Model created in COMSOL Multiphysics 6.4

Simulation of Dispersion and Hyperbolic Wave in Metal–Dielectric Layered Metamaterial

Introduction

Optical metamaterials are artificially engineered structures composed of subwavelength building blocks that offer unique electromagnetic phenomena. Such structures exhibit anisotropic dispersion and their electrical properties (that is, permittivity, permeability, conductivity, and so on) can be controlled by changing the shape, geometry, size, orientation, and material properties of the composing unit cell. Metamaterials enable manipulation of light in unprecedented ways with extreme control over the properties of electromagnetic fields that cannot be obtained with materials available in nature.

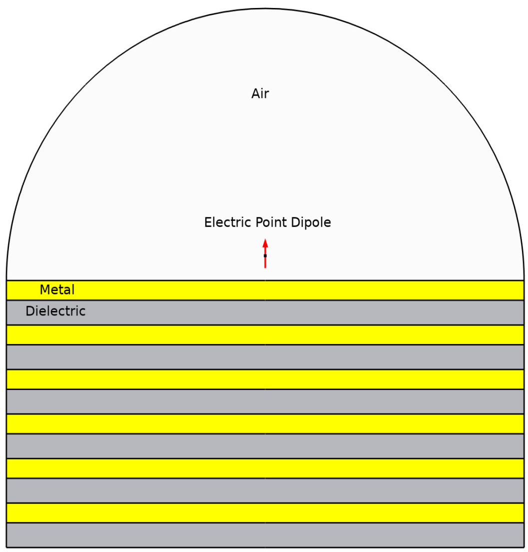


Figure 1: Schematic drawing of an electric point dipole located in air near a metamaterial structure. The structure is composed of periodically organized metal and dielectric layers with subwavelength thickness and periodicity.

One class of extremely anisotropic metamaterials displays hyperbolic dispersion thanks to the resonance effect arising from the equivalent electrical LC circuit, due to the opposite signs of the electric or magnetic tensor components along the orthogonal optical axes. These structures are commonly referred to as hyperbolic metamaterials. That can be constructed by periodically organized (i) metal–dielectric layers and (ii) metallic nanorods embedded in a dielectric, with subwavelength periodicity and dimension. The hyperbolic waves propagating within the metamaterial structure are extremely confined and their wavelengths can be a hundred times smaller than the free-space wavelength. Such distinctive electromagnetic features of hyperbolic metamaterials are useful in novel nanophotonic applications including enhanced superlensing effect, subdiffraction imaging, sensing, negative refraction, canalization of light, energy harvesting, and quantum and thermal engineering with superior performances.

This model discusses how to simulate hyperbolic waves propagating in a hyperbolic metamaterial constructed of periodically organized metal–dielectric layers, using an electric point dipole located in the air above the structure. It also demonstrates the procedure to compute the effective relative permittivity tensor components of the metamaterial.

Model Definition

Figure 1 shows the schematic of the model. A linearly (vertically) polarized electric point dipole source is located in air near a hyperbolic metamaterial composed of periodically oriented subwavelength metal-dielectric layers. The evanescent fields radiated by the dipole couple to the structure and excite two types of waves: (i) surface plasmon polariton that propagates along the metal–air interface and (ii) hyperbolic wave that propagates within the metamaterial.

Imagine a nonmagnetic metamaterial as is shown in Figure 1. The effective permittivity of such a structure is a diagonal tensor. Within the subwavelength regime, the diagonal components can be calculated using the effective medium theory as

$$\varepsilon_{rr} = \varepsilon_{\mathrm{m}} F_{\mathrm{m}} + \varepsilon_{\mathrm{d}}(1 - F_{\mathrm{m}}), \tag{1}$$

$$\varepsilon_{zz} = \left\{ \frac{F_{\mathrm{m}}}{\varepsilon_{\mathrm{m}}} + \frac{1 - F_{\mathrm{m}}}{\varepsilon_{\mathrm{d}}} \right\}^{-1}, \tag{2}$$

with $F_{\mathrm{m}} = t_{\mathrm{m}}/(t_{\mathrm{m}} + t_{\mathrm{d}})$ being the filling ratio of metal.

Here, ε_{rr} and ε_{zz} are the tangential and normal components of the effective relative permittivity, respectively; t_{m} and t_{d} are thicknesses of metal and dielectric layers, respectively; and ε_{m} and ε_{d} are relative permittivities of metal and dielectric, respectively.

Equation 1 and Equation 2 show that the anisotropic dispersion of the metamaterial depends on the thickness of the metal–dielectric layers or the filling ratio. The values of ε_{rr} and ε_{zz} can be positive or negative depending on the layer thickness, and the resulting metamaterial can exhibit three different topologies: (i) dielectric ($\varepsilon_{rr} > 0$, $\varepsilon_{zz} > 0$), (ii) hyperbolic ($\varepsilon_{rr} < 0$, $\varepsilon_{zz} > 0$), and (iii) elliptic ($\varepsilon_{rr} < 0$, $\varepsilon_{zz} < 0$).

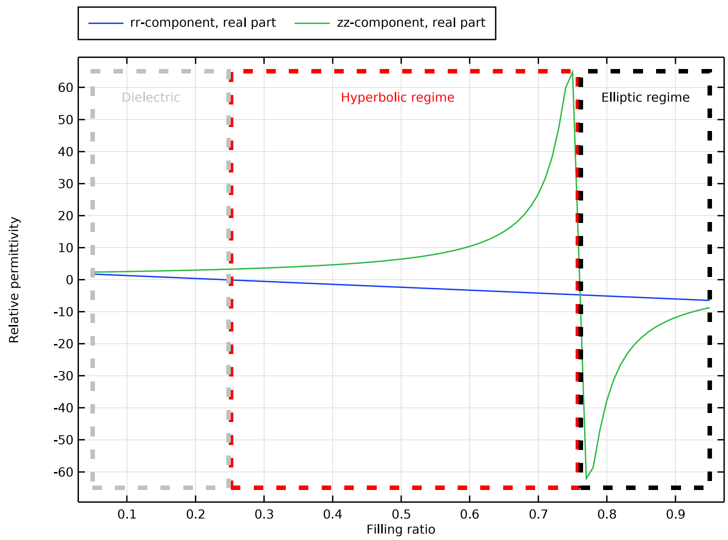


Figure 2: Real part of diagonal components of the effective relative permittivity of metamaterial as a function of filling ratio of metal. The metamaterial is composed of silver and silicon dioxide.

For further demonstration, assume a metamaterial composed of silver (metal) and silicon dioxide (dielectric). Figure 2 shows the real parts of ε_{rr} (green line) and ε_{zz} (blue line) versus the metal filling ratio, F_{m}, indicating the dielectric (gray box), hyperbolic (red box), and elliptic (magenta box) regimes. Here, ε_{zz} displays a resonance behavior as it depends on the electromagnetic coupling between the adjacent metal layers, whereas ε_{rr} shows a smooth variation. Within the hyperbolic regime, the metamaterial exhibits metallic (that is, inductive, $\varepsilon_{rr} < 0$) and dielectric (that is, capacitive, $\varepsilon_{zz} > 0$) response along the tangential and vertical directions, respectively. In the case of larger F_{m}, the value of ε_{zz} is dominated by the increased volume of metal and becomes negative, corresponding to an elliptic topology. When F_{m} is very small, the influence of the metal is negligible on the metamaterial properties and it behaves like an anisotropic dielectric media.

Another approach to calculate the anisotropic relative permittivity tensor ε of the metamaterial is to solve the constitutive relation. It can be written in terms of the electric displacement field $\mathbf{D}$ and the electric field $\mathbf{E}$ as

$$\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}\,. \tag{3}$$

Equation 3 permits to calculate the radial and vertical components of ε as

$$\varepsilon_{rr} = \frac{D_{rr}}{\varepsilon_0 E_{rr}}, \varepsilon_{zz} = \frac{D_{zz}}{\varepsilon_0 E_{zz}}. \quad (4)$$

For a more detailed discussion on the dispersion of hyperbolic metamaterials, see Ref. 1.

This model simulates a hyperbolic metamaterial in a 2D axisymmetric geometry using an Electromagnetic Waves, Frequency Domain interface. The metamaterial is constructed using silver and silicon dioxide thin layers with 40% metal filling ratio, with material properties taken from the built-in Optical Material Library. The Electric Point Dipole feature is used to excite the waves in the metamaterial and perfectly matched layers are employed to absorb the waves reaching the domain boundaries to minimize unexpected reflections. A Wavelength Domain study step is used to solve for the domain fields. Finally, an additional Wavelength Domain study step is performed to calculate the effective relative permittivity tensor components of the metamaterial versus wavelength. This calculation is performed following the effective medium theory, as demonstrated in Equation 1 and Equation 2, and by solving the constitutive relation as shown in Equation 4.

Results and Discussion

Figure 3 shows the electric field norm of the waves excited in the metamaterial using an electric point dipole located above it. The fields propagate within narrow channels thus resulting in a hyperbolic pattern which is different from the case in conventional materials. Fields in the surrounding air domain are not plotted here for better visualization of the hyperbolic waves.

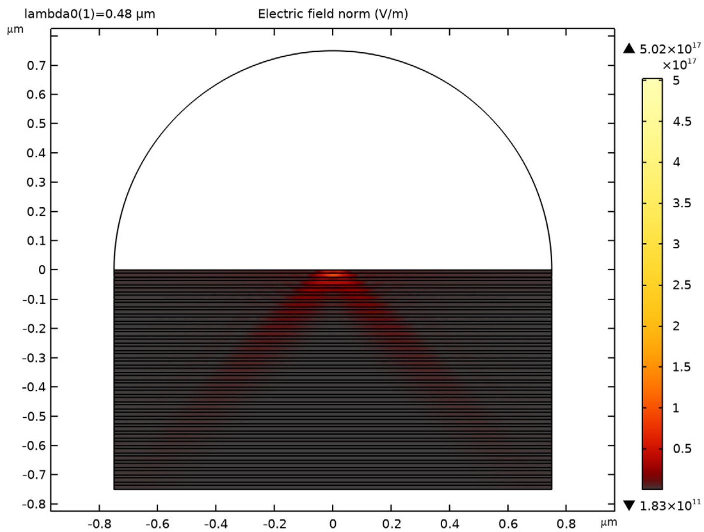


Figure 3: Electric field norm of hyperbolic wave propagating in a hyperbolic metamaterial made of silver and silicon dioxide layers with thickness 10 nm and 15 nm, respectively. Operation wavelength is 480 nm.

Figure 4 shows the instantaneous electric field norm of the generated waves within the metamaterial and the surrounding air. It confirms the excitation of surface plasmon polariton that propagates along the metal–air interface in the tangential direction apart from the hyperbolic waves.

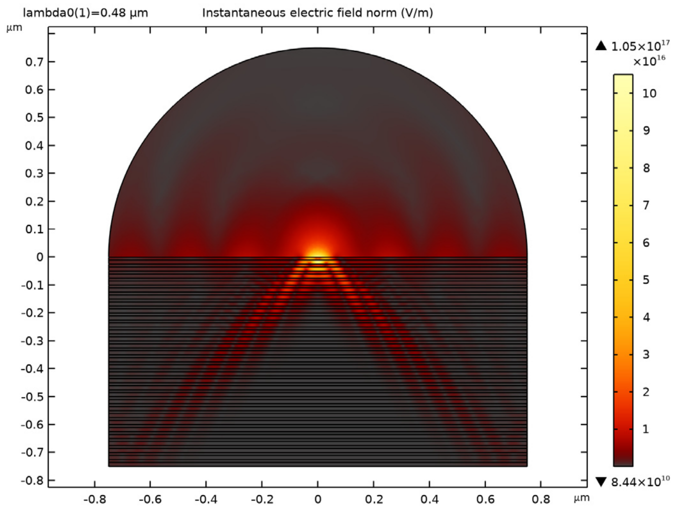


Figure 4: Instantaneous electric field norm of the excited wave in the complete simulation domain. Surface plasmon polariton is excited at the metal–air interface in addition to hyperbolic waves.

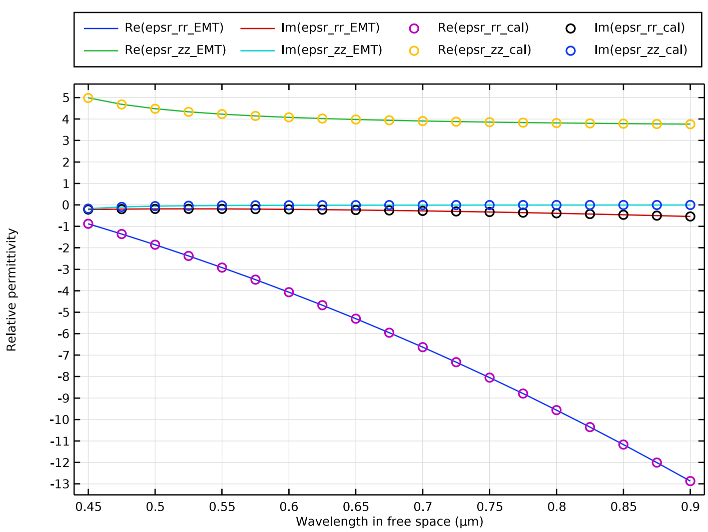


Figure 5: Real part of diagonal components of the metamaterial effective permittivity tensor calculated using effective medium theory (solid lines) and from simulation (markers).

Figure 5 shows the real part of the tangential (ε_{rr}) and vertical (ε_{zz}) components versus the operation wavelength. Results calculated using simulation (markers) and effective medium theory (solid lines) display excellent agreement.

Reference

1. Z. Guo, H. Jiang, and H. Chen, "Hyperbolic Metamaterials: From Dispersion Manipulation to Applications," *J. Appl. Phys.*, vol. 127, no. 7, p. 071101, 2016.

Application Library path: `Wave_Optics_Module/Gratings_and_Metamaterials/dipole_near_hyperbolic_metamaterial`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Optics** > **Wave Optics** > **Electromagnetic Waves, Frequency Domain (ewfd)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Wavelength Domain**.

6 Click **Done**.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **µm**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
W	2[um]	2E-6 m	Geometry width
lda0	480[nm]	4.8E-7 m	Wavelength
td	15[nm]	1.5E-8 m	Thickness, dielectric
tm	10[nm]	1E-8 m	Thickness, metal
P	td+tm	2.5E-8 m	Periodicity
tPML	250[nm]	2.5E-7 m	Thickness, PML
d	25[nm]	2.5E-8 m	Dipole-metamaterial distance
nlayer	30	30	Number of layers
hMTM	nlayer*P	7.5E-7 m	Thickness, metamaterial

GEOMETRY 1

The geometry consists of a metamaterial composed of periodically organized metal-dielectric thin layers, and air on top of it. An electric point dipole is located in air above the metamaterial.

Top Air Domain

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, type `Top Air Domain` in the **Label** text field.

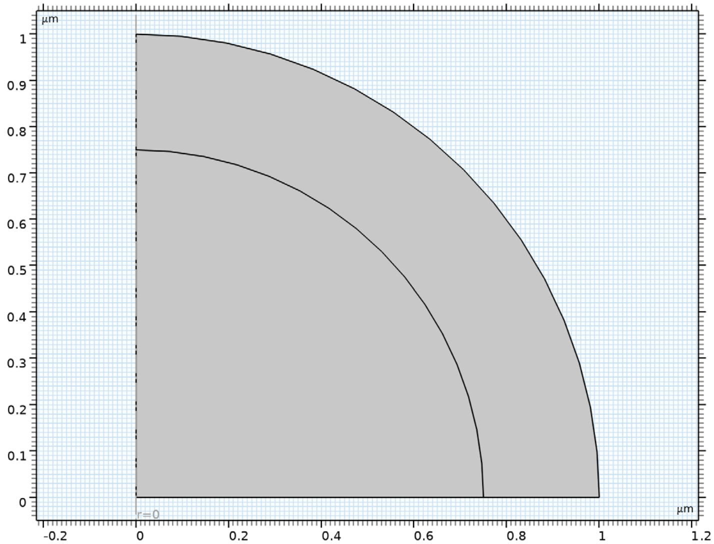
3 Locate the **Size and Shape** section. In the **Radius** text field, type `W/2`.

4 In the **Sector angle** text field, type `90`.

5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (µm)
Layer 1	tPML

6 Click **Build Selected**.



Thin Metal Layer

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, type `Thin Metal Layer` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Width** text field, type `W/2`.

4 In the **Height** text field, type `tm`.

5 Locate the **Position** section. In the **z** text field, type `-tm`.

6 Click to expand the **Layers** section. In the table, enter the following settings:

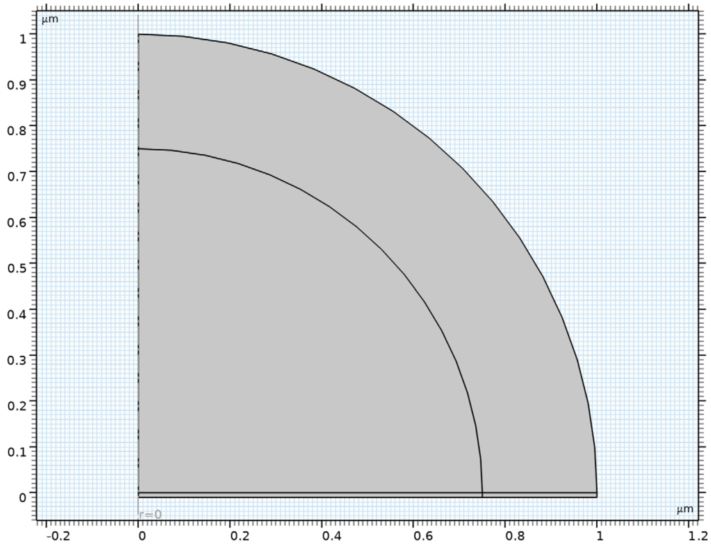
Layer name	Thickness (µm)
Layer 1	tPML

7 Clear the **Layers on bottom** checkbox.

8 Select the **Layers to the right** checkbox.

9 Click **Build Selected**.

10 Click the **Zoom Extents** button in the **Graphics** toolbar.



Thin Dielectric Layer

1 Right-click **Thin Metal Layer** and choose **Duplicate**.

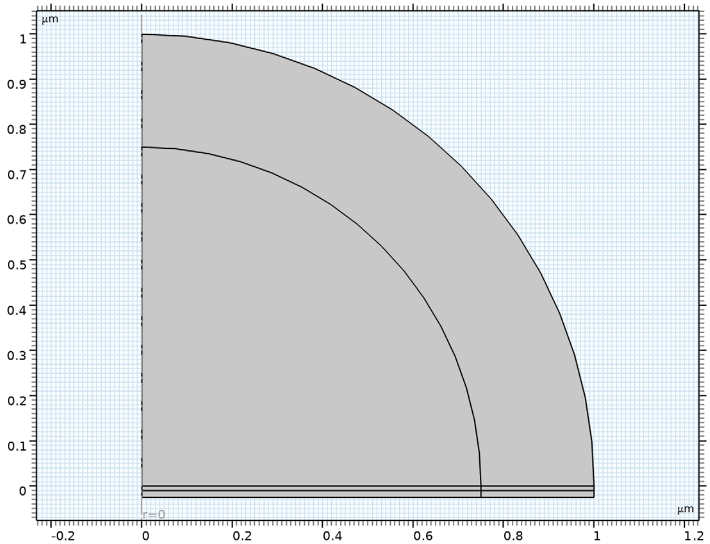
2 In the **Settings** window for **Rectangle**, type `Thin Dielectric Layer` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Height** text field, type `td`.

4 Locate the **Position** section. In the **z** text field, type `-tm-td`.

5 Click **Build Selected**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.



Array 1 - Metal Layers

1 In the **Geometry** toolbar, click **Transforms** and choose **Array**.

2 In the **Settings** window for **Array**, type `Array 1 - Metal Layers` in the **Label** text field.

3 Select the object **r1** only.

4 Locate the **Size** section. From the **Array type** list, choose **Linear**.

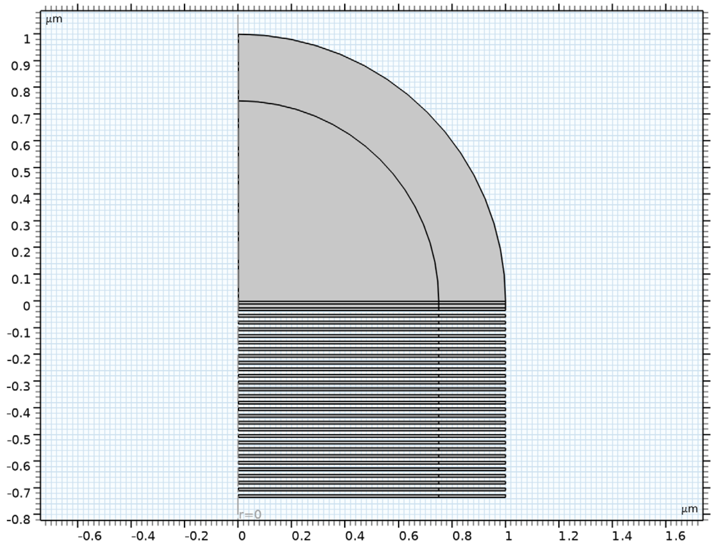
5 In the **Size** text field, type `nlayer`.

6 Locate the **Displacement** section. In the **z** text field, type `-P`.

7 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.

8 Click **Build Selected**.

9 Click the **Zoom Extents** button in the **Graphics** toolbar.



Array 2 - Dielectric Layers

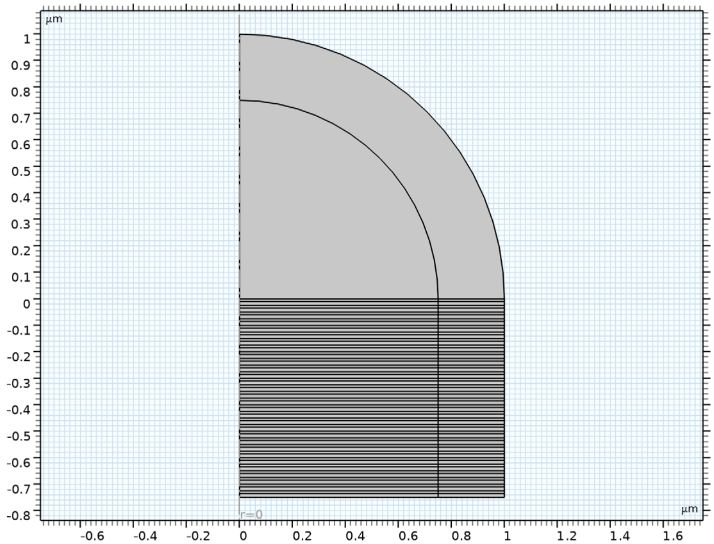
1 Right-click **Array 1 - Metal Layers** and choose **Duplicate**.

2 In the **Settings** window for **Array**, type `Array 2 - Dielectric Layers` in the **Label** text field.

3 Select the object **r2** only.

4 Click **Build Selected**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



Bottom Domain

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, type `Bottom Domain` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Width** text field, type `W/2`.

4 In the **Height** text field, type `tPML`.

5 Locate the **Position** section. In the **z** text field, type `-hMTM-tPML`.

6 Locate the **Layers** section. In the table, enter the following settings:

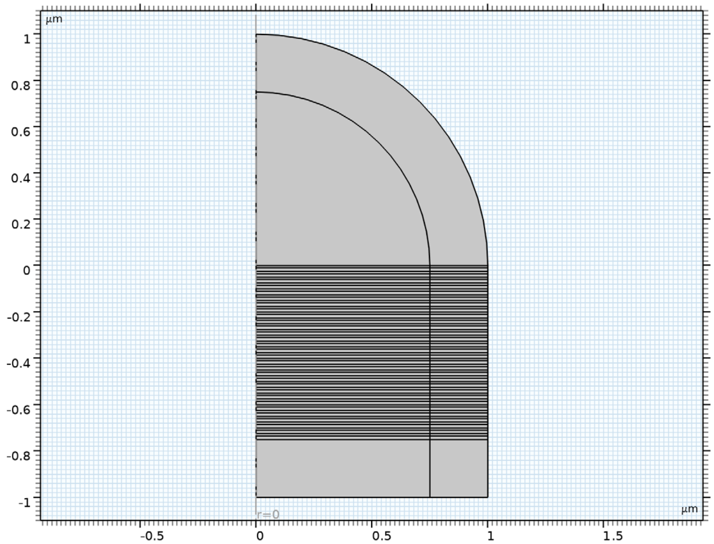
Layer name	Thickness (µm)
Layer 1	tPML

7 Clear the **Layers on bottom** checkbox.

8 Select the **Layers to the right** checkbox.

9 Click **Build Selected**.

10 Click the **Zoom Extents** button in the **Graphics** toolbar.



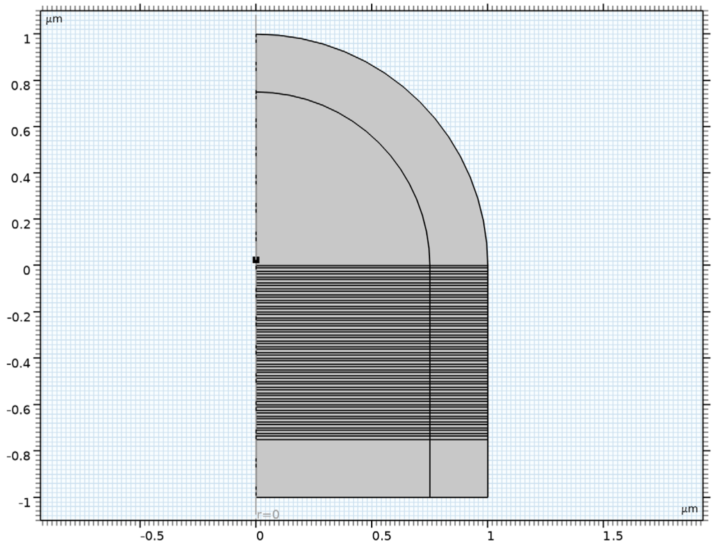
Electric Point Dipole

1 In the **Geometry** toolbar, click **Point**.

2 In the **Settings** window for **Point**, type `Electric Point Dipole` in the **Label** text field.

3 Locate the **Point** section. In the **z** text field, type `d`.

4 Click **Build All Objects**.



MATERIALS

Pick silver as the metal and silicon dioxide as the dielectric from the Optical material library.

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Right-click and choose **Add to Component 1 (comp1)**.

5 In the tree, select **Optical** > **Inorganic Materials** > **Ag - Silver** > **Experimental data: thin film** > **Ag (Silver) (Ciesielski et al. 2017: Ag/SiO2; n,k 0.191-20.9 um)**.

6 Right-click and choose **Add to Component 1 (comp1)**.

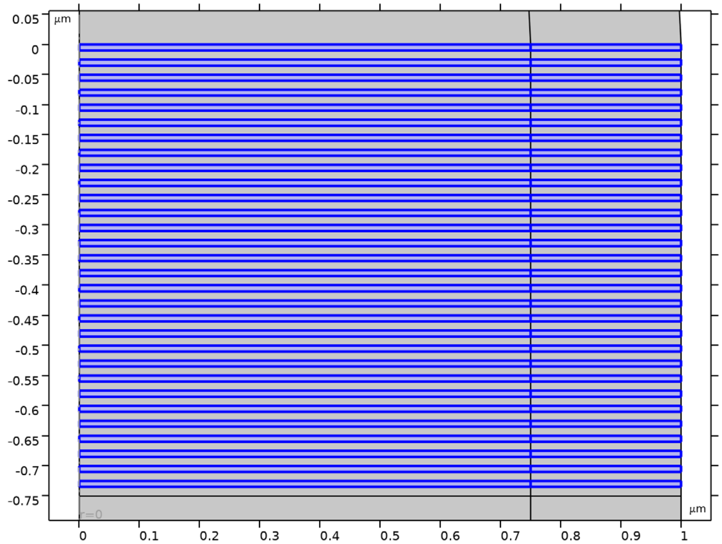
MATERIALS

Ag (Silver) (Ciesielski et al. 2017: Ag/SiO2; n,k 0.191-20.9 um) (mat2)

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Materials** click **Ag (Silver) (Ciesielski et al. 2017: Ag/SiO2; n,k 0.191-20.9 um) (mat2)**.

2 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Array 1 - Metal Layers**.



ADD MATERIAL

1 Go to the **Add Material** window.

2 In the tree, select **Optical** > **Inorganic Materials** > **O - Oxygen and oxides** > **Thin film** > **SiO2 (Silicon dioxide, Silica, Quartz) (Gao et al. 2013: Thin film; n,k 0.252-1.25 um)**.

3 Right-click and choose **Add to Component 1 (comp1)**.

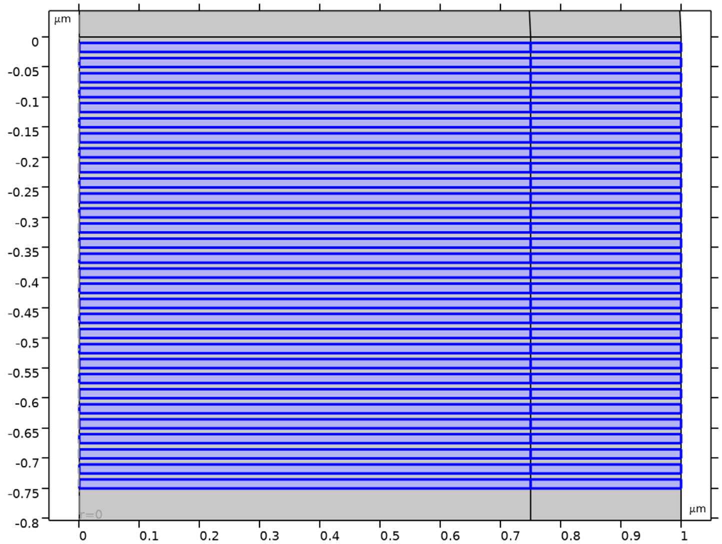
4 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

SiO2 (Silicon dioxide, Silica, Quartz) (Gao et al. 2013: Thin film; n,k 0.252-1.25 um) (mat3)

1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

2 From the **Selection** list, choose **Array 2 - Dielectric Layers**.



DEFINITIONS

Material Relative Permittivity

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Material Relative Permittivity` in the **Label** text field.

3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
epsr_air	(mat1.rfi.n11-i* mat1.rfi.ki11)^2		Relative permittivity, air
epsr_m	(mat2.rfi.nr(ewfd.lambda0)-i* mat2.rfi.ni(ewfd.lambda0))^2		Relative permittivity, metal
epsr_d	(mat3.rfi.nr(ewfd.lambda0)-i* mat3.rfi.ni(ewfd.lambda0))^2		Relative permittivity, dielectric
epsr_ave	(epsr_air+epsr_d)/2		Average relative permittivity

The corresponding materials are dispersive and their relative permittivities are calculated as a function of wavelength.

Metamaterial Domains

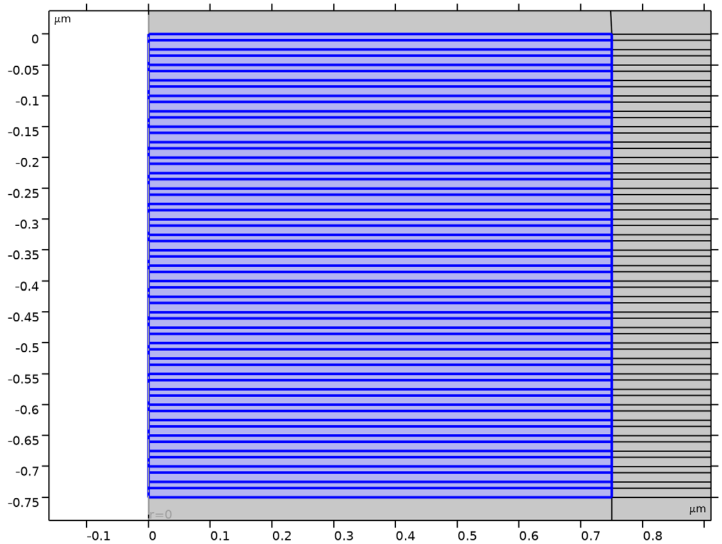
1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Metamaterial Domains` in the **Label** text field.

3 Locate the **Input Entities** section. Click **Paste Selection**.

4 In the **Paste Selection** dialog, type `2-61` in the **Selection** text field.

5 Click **OK**.

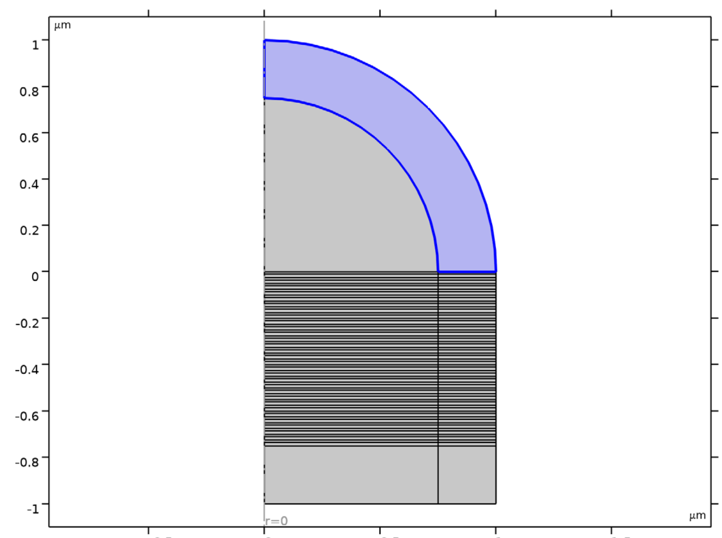


This selection will be used to define a user-controlled mesh in the metamaterial domain.

Perfectly Matched Layer 1 (pml1)

1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 Select Domain 63 only.



ARTIFICIAL DOMAINS

Perfectly Matched Layer 2 (pml2)

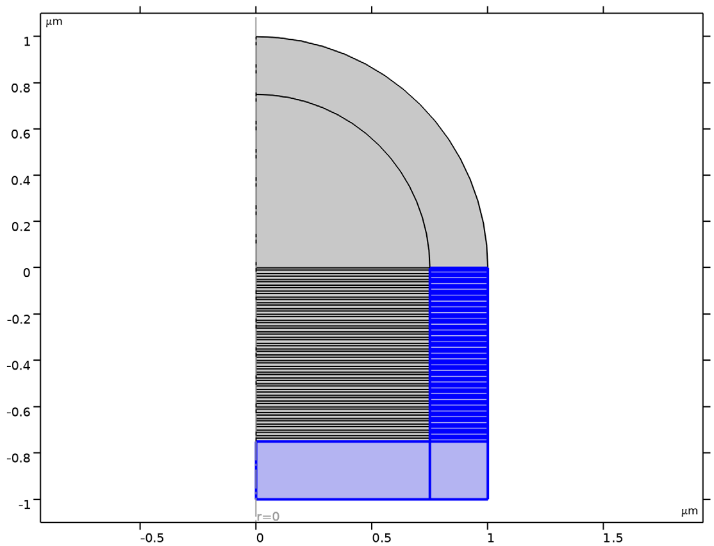
1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 In the **Settings** window for **Perfectly Matched Layer**, locate the **Domain Selection** section.

3 Click **Paste Selection**.

4 In the **Paste Selection** dialog, type `1,64-124` in the **Selection** text field.

5 Click **OK**.



6 In the **Settings** window for **Perfectly Matched Layer**, locate the **Geometry** section.

7 From the **Type** list, choose **Cylindrical**.

8 Locate the **Scaling** section. From the **Typical wavelength from** list, choose **User defined**.

9 In the **Typical wavelength** text field, type `lda0`.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (ewfd)**.

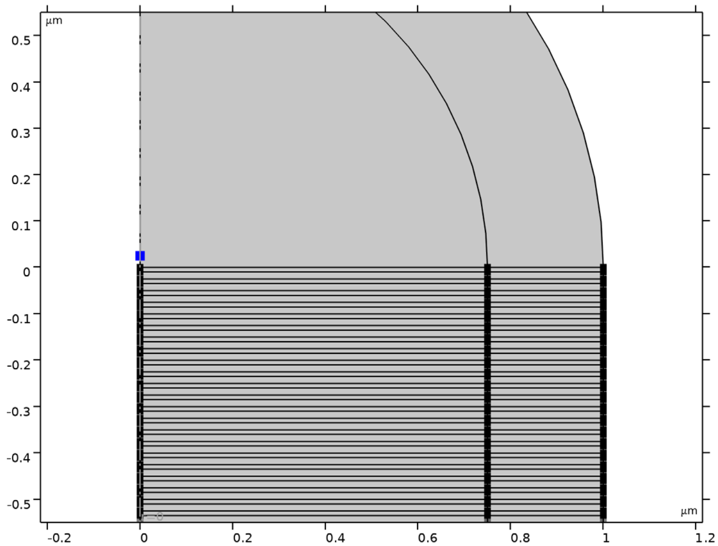
2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **In-plane vector**, as only the in-plane polarization will be included in the simulation.

Electric Point Dipole 1

1 In the **Physics** toolbar, click **Points** and choose **Electric Point Dipole**.

2 Select Point 63 only.



3 In the **Settings** window for **Electric Point Dipole**, locate the **Dipole Parameters** section.

4 In the **p** text field, type `1`.

The electric point dipole is vertically polarized.

MESH 1

Waves propagating inside the metamaterials possess very large wavenumber. Therefore, User-controlled mesh with reduced maximum element size is used to properly resolve the waves.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.

3 From the list, choose **User-controlled mesh**.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size Parameters** section.

3 In the **Maximum element size** text field, type `0.06`.

Size 1

1 In the **Model Builder** window, click **Size 1**.
2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.
3 From the **Selection** list, choose **Metamaterial Domains**.
4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type `0.01`.

   Adequate choice of Maximum element size is important to properly resolve the electromagnetic waves inside the metamaterial. A value of 10 nm is sufficient for this example model.
5 Click **Build All**.

STUDY 1

Step 1: Wavelength Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Wavelength Domain**.
2 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.
3 In the **Wavelengths** text field, type `lda0`.
4 In the **Study** toolbar, click **Compute**.

RESULTS

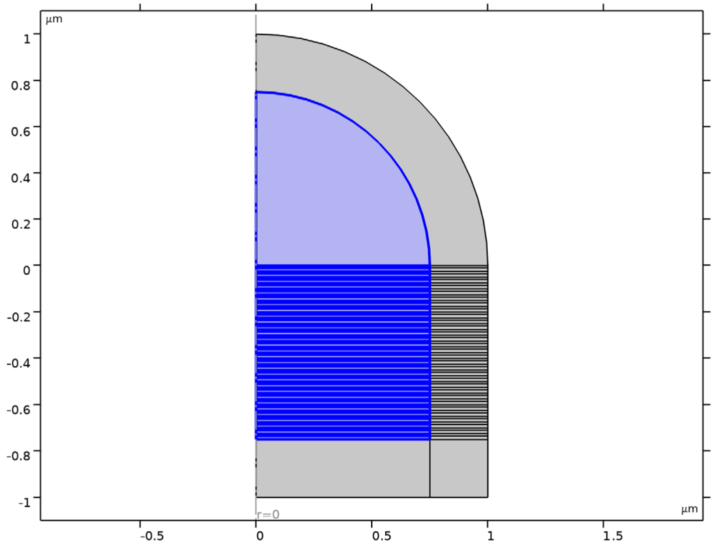
Study 1/Solution 1 (sol1)

In the **Model Builder** window, expand the **Results > Datasets** node, then click **Study 1/ Solution 1 (sol1)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.
2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 Click **Paste Selection**.
5 In the **Paste Selection** dialog, type `2-62` in the **Selection** text field, to exclude the PML domains.

6 Click **OK**.



Revolution 2D 1

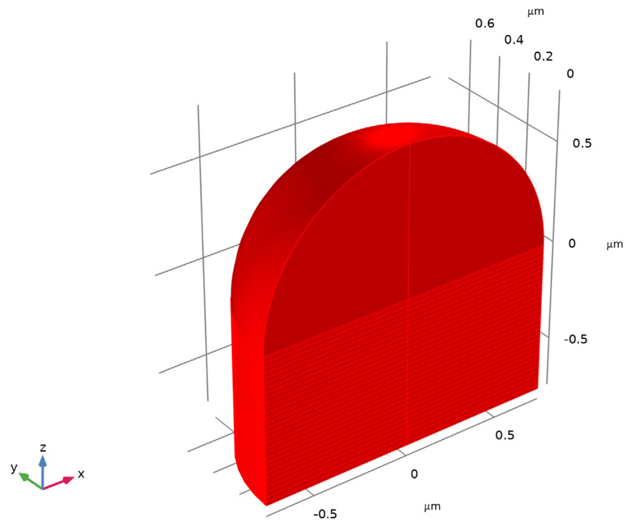
1 In the **Model Builder** window, under **Results** > **Datasets** click **Revolution 2D 1**.

2 In the **Settings** window for **Revolution 2D**, click to expand the **Revolution Layers** section.

3 In the **Start angle** text field, type `0`.

4 In the **Revolution angle** text field, type `180`.

5 Click **Plot**.



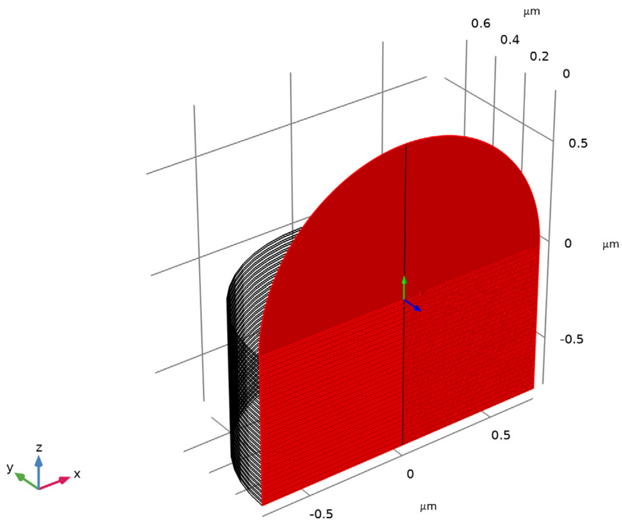
Cut Plane 1

1 In the **Results** toolbar, click **Cut Plane**.

2 In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.

3 From the **Plane** list, choose **XZ-planes**.

4 Click **Plot**.



The fields will be plotted over this plane.

Electric Field (ewfd)

1 In the **Model Builder** window, under **Results** click **Electric Field (ewfd)**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Plane 1**.

Filter 1

1 In the **Model Builder** window, expand the **Electric Field (ewfd)** node.

2 Right-click **Surface 1** and choose **Filter**.

3 In the **Settings** window for **Filter**, locate the **Element Selection** section.

4 In the **Logical expression for inclusion** text field, type `z<=0`. This enables to visualize the fields inside the metamaterial only.

Surface 1

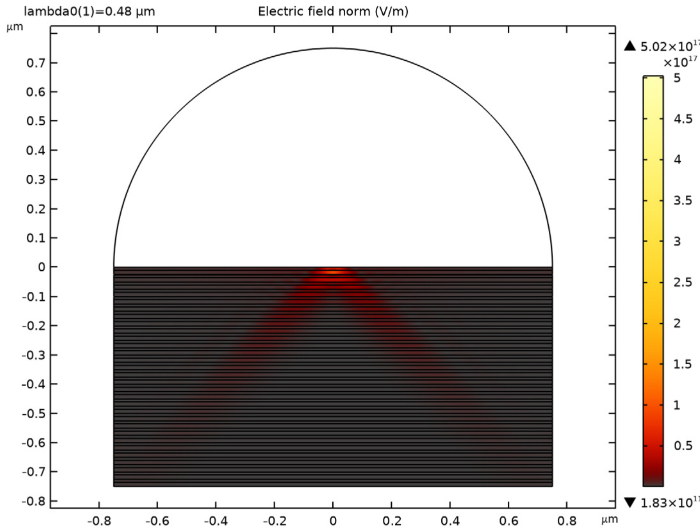
1 In the **Model Builder** window, click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Coloring and Style** section.

3 From the **Color table** list, choose **GrayBody**.

4 From the **Color table transformation** list, choose **Nonlinear**.

5 In the **Color calibration parameter** text field, type `-1`, for better visualization of the fields.



The fields propagating inside the metamaterial exhibit a hyperbolic shape. This is because of the inductive-capacitive (LC) resonance arising from the metallic and dielectric response along the orthogonal optical axes.

Instantaneous Electric Field norm (ewfd)

1 In the **Model Builder** window, right-click **Electric Field (ewfd)** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Instantaneous Electric Field norm (ewfd)` in the **Label** text field.

Surface 1

1 In the **Model Builder** window, expand the **Instantaneous Electric Field norm (ewfd)** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

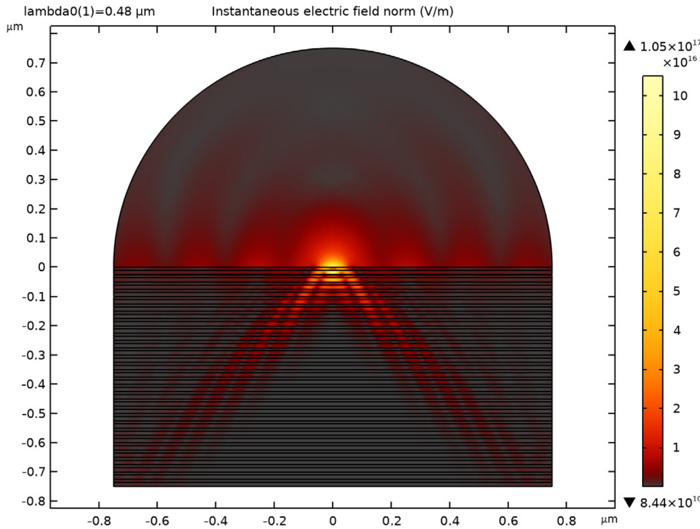
3 In the **Expression** text field, type `ewfd.normEi`.

Filter 1

1 In the **Model Builder** window, expand the **Surface 1** node.

2 Right-click **Filter 1** and choose **Delete**, to visualize the fields in both air and metamaterial.

3 In the **Instantaneous Electric Field norm (ewfd)** toolbar, click **Plot**.



Instantaneous Electric Field norm (ewfd)

Now, the radial and vertical components of the electromagnetic waves will be plotted.

Electric Field Components (ewfd)

1 In the **Model Builder** window, right-click **Instantaneous Electric Field norm (ewfd)** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Field Components (ewfd)` in the **Label** text field.

3 Click to expand the **Plot Array** section. From the **Array type** list, choose **Linear**.

Surface 1

1 In the **Model Builder** window, expand the **Electric Field Components (ewfd)** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewfd.Er`, to plot the radial component.

4 Locate the **Coloring and Style** section. From the **Color table** list, choose **ThermalWaveDark**.

5 From the **Scale** list, choose **Linear symmetric**.

6 Click to expand the **Range** section. Select the **Manual color range** checkbox.

7 In the **Minimum** text field, type `-1.1E16`.

8 In the **Maximum** text field, type `1.1E16`.

It helps to properly visualize the field profile.

Surface 2

1 In the **Model Builder** window, right-click **Surface 1** and choose **Duplicate**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewfd.Ez`, to plot the vertical component.

4 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.

Electric Field Components (ewfd)

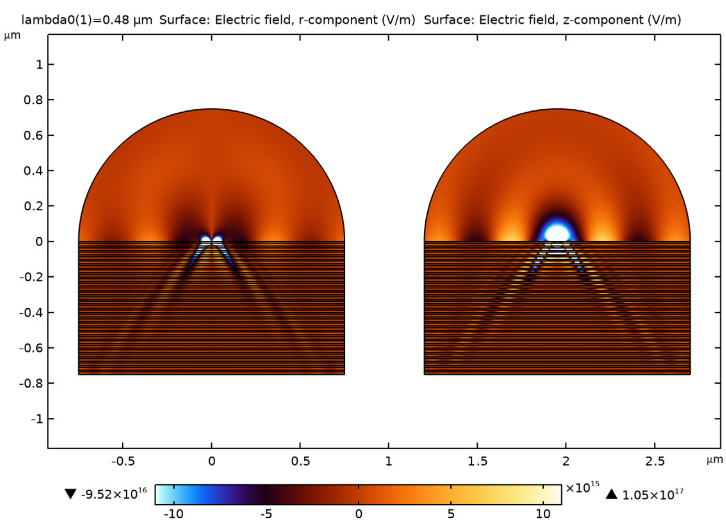
1 In the **Model Builder** window, click **Electric Field Components (ewfd)**.

2 In the **Settings** window for **2D Plot Group**, locate the **Color Legend** section.

3 From the **Position** list, choose **Bottom**.

4 In the **Electric Field Components (ewfd)** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



DEFINITIONS

This part of the step-by-step instructions demonstrates how to calculate the effective relative permittivity tensor components of the metamaterial from the simulated data, and compares with the ones computed using the effective medium theory.

Relative Permittivity, Effective Medium Theory

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Relative Permittivity, Effective Medium Theory` in the **Label** text field.

3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
f	tm/(tm+td)		Filling ratio, metal
epsr_rr_EMT	f*epsr_m+(1-f)*epsr_d		Effective relative permittivity, rr-component
epsr_zz_EMT	((f/epsr_m)+((1-f)/epsr_d))^-1		Effective relative permittivity, zz-component

Effective medium theory is valid when the thickness of metal/dielectric layers, and periodicity of the unit cell are within the subwavelength regime.

Metal Thickness Variable

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Metal Thickness Variable` in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Array 1 - Metal Layers**.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
t	tm	m	Metal thickness variable

This variable will be used in the expression to calculate the effective relative permittivity.

Dielectric Thickness Variable

1 Right-click **Metal Thickness Variable** and choose **Duplicate**.

2 In the **Settings** window for **Variables**, type `Dielectric Thickness Variable` in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Selection** list, choose **Array 2 - Dielectric Layers**.

4 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
t	td	m	Dielectric thickness variable

This variable will also be used in the expression to calculate the effective relative permittivity.

Metal-dielectric Interior Boundaries

1 In the **Definitions** toolbar, click **Adjacent**.

2 In the **Settings** window for **Adjacent**, type `Metal-dielectric Interior Boundaries` in the **Label** text field.

3 Locate the **Input Entities** section. Under **Input selections**, click **Add**.

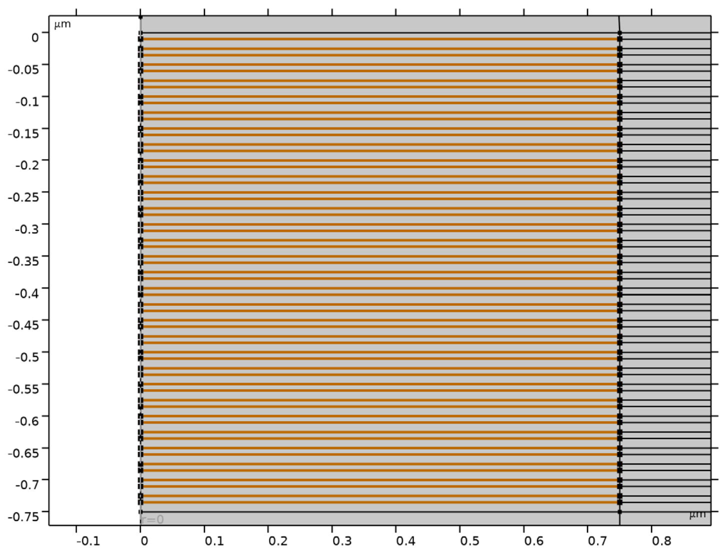
4 In the **Add** dialog, select **Metamaterial Domains** in the **Input selections** list.

5 Click **OK**.

6 In the **Settings** window for **Adjacent**, locate the **Output Entities** section.

7 From the **Exterior boundaries** list, choose **None**.

8 Select the **Interior boundaries** checkbox.



Average 1 (aveop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Average**.

2 In the **Settings** window for **Average**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Metal-dielectric Interior Boundaries**.

Calculated Effective Relative Permittivity

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, type `Calculated Effective Relative Permittivity` in the **Label** text field.

3 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
`meanDr`	`(up(ewfd.Dr)*up(t)+down(ewfd.Dr)*down(t))/(tm+td)`	C/m^2	`Average electric displacement field, r-component`
`meanEr`	`(up(ewfd.Er)*up(t)+down(ewfd.Er)*down(t))/(tm+td)`	V/m	`Average electric field, r-component`
`meanDz`	`(up(ewfd.Dz)*up(t)+down(ewfd.Dz)*down(t))/(tm+td)`	C/m^2	`Average electric displacement field, z-component`
`meanEz`	`(up(ewfd.Ez)*up(t)+down(ewfd.Ez)*down(t))/(tm+td)`	V/m	`Average electric field, z-component`
`epsr_rr_cal`	`aveop1(meanDr/(epsilon0_const*meanEr))`		`Effective relative permittivity, rr-component (calculated)`
`epsr_zz_cal`	`aveop1(meanDz/(epsilon0_const*meanEz))`		`Effective relative permittivity, zz-component (calculated)`

The expressions used to calculate the average electric displacement field and electric field components use up and down operators. These operators are performed on the metal-dielectric interfaces within the metamaterial, and helps to evaluate fields with discontinuity on each side of the interfaces. Then, the average operator is used to perform integration of the constitutive relation `D=epsilon0_const*E` on the boundaries to calculate the metamaterial relative permittivity.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Wavelength Domain**.

4 Right-click and choose **Add Study**.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Wavelength Domain

1 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.

2 In the **Wavelengths** text field, type `range(450[nm],25[nm],900[nm])`.

3 In the **Model Builder** window, click **Study 2**.

4 In the **Settings** window for **Study**, locate the **Study Settings** section.

5 Clear the **Generate default plots** checkbox.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Effective Relative Permittivity

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Effective Relative Permittivity` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

Global 1

1 Right-click **Effective Relative Permittivity** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`real(epsr_rr_EMT)`	1	
`real(epsr_zz_EMT)`	1	
`imag(epsr_rr_EMT)`	1	
`imag(epsr_zz_EMT)`	1	

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `ewfd.lambda0`.

6 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

7 In the table, enter the following settings:

Legends
`Re(epsr_rr_EMT)`
`Re(epsr_zz_EMT)`
`Im(epsr_rr_EMT)`
`Im(epsr_zz_EMT)`

Global 2

1 Right-click **Global 1** and choose **Duplicate**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	**Unit**	**Description**
`real(epsr_rr_cal)`	1	
`real(epsr_zz_cal)`	1	
`imag(epsr_rr_cal)`	1	
`imag(epsr_zz_cal)`	1	

4 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Re(epsr_rr_cal)`
`Re(epsr_zz_cal)`
`Im(epsr_rr_cal)`
`Im(epsr_zz_cal)`

Effective Relative Permittivity

1 In the **Model Builder** window, click **Effective Relative Permittivity**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section.

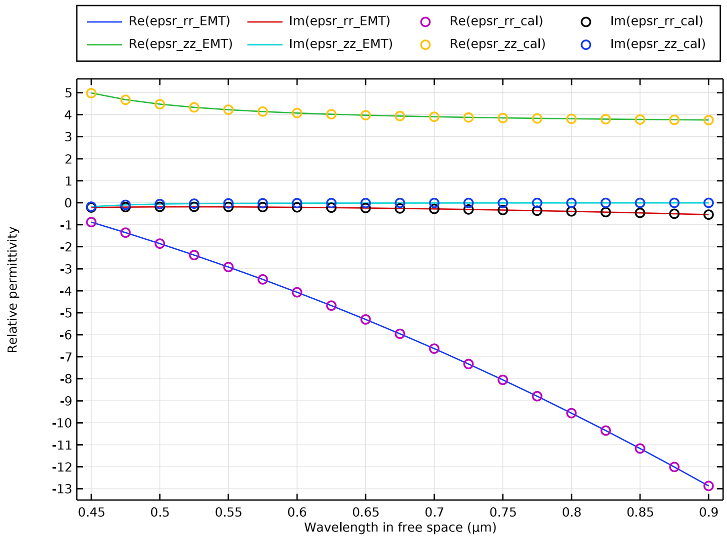
5 Select the **y-axis label** checkbox. In the associated text field, type `Relative permittivity`.

6 Locate the **Legend** section. From the **Layout** list, choose **Outside graph axis area**.

7 From the **Position** list, choose **Top**.

8 In the **Number of rows** text field, type 2.

9 In the **Effective Relative Permittivity** toolbar, click **Plot**.



Metamaterial relative permittivity tensor components calculated from the simulation and effective medium theory show excellent agreement.