Model created in COMSOL Multiphysics 6.4

RCS of a Metallic Sphere Using the Boundary Element Method

Introduction

This model illustrates the process of evaluating the radar cross section (RCS) of a metallic sphere through the utilization of the boundary element method (BEM). By taking advantage of a vertical symmetry plane that is parallel to the polarization of an incident background field, the model reduces computational expenses. The computed RCS values are compared with analytical values within the Mie RCS region. See Table 1 below for a discussion of the characteristic sphere size a for the three RCS scattering regions.

TABLE 1: RADAR TERMINOLOGY SCATTERING REGIONS.

Region	**Sphere size**
Rayleigh	$r_0 << \lambda_0$
Mie	Between Rayleigh and optical region
Optical	$r_0 >> \lambda_0$, conventionally $2\pi r_0/\lambda_0 > 10$

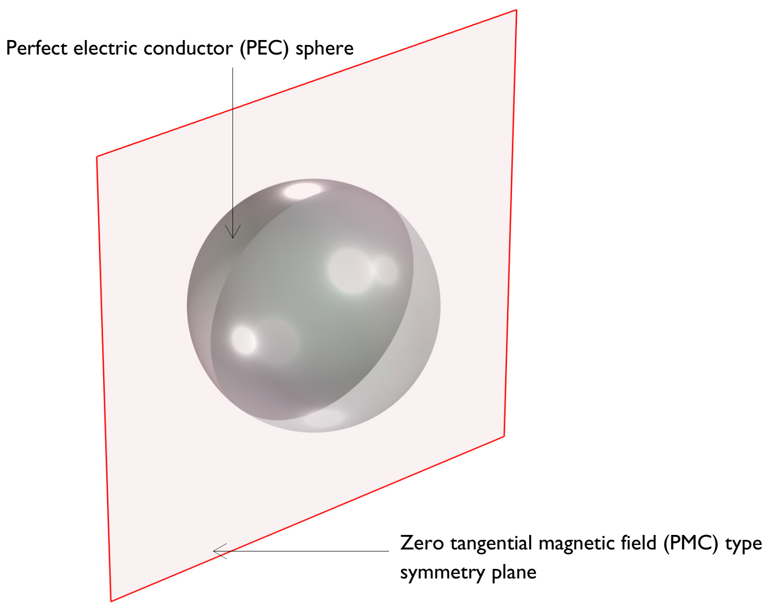


Figure 1: Only the half of the metallic sphere is modeled using the symmetric plane.

This classic benchmark problem in computational electromagnetics is about computing the RCS of a perfectly conducting sphere in free space, illuminated by a linearly polarized plane wave. The simulation result can be directly compared to the analytically solution presented in many electromagnetics textbooks to verify the simulation accuracy.

The analytical solution to the RCS of a perfectly conducting sphere is given by

$$\sigma = \frac{S}{\rho^2}\left|\sum_{n=1}^{\infty}(-1)^n(2n+1)(a_n+b_n)\right|^2$$

where $S = \pi r_0^2$ is the cross sectional area of the sphere, $\rho = 2\pi r_0 f/c$ is the normalized frequency, r_0 is the sphere radius, f is the frequency, and c is the speed of light.

In addition, the coefficients a_n and b_n are given by

$$a_n = \frac{j_n(\rho)}{h_n^{(2)}(\rho)},$$

and

$$b_n = \frac{\frac{d}{d\rho}\rho j_n(\rho)}{\frac{d}{d\rho}\rho h_n^{(2)}(\rho)}$$

where j_n is the spherical Bessel function of the first kind and $h_n^{(2)}$ is the spherical Hankel function of the second kind.

Model Definition

The linearly z-polarized plane wave in the background field defines the incident field on the metallic spherical object. The conductive sphere is set up using the default perfect electric conductor (PEC) boundary condition. For the simulation, the sphere is divided in half, and only this half size is utilized. The boundary for far-field calculation is applied to the sphere's exterior boundaries but excludes the cut plane due to the symmetry plane. As the symmetry plane aligns parallel to the polarization of the background field, a zero tangential magnetic field (PMC) type of symmetry plane is employed. Unlike the conventional finite element method (FEM), the boundary element method (BEM) does not necessitate an air-domain enclosing the sphere and incorporating absorbing features.

Results and Discussion

Figure 2 shows correspondence between the computed and analytical values of RCS in the Mie region, with both values scaled by the factor $2\pi {r_0}^2$, where r_0 represents the radius of the sphere.

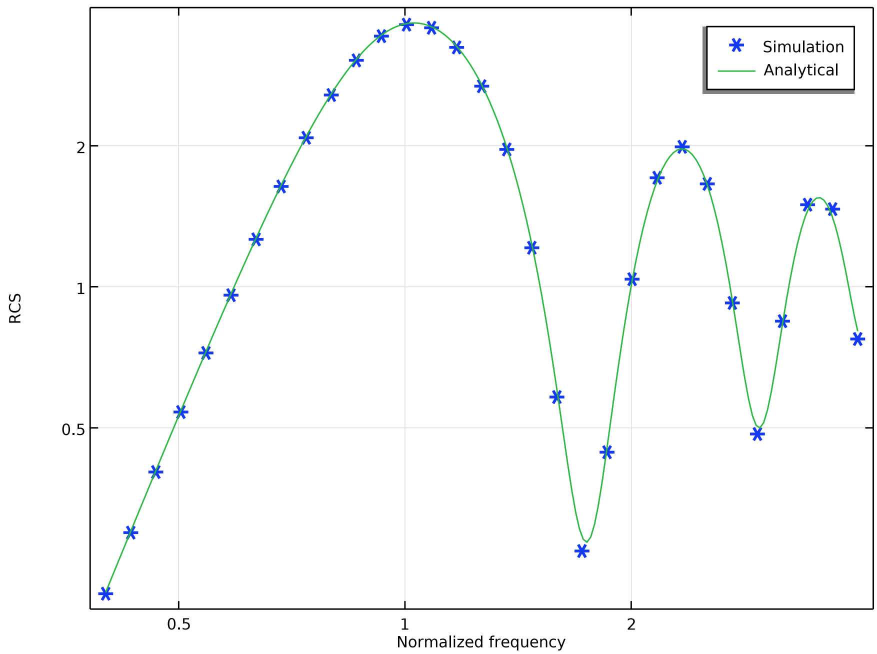


Figure 2: RCS comparison between the computed and analytical values.

In Figure 3, the computed field solution outside the scattering object is assessed using a grid dataset. Initially, the plot of the metallic surface depicts only half the size of the sphere, which corresponds to the actual computation area. However, by employing a mirror dataset, the entire sphere can be visualized. Additionally, the material appearance subfeature provides a glossy metallic look, enhancing the visual effects.

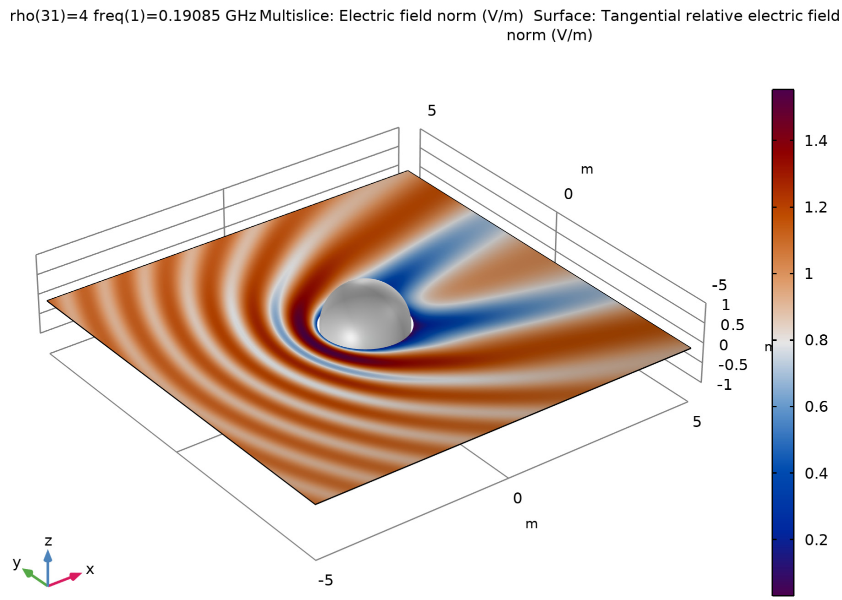


Figure 3: Adjusted multislice plot of the electric field norm on a grid 3D dataset with a metallic sphere.

Notes About the COMSOL Implementation

This model uses the Electromagnetic Waves, Boundary Elements interface to demonstrate its functionalities. In principle, the Electromagnetic Waves, Frequency Domain interface can be used to model the same structure and achieve the same results. Both BEM and FEM solve the full Maxwell equations but FEM requires a finite simulation domain with volumetric meshing while BEM can model infinite domains and only require boundary meshing. Although the degrees of freedom in a BEM model are generally fewer compared to FEM, the memory and computation time requirements are not necessarily smaller. Therefore, one method could be more efficient than the other, depending on the type of problem to be solved.

Reference

1. J.A. Stratton, *Electromagnetic Theory*, Adams Press, 2013.

Application Library path: `Wave_Optics_Module/Optical_Scattering/rcs_sphere_bem`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics** > **Wave Optics** > **Electromagnetic Waves, Boundary Elements (ebem)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies** > **Frequency Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r	1[um]	1E-6 m	Radius of the sphere
cir	2*pi*r	6.2832E-6 m	Circumference of the sphere
rho	1	1	Ratio of circumference to wavelength

DEFINITIONS

To compute the RCS using the analytical expression, the spherical Bessel functions of the first and second kinds and the spherical Hankel function of the second kind need to be defined. Create a **Variables** node for each function to avoid clustering all the variables in a single node. For the frequency range of interest in this model, truncate the summation in the analytical expression of the RCS at `n=6`.

Spherical Bessel Function of the First Kind

1. In the **Model Builder** window, expand the **Component 1 (comp1)** > **Definitions** node.
2. Right-click **Definitions** and choose **Variables**.
3. In the **Settings** window for **Variables**, type `Spherical Bessel Function of the First Kind` in the **Label** text field.
4. Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
sbesselj1	sqrt(pi/(2*rho))*besselj(1.5, rho)		Order 1
sbesselj2	sqrt(pi/(2*rho))*besselj(2.5, rho)		Order 2
sbesselj3	sqrt(pi/(2*rho))*besselj(3.5, rho)		Order 3
sbesselj4	sqrt(pi/(2*rho))*besselj(4.5, rho)		Order 4
sbesselj5	sqrt(pi/(2*rho))*besselj(5.5, rho)		Order 5
sbesselj6	sqrt(pi/(2*rho))*besselj(6.5, rho)		Order 6

Spherical Bessel Function of the Second Kind

1. Right-click **Definitions** and choose **Variables**.
2. In the **Settings** window for **Variables**, type `Spherical Bessel Function of the Second Kind` in the **Label** text field.
3. Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
sbessely1	sqrt(pi/(2*rho))*bessely(1.5, rho)		Order 1
sbessely2	sqrt(pi/(2*rho))*bessely(2.5, rho)		Order 2

Name	Expression	Unit	Description
sbessely3	sqrt(pi/(2*rho))*bessely(3.5, rho)		Order 3
sbessely4	sqrt(pi/(2*rho))*bessely(4.5, rho)		Order 4
sbessely5	sqrt(pi/(2*rho))*bessely(5.5, rho)		Order 5
sbessely6	sqrt(pi/(2*rho))*bessely(6.5, rho)		Order 6

Spherical Hankel Function of the Second Kind

1 Right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Spherical Hankel Function of the Second Kind` in the **Label** text field.

3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
shankel1	sbesselj1-1j*sbessely1		Order 1
shankel2	sbesselj2-1j*sbessely2		Order 2
shankel3	sbesselj3-1j*sbessely3		Order 3
shankel4	sbesselj4-1j*sbessely4		Order 4
shankel5	sbesselj5-1j*sbessely5		Order 5
shankel6	sbesselj6-1j*sbessely6		Order 6

Coefficient a

1 Right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Coefficient a` in the **Label** text field.

3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
a1	sbesselj1/shankel1		
a2	sbesselj2/shankel2		
a3	sbesselj3/shankel3		
a4	sbesselj4/shankel4		
a5	sbesselj5/shankel5		
a6	sbesselj6/shankel6		

Computing the coefficient b requires taking the derivative with respect to rho. This can be done conveniently by using the built-in d(f,x) operator.

Coefficient b

1 Right-click **Definitions** and choose **Variables**.
2 In the **Settings** window for **Variables**, type Coefficient b in the **Label** text field.
3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
b1	-d(rho*sbesselj1,rho)/d(rho*shankel1,rho)		
b2	-d(rho*sbesselj2,rho)/d(rho*shankel2,rho)		
b3	-d(rho*sbesselj3,rho)/d(rho*shankel3,rho)		
b4	-d(rho*sbesselj4,rho)/d(rho*shankel4,rho)		
b5	-d(rho*sbesselj5,rho)/d(rho*shankel5,rho)		
b6	-d(rho*sbesselj6,rho)/d(rho*shankel6,rho)		

RCS

1 Right-click **Definitions** and choose **Variables**.
2 In the **Settings** window for **Variables**, type RCS in the **Label** text field.
3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
RCS	1/(rho)^2*abs(-3*(a1+b1)+5*(a2+b2)-7*(a3+b3)+9*(a4+b4)-11*(a5+b5)+13*(a6*b6))^2		Analytically calculated RCS

With the calculated coefficients, the RCS can be evaluated according to the analytical expression.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
2 In the **Settings** window for **Geometry**, locate the **Units** section.
3 From the **Length unit** list, choose **µm**.

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type `r`.

4 In the **Model Builder** window, click **Geometry 1**.

5 In the **Settings** window for **Geometry**, locate the **Reduction for Symmetry Boundaries** section.

6 Select the **zx-plane: remove y<0** checkbox.

7 In the **Geometry** toolbar, click **Build All**.

MATERIALS

Air

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.

Apply the air material to all voids, since we are modeling the scattering of a metallic sphere in air.

3 Locate the **Geometric Entity Selection** section. From the **Selection** list, choose **All voids**.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	1	1	Refractive index

ELECTROMAGNETIC WAVES, BOUNDARY ELEMENTS (EBEM)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Boundary Elements (ebem)**.

2 In the **Settings** window for **Electromagnetic Waves, Boundary Elements**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **All voids**.

The interior of the PEC sphere will not be included in the simulation, so only **Infinite void** is selected in the **Domain Selection**.

4 Locate the **Formulation** section. From the list, choose **Scattered field**.

5 From the **Background wave type** list, choose **Linearly polarized plane wave**.

6 Click to expand the **Symmetry** section. From the **Condition for the y = y_0 plane** list, choose **Zero tangential magnetic field (PMC)**.

Far-Field Calculation 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Far-Field Calculation**.

Use this node to enable the evaluation of far-field quantities such as the RCS in the postprocessing.

2 In the **Settings** window for **Far-Field Calculation**, locate the **Far-Field Calculation** section.

3 From the **Symmetry settings** list, choose **From symmetry plane(s)**.

STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
rho (Ratio of circumference to wavelength)	`10^{range(log10(0.4),1/30,log10(4))}`	

The normalized frequency is swept in a logarithmic scale. In the postprocessing, the RCS will be plotted in logarithmic scale as well.

Step 1: Frequency Domain

1 In the **Model Builder** window, click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `c_const*rho/cir`.

4 In the **Study** toolbar, click **Compute**.

RESULTS

Global Evaluation Sweep 1

1 In the **Results** toolbar, click **More Derived Values** and choose **Other** > **Global Evaluation Sweep**.

By default, the analytically calculated RCS variable will be evaluated at the simulated frequencies. However, a finer frequency resolution can be achieved without actually running the simulation by utilizing a **Global Evaluation Sweep**, where the parameter `rho` is swept at a much finer spacing.This way, the analytically calculated RCS will show a smooth curve.

2 In the **Settings** window for **Global Evaluation Sweep**, locate the **Parameters** section.

3 In the table, enter the following settings:

Parameter name	Parameter value list
`rho`	`10^{range(log10(0.4),1/200,log10(4))}`

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`RCS`		`Analytically calculated RCS`

5 Click **Evaluate**.

Simulated and Analytically Calculated RCS

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Simulated and Analytically Calculated RCS` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **None**.

4 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (sol2)**.

5 Locate the **Plot Settings** section.

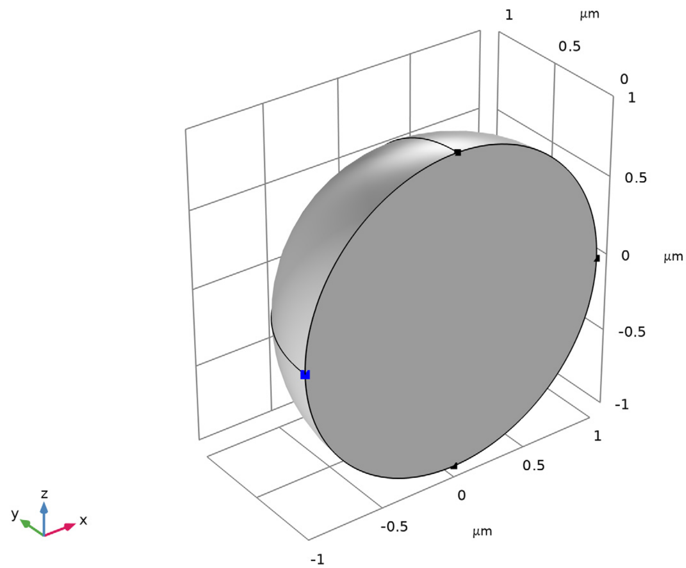
6 Select the **x-axis label** checkbox. In the associated text field, type `Normalized frequency`.

7 Select the **y-axis label** checkbox. In the associated text field, type `RCS`.

Point Graph 1

1 Right-click **Simulated and Analytically Calculated RCS** and choose **Point Graph**.

2 Select Point 1 only.



3 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `ebem.bRCS3D/(pi*r^2)`.

5 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **Outer solutions**.

6 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

7 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

8 Click to expand the **Legends** section. Select the **Show legends** checkbox.

9 From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
`Simulation`

Table Graph 1

1 In the **Model Builder** window, right-click **Simulated and Analytically Calculated RCS** and choose **Table Graph**.

2 In the **Settings** window for **Table Graph**, click to expand the **Legends** section.

3 Select the **Show legends** checkbox.

4 From the **Legends** list, choose **Manual**.

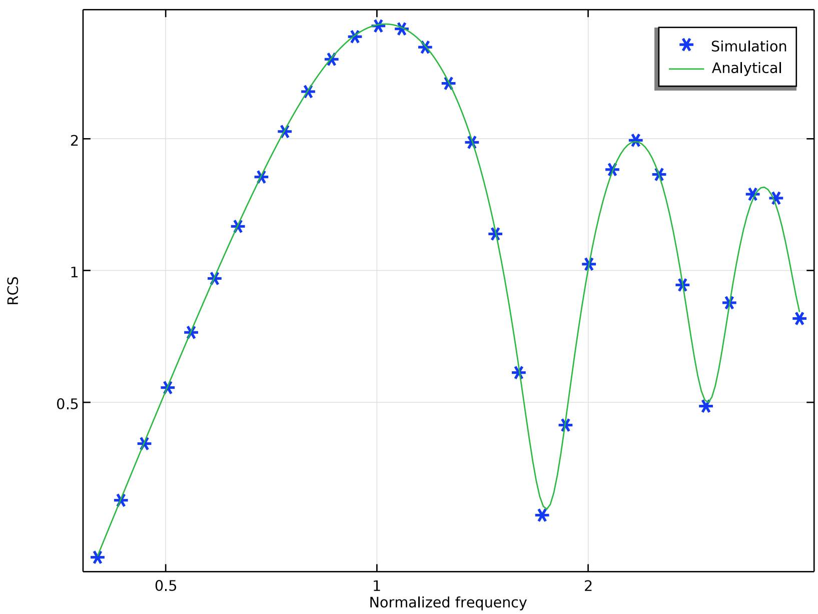
5 In the table, enter the following settings:

Legends
`Analytical`

6 In the **Simulated and Analytically Calculated RCS** toolbar, click **Plot**.

7 Click the **x-Axis Log Scale** button in the **Graphics** toolbar.

8 Click the **y-Axis Log Scale** button in the **Graphics** toolbar.



As expected, the simulation and the analytical calculation agree very well.

Unlike FEM, where it is only possible to visualize field distributions within the truncated simulation domain, BEM allows the evaluation of fields anywhere by utilizing the automatically added **Grid 3D** dataset as shown in the following.

Grid 3D 1

1 In the **Model Builder** window, expand the **Results > Datasets** node, then click **Grid 3D 1**.

2 In the **Settings** window for **Grid 3D**, locate the **Parameter Bounds** section.

3 Find the **First parameter** subsection. In the **Minimum** text field, type `-5`.

4 In the **Maximum** text field, type `5`.

5 Find the **Second parameter** subsection. In the **Minimum** text field, type `-5`.

6 In the **Maximum** text field, type `5`.

7 Find the **Third parameter** subsection. In the **Minimum** text field, type `0`.

8 In the **Maximum** text field, type `0`.

9 Click to expand the **Grid** section. In the **x resolution** text field, type `200`.

10 In the **y resolution** text field, type `200`.

11 In the **z resolution** text field, type `2`.

Increasing the resolution ensures that the details of the field distribution are well resolved.

Multislice 1

1 In the **Model Builder** window, expand the **Results > Electric Field, Domains (ebem)** node, then click **Multislice 1**.

2 In the **Settings** window for **Multislice**, locate the **Multiplane Data** section.

3 Find the **x-planes** subsection. In the **Planes** text field, type `0`.

4 Find the **y-planes** subsection. In the **Planes** text field, type `0`.

5 Locate the **Coloring and Style** section. From the **Color table** list, choose **Opadometa**.

Remove the staircase shape in the field plot by using a **Filter** subfeature.

Filter 1

1 In the **Model Builder** window, right-click **Multislice 1** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `x^2+y^2>r*1.1*1[um]`.

4 In the **Electric Field, Domains (ebem)** toolbar, click **Plot**.

Line 1

In the **Model Builder** window, under **Results > Electric Field, Domains (ebem)** right-click **Line 1** and choose **Delete**.

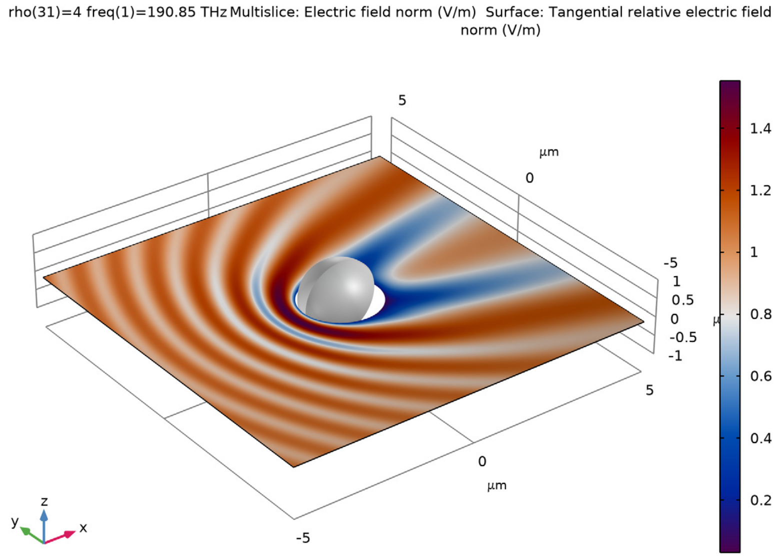
Material Appearance 1

1 Right-click **Surface 1** and choose **Material Appearance**.

2 In the **Settings** window for **Material Appearance**, locate the **Appearance** section.

3 From the **Appearance** list, choose **Custom**.

4 From the **Material type** list, choose **Aluminum (anodized)**.



Make a complete sphere using a **Mirror 3D** dataset.

Mirror 3D 1

1 In the **Results** toolbar, click **More Datasets** and choose **Mirror 3D**.

2 In the **Settings** window for **Mirror 3D**, locate the **Plane Data** section.

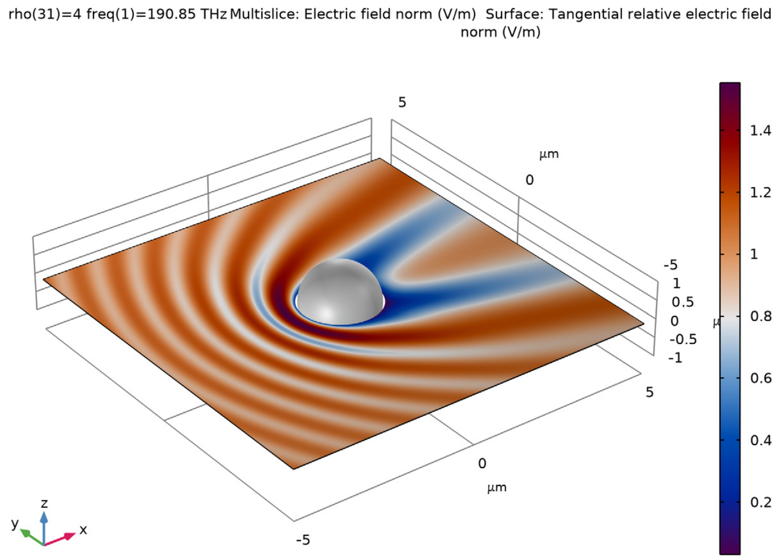
3 From the **Plane** list, choose **ZX-planes**.

Surface 1

1 In the **Model Builder** window, under **Results** > **Electric Field, Domains (ebem)** click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Mirror 3D 1**.



4 In the **Electric Field, Domains (ebem)** toolbar, click **Plot**.