



Model created in COMSOL Multiphysics 6.4

# Fresnel Equations

## Introduction

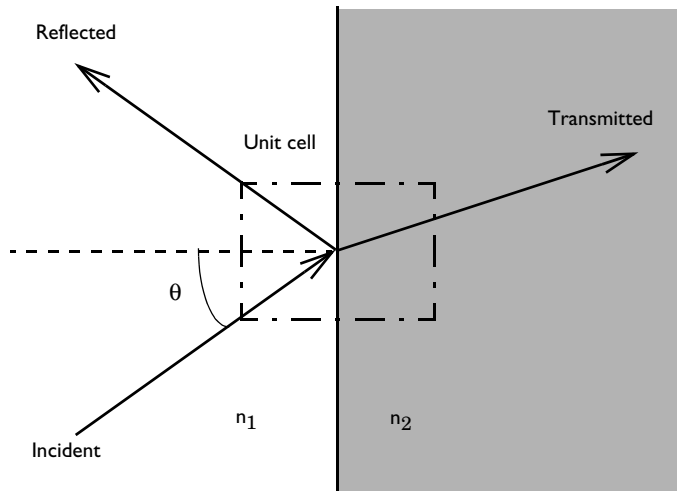
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A plane electromagnetic wave propagating through free space is incident at an angle upon an infinite dielectric medium. This model computes the reflection and transmission coefficients and compares the results to the Fresnel equations.

## Model Definition

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A plane wave propagating through free space ( $n = 1$ ) as shown in [Figure 1](#) is incident upon an infinite dielectric medium ( $n = 1.5$ ) and is partially reflected and partially transmitted. If the electric field is *p-polarized* — that is, if the electric field vector is in the same plane as the Poynting vector and the surface normal — then there are no reflections at an incident angle of roughly  $56^\circ$ , known as the *Brewster angle*.



*Figure 1: A plane wave propagating through free space incident upon an infinite dielectric medium.*

Although, by assumption, space extends to infinity in all directions, it is sufficient to model a small unit cell, as shown in [Figure 1](#); a Floquet-periodic boundary condition applies on the top and bottom unit-cell boundaries because the solution is periodic along the interface. This model uses a 3D unit cell and applies also a periodic boundary condition in the out-of-plane direction. However, this periodic boundary condition acts like a continuity condition. The angle of incidence ranges between  $0$ – $90^\circ$  for both polarizations.

To simplify the process of adding the boundary conditions, a Periodic Structure node is used. It automatically adds and configures its subnodes — the ports and the periodic conditions.

For comparison, Ref. 1 and Ref. 2 provide analytic expressions for the reflectance and transmittance<sup>1</sup>. Reflection and transmission coefficients for s-polarization and p-polarization are defined respectively as

$$r_s = \frac{n_1 \cos \theta_{\text{incident}} - n_2 \cos \theta_{\text{transmitted}}}{n_1 \cos \theta_{\text{incident}} + n_2 \cos \theta_{\text{transmitted}}}$$

$$t_s = \frac{2n_1 \cos \theta_{\text{incident}}}{n_1 \cos \theta_{\text{incident}} + n_2 \cos \theta_{\text{transmitted}}}$$

$$r_p = \frac{n_2 \cos \theta_{\text{incident}} - n_1 \cos \theta_{\text{transmitted}}}{n_1 \cos \theta_{\text{transmitted}} + n_2 \cos \theta_{\text{incident}}}$$

$$t_p = \frac{2n_1 \cos \theta_{\text{incident}}}{n_1 \cos \theta_{\text{transmitted}} + n_2 \cos \theta_{\text{incident}}}$$

Reflectance and transmittance are defined as

$$R = |r|^2$$

$$T = \frac{n_2 \cos \theta_{\text{transmitted}}}{n_1 \cos \theta_{\text{incident}}} |t|^2$$

The Brewster angle at which  $r_p = 0$  is defined as

$$\theta_B = \text{atan} \frac{n_2}{n_1}$$

In the first part of the simulation, the Electromagnetic Waves, Frequency Domain interface will be used. The second part instead uses the Electromagnetic Waves, Beam Envelopes interface. Using the Electromagnetic Waves, Beam Envelopes interface, the model is simplified, replacing the glass substrate with an Impedance boundary condition. Furthermore, as the wave vectors for the incident and reflected waves are known, the simulation can be performed with a very coarse mesh.

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1. Note that depending on the sign convention for what is defined as a positive polarization for the reflected wave with in-plane (p-polarization), the sign for the reflection coefficient  $r_p$  can differ for different authors. However, the total field solution will always be the same.

## Results and Discussion

Figure 2 is a combined plot of the y component of the electric-field distribution and the power flow visualized as an arrow plot for the TE case.

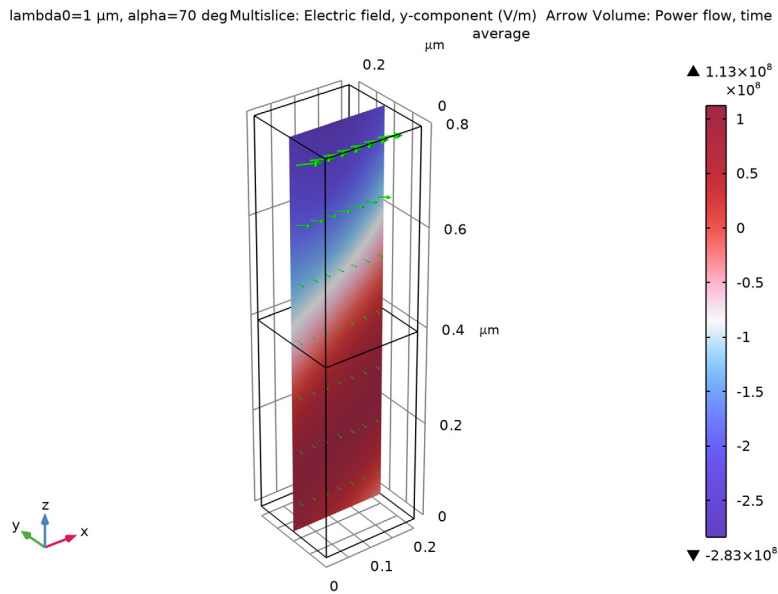
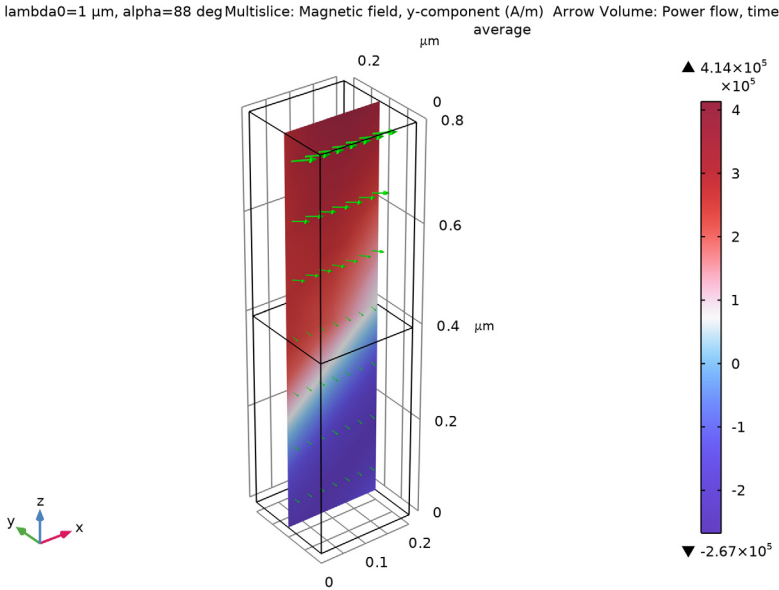


Figure 2: Electric field,  $E_y$  (slice) and power flow (arrows) for TE incidence at  $70^\circ$  inside the unit cell.

For the TM case, [Figure 3](#) visualizes the  $y$  component of the magnetic-field distribution instead, again in combination with the power flow.



*Figure 3: Magnetic field,  $H_y$  (slice) and power flow (arrows) for TM incidence at  $70^\circ$  inside the unit cell.*

Note that the sum of reflectance and transmittance in [Figure 4](#) and [Figure 5](#) equals 1, showing conservation of power. [Figure 5](#) also shows that the reflectance around  $56^\circ$  — the Brewster angle in the TM case — is close to zero.

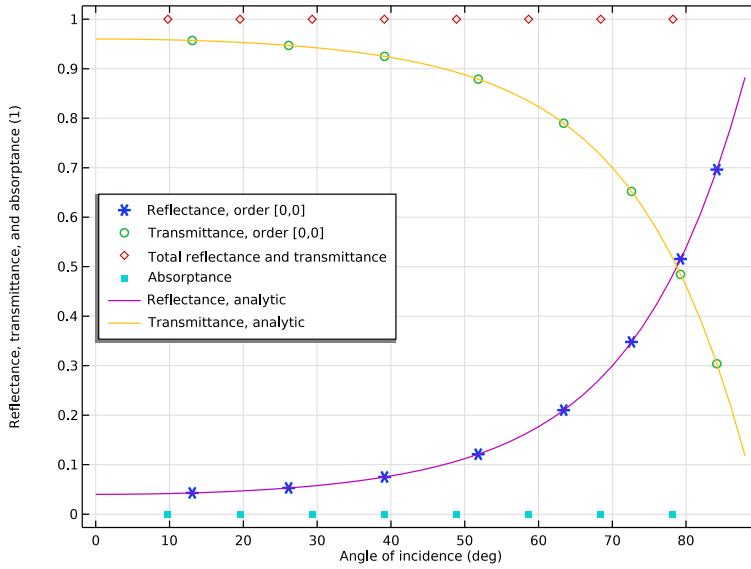


Figure 4: The reflectance and transmittance for TE incidence agree well with the analytic solutions.

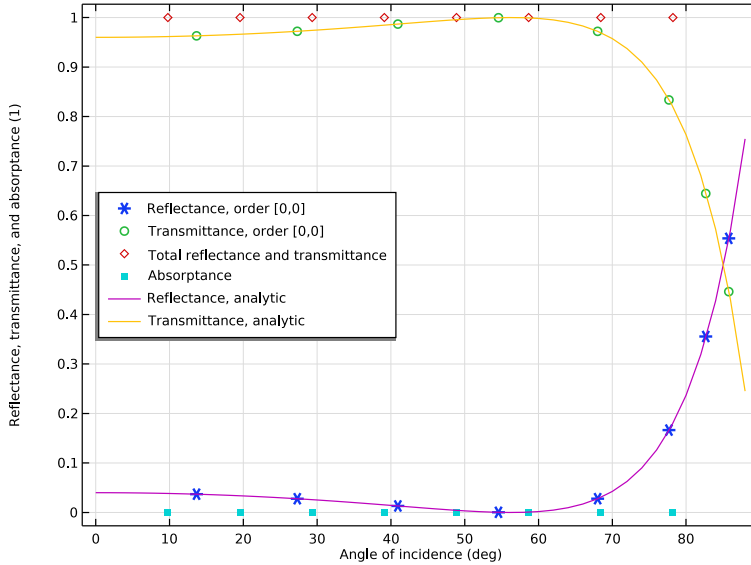


Figure 5: The reflectance and transmittance for TM incidence agree well with the analytic solutions. The Brewster angle is also observed at the expected location.

Figure 6 shows a TE field plot when the Electromagnetic Waves, Beam Envelopes interface is used. Here, the simulation is only performed for the air domain. The bottom glass domain is represented by an Impedance boundary condition, implemented for nonnormal propagation in the exterior (glass) material. This particular implementation of the Impedance boundary condition for the Electromagnetic Waves, Beam Envelopes interface is useful for truncating the simulation domain at an interface between two dielectric materials.

Since the wave vectors for the solution are known, the mesh consists of only one mesh element in the  $z$  direction.

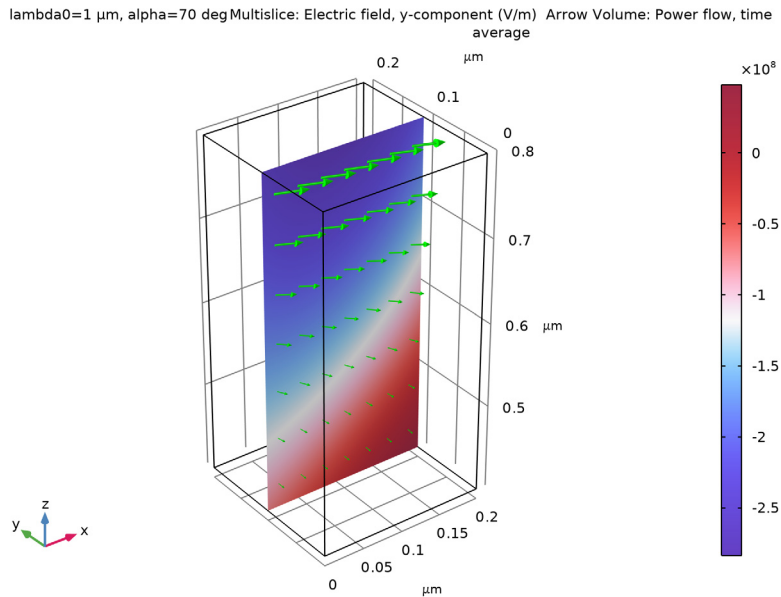


Figure 6: Electric field,  $E_y$  (slice) and power flow (arrows) for TE incidence at  $70^\circ$  inside the unit cell when the Electromagnetic Waves, Beam Envelopes interface is used.

Figure 7 shows that the TE reflectance results for the Electromagnetic Waves, Beam Envelopes interface agree well with the simulation using the Electromagnetic Waves, Frequency Domain interface and the analytical results.

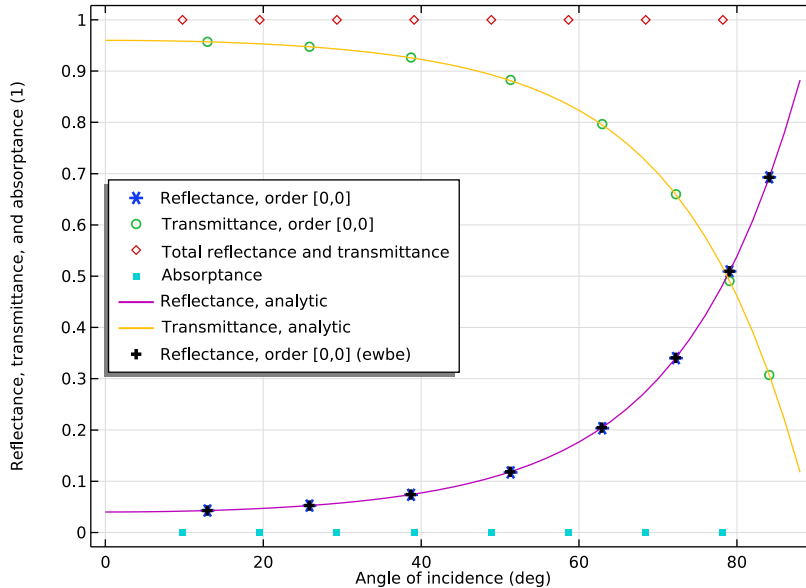


Figure 7: Same plot as Figure 5, but with the TE reflectance curve from the Electromagnetic Waves, Beam Envelopes interface also included.

## References


1. J.D. Jackson, *Classical Electrodynamics*, 3rd Ed., Wiley, 1999.
2. B.E.A. Saleh and M.C. Teich, *Fundamentals of Photonics*, Wiley, 1991.

**Application Library path:** Wave\_Optics\_Module/Verification\_Examples/fresnel\_equations




## Modeling Instructions

From the **File** menu, choose **New**.

## NEW

In the **New** window, click  **Model Wizard**.

## MODEL WIZARD

- 1 In the **Model Wizard** window, click  **3D**.
- 2 In the **Select Physics** tree, select **Optics > Wave Optics > Electromagnetic Waves, Frequency Domain (ewfd)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Wavelength Domain**.
- 6 Click  **Done**.

## GLOBAL DEFINITIONS

Define some parameters that are useful when setting up the geometry and the study.

### *Parameters I*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
n_air	1	1	Refractive index, air
n_slab	1.5	1.5	Refractive index, slab
lda0	1[um]	1E-6 m	Wavelength
alpha	70[deg]	1.2217 rad	Angle of incidence
beta	$\text{asin}(n_{\text{air}} \cdot \sin(\alpha) / n_{\text{slab}})$	0.67701 rad	Refraction angle
alpha_brewster	$\text{atan}(n_{\text{slab}} / n_{\text{air}})$	0.98279 rad	Brewster angle, TM only
r_s	$(n_{\text{air}} \cdot \cos(\alpha) - n_{\text{slab}} \cdot \cos(\beta)) / (n_{\text{air}} \cdot \cos(\alpha) + n_{\text{slab}} \cdot \cos(\beta))$	-0.54735	Reflection coefficient, TE
r_p	$(n_{\text{slab}} \cdot \cos(\alpha) - n_{\text{air}} \cdot \cos(\beta)) / (n_{\text{air}} \cdot \cos(\beta) + n_{\text{slab}} \cdot \cos(\alpha))$	-0.20613	Reflection coefficient, TM
t_s	$(2 \cdot n_{\text{air}} \cdot \cos(\alpha)) / (n_{\text{air}} \cdot \cos(\alpha) + n_{\text{slab}} \cdot \cos(\beta))$	0.45265	Transmission coefficient, TE
t_p	$(2 \cdot n_{\text{air}} \cdot \cos(\alpha)) / (n_{\text{air}} \cdot \cos(\beta) + n_{\text{slab}} \cdot \cos(\alpha))$	0.52925	Transmission coefficient, TM


The angle of incidence is updated while running the parametric sweep. The refraction (transmitted) angle is defined by Snell's law with the updated angle of incidence. The Brewster angle exists only for TM incidence, p-polarization (polarization is in the plane of incidence).

## GEOMETRY I



- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **µm**.

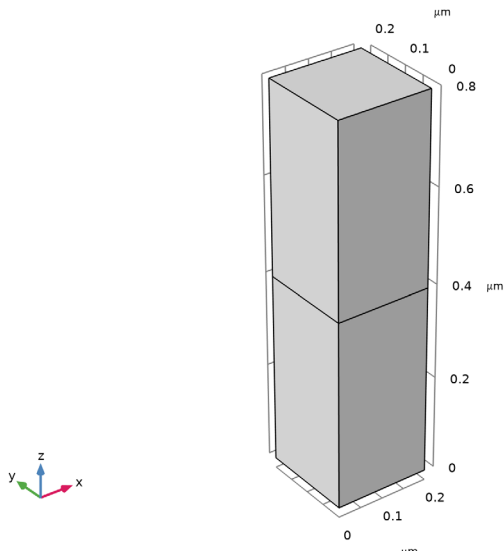
First, create a block composed of two domains. Use layers to split the block.

Block 1 (blk1)

- 1 In the **Geometry** toolbar, click  **Block**.
- 2 In the **Settings** window for **Block**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type 0.2.
- 4 In the **Depth** text field, type 0.2.
- 5 In the **Height** text field, type 0.8.
- 6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness ( $\mu\text{m}$ )
Layer 1	0.4

- 7 Click  **Build All Objects**.
- 8 Click the  **Zoom Extents** button in the **Graphics** toolbar.



Choose wireframe rendering to get a better view of each boundary.

- 9 Click the  **Wireframe Rendering** button in the **Graphics** toolbar.

## MATERIALS

Now set up the material properties based on refractive index. The top half is filled with air.

### Air

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, type Air in the **Label** text field.
- 3 Select Domain 2 only.
- 4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	$n_{iso}$ ; $n_{ii} = n_{iso}$ , $n_{ij} = 0$	$n_{air}$		Refractive index
Refractive index, imaginary part	$k_{iiso}$ ; $k_{iii} =$ $k_{iiso}$ , $k_{ijj} = 0$	0		Refractive index

### Glass

The bottom half is glass.

- 1 Right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, type Glass in the **Label** text field.
- 3 Select Domain 1 only.
- 4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	$n_{iso}$ ; $n_{ii} = n_{iso}$ , $n_{ij} = 0$	$n_{slab}$		Refractive index
Refractive index, imaginary part	$k_{iiso}$ ; $k_{iii} =$ $k_{iiso}$ , $k_{ijj} = 0$	0		Refractive index

## ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD, TE)

Set up the physics based on the direction of propagation and the E-field polarization. First, assume a TE-polarized wave which is equivalent to s-polarization or perpendicular polarization.  $E_x$  and  $E_z$  are zero while  $E_y$  is dominant.

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (ewfd)**.
- 2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, type Electromagnetic Waves, Frequency Domain (ewfd, TE) in the **Label** text field.

### Periodic Structure 1

- 1 In the **Physics** toolbar, click  **Domains** and choose **Periodic Structure**.

- 2 In the **Settings** window for **Periodic Structure**, locate the **Port Mode Settings** section.
- 3 In the  $\alpha_1$  text field, type alpha.

The **Periodic Structure** node automatically adds and configures its port and periodic condition subnodes.

#### **STUDY 1 (EWFD, TE)**

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Study 1 (ewfd, TE) in the **Label** text field.

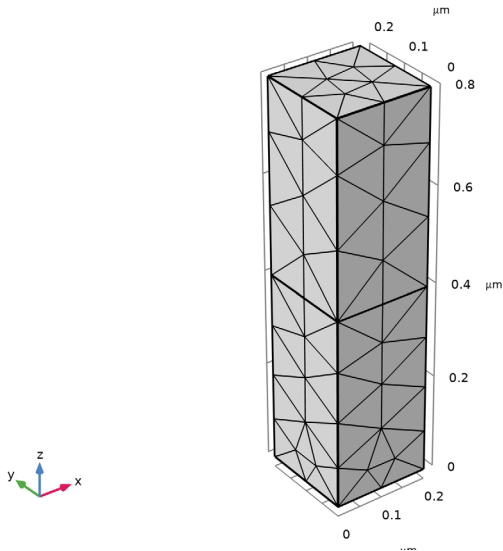
##### *Step 1: Wavelength Domain*

- 1 In the **Model Builder** window, under **Study 1 (ewfd, TE)** click **Step 1: Wavelength Domain**.
- 2 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.
- 3 In the **Wavelengths** text field, type 1da0.
- 4 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 5 Click **+ Add**.
- 6 In the table, enter the following settings:


<b>Parameter name</b>	<b>Parameter value list</b>	<b>Parameter unit</b>
alpha (Angle of incidence)	range(0,2[deg],88[deg])	deg

## MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.



## STUDY 1 (EWFD, TE)

In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Electric Field (ewfd, TE)*

1 In the **Settings** window for **3D Plot Group**, type **Electric Field (ewfd, TE)** in the **Label** text field.

The default plot is the E-field norm for the last solution, which corresponds to almost tangential incidence. Replace the expression with  $E_y$ , add an arrow plot of the power flow (Poynting vector), and choose a more interesting angle of incidence for the plot.

### *Multislice 1*


- 1 In the **Model Builder** window, expand the **Electric Field (ewfd, TE)** node, then click **Multislice 1**.
- 2 In the **Settings** window for **Multislice**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Frequency Domain (ewfd, TE) > Electric > Electric field - V/m > ewfd.Ey - Electric field, y-component**.


- 3 Locate the **Multiplane Data** section. Find the **X-planes** subsection. In the **Planes** text field, type 0.
- 4 Find the **Z-planes** subsection. In the **Planes** text field, type 0.
- 5 Locate the **Coloring and Style** section. From the **Color table** list, choose **WaveLight**.

*Arrow Volume 1*

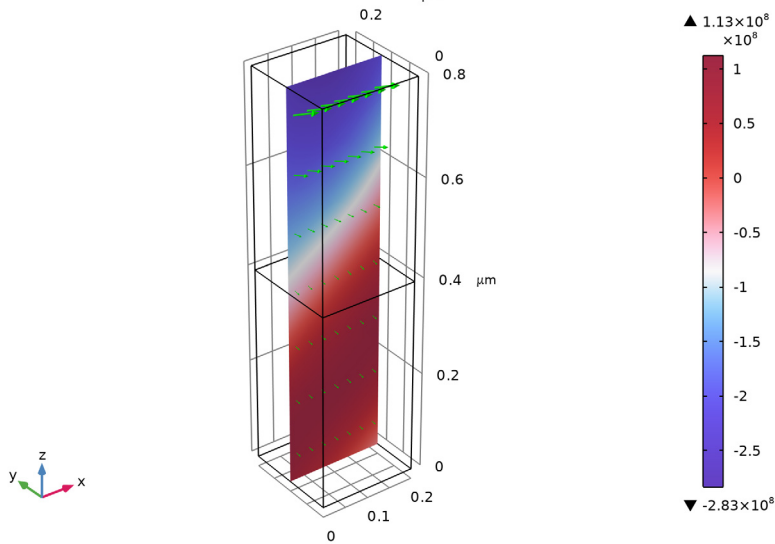
- 1 In the **Model Builder** window, right-click **Electric Field (ewfd, TE)** and choose **Arrow Volume**.
- 2 In the **Settings** window for **Arrow Volume**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Frequency Domain (ewfd, TE) > Energy and power > ewfd.Poavx, ...,ewfd.Poavz - Power flow, time average**.
- 3 Locate the **Arrow Positioning** section. Find the **Y grid points** subsection. In the **Points** text field, type 1.
- 4 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.

*Electric Field (ewfd, TE)*

- 1 In the **Model Builder** window, click **Electric Field (ewfd, TE)**.
- 2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (alpha (deg))** list, choose **70**.
- 4 In the **Electric Field (ewfd, TE)** toolbar, click  **Plot**.

5 Click the  **Zoom Extents** button in the **Graphics** toolbar.

lambda0=1  $\mu\text{m}$ , alpha=70 deg Multislice: Electric field, y-component (V/m) Arrow Volume: Power flow, time average



The plot should look like that in [Figure 2](#).

### Reflectance, Transmittance, and Absorptance (ewfd, TE)

A 1D plot with the reflection and transmission versus the angle of incidence is added by default.

- 1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorptance (ewfd)**.
- 2 In the **Settings** window for **ID Plot Group**, type Reflectance, Transmittance, and Absorptance (ewfd, TE) in the **Label** text field.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 4 Locate the **Plot Settings** section. Select the **x-axis label** checkbox.
- 5 Locate the **Legend** section. From the **Position** list, choose **Middle left**.

### Global 1

- 1 In the **Model Builder** window, expand the **Reflectance, Transmittance, and Absorptance (ewfd, TE)** node, then click **Global 1**.
- 2 In the **Settings** window for **Global**, click to expand the **Coloring and Style** section.
- 3 Find the **Line style** subsection. From the **Line** list, choose **None**.

- 4 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
- 5 From the **Positioning** list, choose **Interpolated**.

*Global 2*

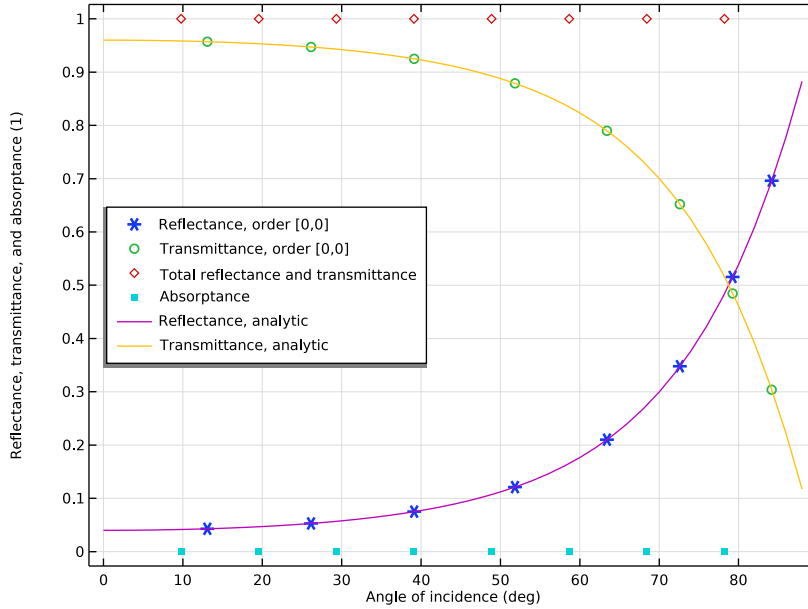
- 1 In the **Model Builder** window, right-click **Reflectance, Transmittance, and Absorptance (ewfd, TE)** and choose **Global**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
$\text{abs}(r_s)^2$		Reflectance, analytic
$\frac{n_{\text{slab}} \cdot \cos(\beta)}{(n_{\text{air}} \cdot \cos(\alpha)) \cdot \text{abs}(t_s)^2}$		Transmittance, analytic

- 4 Locate the **x-Axis Data** section. From the **Unit** list, choose °.
- 5 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.
- 6 In the table, enter the following settings:


Legends
Reflectance, analytic
Transmittance, analytic

**7** In the **Reflectance, Transmittance, and Absorptance (ewfd, TE)** toolbar, click  **Plot**.



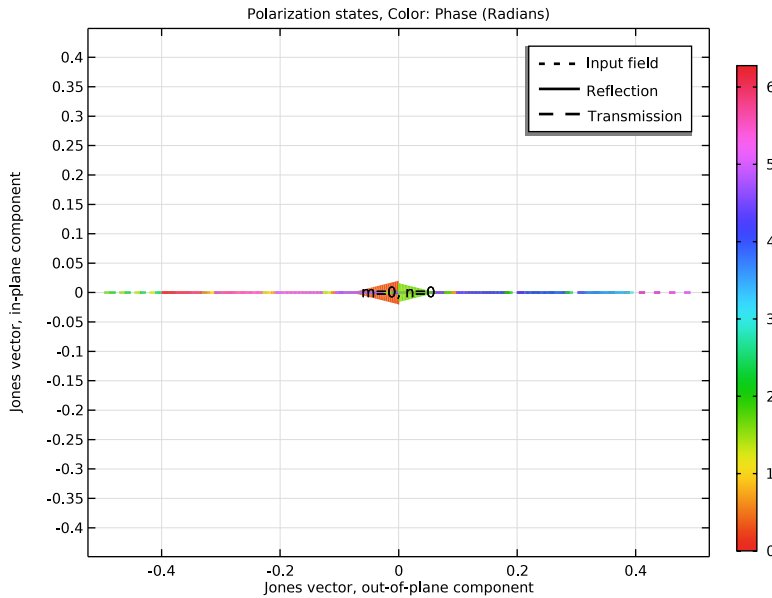
Compare the resulting plots with [Figure 4](#).

#### *Polarization Plot (ewfd, TE)*

- 1** In the **Model Builder** window, under **Results** click **Polarization Plot (ewfd)**.
- 2** In the **Settings** window for **ID Plot Group**, type **Polarization Plot (ewfd, TE)** in the **Label** text field.
- 3** In the **Polarization Plot (ewfd, TE)** toolbar, click  **Plot**.

The polarization plot shows that the field is linearly polarized, with a pure out-of-plane polarization. The out-of-plane component of the Jones vector represents the amplitude of a wave polarized in the  $y$  direction, whereas the in-plane component represents the

amplitude of a wave with a polarization that is orthogonal to both the out-of-plane direction and the propagation direction, resulting in a polarization in the  $x - z$  plane.



### ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD, TE) (EWFD)

The following instructions are for the case of a TM-polarized wave (p-polarization or parallel polarization). In this case,  $E_y$  is zero while  $E_x$  and  $E_z$  characterize the wave. In other words,  $H_y$  is dominant while  $H_x$  and  $H_z$  are zero.

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electromagnetic Waves, Frequency Domain (ewfd, TE) (ewfd)** and choose **Copy**.

### ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD2, TM)



- 1 In the **Model Builder** window, right-click **Component 1 (comp1)** and choose **Paste Electromagnetic Waves, Frequency Domain**.
- 2 In the **Messages from Paste** dialog, click **OK**.
- 3 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, type **Electromagnetic Waves, Frequency Domain (ewfd2, TM)** in the **Label** text field.

#### Periodic Structure 1

- 1 In the **Model Builder** window, expand the **Component 1 (comp1) > Electromagnetic Waves, Frequency Domain (ewfd2, TM) (ewfd2)** node, then click **Periodic Structure 1**.

- 2 In the **Settings** window for **Periodic Structure**, locate the **Port Mode Settings** section.
- 3 From the list, choose **P**. This will change the port mode fields to be polarized parallel to the plane of incidence.

#### ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

#### STUDY 2 (EWFD2, TM)

In the **Settings** window for **Study**, type Study 2 (ewfd2, TM) in the **Label** text field.

#### STUDY 1 (EWFD, TE)


*Step 1: Wavelength Domain*

In the **Model Builder** window, under **Study 1 (ewfd, TE)** right-click

**Step 1: Wavelength Domain** and choose **Copy**.

#### STUDY 2 (EWFD2, TM)

In the **Model Builder** window, right-click **Study 2 (ewfd2, TM)** and choose **Paste Wavelength Domain**.

- 1 In the **Settings** window for **Wavelength Domain**, locate the **Physics and Variables Selection** section.
- 2 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electromagnetic Waves, Frequency Domain (ewfd, TE) (ewfd)**.
- 3 In the **Study** toolbar, click  **Compute**.

#### RESULTS

*Magnetic Field (ewfd2, TM)*

In the **Settings** window for **3D Plot Group**, type Magnetic Field (ewfd2, TM) in the **Label** text field.


*Multislice 1*

- 1 In the **Model Builder** window, expand the **Magnetic Field (ewfd2, TM)** node, then click **Multislice 1**.

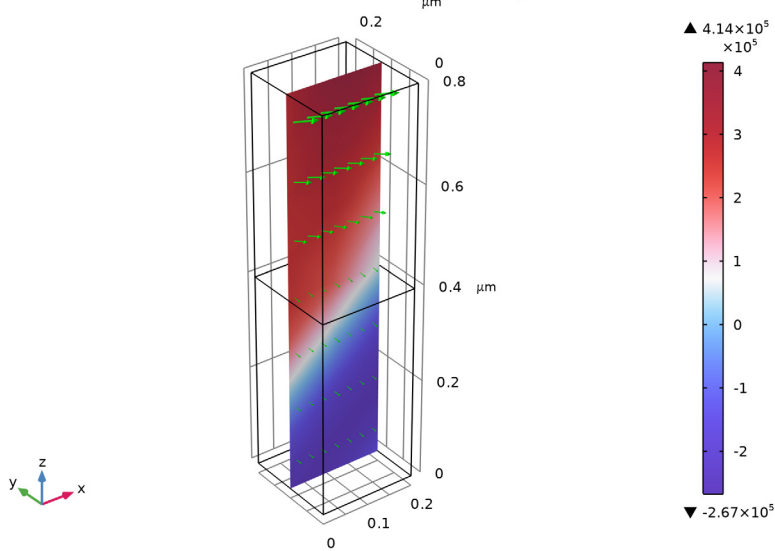
- 2 In the **Settings** window for **Multislice**, locate the **Expression** section.
- 3 In the **Expression** text field, type  $ewfd2 \cdot Hy$ .
- 4 Locate the **Multipane Data** section. Find the **X-planes** subsection. In the **Planes** text field, type 0.
- 5 Find the **Z-planes** subsection. In the **Planes** text field, type 0.
- 6 Locate the **Coloring and Style** section. From the **Color table** list, choose **WaveLight**.

*Arrow Volume 1*

- 1 In the **Model Builder** window, right-click **Magnetic Field (ewfd2, TM)** and choose **Arrow Volume**.
- 2 In the **Settings** window for **Arrow Volume**, locate the **Expression** section.
- 3 In the **X-component** text field, type  $ewfd2 \cdot Poavx$ .
- 4 In the **Y-component** text field, type  $ewfd2 \cdot Poavy$ .
- 5 In the **Z-component** text field, type  $ewfd2 \cdot Poavz$ .
- 6 Locate the **Arrow Positioning** section. Find the **Y grid points** subsection. In the **Points** text field, type 1.
- 7 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.

8 In the **Magnetic Field (ewfd2, TM)** toolbar, click  **Plot**.

lambda0=1 μm, alpha=88 deg Multislice: Magnetic field, y-component (A/m) Arrow Volume: Power flow, time average



This reproduces [Figure 3](#).

*Reflectance, Transmittance, and Absorptance (ewfd2, TM)*

- 1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorptance (ewfd2)**.
- 2 In the **Settings** window for **ID Plot Group**, type Reflectance, Transmittance, and Absorptance (ewfd2, TM) in the **Label** text field.
- 3 Locate the **Title** section. From the **Title type** list, choose **None**.
- 4 Locate the **Plot Settings** section. Select the **x-axis label** checkbox.
- 5 Locate the **Legend** section. From the **Position** list, choose **Middle left**.

*Global 1*

- 1 In the **Model Builder** window, expand the **Reflectance, Transmittance, and Absorptance (ewfd2, TM)** node, then click **Global 1**.
- 2 In the **Settings** window for **Global**, locate the **Coloring and Style** section.
- 3 Find the **Line style** subsection. From the **Line** list, choose **None**.
- 4 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
- 5 From the **Positioning** list, choose **Interpolated**.

*Global 2*

- 1 In the **Model Builder** window, right-click **Reflectance, Transmittance, and Absorptance (ewfd2, TM)** and choose **Global**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

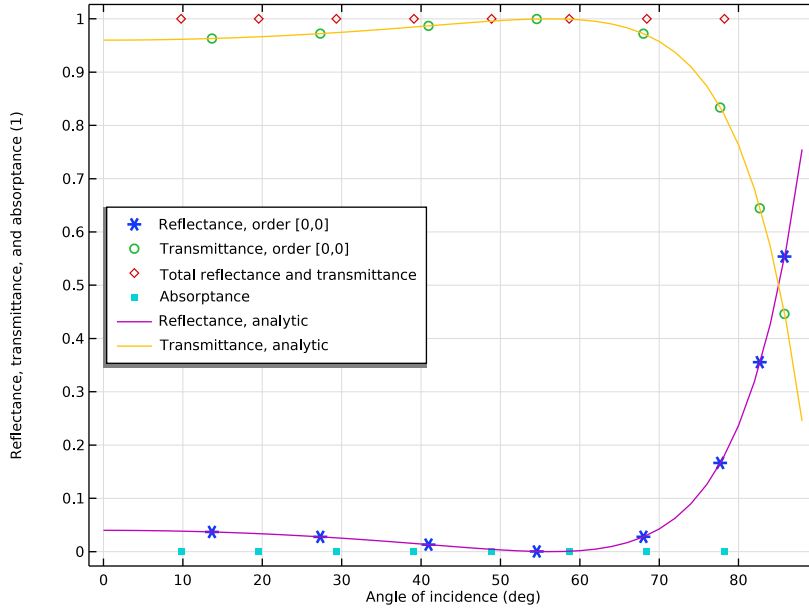
Expression	Unit	Description
$\text{abs}(r_p)^2$		Reflectance, analytic
$n_{\text{slab}} \cdot \cos(\beta) / (n_{\text{air}} \cdot \cos(\alpha)) \cdot \text{abs}(t_p)^2$		Transmittance, analytic

- 4 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.
- 5 In the table, enter the following settings:

Legends
Reflectance, analytic
Transmittance, analytic

- 6 Locate the **x-Axis Data** section. From the **Unit** list, choose  $^\circ$ .

7 In the **Reflectance, Transmittance, and Absorptance (ewfd2, TM)** toolbar, click  **Plot**.



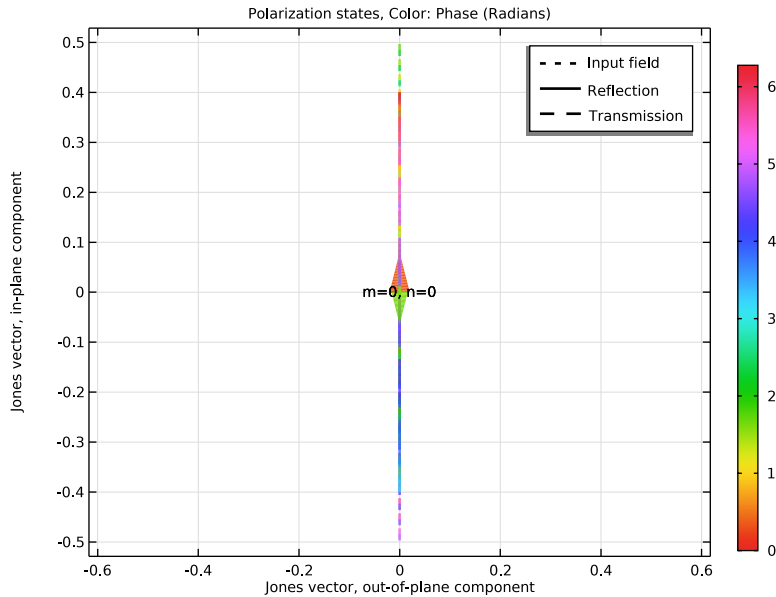
The plot should look like [Figure 5](#). The Brewster angle is observed around 56 degrees, which is close to the analytic value.

#### Polarization Plot (ewfd2, TM)

- 1 In the **Model Builder** window, under **Results** click **Polarization Plot (ewfd2)**.
- 2 In the **Settings** window for **ID Plot Group**, type Polarization Plot (ewfd2, TM) in the **Label** text field.

3 In the **Polarization Plot (ewfd2, TM)** toolbar, click  **Plot**.



Also in this case, the polarization plot shows that the field is linearly polarized, in this case with a pure in-plane polarization.



### COMPONENT I (COMP I)

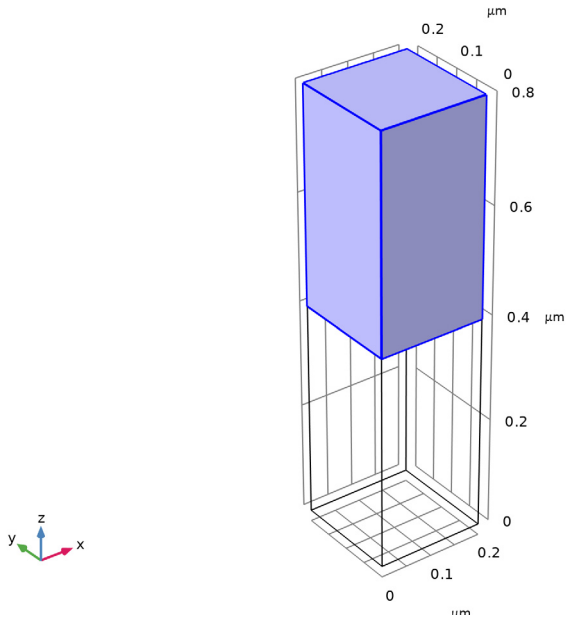
Now, add a simulation using the Electromagnetic Waves, Beam Envelopes interface. In this simulation, the Impedance boundary condition is added to the interface between the air and the glass domain and the simulation is only performed for the air domain. This example demonstrates that the Impedance boundary condition for the Electromagnetic Waves, Beam Envelopes interface can be used for truncating the simulation domain at a boundary between two dielectric material domains.

### ADD PHYSICS

- 1 In the **Home** toolbar, click  **Add Physics** to open the **Add Physics** window.
- 2 Go to the **Add Physics** window.
- 3 In the tree, select **Optics > Wave Optics > Electromagnetic Waves, Beam Envelopes (ewbe)**.
- 4 Click the **Add to Component I** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Physics** to close the **Add Physics** window.

## ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE, TE)

- 1 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, type **Electromagnetic Waves, Beam Envelopes (ewbe, TE)** in the **Label** text field.
- 2 Select Domain 2 only, to only perform the simulation for the air domain.




- 3 Locate the **Wave Vectors** section. Specify the  $\mathbf{k}_1$  vector as

$\text{ewbe.k0} \cdot \sin(\alpha)$	x
0	y
$-\text{ewbe.k0} \cdot \cos(\alpha)$	z

- 4 Specify the  $\mathbf{k}_2$  vector as

$\text{ewbe.k1x}$	x
$\text{ewbe.k1y}$	y
$-\text{ewbe.k1z}$	z

### Port 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Port**.
- 2 Select Boundary 7 only.
- 3 In the **Settings** window for **Port**, locate the **Port Properties** section.

- 4 From the **Type of port** list, choose **Periodic**.


For the first port, wave excitation is **on** by default.

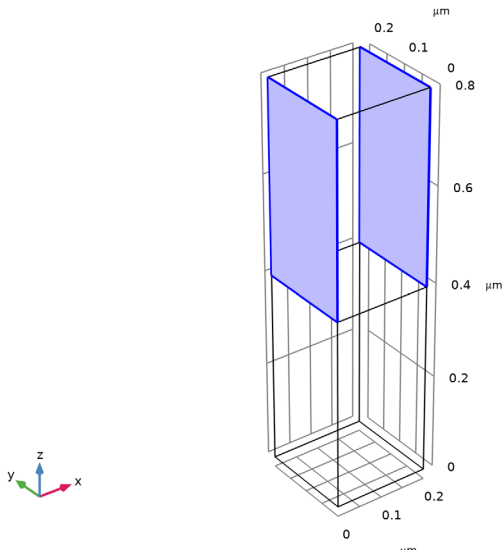
- 5 Locate the **Port Mode Settings** section. From the **Polarization** list, choose **Linear polarization**, to give a polarization in the  $y$ -direction (orthogonal to the plane of incidence).

- 6 In the  $\alpha_1$  text field, type alpha.


The maximum frequency in the setting window will be used only when **Compute Diffraction Order** button is clicked to generate **Diffraction Order** features representing higher order modes. In this model, no diffraction is expected from the given geometry, so this setting is ineffective.

#### *Periodic Condition 1*

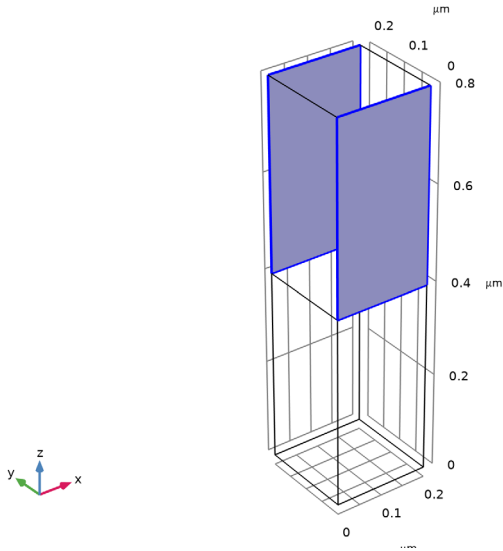
- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Periodic Condition**.
- 2 Select Boundaries 4 and 11 only.
- 3 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.
- 4 From the **Type of periodicity** list, choose **Floquet periodicity**.
- 5 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.



#### *Periodic Condition 2*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Periodic Condition**.
- 2 Select Boundaries 5 and 9 only.

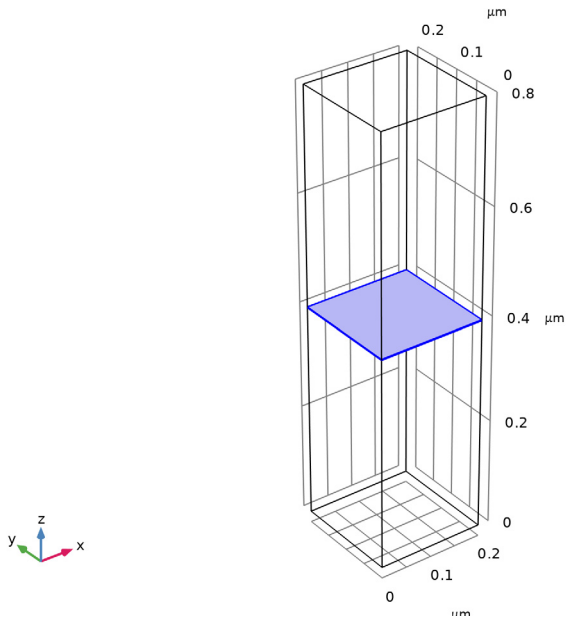
- 3 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.
- 4 From the **Type of periodicity** list, choose **Floquet periodicity**.
- 5 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.



*Impedance Boundary Condition I*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Impedance Boundary Condition**.

2 Select Boundary 6 only.



3 In the **Settings** window for **Impedance Boundary Condition**, locate the **Propagation Direction** section.

4 From the list, choose **From wave vector**, to make the **Impedance Boundary Condition** also work for nonnormal propagation in the exterior (glass) material.

## MATERIALS

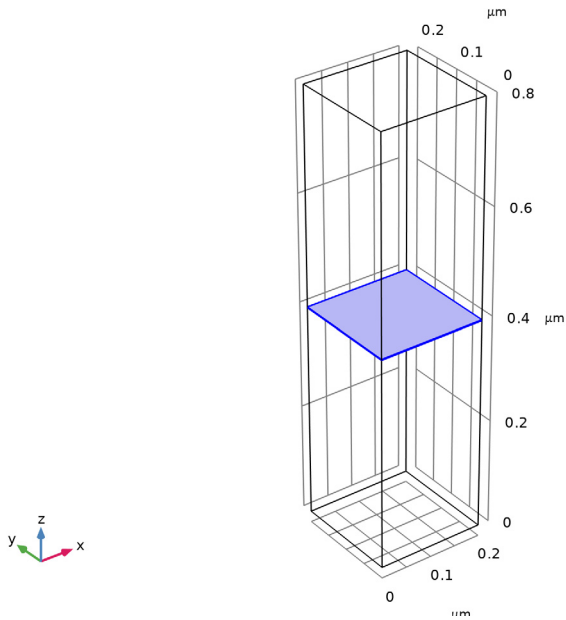
### *Glass (Boundary)*

1 In the **Model Builder** window, under **Component 1 (comp1) > Materials** right-click **Glass (mat2)** and choose **Duplicate**.

2 In the **Settings** window for **Material**, type **Glass (Boundary)** in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 6 only.



#### MESH 1

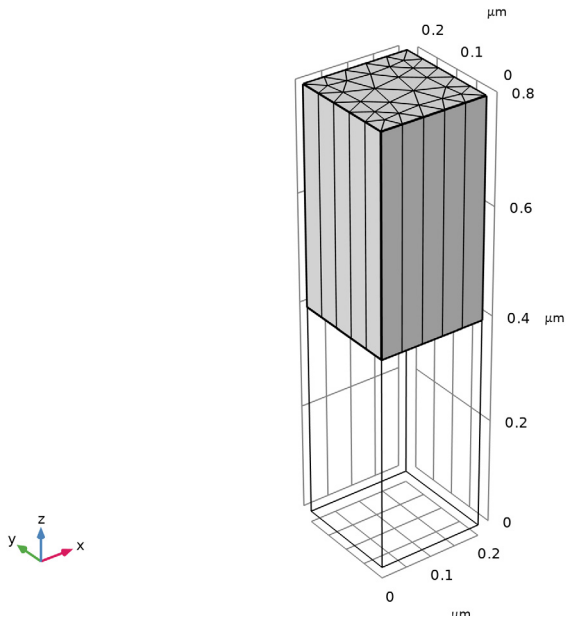
For the Electromagnetic Waves, Beam Envelopes interface a single mesh element is required, as the prescribed wave vectors match the full-wave solution.

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
- 3 In the table, clear the **Use** checkbox for **Electromagnetic Waves, Beam Envelopes (ewbe, TE) (ewbe)**.


#### MESH 2

- 1 In the **Mesh** toolbar, click **Add Mesh** and choose **Add Mesh**.
- 2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
- 3 In the table, clear the **Use** checkboxes for **Electromagnetic Waves, Frequency Domain (ewfd, TE) (ewfd)** and **Electromagnetic Waves, Frequency Domain (ewfd2, TM) (ewfd2)**.
- 4 Locate the **Electromagnetic Waves, Beam Envelopes (ewbe, TE) (ewbe)** section. In the  $N_T$  text field, type 1.
- 5 In the  $N_L$  text field, type 1.

6 Click  **Build All**.



#### ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Click the **Add Study** button in the window toolbar.

#### STUDY 3 (EWBE, TE)

In the **Settings** window for **Study**, type Study 3 (ewbe, TE) in the **Label** text field.

#### STUDY 2 (EWFD2, TM)

*Step 1: Wavelength Domain*

In the **Model Builder** window, under **Study 2 (ewfd2, TM)** right-click



**Step 1: Wavelength Domain** and choose **Copy**.

#### STUDY 3 (EWBE, TE)

In the **Model Builder** window, right-click **Study 3 (ewbe, TE)** and choose **Paste Wavelength Domain**.

- 1 In the **Settings** window for **Wavelength Domain**, locate the **Physics and Variables Selection** section.
- 2 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electromagnetic Waves, Frequency Domain (ewfd2, TM) (ewfd2)**.
- 3 Click to expand the **Mesh Selection** section. In the table, enter the following settings:

Component	Mesh
Geometry 1	Mesh 2

- 4 In the **Study** toolbar, click  **Add Study** to close the **Add Study** window.
- 5 In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Electric Field(ewbe, TE)*

- 1 In the **Settings** window for **3D Plot Group**, type **Electric Field(ewbe, TE)** in the **Label** text field.
- 2 Locate the **Data** section. From the **Parameter value (alpha (deg))** list, choose **70**.


### *Electric Field*

- 1 In the **Model Builder** window, expand the **Electric Field(ewbe, TE)** node, then click **Electric Field**.
- 2 In the **Settings** window for **Multislice**, locate the **Expression** section.
- 3 In the **Expression** text field, type **ewbe.Ey**.
- 4 Locate the **Multipane Data** section. Find the **X-planes** subsection. In the **Planes** text field, type **0**.
- 5 Find the **Z-planes** subsection. In the **Planes** text field, type **0**.
- 6 Locate the **Coloring and Style** section. From the **Color table** list, choose **WaveLight**.

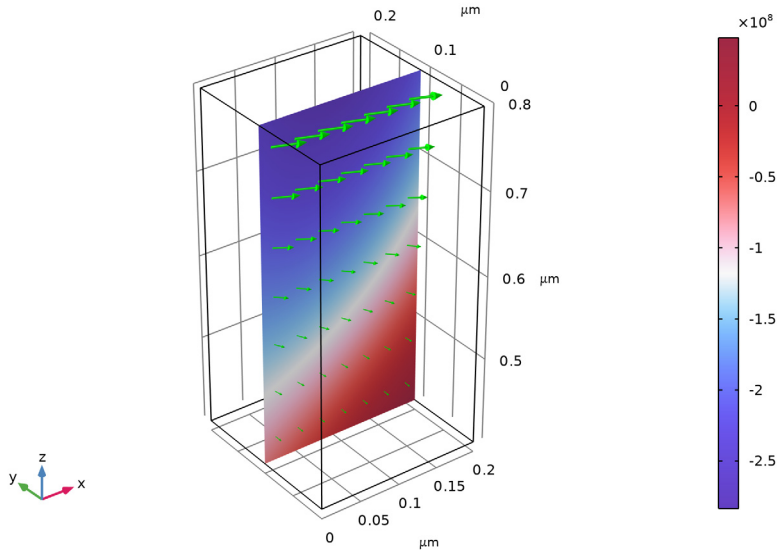
### *Arrow Volume 1*

- 1 In the **Model Builder** window, right-click **Electric Field(ewbe, TE)** and choose **Arrow Volume**.
- 2 In the **Settings** window for **Arrow Volume**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Beam Envelopes (ewbe, TE) > Energy and power > ewbe.Poavx,..., ewbe.Poavz - Power flow, time average**.
- 3 Locate the **Arrow Positioning** section. Find the **Y grid points** subsection. In the **Points** text field, type **1**.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.

5 In the **Electric Field(ewbe, TE)** toolbar, click  **Plot**.

lambda0=1  $\mu\text{m}$ , alpha=70 deg Multislice: Electric field, y-component (V/m) Arrow Volume: Power flow, time average



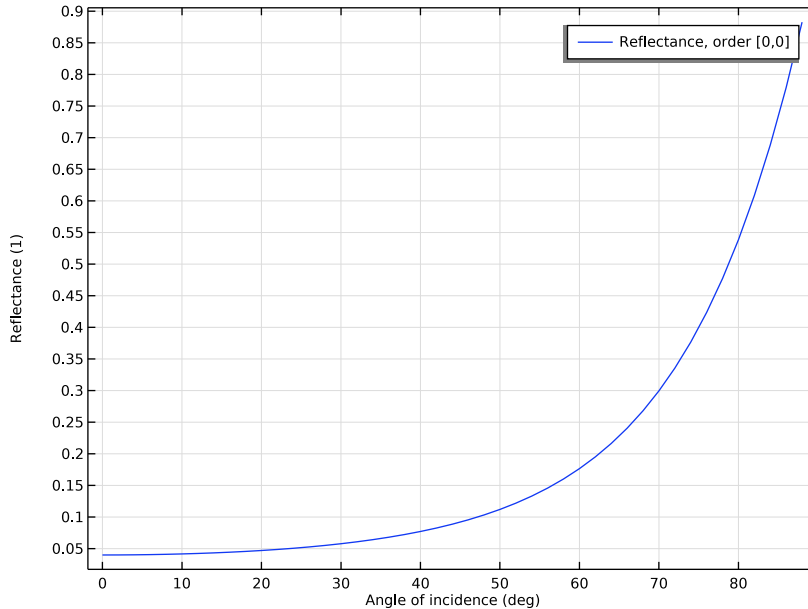
*Electric Field, Second Wave (ewbe)*

Here, the norm of the electric field of the second wave is not of interest. So, delete this plot group.

In the **Model Builder** window, under **Results** right-click **Electric Field, Second Wave (ewbe)** and choose **Delete**.

### Reflectance (ewbe, TE)

1 In the **Settings** window for **ID Plot Group**, type Reflectance (ewbe, TE) in the **Label** text field.



### Global 1

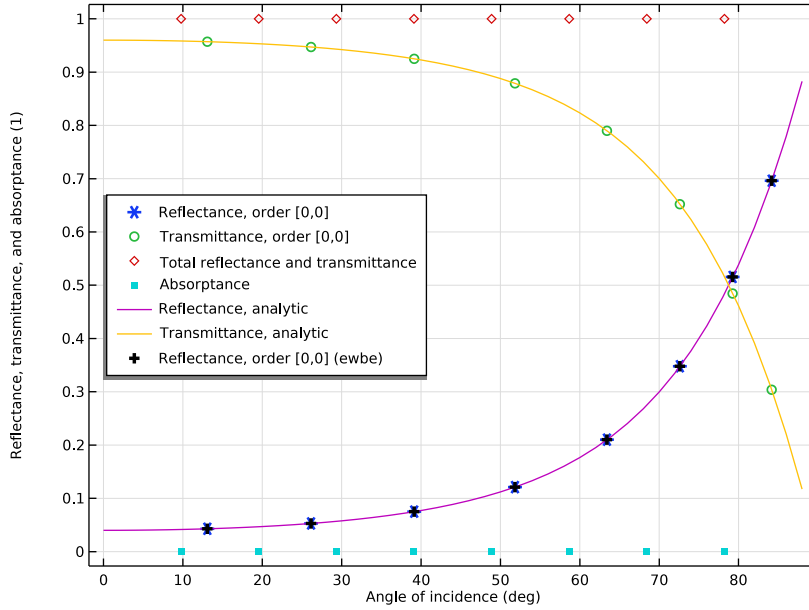
We will now compare this plot to the other TE reflectance plot.

### Global 3

- 1 In the **Model Builder** window, under **Results > Reflectance, Transmittance, and Absorptance (ewfd, TE)** right-click **Global 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Global**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 3 (ewbe, TE)/Solution 3 (sol3)**.
- 4 Click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Beam Envelopes (ewbe, TE) > Ports > Reflectance, by order > ewbe.Rorder\_0\_0 - Reflectance, order [0,0]**.
- 5 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
ewbe.Rorder_0_0	1	Reflectance, order [0,0] (ewbe)

6 In the **Reflectance, Transmittance, and Absorptance (ewfd, TE)** toolbar, click  **Plot**.

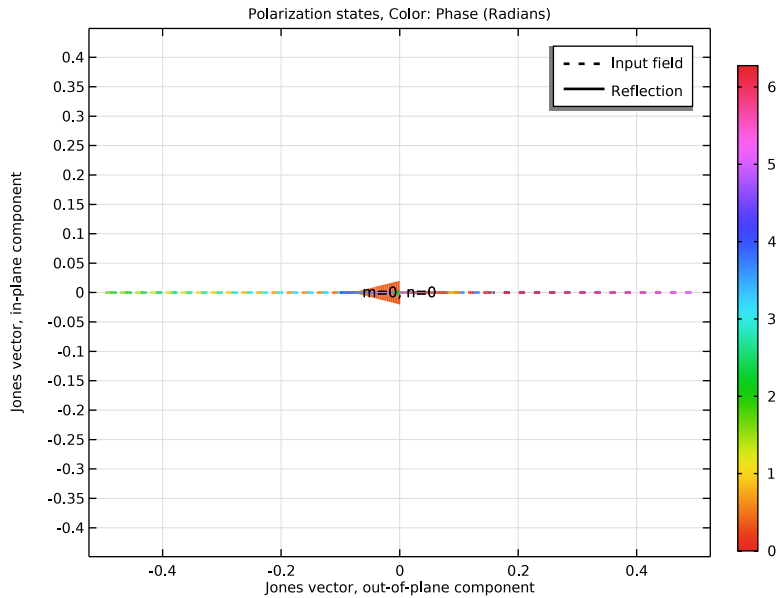


Notice that the reflectance curve for the Electromagnetic Waves, Beam Envelopes interface reproduces the other reflectance curves.

#### Polarization Plot (ewbe, TE)

1 In the **Model Builder** window, under **Results** click **Polarization Plot (ewbe)**.

- 2 In the **Settings** window for **ID Plot Group**, type **Polarization Plot (ewbe, TE)** in the **Label** text field.



### **ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE, TE) (EWBE)**

The following instructions are for the p-polarization (TM) case for the Electromagnetic Waves, Beam Envelopes interface.

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electromagnetic Waves, Beam Envelopes (ewbe, TE) (ewbe)** and choose **Copy**.

### **ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE2, TM)**



- 1 In the **Model Builder** window, right-click **Component 1 (comp1)** and choose **Paste Electromagnetic Waves, Beam Envelopes**.
- 2 In the **Messages from Paste** dialog, click **OK**.
- 3 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, type **Electromagnetic Waves, Beam Envelopes (ewbe2, TM)** in the **Label** text field.

#### *Port 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1) > Electromagnetic Waves, Beam Envelopes (ewbe2, TM) (ewbe2)** node, then click **Port 1**.
- 2 In the **Settings** window for **Port**, locate the **Port Mode Settings** section.

3 From the list, choose **P**, to give a polarization that is parallel to the plane of incidence.

#### ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

#### STUDY 4 (EWBE2, TM)

In the **Settings** window for **Study**, type Study 4 (ewbe2, TM) in the **Label** text field.

#### STUDY 3 (EWBE, TE)


*Step 1: Wavelength Domain*

In the **Model Builder** window, under **Study 3 (ewbe, TE)** right-click

**Step 1: Wavelength Domain** and choose **Copy**.

#### STUDY 4 (EWBE2, TM)

In the **Model Builder** window, right-click **Study 4 (ewbe2, TM)** and choose **Paste Wavelength Domain**.

- 1 In the **Settings** window for **Wavelength Domain**, locate the **Physics and Variables Selection** section.
- 2 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electromagnetic Waves, Beam Envelopes (ewbe, TE) (ewbe)**.
- 3 In the **Study** toolbar, click  **Compute**.

#### RESULTS

*Magnetic Field (ewbe2, TM)*


- 1 In the **Settings** window for **3D Plot Group**, type Magnetic Field (ewbe2, TM) in the **Label** text field.
- 2 Locate the **Data** section. From the **Parameter value (alpha (deg))** list, choose **70**.

*Electric Field*

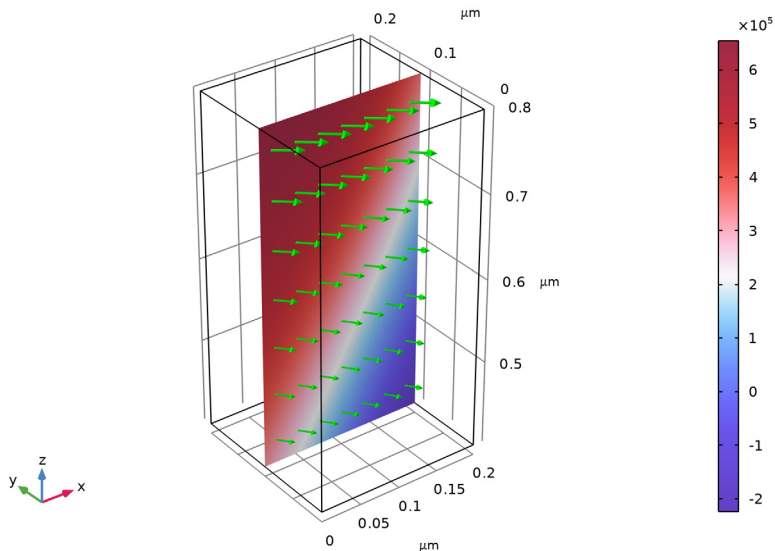
- 1 In the **Model Builder** window, expand the **Magnetic Field (ewbe2, TM)** node, then click **Electric Field**.
- 2 In the **Settings** window for **Multislice**, locate the **Expression** section.

- 3 In the **Expression** text field, type `ewbe2.Hy`.
- 4 Locate the **Multiplane Data** section. Find the **X-planes** subsection. In the **Planes** text field, type 0.
- 5 Find the **Z-planes** subsection. In the **Planes** text field, type 0.
- 6 Locate the **Coloring and Style** section. From the **Color table** list, choose **WaveLight**.

*Arrow Volume 1*

- 1 In the **Model Builder** window, right-click **Magnetic Field (ewbe2, TM)** and choose **Arrow Volume**.
- 2 In the **Settings** window for **Arrow Volume**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Beam Envelopes (ewbe2, TM) > Energy and power > ewbe2.Poavx, ...,ewbe2.Poavz - Power flow, time average**.
- 3 Locate the **Arrow Positioning** section. Find the **Y grid points** subsection. In the **Points** text field, type 1.
- 4 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.
- 5 In the **Magnetic Field (ewbe2, TM)** toolbar, click  **Plot**.

lambda0=1 μm, alpha=70 deg Multislice: Magnetic field, y-component (A/m) Arrow Volume: Power flow, time average



### Electric Field, Second Wave (ewbe2)

Again, the norm of the electric field of the second wave is not of interest. So, delete this plot group.


In the **Model Builder** window, under **Results** right-click **Electric Field, Second Wave (ewbe2)** and choose **Delete**.

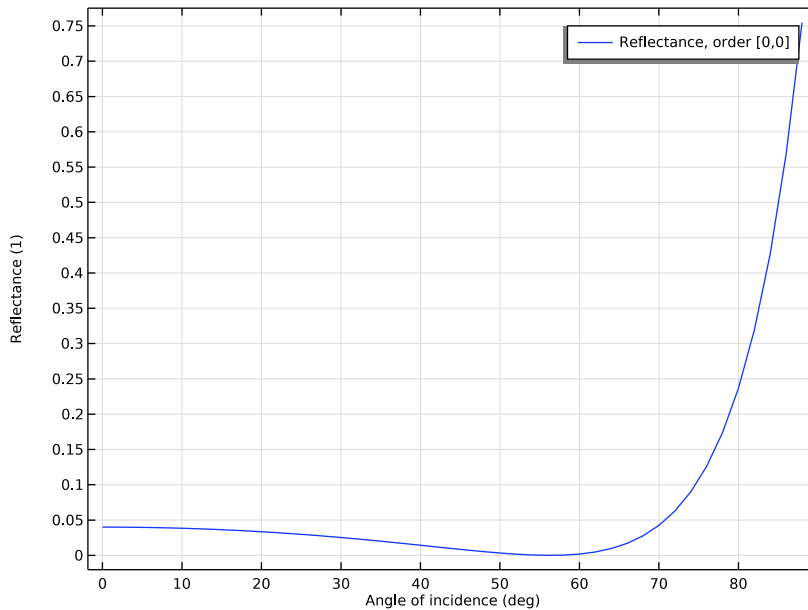
### Reflectance (ewbe2, TM)

In the **Settings** window for **ID Plot Group**, type Reflectance (ewbe2, TM) in the **Label** text field.

### Global 1

**1** In the **Model Builder** window, expand the **Reflectance (ewbe2, TM)** node, then click **Global 1**.

**2** In the **Reflectance (ewbe2, TM)** toolbar, click  **Plot**.



### Global 3

Now compare this plot to the other TM reflectance plot.

**1** In the **Model Builder** window, under **Results > Reflectance, Transmittance, and Absorptance (ewfd2, TM)** right-click **Global 1** and choose **Duplicate**.

**2** In the **Settings** window for **Global**, locate the **Data** section.

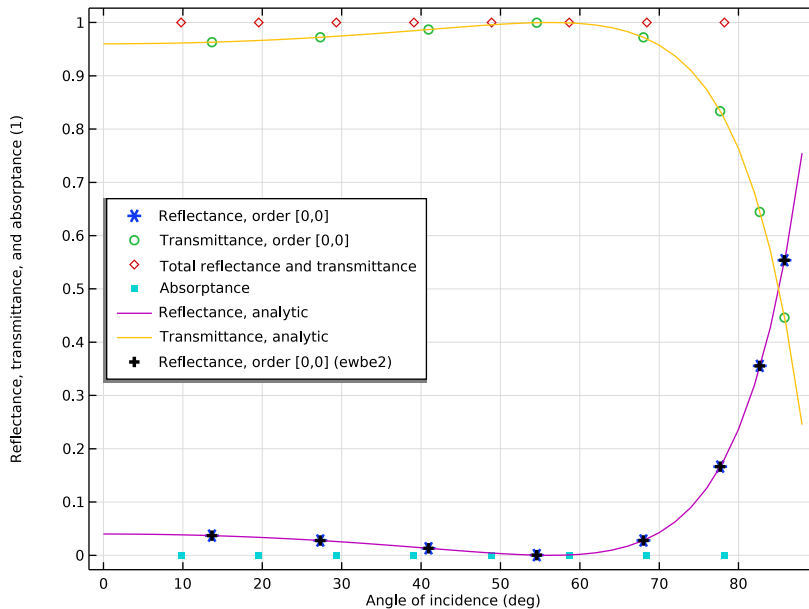
3 From the **Dataset** list, choose **Study 4 (ewbe2, TM)/Solution 4 (sol4)**.

4 Click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Beam Envelopes (ewbe2, TM) > Ports > Reflectance, by order > ewbe2.Rorder\_0\_0 - Reflectance, order [0,0]**.

5 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
ewbe2.Rorder_0_0	1	Reflectance, order [0,0] (ewbe2)

6 In the **Reflectance, Transmittance, and Absorptance (ewfd2, TM)** toolbar, click  **Plot**.



Again, the reflectance curve for the Electromagnetic Waves, Beam Envelopes interface reproduces well the other reflectance curves.

*Polarization Plot (ewbe2, TM)*

1 In the **Model Builder** window, under **Results** click **Polarization Plot (ewbe2)**.

2 In the **Settings** window for **ID Plot Group**, type Polarization Plot (ewbe2, TM) in the **Label** text field.

