Model created in COMSOL Multiphysics 6.4

Gaussian Beam Propagation Through an Optical Prism

Introduction

Optical prisms are transparent optical elements with polished surfaces that are designed to disperse or refract light. As light hits a prism boundary at an oblique angle of incidence, the ray path is deviated from its original direction after passing through the prism boundaries. The amount of deviation depends on the refractive indices of the prism and the surrounding medium. For instance, when light propagates from a low index medium to a prism with high refractive index, as it passes the interface between the two media, the transmitted light moves toward the normal of the interface.

Triangular prisms are the most familiar type of optical prism. Their geometry is simple, consisting of triangular base and rectangular sides. The most widely used prism materials include glass, fluorite, and acrylic. To make optical prisms transparent to light and minimize reflection, anti–reflection coatings are applied on the prism boundaries. Optical prisms are used in a wide variety of practical applications such as spectroscopy design, binoculars, cameras, fiber optics communication, periscopes, microscopes, among many others.

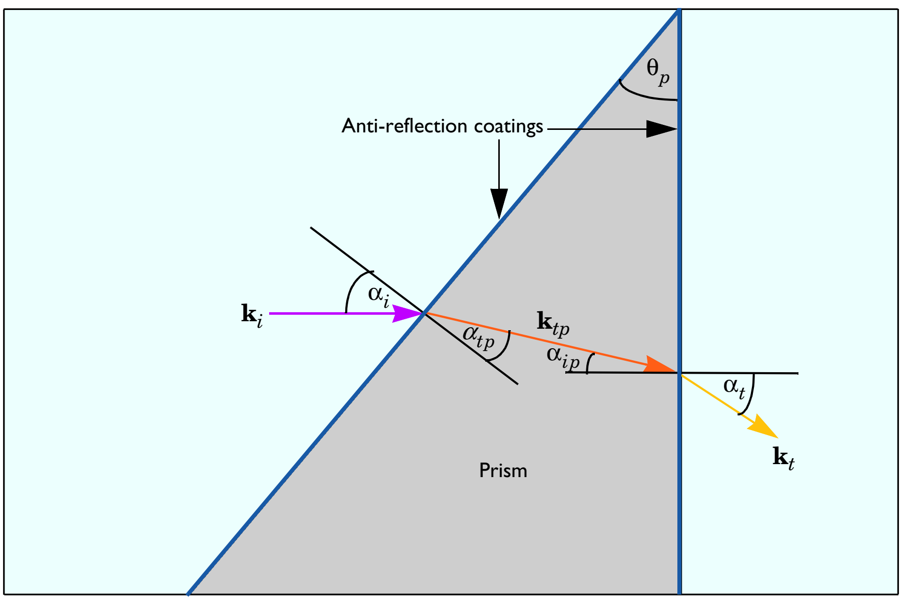


Figure 1: Schematic of a right–angled triangular prism indicating the refraction of light.

Model Definition

This model demonstrates the refraction of light through an optical prism. Figure 1 shows the schematic. A right–angled triangular optical prism with acute angles α_p and $90° - \alpha_p$ is surrounded by air. The prism boundaries are coated with anti-reflection coatings. A light ray with wave vector $\mathbf{k}_i$ is incident at an angle α_i. As light propagates from a low refractive index material (air) to a high index material (prism), the transmitted light moves closer to the boundary normal where angle of transmission is α_{tp}. This ray with wave vector $\mathbf{k}_{tp}$ hits the second prism boundary at an angle α_{ip} and exits in the air medium at an angle α_t.

The anti-reflection (AR) coatings at the prism boundaries are defined using a *Transition Boundary Condition* that models a thin layer with refractive index n_{AR} and thickness d_{AR}. These two parameters are calculated from the propagation directions of the light and the refractive indices of the surrounding media. Note that for oblique angles of incidence, the refractive index and layer thickness of the AR coatings are different for s- and p-polarized lights.

Assuming lossless media, as shown in Figure 1, the refractive index and thickness of the AR coating layer, lying between the incident (air) medium and the prism, for s- and p-polarized incident lights can be derived from Ref. 1 as

$$n^s_{AR1} = \sqrt{(\sin\alpha_i)^2 + n_p\cos\alpha_i\cos\alpha_{tp}},$$

$$n^p_{AR1} = \sqrt{\frac{n_p\left(1\left(-\sqrt{1-\frac{4}{n_p}\cos\alpha_i\cos\alpha_{tp}(\sin\alpha_i)^2}\right)\right)}{2\cos\alpha_i\cos\alpha_{tp}}},$$

$$\theta^s_1 = \mathrm{asin}\left(\frac{\sin\alpha_i}{n^s_{AR1}}\right),$$

$$\theta^p_1 = \mathrm{asin}\left(\frac{\sin\alpha_i}{n^p_{AR1}}\right),$$

$$d^s_{AR1} = \frac{\lambda_0}{4n^s_{AR1}\cos\theta^s_1},$$

$$d^p_{AR1} = \frac{\lambda_0}{4n^p_{AR1}\cos\theta^p_1},$$

where n_p is the refractive index of the prism material. Now, the refractive index and thickness of the AR coating lying between the prism and exiting media are expressed as

$$n^s_{AR2} = \sqrt{(n_p \sin\alpha_{ip})^2 + n_p \cos\alpha_{ip} \cos\alpha_t},$$

$$n^p_{AR2} = \sqrt{\frac{n_p(1 + \sqrt{1 - 4n_p \cos\alpha_{ip} \cos\alpha_t (\sin\alpha_{ip})^2})}{2\cos\alpha_{ip} \cos\alpha_t}},$$

$$\theta^s_2 = \operatorname{asin}(n^s_{AR2} \sin\alpha_{ip}),$$

$$\theta^p_2 = \operatorname{asin}(n^p_{AR2} \sin\alpha_{ip}),$$

$$d^s_{AR2} = \frac{\lambda_0}{4n^s_{AR2} \cos\theta^s_2},$$

$$d^p_{AR2} = \frac{\lambda_0}{4n^p_{AR2} \cos\theta^p_2}.$$

This model simulates the refraction of s- and p-polarized Gaussian beams passing through a right-angle triangular optical prism in a 2D geometry using the Electromagnetic Waves, Frequency Domain interface. A Matched Boundary Condition feature is used to launch the Gaussian beam and to properly absorb the reflected and refracted waves with known wave directions. A Transition Boundary Condition feature is used to model the anti–reflection coatings applied to the prism boundaries using the above equations. Wavelength Domain study steps are used to solve for the domain fields. Reflectance and transmittance are also calculated using an integration operator.

Results and Discussion

Figure 2 show the nonzero electric field component of an s-polarized Gaussian beam. The anti-reflection coatings at the prism boundaries almost eliminate all reflection at the prism boundaries.

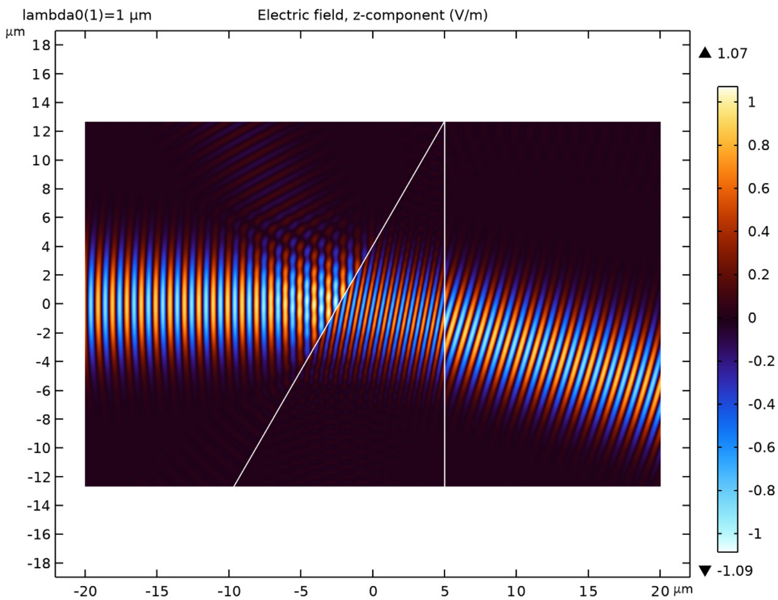


Figure 2: Electric field component of an s-polarized Gaussian beam propagating from left to right through a right–angled triangular optical prism.

Figure 3 shows the dominant electric field component of a p-polarized Gaussian beam.

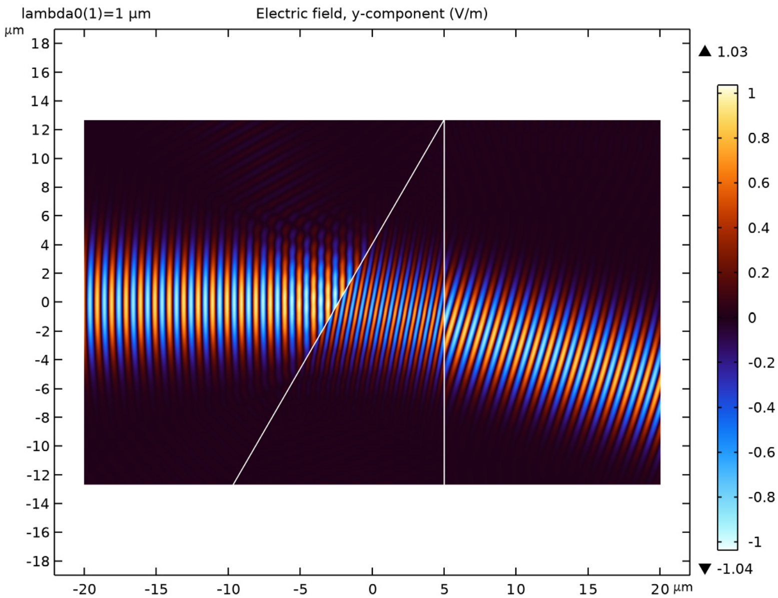


Figure 3: Dominant electric field component of a p-polarized Gaussian beam propagating from left to right through a right angle triangular optical prism.

Table 1 shows the calculated reflectance and transmittance for the s- and p-polarized Gaussian beams.

TABLE 1: REFLECTANCE AND TRANSMITTANCE.

Reflectance, s-polarized wave	**Transmittance, s-polarized wave**	**Reflectance, p-polarized wave**	**Transmittance, p-polarized wave**
0.0105	0.9856	0.0089	0.9893

Reference

1. M. Born and E. Worlf, *Principle of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, *Elsevier*, 2013.

Application Library path: `Wave_Optics_Module/Beam_Propagation/gaussian_beam_propagation_optical_prism`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics** > **Wave Optics** > **Electromagnetic Waves, Frequency Domain (ewfd)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Wavelength Domain**.

6 Click **Done**.

GEOMETRY 1

The geometry is very simple, consisting of a rectangle and a polygon.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **µm**.

GLOBAL DEFINITIONS

Wave and Geometric Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Wave and Geometric Parameters` in the **Label** text field.

3 Locate the **Parameters** section. Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `gaussian_beam_propagation_optical_prism_parameters_geom.txt`.

Anti-Reflection Coatings Parameters

1 In the **Home** toolbar, click **Parameters** and choose **Add** > **Parameters**.

2 In the **Settings** window for **Parameters**, type `Anti-Reflection Coatings Parameters` in the **Label** text field.

3 Locate the **Parameters** section. Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `gaussian_beam_propagation_optical_prism_parameters_ar.txt`.

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

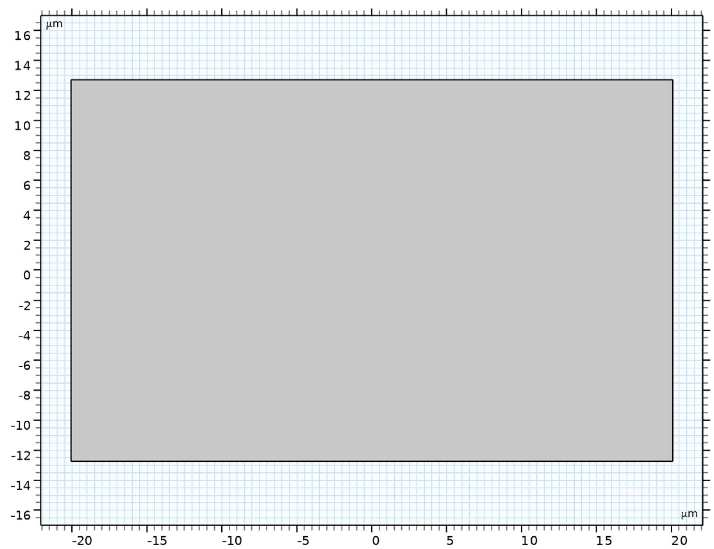
2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `W`.

4 In the **Height** text field, type `H`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 Click **Build Selected**.



Prism

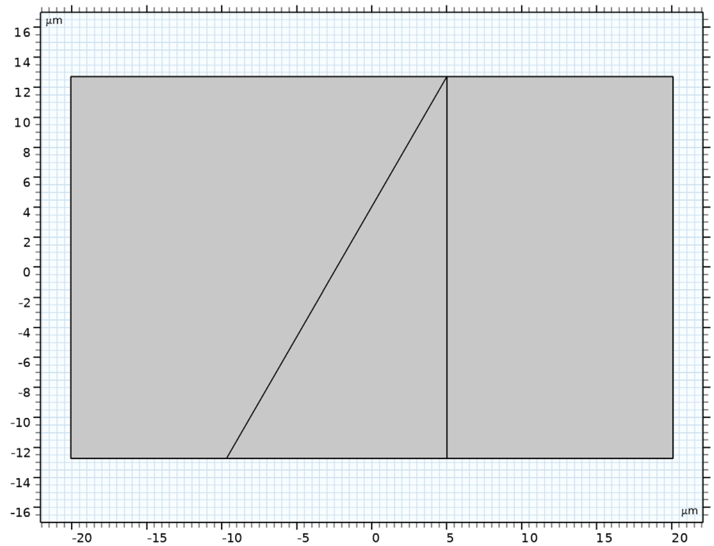
1 In the **Geometry** toolbar, click **Polygon**.

2 In the **Settings** window for **Polygon**, type `Prism` in the **Label** text field.

3 Locate the **Coordinates** section. In the table, enter the following settings:

x (µm)	y (µm)
`-H*tan(anglePrism)+dOffset`	`-H/2`
`5[um]`	`H/2`
`5[um]`	`-H/2`

4 Click **Build All Objects**.



ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Right-click and choose **Add to Component 1 (comp1)**.

5 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

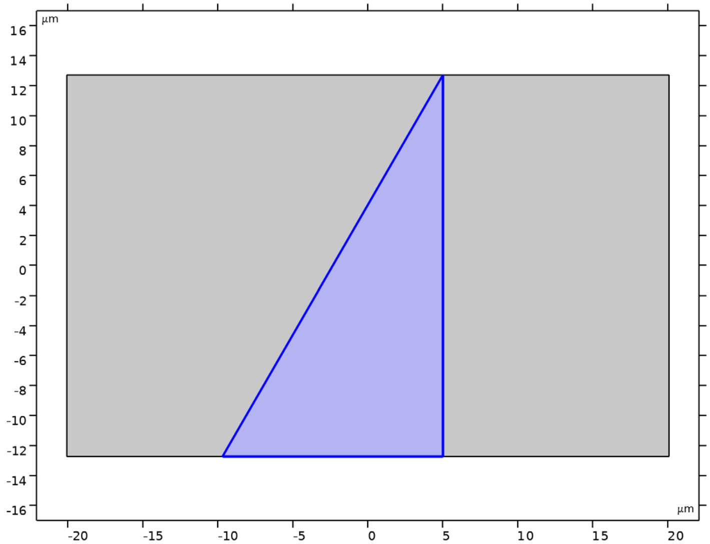
MATERIALS

Prism

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Prism` in the **Label** text field.

3 Select Domain 2 only.



4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	np	1	Refractive index

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

First, simulate the model for an s-polarized Gaussian beam.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (ewfd)**.

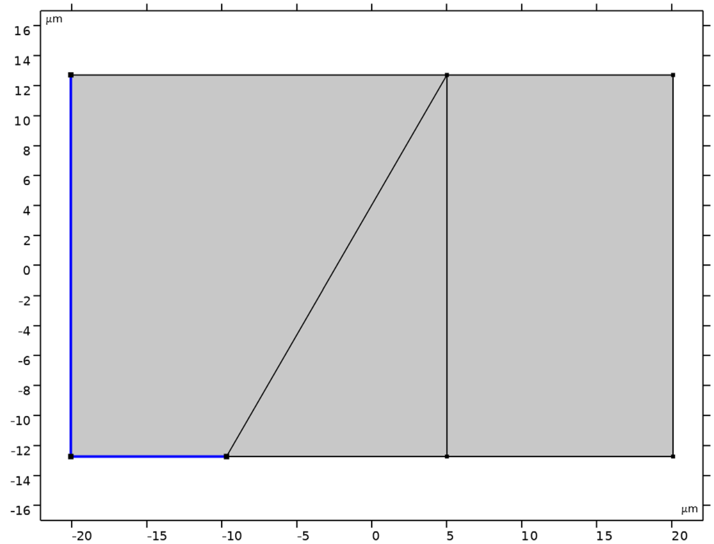
2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **Out-of-plane vector**, as the beam possesses out-of-plane electric field component only.

Matched Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundaries 1 and 2 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Boundary Selection** section.

4 Click **Create Selection**.

5 In the **Create Selection** dialog, type `Input Boundaries` in the **Selection name** text field.

6 Click **OK**.

Later, this selection will be used to define the integration operator to calculate the input power of the Gaussian beam.

7 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

8 From the **Incident field** list, choose **Gaussian beam**.

9 In the w_0 text field, type `w0`.

10 In the p_0 text field, type `W/2`.

Now, define the electric field vector of the s-polarized beam propagating along the x direction.

11 Specify the $\mathbf{E}_{g0}$ vector as

1	z

12 Specify the $\mathbf{k}_{\mathrm{i,dir}}$ vector as

1	x
0	y

Reference Point 1

The reference point and the incident wave direction together define the optical axis of the Gaussian beam.

1 In the **Physics** toolbar, click **Attributes** and choose **Reference Point**.

2 In the **Settings** window for **Reference Point**, locate the **Reference Point** section.

3 From the **Definition** list, choose **User defined**.

4 Specify the $\mathbf{r}_0$ vector as

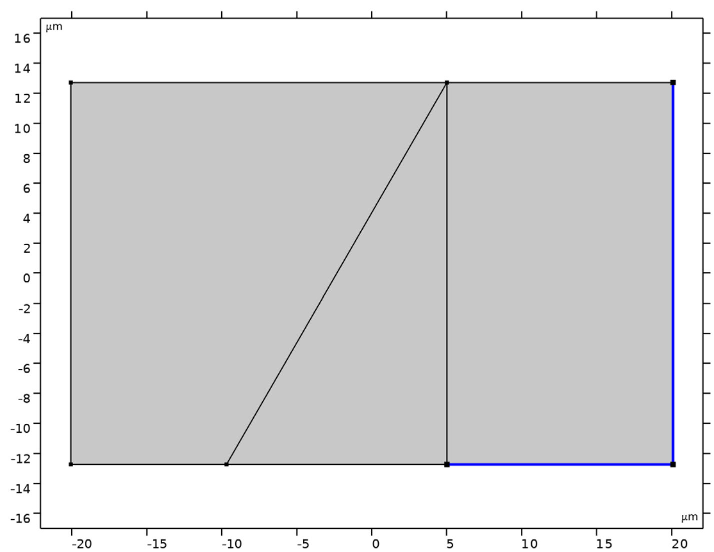
-W/2	x

Matched Boundary Condition 2

Add a matched boundary condition to absorb the transmitted wave from the prism.

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundaries 7 and 9 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Boundary Selection** section.

4 Click **Create Selection**.

5 In the **Create Selection** dialog, type `Output Boundaries` in the **Selection name** text field.

6 Click **OK**.

Later, this selection will be used to define the integration operator to calculate the transmitted power of the Gaussian beam.

7 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

8 Find the **Scattered field** subsection. Specify the $\mathbf{k}_{s,dir}$ vector as

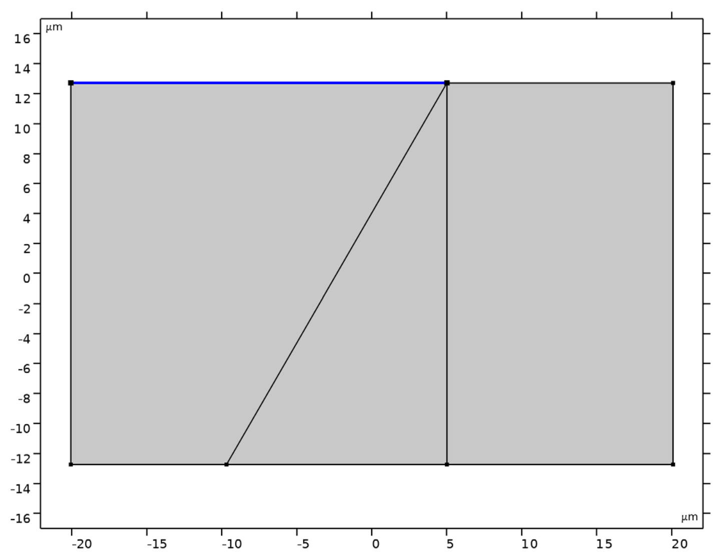
kxScaDir	x
kyScaDir	y

Matched Boundary Condition 3

Add a matched boundary condition to absorb the reflected wave from the first prism boundary.

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 3 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Boundary Selection** section.

4 Click **Create Selection**.

5 In the **Create Selection** dialog, type `Reflection Boundaries` in the **Selection name** text field.

6 Click **OK**.

Later, this selection will be used to define the integration operator to calculate the reflected power of the beam from the prism.

7 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

8 Find the **Scattered field** subsection. Specify the $\mathbf{k}_{s,dir}$ vector as

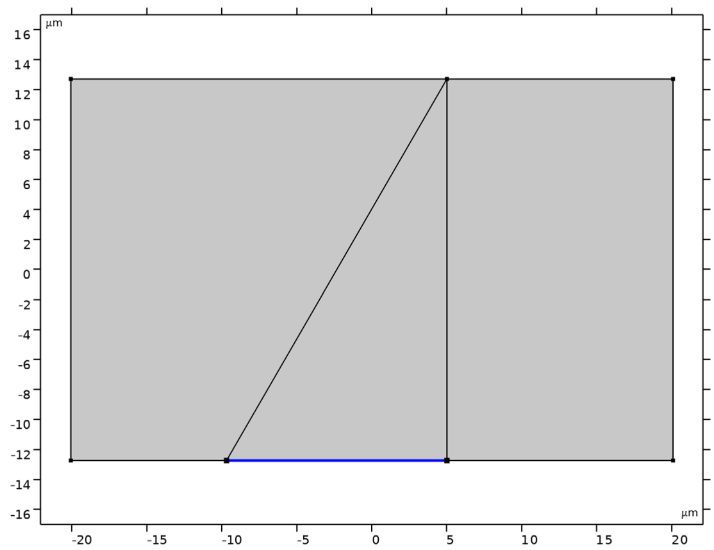
`-cos(2*alphaInc)`	x
`sin(2*alphaInc)`	y

Matched Boundary Condition 4

Add another matched boundary condition to the truncated prism boundary to absorb the transmitted wave inside the prism.

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 4 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

4 Find the **Scattered field** subsection. Specify the $\mathbf{k}_{s,dir}$ vector as

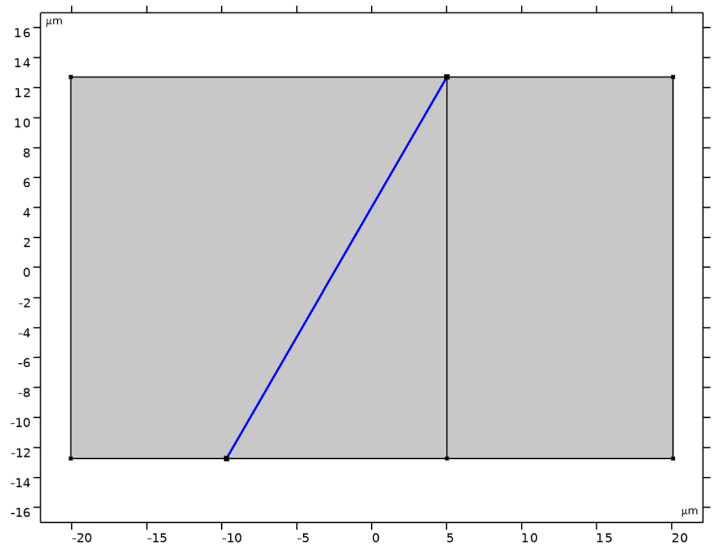
cos(alphaIncPrism)	x
-sin(alphaIncPrism)	y

Transition Boundary Condition 1

Add a transition boundary condition to model the anti-reflection coating on the incident prism boundary.

1 In the **Physics** toolbar, click **Boundaries** and choose **Transition Boundary Condition**.

2 Select Boundary 5 only.



3 In the **Settings** window for **Transition Boundary Condition**, locate the **Transition Boundary Condition** section.

4 From the n list, choose **User defined**. In the associated text field, type `n1_s`.

5 From the k list, choose **User defined**, and keep the default value 0.

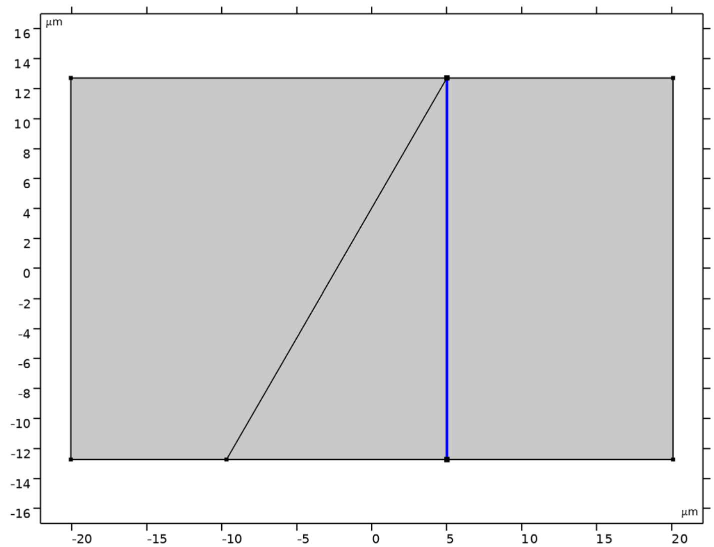
6 In the d text field, type `d1_s`.

Transition Boundary Condition 2

Add another transition boundary condition to model the anti-reflection coating on the transmission-side prism boundary.

1 In the **Physics** toolbar, click **Boundaries** and choose **Transition Boundary Condition**.

2 Select Boundary 6 only.



3 In the **Settings** window for **Transition Boundary Condition**, locate the **Transition Boundary Condition** section.

4 From the *n* list, choose **User defined**. In the associated text field, type `n2_s`.

5 From the *k* list, choose **User defined**, and keep the default value 0.

6 In the *d* text field, type `d2_s`.

7 In the **Model Builder** window, right-click **Electromagnetic Waves, Frequency Domain (ewfd)** and choose **Copy**, to simulate for the p-polarized Gaussian beam.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN 2 (EWFD2)

1 In the **Model Builder** window, right-click **Component 1 (comp1)** and choose **Paste Electromagnetic Waves, Frequency Domain**.

2 In the **Messages from Paste** dialog, click **OK**.

Now, set up the physics interface settings to simulate for the p-polarized Gaussian beam.

3 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Components** section.

4 From the **Electric field components solved for** list, choose **In-plane vector**, as the beam possesses in-plane electric field components only.

Matched Boundary Condition 1

1. In the **Model Builder** window, expand the **Electromagnetic Waves, Frequency Domain 2 (ewfd2)** node, then click **Matched Boundary Condition 1**.
2. In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.
3. Specify the $\mathbf{E}_{g0}$ vector as

1	y
0	z

Transition Boundary Condition 1

1. In the **Model Builder** window, click **Transition Boundary Condition 1**.
2. In the **Settings** window for **Transition Boundary Condition**, locate the **Transition Boundary Condition** section.
3. In the n text field, type `n1_p`.
4. In the d text field, type `d1_p`.

Transition Boundary Condition 2

1. In the **Model Builder** window, click **Transition Boundary Condition 2**.
2. In the **Settings** window for **Transition Boundary Condition**, locate the **Transition Boundary Condition** section.
3. In the n text field, type `n2_p`.
4. In the d text field, type `d2_p`.

DEFINITIONS

Define the operators and variables to calculate the reflectance and transmittance.

Integration 1 (intop1)

1. In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.
2. In the **Settings** window for **Integration**, locate the **Source Selection** section.
3. From the **Geometric entity level** list, choose **Boundary**.
4. From the **Selection** list, choose **Input Boundaries**.

Integration 2 (intop2)

1. Right-click **Integration 1 (intop1)** and choose **Duplicate**.
2. In the **Settings** window for **Integration**, locate the **Source Selection** section.
3. From the **Selection** list, choose **Reflection Boundaries**.

Integration 3 (intop3)

1 Right-click **Integration 2 (intop2)** and choose **Duplicate**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Selection** list, choose **Output Boundaries**.

Reflectance and Transmittance Calculation

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Reflectance and Transmittance Calculation` in the **Label** text field.

3 Locate the **Variables** section. Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `gaussian_beam_propagation_optical_prism_variables.txt`.

STUDY 1

Wavelength Domain (s-polarization)

1 In the **Model Builder** window, under **Study 1** click **Step 1: Wavelength Domain**.

2 In the **Settings** window for **Wavelength Domain**, type `Wavelength Domain (s-polarization)` in the **Label** text field.

3 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electromagnetic Waves, Frequency Domain 2 (ewfd2)**.

4 Locate the **Study Settings** section. In the **Wavelengths** text field, type `lda0`.

Wavelength Domain (p-polarization)

1 Right-click **Study 1 > Step 1: Wavelength Domain (s-polarization)** and choose **Duplicate**.

2 In the **Settings** window for **Wavelength Domain**, type `Wavelength Domain (p-polarization)` in the **Label** text field.

3 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electromagnetic Waves, Frequency Domain (ewfd)**.

4 In the **Solve for** column of the table, under **Component 1 (comp1)**, select the checkbox for **Electromagnetic Waves, Frequency Domain 2 (ewfd2)**.

5 In the **Study** toolbar, click **Compute**.

RESULTS

Global Evaluation 1

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

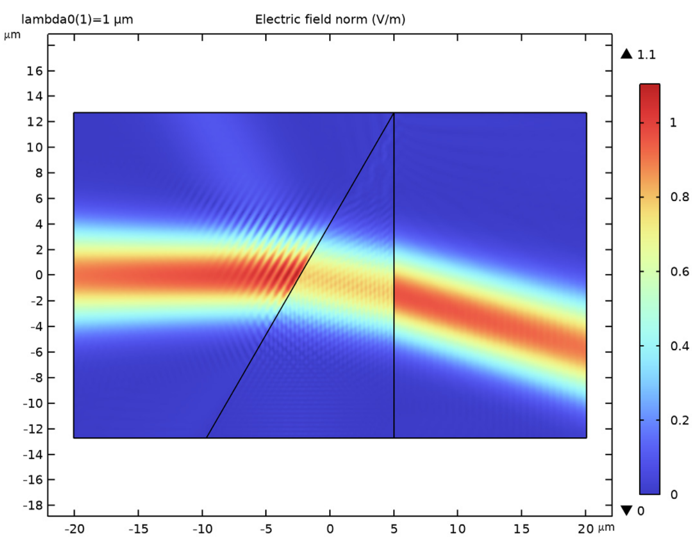
Expression	Unit	Description
`R_s`	1	`Reflectance, s-polarized wave`
`T_s`	1	`Transmittance, s-polarized wave`
`R_p`	1	`Reflectance, p-polarized wave`
`T_p`	1	`Transmittance, p-polarized wave`

4 Click **Evaluate**. The results should resemble those in Table 1.

Electric Field Norm, s-polarized Beam

1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd)**.

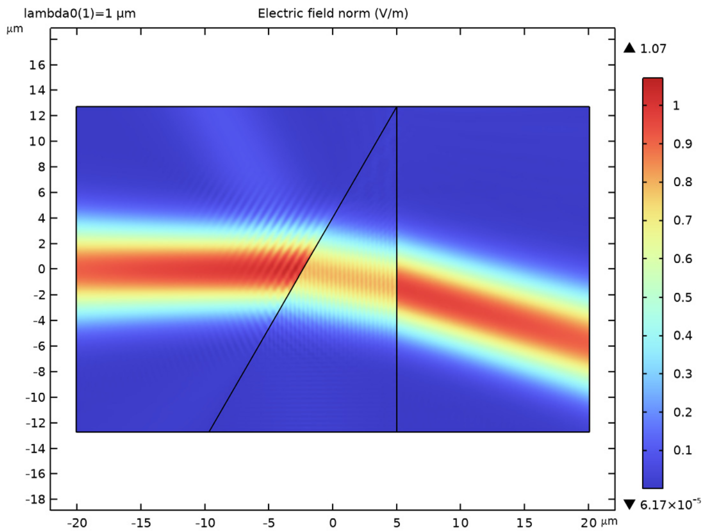
2 In the **Settings** window for **2D Plot Group**, type `Electric Field Norm, s-polarized Beam` in the **Label** text field.



Electric Field Norm, p-polarized Beam

1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd2)**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Field Norm, p-polarized Beam` in the **Label** text field.



Electric Field Component, s-polarized Beam

1 Right-click **Electric Field Norm, p-polarized Beam** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Field Component, s-polarized Beam` in the **Label** text field.

Surface 1

1 In the **Model Builder** window, expand the **Electric Field Component, s-polarized Beam** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewfd.Ez`, to plot the nonzero electric field component.

4 Locate the **Coloring and Style** section. From the **Color table** list, choose **ThermalWaveDark**.

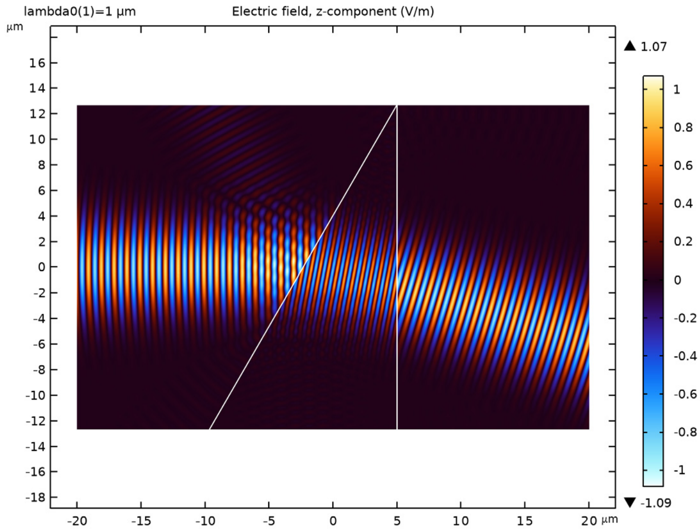
Electric Field Component, s-polarized Beam

1 Click the  **Zoom Extents** button in the **Graphics** toolbar.

2 In the **Model Builder** window, click **Electric Field Component, s-polarized Beam**.

3 In the **Settings** window for **2D Plot Group**, locate the **Plot Settings** section.

4 From the **Color** list, choose **White**, for better visualization of the prism boundaries.



Electric Field Component, p-polarized Beam

1 Right-click **Electric Field Component, s-polarized Beam** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Field Component, p-polarized Beam` in the **Label** text field.

Surface 1

1 In the **Model Builder** window, expand the **Electric Field Component, p-polarized Beam** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewfd2.Ey`, to plot the dominant electric field component.