norm for the Otto and the Kretschmann configurations.

- 1 In the **Results** toolbar, click  **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, type Instantaneous Electric Field Norm in the **Label** text field.
- 3 Locate the **Data** section. From the **Parameter value (angle (deg))** list, choose **49**.

Surface 1

- 1 Right-click **Instantaneous Electric Field Norm** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type `ewfd.normEi`.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **GrayBody**.

Arrow Surface 1


- 1 In the **Model Builder** window, right-click **Instantaneous Electric Field Norm** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.
- 3 In the **X-component** text field, type `ewfd.Ex`.


- 4 In the **Y-component** text field, type `ewfd.Ey`.
- 5 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type 30.
- 6 Find the **Y grid points** subsection. In the **Points** text field, type 30.
- 7 Locate the **Coloring and Style** section.
- 8 Select the **Scale factor** checkbox. In the associated text field, type 5E-7.
- 9 From the **Color** list, choose **White**.

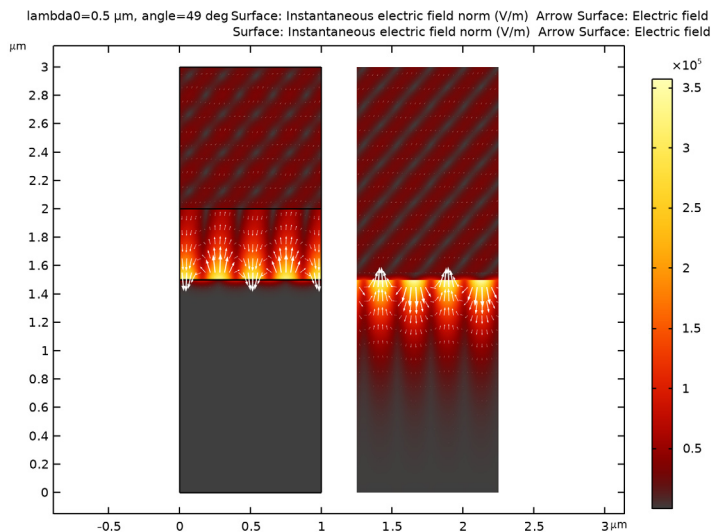
Surface 2

- 1 In the **Model Builder** window, under **Results > Instantaneous Electric Field Norm** right-click **Surface 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Surface**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study, Kretschmann/Solution 2 (sol2)**.
- 4 From the **Parameter value (angle (deg))** list, choose **49**.
- 5 Locate the **Expression** section. In the **Expression** text field, type `ewfd2.normEi`.
- 6 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.

Arrow Surface 2


- 1 In the **Model Builder** window, under **Results > Instantaneous Electric Field Norm** right-click **Arrow Surface 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study, Kretschmann/Solution 2 (sol2)**.
- 4 From the **Parameter value (angle (deg))** list, choose **49**.
- 5 Locate the **Expression** section. In the **X-component** text field, type `ewfd2.Ex`.
- 6 In the **Y-component** text field, type `ewfd2.Ey`.
- 7 In the **Instantaneous Electric Field Norm** toolbar, click  **Plot**.

8 Click the  **Zoom Extents** button in the **Graphics** toolbar.



STUDY, ANGLE PHASE MATCHING

Now, simulate to calculate the angle phase matching.


In the **Study** toolbar, click  **Compute**.

RESULTS

Study, Angle Phase Matching/Parametric Solutions 1 (sol6)

In the **Model Builder** window, under **Results** > **Datasets** click **Study, Angle Phase Matching/Parametric Solutions 1 (sol6)**.

Selection

- 1 In the **Results** toolbar, click  **Attributes** and choose **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domains 7 and 8 only.

Electric Field (ewfd3)

Create a plot of the electric field to visualize the surface plasmon polariton wave.

- 1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd3)**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Parameter value (lda0 (nm))** list, choose **500**.

Surface 1

1 In the **Model Builder** window, expand the **Electric Field (ewfd3)** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type $ewfd3 \cdot Ey$.

4 Locate the **Coloring and Style** section. From the **Color table** list, choose **Wave**.

5 From the **Scale** list, choose **Linear symmetric**.

Electric Field (ewfd3)

Now, add the electric field polarization.

Arrow Surface 1

1 In the **Model Builder** window, right-click **Electric Field (ewfd3)** and choose **Arrow Surface**.

2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.

3 In the **X-component** text field, type $ewfd3 \cdot Ex$.

4 In the **Y-component** text field, type $ewfd3 \cdot Ey$.

5 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type 40.

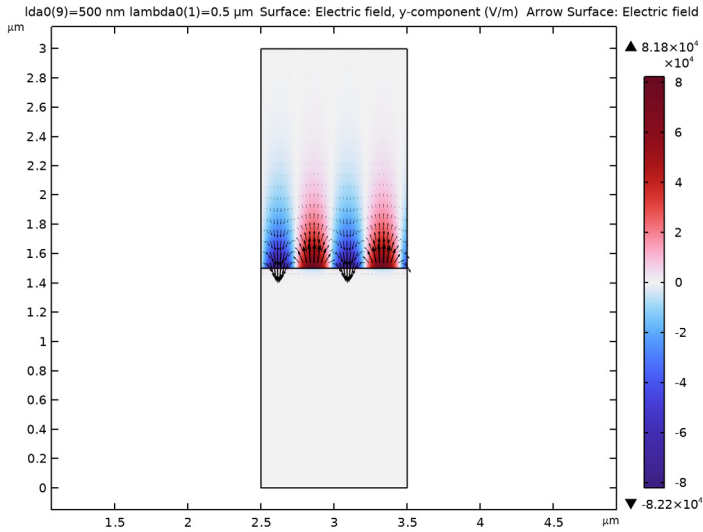
6 Find the **Y grid points** subsection. In the **Points** text field, type 40.

7 Locate the **Coloring and Style** section.

8 Select the **Scale factor** checkbox. In the associated text field, type $2E-6$.

9 From the **Color** list, choose **Black**.

10 In the **Electric Field (ewfd3)** toolbar, click  **Plot**.



Reflectance, Transmittance, and SPP Coupling (ewfd3)

Create a plot of the reflectance, transmittance, and SPP coupling versus the wavelength.

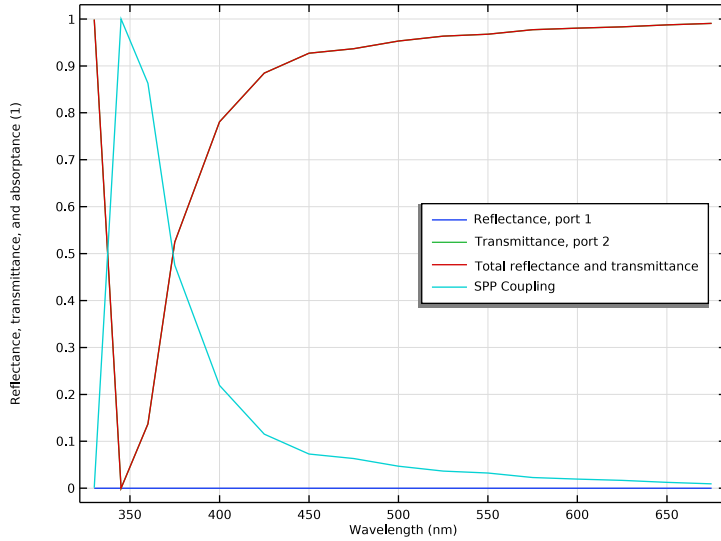
- 1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorptance (ewfd3)**.
- 2 In the **Settings** window for **ID Plot Group**, type Reflectance, Transmittance, and SPP Coupling (ewfd3) in the **Label** text field.
- 3 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

Global 1

- 1 In the **Model Builder** window, expand the **Reflectance, Transmittance, and SPP Coupling (ewfd3)** node, then click **Global 1**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
ewfd3.Atotal	1	SPP Coupling


4 In the **Reflectance, Transmittance, and SPP Coupling (ewfd3)** toolbar, click  **Plot**.



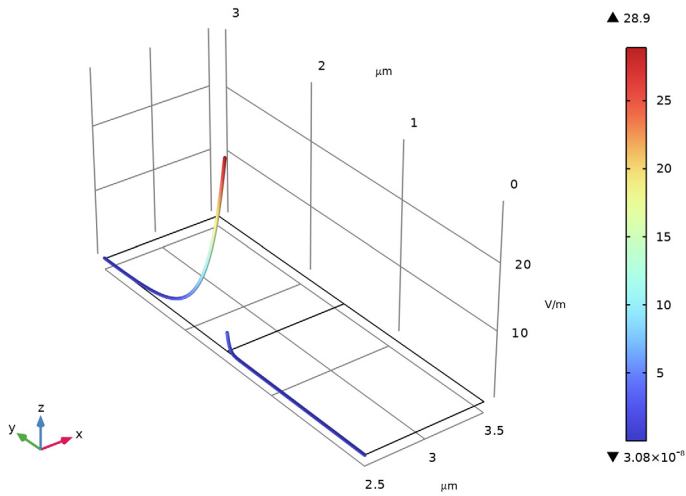
Electric Mode Field, Port 1 (ewfd3)

Visualize the port mode field.

- 1 In the **Model Builder** window, under **Results** click **Electric Mode Field, Port 1 (ewfd3)**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (lda0 (nm))** list, choose **500**.


4 In the **Electric Mode Field, Port 1 (ewfd3)** toolbar, click  **Plot**.

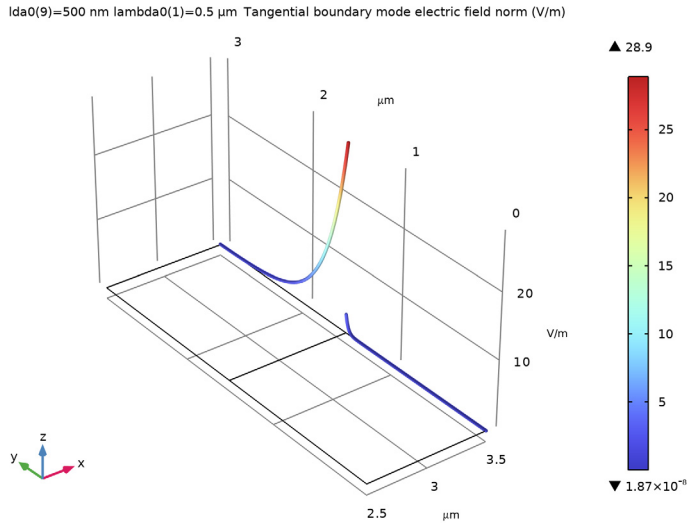
Ida0(9)=500 nm lambda0(1)=0.5 μm Tangential boundary mode electric field norm (V/m)



Electric Mode Field, Port 2 (ewfd3)


- 1 In the **Model Builder** window, click **Electric Mode Field, Port 2 (ewfd3)**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (Ida0 (nm))** list, choose **500**.

4 In the **Electric Mode Field, Port 2 (ewfd3)** toolbar, click  **Plot**.




Evaluation Group 1

Now, evaluate the tangential wavenumber of the incident light and the propagation constant of the surface plasmon polariton wave to determine the requirement for angle phase matching, and create a plot to reproduce [Figure 6](#).

- 1 In the **Results** toolbar, click  **Evaluation Group**.
- 2 In the **Settings** window for **Evaluation Group**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study, Angle Phase Matching/Parametric Solutions 1 (sol6)**.


Global Evaluation Sweep 1

- 1 In the **Evaluation Group 1** toolbar, click  **More Evaluations** and choose **Global Evaluation Sweep**.
- 2 In the **Settings** window for **Global Evaluation Sweep**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study, Angle Phase Matching/Parametric Solutions 1 (sol6)**.
- 4 From the **Parameter value (Ida0 (nm))** list, choose **500**.
- 5 Locate the **Parameters** section. In the table, enter the following settings:

Parameter name	Parameter value list
angle	range(40, 1, 60)

6 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
lda0	μm	Wavelength
ewfd3.k0*n_prism*sin(angle*pi/180)	rad/m	Tangential wavenumber
ewfd3.beta_1	rad/m	Propagation constant

7 In the **Evaluation Group 1** toolbar, click  **Evaluate**.

EVALUATION GROUP 1


- 1 Go to the **Evaluation Group 1** window.
- 2 Click the **Table Graph** button in the window toolbar.

RESULTS

Table Graph 1

- 1 In the **Settings** window for **Table Graph**, locate the **Data** section.
- 2 From the **x-axis data** list, choose **angle**.
- 3 From the **Plot columns** list, choose **Manual**.
- 4 In the **Columns** list, choose **Tangential wavenumber (rad/m)** and **Propagation constant (rad/m)**.
- 5 Click to expand the **Legends** section. Select the **Show legends** checkbox.

Angle Phase Matching

- 1 In the **Model Builder** window, under **Results** click **ID Plot Group 12**.
- 2 In the **Settings** window for **ID Plot Group**, type Angle Phase Matching in the **Label** text field.
- 3 Locate the **Plot Settings** section.
- 4 Select the **x-axis label** checkbox. In the associated text field, type angle [deg].
- 5 Select the **y-axis label** checkbox. In the associated text field, type Wavenumber [rad/m].
- 6 In the **Angle Phase Matching** toolbar, click  **Plot**, to reproduce [Figure 6](#).