Model created in COMSOL Multiphysics 6.4

Gaussian Beam Incident at the Brewster Angle

Introduction

For a plane wave incident at an interface between two different media, there exists an angle of incidence for which there is no reflectance if the incident wave is polarized in the plane of incidence. The angle, for which the reflectance is zero, is called *the Brewster angle*.

Figure 1 shows an incident wave being reflected and refracted at the interface between the two media. The polarization component polarized in the plane of incident (the plane spanned by the wave vector of the incident wave and the normal to the interface) is not reflected. This polarization component is called the p-polarization.

The polarization component orthogonal to the plane of incidence (the s-polarization) is both reflected and refracted.

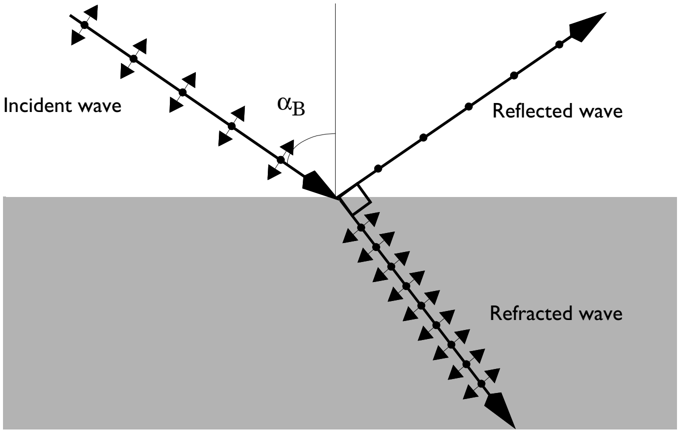


Figure 1: The figure shows the incident, reflected, and refracted waves. At the Brewster angle α_B *the wave polarized in the plane of incidence is only refracted and not reflected.*

At the Brewster angle, the incident p-polarized wave creates a polarization in the second medium (where the refracted wave is propagating) with the components in the propagation direction of the reflected wave. Because this is a longitudinal polarization for the reflected wave, and not a transverse polarization, it is clear that this polarization cannot excite a reflected wave.

Referring to the angles defined in Figure 1, write Snell's law as

$$n_1 \sin\alpha_B = n_2 \sin\left(\Pi - \alpha_B - \frac{\Pi}{2}\right) = n_2 \cos\alpha_B \tag{1}$$

where n_1 and n_2 are the refractive indices above and below the interface, respectively.

Equation 1 results in the Brewster angle definition

$$\tan\alpha_B = \frac{n_2}{n_1}$$

From the Fresnel Equations Application Libraries example, the reflectance for the s-polarization at the Brewster angle is given by

$$R_s = \left|\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}\right|^2 = \left|\frac{n_1 \cos\alpha_B - n_2 \sin\alpha_B}{n_1 \cos\alpha_B + n_2 \sin\alpha_B}\right|^2 = \left|\frac{n_1^2 - n_2^2}{n_1^2 + n_2^2}\right|^2 \tag{2}$$

This model does not use plane waves, but Gaussian beams (see for instance the Second Harmonic Generation of a Gaussian Beam Application Libraries model for a discussion about Gaussian beams). However, because the spot size for the beam is much larger than the wavelength, the plane wave relations above are good approximations also for the Gaussian beams.

Model Definition

This model demonstrates how the Matched Boundary Condition feature can be used to absorb waves that propagate toward a boundary in a direction different from the boundary's normal direction. Here, a Scattering Boundary Condition feature is not an option, as that feature only absorbs waves propagating at or close to the normal direction to the boundary normal. A second alternative would be to use a Perfectly Matched Layer (PML) domain. However, in that case, extra degrees of freedom would have to be included for the PML domain. Thus, the Matched Boundary Condition feature is the best feature to use for absorbing beams propagating in directions that are not in the normal direction to the boundary.

Results and Discussion

First, the results are computed for s-polarization, where the polarization is orthogonal to the plane of incidence (out-of-plane polarization). As shown in Figure 2, there are both a refracted and a reflected beam. The incident beam and the reflected beam form an inference pattern. Thanks to the fine mesh used in the model, the interference pattern is resolved. Equation 2 is also used to verify that the reflectance is correct. It should be close to 14.8%.

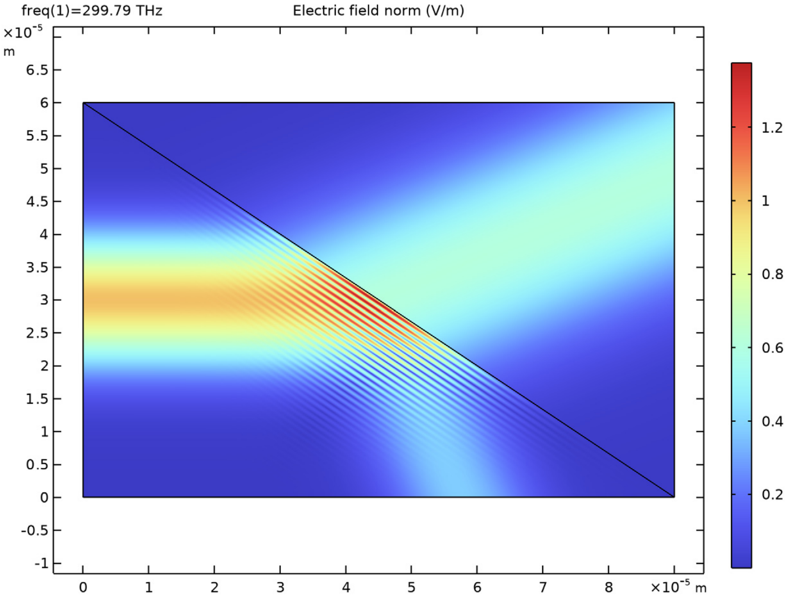


Figure 2: The incident, transmitted (refracted) and reflected Gaussian beams for s-polarization (out-of-plane polarization).

Figure 3 shows the results for p-polarization (in-plane polarization). As expected, when the beam is incident at the Brewster angle, there is no reflected beam, but only a refracted (transmitted) beam.

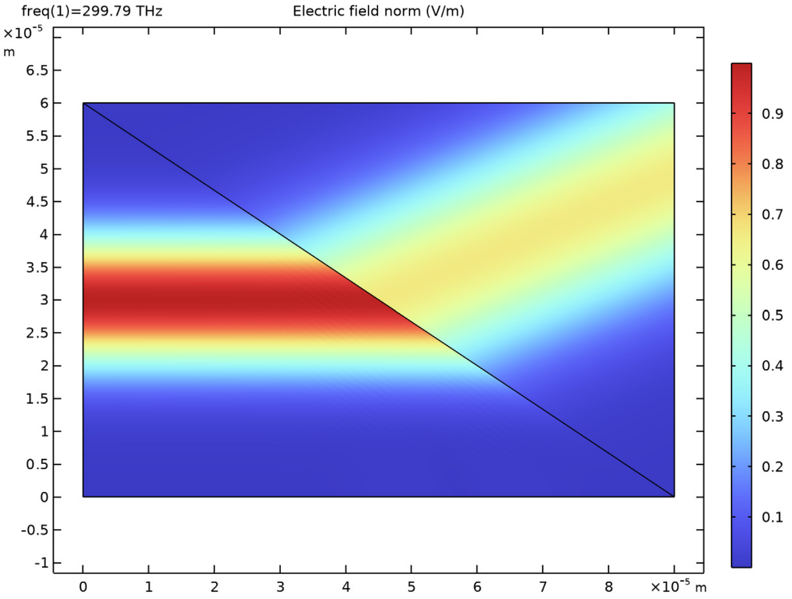


Figure 3: The incident and transmitted Gaussian beams for p-polarization (in-plane polarization). For this polarization, with Brewster angle incidence, the reflected beam is gone.

Application Library path: `Wave_Optics_Module/Beam_Propagation/brewster_interface`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2. In the **Select Physics** tree, select **Optics** > **Wave Optics** > **Electromagnetic Waves, Beam Envelopes (ewbe)**.
3. Click **Add**.
4. Click **Study**.
5. In the **Select Study** tree, select **General Studies** > **Frequency Domain**.
6. Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

The parameters for the model will be read from a file.

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, locate the **Parameters** section.
3. Click **Load from File**.
4. Browse to the model's Application Libraries folder and double-click the file `brewster_interface.txt`.

GEOMETRY 1

Define the geometry as a rectangle with a diagonal boundary.

Rectangle 1 (r1)

1. In the **Geometry** toolbar, click **Rectangle**.
2. In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
3. In the **Width** text field, type `b`.
4. In the **Height** text field, type `a`.

Line Segment 1 (ls1)

1. In the **Geometry** toolbar, click **More Primitives** and choose **Line Segment**.
2. On the object **r1**, select Point 4 only.
3. In the **Settings** window for **Line Segment**, locate the **Endpoint** section.
4. Click to select the **Activate Selection** toggle button for **End vertex**.
5. On the object **r1**, select Point 2 only.
6. Click **Build All Objects**.

7 Click the **Zoom Extents** button in the **Graphics** toolbar.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

The leftmost part consists of air and the rightmost part will be glass.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Click the **Add to Component** button in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

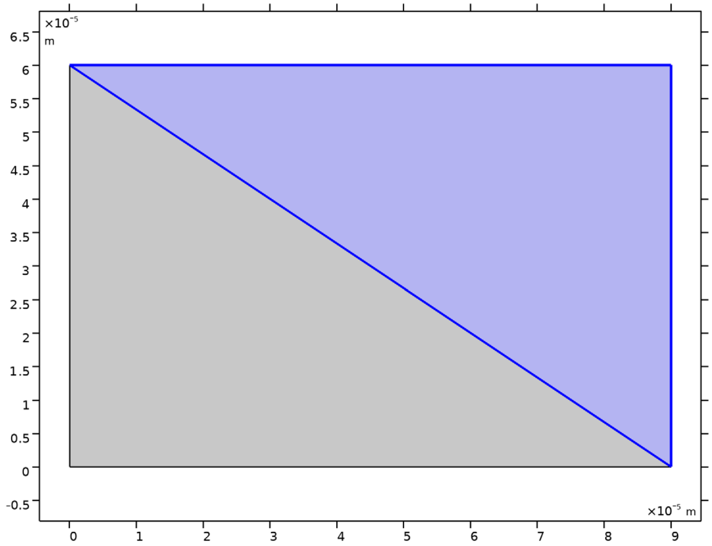
Glass

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Glass` in the **Label** text field.

3 Select Domain 2 only.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.



Define the refractive index for glass, using the parameter `n2`.

5 Locate the **Material Contents** section. In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	n2	1	Refractive index

DEFINITIONS

Set up expressions for the wave vector components for the two waves, with different expressions in the two domains.

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 1 only.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
k1x	k1x_air	1/m	Wave vector, first wave, x-component
k1y	k1y_air	1/m	Wave vector, first wave, y-component
k2x	k2x_air	1/m	Wave vector, second wave, x-component
k2y	k2y_air	1/m	Wave vector, second wave, y-component

Variables 2

1 In the **Definitions** toolbar, click a= **Local Variables**.

2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 2 only.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
k1x	k1x_glass	1/m	Wave vector, first wave, x-component
k1y	k1y_glass	1/m	Wave vector, first wave, y-component
k2x	k2x_glass	1/m	Wave vector, second wave, x-component
k2y	k2y_glass	1/m	Wave vector, second wave, y-component

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.

2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **Out-of-plane vector**.

4 Locate the **Wave Vectors** section. Specify the $\mathbf{k}_1$ vector as

k1x	x
k1y	y

5 Specify the $\mathbf{k}_2$ vector as

k2x	x
k2y	y

Matched Boundary Condition 1

1. In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.
2. Select Boundary 1 only.
3. In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.
4. From the **Incident field** list, choose **Gaussian beam**.
5. In the w_0 text field, type `w0`.
6. Specify the $\mathbf{E}_{g0}$ vector as

`0`	x
`0`	y
`1[V/m]`	z

On this boundary, only an incident field is expected, but there should not be any scattered field. Thus, provide that information, with the following setting, to avoid any potential spurious solutions.

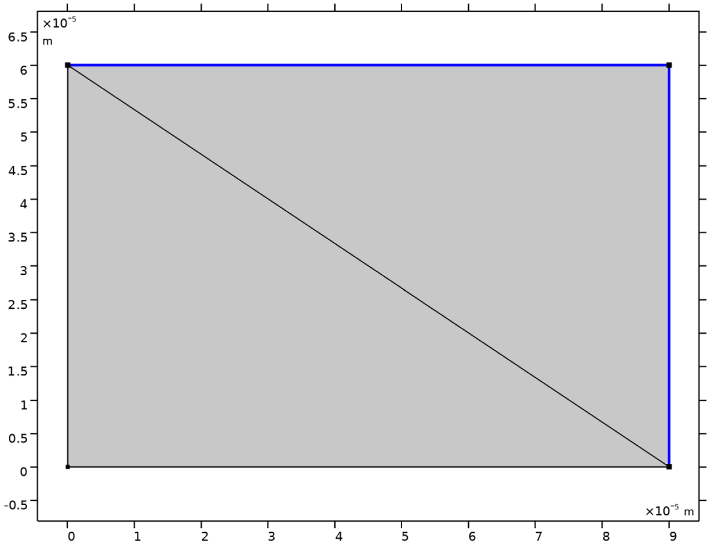
7. Find the **Scattered field** subsection. Select the **No scattered field** checkbox.

On the rightmost boundary, a transmitted Gaussian beam, propagating at an angle to the boundary normal, is expected. Thus, add a Matched Boundary Condition feature that will absorb this transmitted Gaussian beam.

Matched Boundary Condition 2

1. In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundaries 4 and 5 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

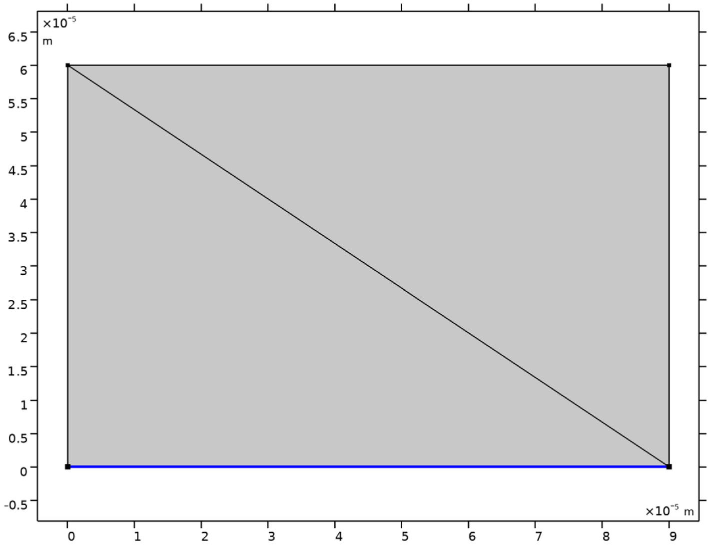
4 From the **Input wave** list, choose **Second wave**.

The reflected wave, propagating toward the bottom boundary, will also propagate at an angle to the normal to the bottom boundary. Thus, add a Matched Boundary Condition feature here, too, to absorb the reflected beam.

Matched Boundary Condition 3

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 2 only.



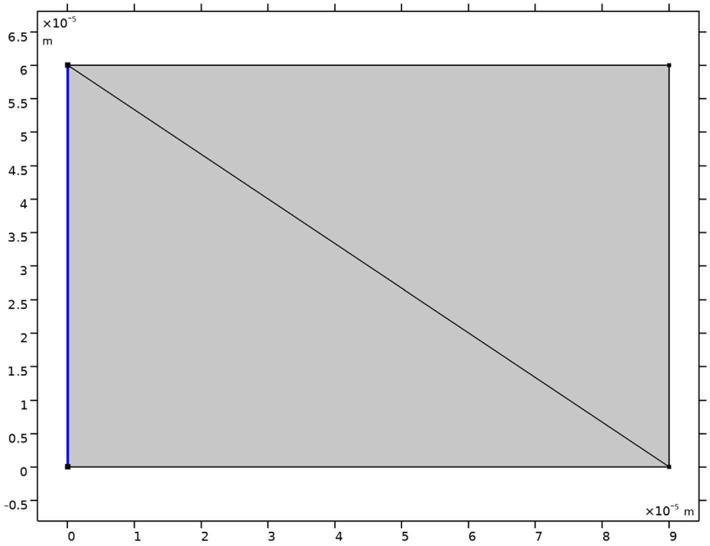
DEFINITIONS

Set up integration operators to calculate the power of the incident, reflected, and transmitted beams.

Integration 1 (intop1)

1. In the **Definitions** toolbar, click 🔗 **Nonlocal Couplings** and choose **Integration**.
2. In the **Settings** window for **Integration**, locate the **Source Selection** section.
3. From the **Geometric entity level** list, choose **Boundary**.

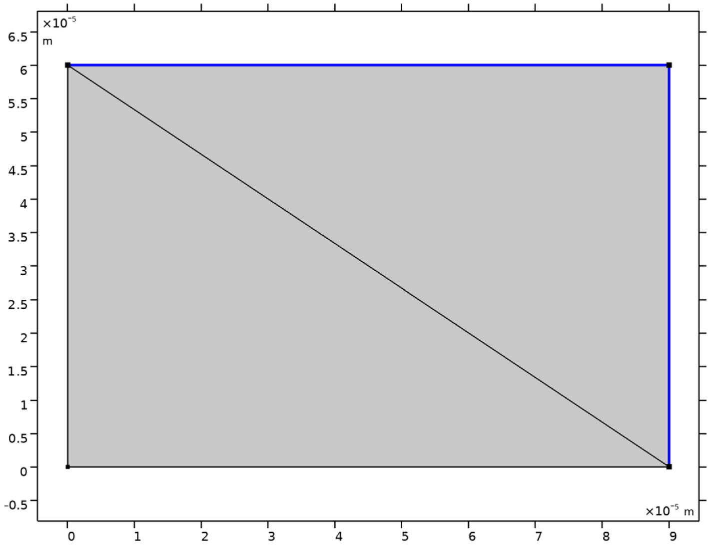
4 Select Boundary 1 only.



Integration 2 (intop2)

1. In the **Definitions** toolbar, click 🔗 **Nonlocal Couplings** and choose **Integration**.
2. In the **Settings** window for **Integration**, locate the **Source Selection** section.
3. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundaries 4 and 5 only.



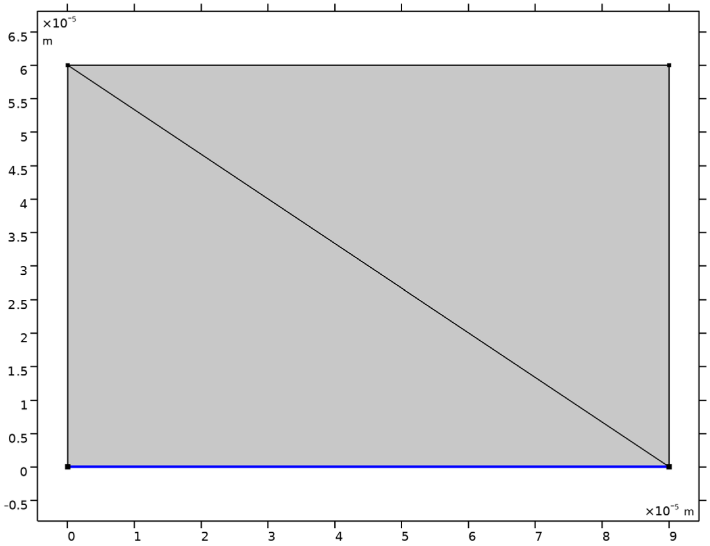
Integration 3 (intop3)

1 In the **Definitions** toolbar, click 🔗 **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 2 only.



Variables 3

Now, define the power variables for the beams, using the previously defined integration operators.

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	**Expression**	**Unit**	**Description**
`Pin`	`-intop1(ewbe.nPoav)`	W/m	`Input power`
`Pt`	`intop2(ewbe.nPoav)`	W/m	`Transmitted power`
`Pr`	`intop3(ewbe.nPoav)`	W/m	`Reflected power`

The minus sign for the input power is used as the power flow and the boundary normal point in the opposite directions.

MESH 1

Let the physics define a triangular mesh where the maximum mesh element size is set to half a wavelength, to resolve the interference pattern created by the incident and the reflected beam.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2. In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Beam Envelopes (ewbe)** section.
3. From the **Mesh type** list, choose **Triangular mesh**.
4. In the $h_{\max}$ text field, type `lda0/2`.

STUDY 1

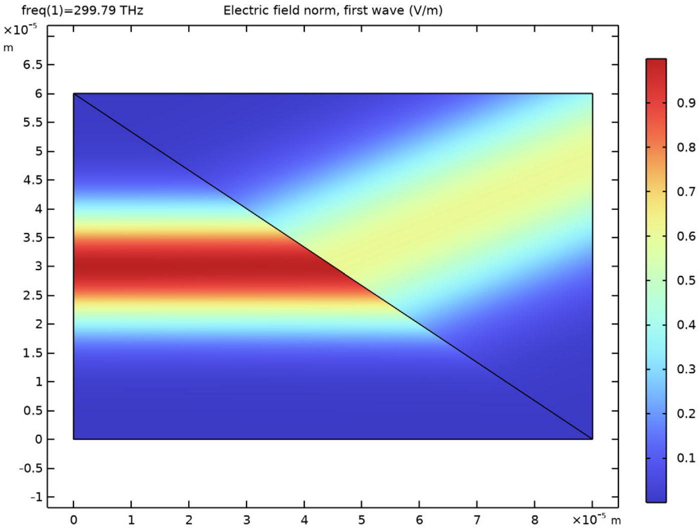
Step 1: Frequency Domain

Define the frequency and compute the solution for the model.

1. In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.
2. In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
3. In the **Frequencies** text field, type `f0`.
4. In the **Study** toolbar, click = **Compute**.

RESULTS

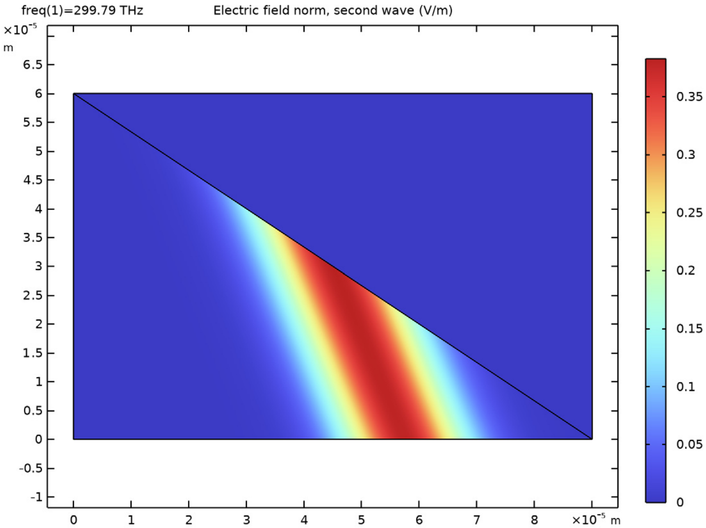
Electric Field, First Wave (ewbe)



This is the electric field norm of the first wave for s-polarization.

Electric Field, Second Wave (ewbe)

1 In the **Model Builder** window, click **Electric Field, Second Wave (ewbe)**.



This is the electric field norm of the second wave for s-polarization.

Electric Field (ewbe)

1 Right-click **Electric Field, Second Wave (ewbe)** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Field (ewbe)` in the **Label** text field.

Electric Field

1 In the **Model Builder** window, expand the **Electric Field (ewbe)** node, then click **Electric Field**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewbe.normE`.

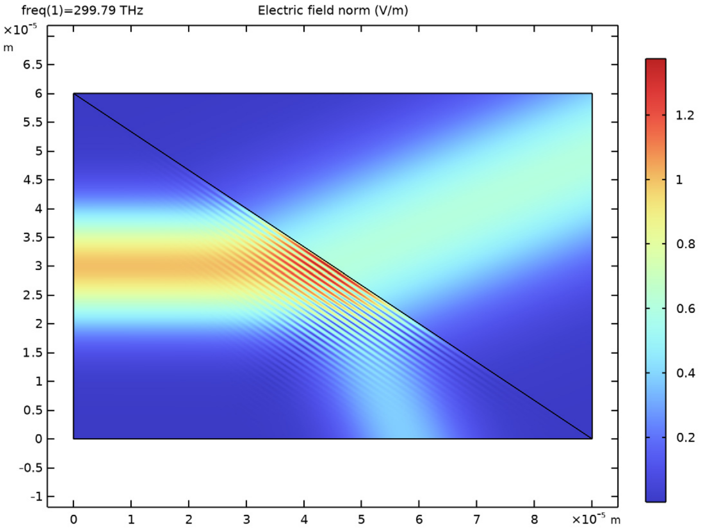
To really resolve the inference pattern to the left of the air-glass interface, set the resolution to extra fine.

4 Click to expand the **Quality** section. From the **Evaluation settings** list, choose **Manual**.

5 From the **Resolution** list, choose **Extra fine**.

6 In the **Electric Field (ewbe)** toolbar, click **Plot**.

7. Click the **Zoom Extents** button in the **Graphics** toolbar. Compare your result with the plot below.



Using the defined variables, compute the reflectance.

Global Evaluation 1

1. In the **Results** toolbar, click **Global Evaluation**.
2. In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.
3. In the table, enter the following settings:

Expression	**Unit**	**Description**
`Pr/Pin`	1	

4. Click **Evaluate**.

TABLE 1

1. Go to the **Table 1** window. Compare the calculated reflectance with the theoretical value for s-polarized plane waves. Notice that the values are almost the same.
2. In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.
3. In the table, enter the following settings:

Expression	**Unit**	**Description**
`((1-n2^2)/(1+n2^2))^2`		

4 Click = **Evaluate**.

Now check that all incident power is either reflected or transmitted.

5 In the table, enter the following settings:

Expression	**Unit**	**Description**
`(Pr+Pt)/Pin`	1	

6 Click = **Evaluate**.

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

In this simulation, set the polarization to be in-plane, that is p-polarization. For this case, there should be no reflected beam.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.

2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **In-plane vector**.

Matched Boundary Condition 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Electromagnetic Waves, Beam Envelopes (ewbe)** click **Matched Boundary Condition 1**.

2 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

3 Specify the $\mathbf{E}_{g0}$ vector as

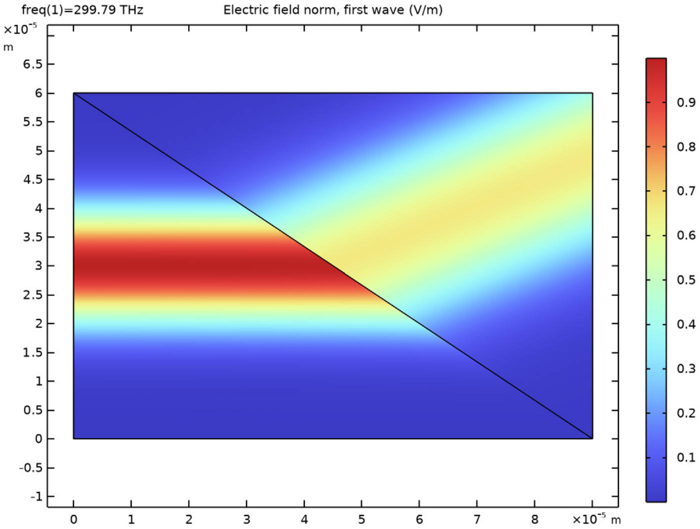
`0`	x
`1[V/m]`	y
`0`	z

STUDY 1

In the **Study** toolbar, click = **Compute**.

RESULTS

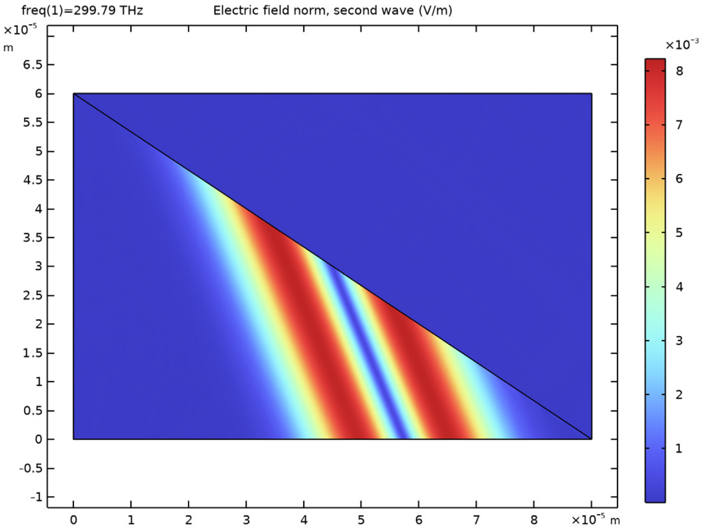
Electric Field, First Wave (ewbe)



This is the electric field norm of the first wave for p-polarization.

Electric Field, Second Wave (ewbe)

1 In the **Model Builder** window, click **Electric Field, Second Wave (ewbe)**.

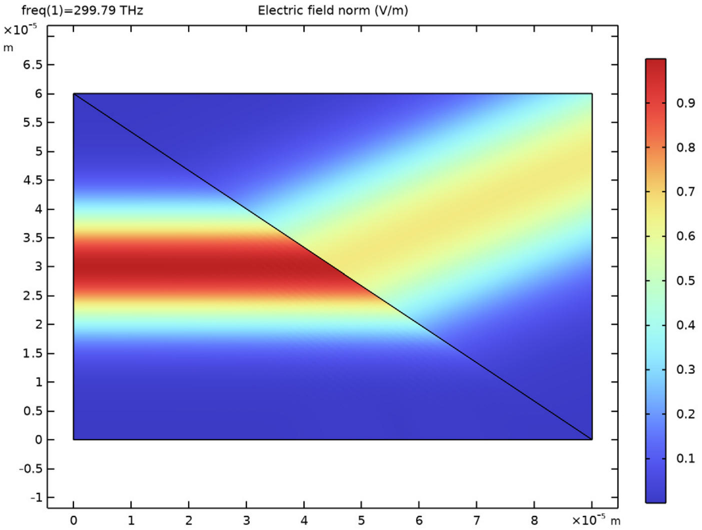


This is the electric field norm of the second wave for p-polarization. Note that the field amplitude is much smaller than the amplitude for the incident wave.

Electric Field (ewbe)

1 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare your result with the plot below. Notice that there is no reflected beam in this case.

2 In the **Model Builder** window, click **Electric Field (ewbe)**.



Global Evaluation 1

Also check numerically that the reflected wave is almost gone for p-polarization at the Brewster angle.

1 In the **Model Builder** window, under **Results** > **Derived Values** click **Global Evaluation 1**.

2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	**Unit**	**Description**
`Pr/Pin`	1	

4 Click = **Evaluate**.