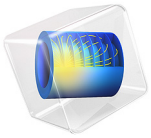

Model created in COMSOL Multiphysics 6.4

Roundness Evaluation of a Deformed Hole

Introduction

During manufacturing processes, or during use, the geometrical shape of structural parts can deviate from what was intended. For instance, a hole can deviate from an ideal circle. In an assembly, it can be important to measure and control this deviation. The process of obtaining this information is often called roundness (or circularity) error evaluation.

There are different methods to measure the roundness error, all based on optimization techniques:

- the Least Squares Circle (LSC)
- the Minimum Zone Circle (MZC)
- the Minimum Circumscribed Circle (MMC)
- the Maximum Inscribed Circle (MIC)

This model illustrates how to reuse the deformed shape data for an optimization analysis to obtain the center coordinates and radii of the circles for each of the methods listed above.

Model Definition

In this model, a pressure load is applied to a plate with a hole in a first study. The boundary condition used are not important; the only purpose is to provide some non-trivial deformation. The hole does no longer remain circular.

Deformed shape data is then exported to a table. Finally, using this deformed shape data, the roundness error can be computed as the difference between the circles with the minimum and maximum radii that fit the deformed shape

$$\Delta z = R_{\max} - R_{\min}$$

To obtain the maximum circle radius and the minimum circle radius, five different techniques are considered here. The first one (LSQ) uses the predefined Least-Squares Fit function, which does not require a special solver to be used. The other four methods require an optimization solver and thus the Optimization Module. These four methods are described below.

LEAST SQUARE CIRCLE (LSC)

This method searches for the radius R_c and the center coordinates (x_c, y_c) of the circle that minimizes the sum of the square deviation from the points of the deformed shape. The objective function can be written as

$$F(x_c, y_c, R_c) = \min\left(\sum_i \left(\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} - R_c\right)^2\right)$$

Solving this problem, the minimum and maximum radii can be obtained as

$$R_{\max} = \max\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$
$$R_{\min} = \min\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$

MINIMUM ZONE CIRCLE (MZC)

This method directly minimizes the roundness error. Two concentric circles are assumed, one with the maximum radius that contains all points of the deformed shape, and the other with the smallest radius that is inscribed in the deformed shape. The objective function is then

$$F(x_c, y_c, R_{\max}, R_{\min}) = \min(R_{\max} - R_{\min})$$
$$R_{\max} = \max\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$
$$R_{\min} = \min\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$

MINIMUM CIRCUMSCRIBED CIRCLE (MMC)

The MCC method searches for the circle with the smallest radius that is circumscribed around the deformed shape. The objective function is

$$F(x_c, y_c, R_{\max}) = \min(R_{\max})$$
$$R_{\max} = \max\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$

Then minimum circle can be obtained as

$$R_{\min} = \min\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$

MAXIMUM INSCRIBED CIRCLE (MIC)

The MIC method searches for the circle with the largest radius that inside the deformed shape. The objective function is

$$F(x_c, y_c, R_{\min}) = \max(R_{\min})$$

$$R_{\min} = \min(\sqrt{(x - x_c)^2 + (y - y_c)^2})$$

Then minimum circle can be obtained as

$$R_{\max} = \max(\sqrt{(x - x_c)^2 + (y - y_c)^2})$$

Results and Discussion

Figure 1 displays the von Mises stress distribution in the part for the assumed boundary condition. The stress concentration around the hole is a result of the ovalization of the hole.

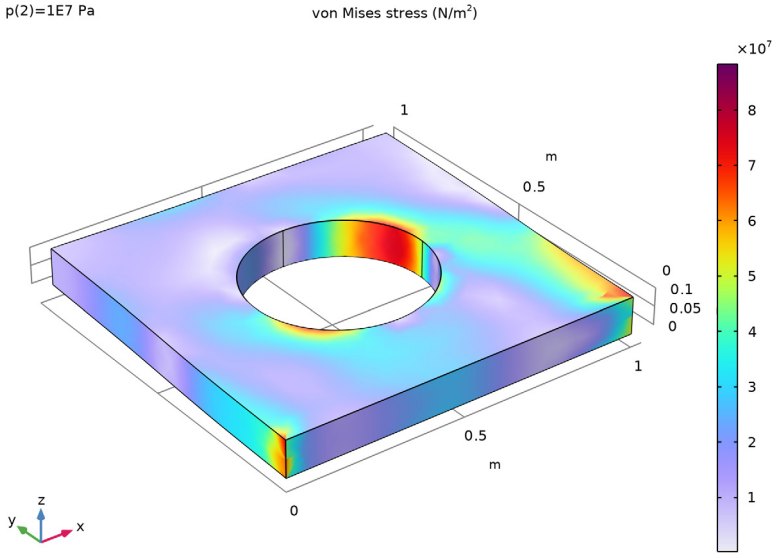

Figure 1: von Mises stress distribution in the plate.

Figure 2 and Figure 3 show the deformed shape of the hole together with the minimum (in blue) and maximum (in green) circle. From top to bottom: the minimum circumscribed circle (MCC) method, the maximum inscribed circle (MIC) method, the

Least-Squares Fit function (LSQ), the Least-Squares Circle (LSC) method, and the minimum zone circle (MZC) method.

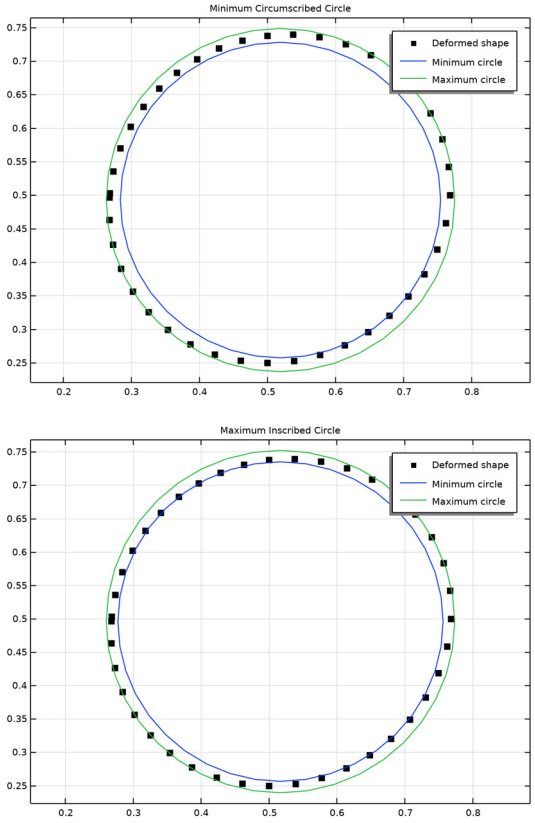

Figure 2: The minimum (blue) and maximum (green) circle obtained using MCC and MIC methods.

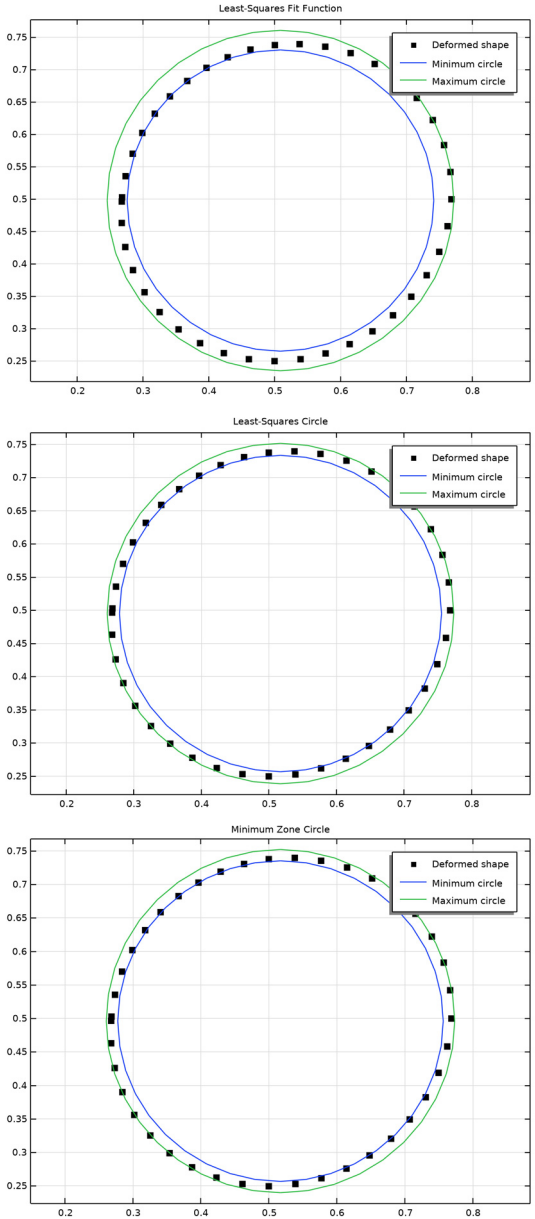

Figure 3: The minimum (blue) and maximum (green) circle using LSQ, LSC, and MZC methods.

In the table Table 1 below you can see the coordinates of the center, the minimum and the maximum radii, and the roundness error (the difference between $R_{\max}$ and $R_{\min}$). The solutions using the different methods differ slightly. The minimum zone circle (MZC) method provides the smallest roundness error. This is not surprising, since that method directly aims at minimizing the roundness error. This may be interpreted as the best result, but there is no unique answer to what is the best fit of the two circles.

TABLE 1: CIRCLE PARAMETERS OBTAINED USING THE DIFFERENT METHODS.

Method	x0 (m)	y0 (m)	Rmin (m)	Rmax (m)	Roundness error (m)
LSQ	0.5085	0.4977	0.2327	0.2630	0.0303
LSC	0.5170	0.4950	0.2383	0.2565	0.0182
MZC	0.5168	0.4960	0.2393	0.2562	0.0169
MCC	0.5189	0.4927	0.2354	0.2560	0.0207
MIC	0.5167	0.4959	0.2393	0.2565	0.0172

Notes About the COMSOL Implementation

To use any of the optimization methods described in this model, you must be able to express the deformed shape as function of a local parameter. Here, the polar angle along the deformed edge is used. To export the deformed shape data along an edge to a table, you need to create a Cut Point 3D dataset combined with a solution dataset restricted to the edge of interest. With such a dataset, you can easily generate a table that has the columns populated with the angle position, the x-coordinates, and the y-coordinates along every point along the selected edge.

Application Library path: `Structural_Mechanics_Module/Sensitivity_and_Optimization/roundness_evaluation`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Structural Mechanics** > **Solid Mechanics (solid)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies** > **Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
p	1e6[Pa]	1E6 Pa	Pressure
x0	0.5[m]	0.5 m	Hole center, x-coordinate
y0	0.5[m]	0.5 m	Hole center, y-coordinate
R0	0.25[m]	0.25 m	Hole radius

GEOMETRY 1

Block 1 (blk1)

1 In the **Geometry** toolbar, click **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Height** text field, type `0.1`.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `R0`.

4 In the **Height** text field, type `0.1`.

5 Locate the **Position** section. In the **x** text field, type `x0`.

6 In the **y** text field, type `y0`.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **blk1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the object **cyl1** only.

6 Click **Build Selected**.

Form Union (fin)

1 In the **Model Builder** window, click **Form Union (fin)**.

2 In the **Settings** window for **Form Union/Assembly**, click **Build Selected**.

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Nylon**.

4 Click the **Add to Component** button in the window toolbar.

5 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

SOLID MECHANICS (SOLID)

Fixed Constraint 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Fixed Constraint**.

2 Select Boundary 2 only.

Boundary Load 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Boundary Load**.

2 Select Boundaries 1 and 5 only.

3 In the **Settings** window for **Boundary Load**, locate the **Force** section.

4 From the **Load type** list, choose **Pressure**.

5 In the *p* text field, type p.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Study Settings** section.

3 Select the **Include geometric nonlinearity** checkbox.

4 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

5 Click **Add**.

6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
p (Pressure)	1e6 1e7	Pa

7 In the **Study** toolbar, click **Compute**.

RESULTS

Study 1/Solution 1 (2) (sol1)

1 In the **Model Builder** window, expand the **Results** > **Datasets** node.

2 Right-click **Results** > **Datasets** > **Study 1/Solution 1 (sol1)** and choose **Duplicate**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Edge**.

4 Select Edges 12, 13, 16, and 19 only.

Cut Point 3D 1

1 In the **Results** toolbar, click **Cut Point 3D**.

2 In the **Settings** window for **Cut Point 3D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution 1 (2) (sol1)**.

4 Locate the **Point Data** section. From the **Entry method** list, choose **Regular grid**.

5 In the **Number of X points** text field, type 20.

6 In the **Number of Y points** text field, type 20.

7 In the **Number of Z points** text field, type 1.

8 From the **Snapping** list, choose **Snap to closest edge**.

9 Click **Plot**.

Deformed Shape

1 In the **Results** toolbar, click **Evaluation Group**.

2 In the **Settings** window for **Evaluation Group**, type Deformed Shape in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 3D 1**.

4 From the **Parameter selection (p)** list, choose **Last**.

5 Locate the **Transformation** section. Select the **Transpose** checkbox.

6 Click to expand the **Format** section. From the **Include parameters** list, choose **Off**.

7 From the **Concatenation** list, choose **Vertical**.

Point Evaluation 1

1 Right-click **Deformed Shape** and choose **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`atan2(y-y0,x-x0)`	`rad`	`angle`

Point Evaluation 2

1 In the **Model Builder** window, right-click **Deformed Shape** and choose **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`x`	`m`	`x-coordinate`

Point Evaluation 3

1 Right-click **Deformed Shape** and choose **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y`	`m`	`y-coordinate`

4 In the **Deformed Shape** toolbar, click **Evaluate**.

GLOBAL DEFINITIONS

Interpolation 1 (int1)

1 In the **Home** toolbar, click **Functions** and choose **Global** > **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 From the **Data source** list, choose **Result table**.

4 Locate the **Data Column Settings** section. In the table, click to select the cell at row number 2 and column number 2.

5 In the **Unit** text field, type `m`.

6 In the table, enter the following settings:

Columns	Type	Settings
	Function values	Function name=int2

7 In the **Name** text field, type `int2`.

8 In the **Unit** text field, type `m`.

Parameters 1

1 In the **Model Builder** window, click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`angle`	`0[rad]`	0 rad	`Angular parameter`

Variables 1

1 In the **Model Builder** window, right-click **Global Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
`xdata`	`int1(angle)`	m	`Deformed shape, x-coordinate`
`ydata`	`int2(angle)`	m	`Deformed shape, x-coordinate`

Least-Squares Fit 1 (lsq1_fun1)

1 In the **Home** toolbar, click f(x) **Functions** and choose **Global** > **Least-Squares Fit**.

2 In the **Settings** window for **Least-Squares Fit**, locate the **Data** section.

3 From the **Data source** list, choose **Result table**.

4 Locate the **Data Column Settings** section. In the table, enter the following settings:

Columns	Type	Settings
	Function values	Name=lsq1_fun2, Expression=b1*x1+b0

5 In the table, click to select the cell at row number 2 and column number 2.

6 Locate the **Parameters** section. In the table, enter the following settings:

Parameter	Values	Scale	Lower bound	Upper bound
r	0	1	0.2	0.3
xc	0	1	0.4	0.6
yc	0	1	0.4	0.6

7 Locate the **Data Column Settings** section. In the **Expression** text field, type `xc+r*cos(x1)`.

8 In the **Unit** text field, type `m`.

9 In the table, click to select the cell at row number 3 and column number 2.

10 In the **Expression** text field, type `yc+r*sin(x1)`.

11 In the **Unit** text field, type `m`.

12 Click **Fit Parameters**.

13 Click to expand the **Plot Parameters** section. Click **Plot**.

14 From the **Function name** list, choose **lsq1_fun2**.

15 Click **Plot**.

Variables 1

1 In the **Model Builder** window, click **Variables 1**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
x_LSQ	lsq1_fun1(pi/2)	m	Center (LSQ), x-coordinate
y_LSQ	lsq1_fun2(0)	m	Center (LSQ), y-coordinate
R_LSQ	lsq1_fun1(0)-x_LSQ	m	Radius (LSQ)

4 Click the **Show More Options** button in the **Model Builder** toolbar.

5 In the **Show More Options** dialog, select **General** > **Variable Utilities** in the tree.

6 In the tree, select the checkbox for the node **General** > **Variable Utilities**.

7 Click **OK**.

DEFINITIONS

Expression Operator 1 (exop1)

1 In the **Definitions** toolbar, click **Variable Utilities** and choose **Expression Operator**.

2 In the **Settings** window for **Expression Operator**, type `dist` in the **Name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `sqrt((x1-x2)^2+(y1-y2)^2)`.

4 In the table, enter the following settings:

Argument	Dimensions	Argument type
x1	m	Expression
y1	m	Expression
x2	m	Expression
y2	m	Expression

Maximum 1 (maxop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Maximum**.

2 In the **Settings** window for **Maximum**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Edge**.

4 Select Edges 12, 13, 16, and 19 only.

Minimum 1 (minop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Minimum**.

2 In the **Settings** window for **Minimum**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Edge**.

4 Select Edges 12, 13, 16, and 19 only.

Variables 2

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Rmin_LSQ	minop1(dist(x,y,x_LSQ, y_LSQ))	m	Radius (LSQ), minimum
Rmax_LSQ	maxop1(dist(x,y,x_LSQ, y_LSQ))	m	Radius (LSQ), maximum

Expression Operator 2 (exop2)

1 In the **Definitions** toolbar, click **Variable Utilities** and choose **Expression Operator**.

2 In the **Settings** window for **Expression Operator**, type `x_plot` in the **Name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `x+R*cos(angle)`.

4 In the table, enter the following settings:

Argument	Dimensions	Argument type
x	m	Expression
R	m	Expression

Expression Operator 3 (exop3)

1 In the **Definitions** toolbar, click **Variable Utilities** and choose **Expression Operator**.

2 In the **Settings** window for **Expression Operator**, type `y_plot` in the **Name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `y+R*sin(angle)`.

4 In the table, enter the following settings:

Argument	Dimensions	Argument type
y	m	Expression
R	m	Expression

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Solid Mechanics (solid)**.

5 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Stationary**.

6 Right-click and choose **Add Study**.

7 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

LEAST-SQUARES FIT FUNCTION

In the **Settings** window for **Study**, type `Least-Squares Fit Function` in the **Label** text field.

Step 1: Stationary

1 In the **Model Builder** window, under **Least-Squares Fit Function** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Values of Dependent Variables** section.

3 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **Study 1, Stationary**.

6 From the **Parameter value (p (Pa))** list, choose **Last**.

7 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

8 Locate the **Study Settings** section. Select the **Include geometric nonlinearity** checkbox.

9 Locate the **Study Extensions** section. Click **Add**.

10 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Angular parameter)	`range(-pi,pi/20,pi)`	`rad`

11 In the **Study** toolbar, click **Compute**.

RESULTS

Least-Squares Fit Function

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Least-Squares Fit Function` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Least-Squares Fit Function/ Solution 2 (sol2)**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Least-Squares Fit Function`.

6 Locate the **Axis** section. Select the **Preserve aspect ratio** checkbox.

Deformed Shape

1 Right-click **Least-Squares Fit Function** and choose **Global**.

2 In the **Settings** window for **Global**, type `Deformed Shape` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
ydata	m	Deformed shape, x-coordinate

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `xdata`.

6 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

7 From the **Color** list, choose **Black**.

8 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

9 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
Deformed shape

Minimum Circle

1 In the **Model Builder** window, right-click **Least-Squares Fit Function** and choose **Global**.

2 In the **Settings** window for **Global**, type `Minimum Circle` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
y_plot(y_LSQ,Rmin_LSQ)	m	Expression Operator 3 (y_plot)

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `x_plot(x_LSQ,Rmin_LSQ)`.

6 Locate the **Legends** section. From the **Legends** list, choose **Manual**.

7 In the table, enter the following settings:

Legends
Minimum circle

Maximum Circle

1 Right-click **Least-Squares Fit Function** and choose **Global**.

2 In the **Settings** window for **Global**, type `Maximum Circle` in the **Label** text field.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_LSQ,Rmax_LSQ)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `x_plot(x_LSQ,Rmax_LSQ)`.

6 In the **Least-Squares Fit Function** toolbar, click **Plot**.

7 Locate the **Legends** section. From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

Legends
`Maximum circle`

Roundness Error

1 In the **Results** toolbar, click **Evaluation Group**.

2 In the **Settings** window for **Evaluation Group**, type `Roundness Error` in the **Label** text field.

3 Locate the **Format** section. From the **Include parameters** list, choose **Off**.

4 From the **Concatenation** list, choose **Vertical**.

LSQ (1)

1 Right-click **Roundness Error** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Least-Squares Fit Function/Solution 2 (sol2)**.

4 From the **Parameter selection (angle)** list, choose **Last**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`x_LSQ`	`m`	`Center, x-coordinate`
`y_LSQ`	`m`	`Center, y-coordinate`
`Rmin_LSQ`	`m`	`Radius, minimum`
`Rmax_LSQ`	`m`	`Radius, maximum`
`Rmax_LSQ-Rmin_LSQ`	`m`	`Roundness error`

6 In the **Label** text field, type `LSQ (1)`.

7 In the **Roundness Error** toolbar, click **Evaluate**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
x_LSC	x0	0.5 m	Center (LSC), x-coordinate
y_LSC	y0	0.5 m	Center (LSC), y-coordinate
R_LSC	R0	0.25 m	Radius (LSC)

DEFINITIONS

Variables 2

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Definitions** click **Variables 2**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Rmin_LSC	minop1(dist(x,y,x_LSC, y_LSC))	m	Radius (LSC), minimum
Rmax_LSC	maxop1(dist(x,y,x_LSC, y_LSC))	m	Radius (LSC), maximum

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Solid Mechanics (solid)**.

5 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Stationary**.

6 Right-click and choose **Add Study**.

7 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

LEAST-SQUARES CIRCLE

In the **Settings** window for **Study**, type `Least-Squares Circle` in the **Label** text field.

Step 1: Stationary

1 In the **Model Builder** window, under **Least-Squares Circle** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Values of Dependent Variables** section.

3 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **Study 1, Stationary**.

6 From the **Parameter value (p (Pa))** list, choose **Last**.

7 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

8 Locate the **Study Settings** section. Select the **Include geometric nonlinearity** checkbox.

9 Locate the **Study Extensions** section. Click + **Add**.

10 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Angular parameter)	`range(-pi,pi/20,pi)`	`rad`

General Optimization

1 In the **Study** toolbar, click **Optimization** and choose **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **IPOPT**.

4 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`(comp1.dist(xdata,ydata,x_LSC,y_LSC)-R_LSC)^2`		Stationary

5 Locate the **Control Variables and Parameters** section. Click + **Add** three times.

6 In the table, enter the following settings:

Parameter	Initial value	Scale	Lower bound	Upper bound	Unit
x_LSC (Center (LSC), x-coordinate)	x0	1	0.8*x0	1.2*x0	m
y_LSC (Center (LSC), y-coordinate)	y0	1	0.8*y0	1.2*y0	m
R_LSC (Radius (LSC))	R0	1	0.8*R0	1.2*R0	m

7 Locate the **Constraints** section. In the table, enter the following settings:

Expression	Lower bound	Upper bound	Evaluate for
comp1.dist(xdata, ydata,x_LSC,y_LSC)	0		Stationary

8 In the **Study** toolbar, click **Compute**.

RESULTS

Least-Squares Circle

1 In the **Model Builder** window, right-click **Least-Squares Fit Function** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Least-Squares Circle` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Least-Squares Circle/ Solution 3 (sol3)**.

4 Locate the **Title** section. In the **Title** text area, type `Least-Squares Circle`.

Minimum Circle

1 In the **Model Builder** window, expand the **Least-Squares Circle** node, then click **Minimum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
y_plot(y_LSC,Rmin_LSC)	m	Expression Operator 3 (y_plot)

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_LSC, Rmin_LSC)`.

Maximum Circle

1 In the **Model Builder** window, click **Maximum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_LSC,Rmax_LSC)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_LSC, Rmax_LSC)`.

5 In the **Least-Squares Circle** toolbar, click **Plot**.

LSC (2)

1 In the **Model Builder** window, right-click **Roundness Error** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, type `LSC (2)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Least-Squares Circle/ Solution 3 (sol3)**.

4 From the **Parameter selection (angle)** list, choose **Last**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`x_LSC`	`m`	`Center, x-coordinate`
`y_LSC`	`m`	`Center, y-coordinate`
`Rmin_LSC`	`m`	`Radius, minimum`
`Rmax_LSC`	`m`	`Radius, maximum`
`Rmax_LSC-Rmin_LSC`	`m`	`Roundness error`

6 In the **Roundness Error** toolbar, click **Evaluate**.

Minimum Zone Circle

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
x_MZC	x0	0.5 m	Center (MZC), x-coordinate
y_MZC	y0	0.5 m	Center (MZC), y-coordinate
R_MZC	R0	0.25 m	Radius (MZC)
Rmin_MZC	0.8*R0	0.2 m	Radius (MZC), minimum
Rmax_MZC	1.1*R0	0.275 m	Radius (MZC), maximum

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Solid Mechanics (solid)**.

5 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Stationary**.

6 Right-click and choose **Add Study**.

7 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

MINIMUM ZONE CIRCLE

In the **Settings** window for **Study**, type `Minimum Zone Circle` in the **Label** text field.

Step 1: Stationary

1 In the **Model Builder** window, under **Minimum Zone Circle** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Values of Dependent Variables** section.

3 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **Study 1, Stationary**.

6 From the **Parameter value (p (Pa))** list, choose **Last**.

7 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

8 Locate the **Study Settings** section. Select the **Include geometric nonlinearity** checkbox.

9 Locate the **Study Extensions** section. Click **Add**.

10 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Angular parameter)	`range(-pi,pi/20,pi)`	`rad`

General Optimization

1 In the **Study** toolbar, click **Optimization** and choose **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **IPOPT**.

4 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`Rmax_MZC-Rmin_MZC`		Stationary

5 Locate the **Control Variables and Parameters** section. Click **Add** four times.

6 In the table, enter the following settings:

Parameter	Initial value	Scale	Lower bound	Upper bound	Unit
x_MZC (Center (MZC), x-coordinate)	`x0`	1	`0.8*x0`	`1.2*x0`	m
y_MZC (Center (MZC), y-coordinate)	`y0`	1	`0.8*y0`	`1.2*y0`	m
Rmin_MZC (Radius (MZC), minimum)	`0.8*R0`	1	`0.8*R0`	`R0`	m
Rmax_MZC (Radius (MZC), maximum)	`1.1*R0`	1	`R0`	`1.2*R0`	m

7 Locate the **Constraints** section. In the table, enter the following settings:

Expression	Lower bound	Upper bound	Evaluate for
`comp1.dist(xdata, ydata,x_MZC,y_MZC)^2-Rmin_MZC^2`	0	1	Stationary
`Rmax_MZC^2-comp1.dist(xdata, ydata,x_MZC,y_MZC)^2`	0	1	Stationary

8 In the **Study** toolbar, click **Compute**.

RESULTS

Minimum Zone Circle

1 In the **Model Builder** window, right-click **Least-Squares Fit Function** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Minimum Zone Circle` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Minimum Zone Circle/ Solution 4 (sol4)**.

4 Locate the **Title** section. In the **Title** text area, type `Minimum Zone Circle`.

Minimum Circle

1 In the **Model Builder** window, expand the **Minimum Zone Circle** node, then click **Minimum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_MZC,Rmin_MZC)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_MZC, Rmin_MZC)`.

Maximum Circle

1 In the **Model Builder** window, click **Maximum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_MZC,Rmax_MZC)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_MZC, Rmax_MZC)`.

5 In the **Minimum Zone Circle** toolbar, click  **Plot**.

MZC (3)

1 In the **Model Builder** window, right-click **Roundness Error** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, type `MZC (3)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Minimum Zone Circle/ Solution 4 (sol4)**.

4 From the **Parameter selection (angle)** list, choose **Last**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
x_MZC	m	Center, x-coordinate
y_MZC	m	Center, y-coordinate
Rmin_MZC	m	Radius, minimum
Rmax_MZC	m	Radius, maximum
Rmax_MZC-Rmin_MZC	m	Roundness error

6 In the **Roundness Error** toolbar, click = **Evaluate**.

Minimum Circumscribed Circle

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
x_MCC	x0	0.5 m	Center (MCC), x-coordinate
y_MCC	y0	0.5 m	Center (MCC), y-coordinate
Rmax_MCC	1.1*R0	0.275 m	Radius (MCC), maximum

DEFINITIONS

Variables 2

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Definitions** click **Variables 2**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Rmin_MCC	minop1(dist(x,y,x_MCC, y_MCC))	m	Radius (MCC), minimum

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Solid Mechanics (solid)**.

5 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Stationary**.

6 Right-click and choose **Add Study**.

7 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

MINIMUM CIRCUMSCRIBED CIRCLE

In the **Settings** window for **Study**, type `Minimum Circumscribed Circle` in the **Label** text field.

Step 1: Stationary

1 In the **Model Builder** window, under **Minimum Circumscribed Circle** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Values of Dependent Variables** section.

3 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **Study 1, Stationary**.

6 From the **Parameter value (p (Pa))** list, choose **Last**.

7 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

8 Locate the **Study Settings** section. Select the **Include geometric nonlinearity** checkbox.

9 Locate the **Study Extensions** section. Click **Add**.

10 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Angular parameter)	`range(-pi,pi/20,pi)`	`rad`

General Optimization

1 In the **Study** toolbar, click **Optimization** and choose **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **IPOPT**.

4 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`Rmax_MCC`	`Radius (MCC), maximum`	Stationary

5 Locate the **Control Variables and Parameters** section. Click **Add** three times.

6 In the table, enter the following settings:

Parameter	Initial value	Scale	Lower bound	Upper bound	Unit
x_MCC (Center (MCC), x-coordinate)	`x0`	1	`0.8*x0`	`1.2*x0`	m
y_MCC (Center (MCC), y-coordinate)	`y0`	1	`0.8*y0`	`1.2*y0`	m
Rmax_MCC (Radius (MCC), maximum)	`1.1*R0`	1	`R0`	`1.2*R0`	m

7 Locate the **Constraints** section. In the table, enter the following settings:

Expression	Lower bound	Upper bound	Evaluate for
`Rmax_MCC-comp1.dist(xdata, ydata,x_MCC,y_MCC)`	0		Stationary

8 In the **Study** toolbar, click **Compute**.

RESULTS

Minimum Circumscribed Circle

1 In the **Model Builder** window, right-click **Least-Squares Fit Function** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Minimum Circumscribed Circle` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Minimum Circumscribed Circle/ Solution 5 (sol5)**.

4 Locate the **Title** section. In the **Title** text area, type `Minimum Circumscribed Circle`.

Minimum Circle

1 In the **Model Builder** window, expand the **Minimum Circumscribed Circle** node, then click **Minimum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_MCC,Rmin_MCC)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_MCC, Rmin_MCC)`.

Maximum Circle

1 In the **Model Builder** window, click **Maximum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_MCC,Rmax_MCC)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_MCC, Rmax_MCC)`.

5 In the **Minimum Circumscribed Circle** toolbar, click **Plot**.

MCC (4)

1 In the **Model Builder** window, right-click **Roundness Error** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, type `MCC (4)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Minimum Circumscribed Circle/ Solution 5 (sol5)**.

4 From the **Parameter selection (angle)** list, choose **Last**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`x_MCC`	`m`	`Center, x-coordinate`
`y_MCC`	`m`	`Center, y-coordinate`
`Rmin_MCC`	`m`	`Radius, minimum`
`Rmax_MCC`	`m`	`Radius, maximum`
`Rmax_MCC-Rmin_MCC`	`m`	`Roundness error`

6 In the **Roundness Error** toolbar, click **Evaluate**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
x_MIC	x0	0.5 m	Center (MIC), x-coordinate
y_MIC	y0	0.5 m	Center (MIC), y-coordinate
Rmin_MIC	0.8*R0	0.2 m	Radius (MIC), minimum

DEFINITIONS

Variables 2

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Definitions** click **Variables 2**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Rmax_MIC	maxop1(dist(x,y,x_MIC, y_MIC))	m	Radius (MIC), maximum

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Solid Mechanics (solid)**.

5 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Stationary**.

6 Right-click and choose **Add Study**.

7 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

MAXIMUM INSCRIBED CIRCLE

In the **Settings** window for **Study**, type `Maximum Inscribed Circle` in the **Label** text field.

Step 1: Stationary

1 In the **Model Builder** window, under **Maximum Inscribed Circle** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Values of Dependent Variables** section.

3 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **Study 1, Stationary**.

6 From the **Parameter value (p (Pa))** list, choose **Last**.

7 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

8 Locate the **Study Settings** section. Select the **Include geometric nonlinearity** checkbox.

9 Locate the **Study Extensions** section. Click + **Add**.

10 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
angle (Angular parameter)	`range(-pi,pi/20,pi)`	`rad`

General Optimization

1 In the **Study** toolbar, click **Optimization** and choose **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **IPOPT**.

4 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`Rmin_MIC`	`Radius (MIC), minimum`	Stationary

5 From the **Type** list, choose **Maximization**.

6 Locate the **Control Variables and Parameters** section. Click + **Add** three times.

7 In the table, enter the following settings:

Parameter	Initial value	Scale	Lower bound	Upper bound	Unit
x_MIC (Center (MIC), x-coordinate)	x0	1	0.8*x0	1.2*x0	m
y_MIC (Center (MIC), y-coordinate)	y0	1	0.8*y0	1.2*y0	m
Rmin_MIC (Radius (MIC), minimum)	0.8*R0	1	0.8*R0	R0	m

8 Locate the **Constraints** section. In the table, enter the following settings:

Expression	Lower bound	Upper bound	Evaluate for
comp1.dist(xdata, ydata,x_MIC,y_MIC)-Rmin_MIC	0		Stationary

9 In the **Study** toolbar, click **Compute**.

RESULTS

Maximum Inscribed Circle

1 In the **Model Builder** window, right-click **Least-Squares Fit Function** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Maximum Inscribed Circle` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Maximum Inscribed Circle/ Solution 6 (sol6)**.

4 Locate the **Title** section. In the **Title** text area, type `Maximum Inscribed Circle`.

Minimum Circle

1 In the **Model Builder** window, expand the **Maximum Inscribed Circle** node, then click **Minimum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
y_plot(y_MIC,Rmin_MIC)	m	Expression Operator 3 (y_plot)

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_MIC, Rmin_MIC)`.

Maximum Circle

1 In the **Model Builder** window, click **Maximum Circle**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`y_plot(y_MIC,Rmax_MIC)`	`m`	`Expression Operator 3 (y_plot)`

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `x_plot(x_MIC, Rmax_MIC)`.

5 In the **Maximum Inscribed Circle** toolbar, click **Plot**.

MIC (5)

1 In the **Model Builder** window, right-click **Roundness Error** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, type `MIC (5)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Maximum Inscribed Circle/ Solution 6 (sol6)**.

4 From the **Parameter selection (angle)** list, choose **Last**.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`x_MIC`	`m`	`Center, x-coordinate`
`y_MIC`	`m`	`Center, y-coordinate`
`Rmin_MIC`	`m`	`Radius, minimum`
`Rmax_MIC`	`m`	`Radius, maximum`
`Rmax_MIC-Rmin_MIC`	`m`	`Roundness error`

6 In the **Roundness Error** toolbar, click **Evaluate**.