



Model created in COMSOL Multiphysics 6.4

# Postbuckling Analysis Using an Incremental Arc Length Method

## Introduction

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Buckling is a phenomenon that can cause sudden failure of a structure.

A linear buckling analysis predicts the critical buckling load. Such an analysis, however, does not give any information about what happens at loads higher than the critical load. It also often overpredicts the load-carrying capacity of the structure. Tracing the solution after the critical load is called a *postbuckling analysis*.

In order to accurately determine the critical buckling load or predict the postbuckling behavior, you can use the nonlinear solver and ramp up the applied load to compute the structure deformation. The buckling load can then be based on when a certain, not acceptable, deformation is reached.

Once the critical buckling load has been reached, it can happen that the structure undergoes a sudden large deformation into a new stable configuration. This is known as a snap-through phenomenon. A snap-through process cannot be simulated using prescribed load in a standard nonlinear static solver because the problem becomes numerically singular. Physically speaking, it is a highly transient problem as the structure “jumps” from one state to another. For simple cases with a single point load, it is often possible to replace the point load with a prescribed displacement and then measure the reaction force instead.

For more general problems, the postbuckling solution must however be tracked using more sophisticated methods, as shown in this example

Figure 1 shows the variation of load versus the displacement for such a difficult case. It illustrates the possible computational problem by using either a load control (path A) or a displacement control (path B).

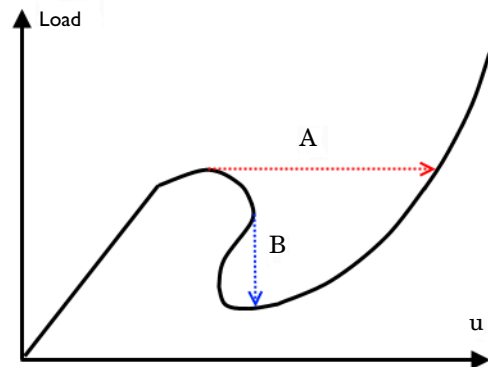


Figure 1: Load versus displacement in snap-through buckling.

The shell structure in this example has a behavior similar to this.

In this example, an incremental arc-length method combined with cubic extrapolation is used for postbuckling analysis of a hinged cylindrical panel subjected to a point load at its center. The results of this example can be compared with the similar Application Library example [Postbuckling Analysis of a Hinged Cylindrical Shell](#) which uses the global equation approach based on the monotonically increasing average displacement.

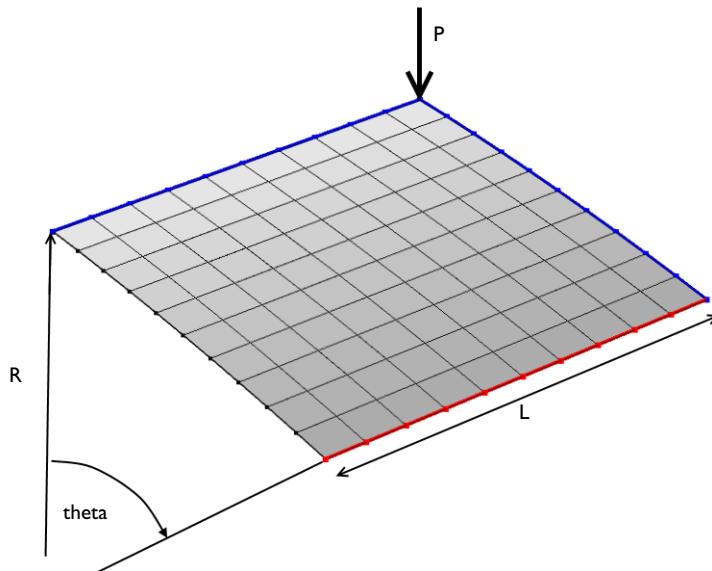
### *Model Definition*

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The model studied here is a benchmark for a hinged cylindrical panel subjected to a point load at its center; see [Ref. 1](#).

- The radius of the cylinder is  $R = 2.54$  m and all edges have a length of  $2L = 0.508$  m. The angular span of the panel is thus 0.2 radians. The panel thickness is  $th = 6.35$  mm.
- The straight edges are hinged.
- In the study the variation of the panel center vertical displacement with respect to the change of the applied load is of interest.

Due to the double symmetry, only one quarter of the geometry is modeled as shown in [Figure 2](#). The blue lines show the symmetry edge conditions, while the red line shows the location of the hinged edge condition.



*Figure 2: Problem description.*

In general, you should be careful with using symmetry in buckling problems, because nonsymmetric solutions may exist.

### *Arc Length Method*

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The *incremental arc length with cubic extrapolation method* used in this example is proposed in Ref. 2, and explained below. Let  $X^0, X^1, X^2, X^3$  be the last four converged solution vectors fulfilling equilibrium, where

$$X^i = \{u_1^k, \dots, u_n^k, v_1^k, \dots, v_n^k, w_1^k, \dots, w_n^k, p^k\}$$

and  $i$  is the index of the equilibrium step. An initial guess vector  $X^4$  for the next solution is given by the cubic extrapolation fit as

$$X^4 = \sum_{i=0}^3 \left( \prod_{j=0, j \neq i}^3 \frac{y_4 - y_j}{y_i - y_j} \right) X^i$$

where  $y$  is the arc length function. The previous arc length functions are given by  $y_0 = 0, y_1 = A_1, y_2 = y_1 + A_2, y_3 = y_2 + A_3, y_4 = y_3 + \Delta A$ . Here, we choose  $\Delta A = A_3$ . The secant arc lengths in the parametric space  $A_i$  are given by the Euclidean norm of differences as

$$A_i = |X^i - X^{i-1}|$$

The vector  $X^4$  provides the load value for next iteration as well as the initial guess for the displacements. With this method, it is possible to trace the equilibrium path automatically, including the snap-through behavior.

The  $X^i$  vectors in this method contain a combination of displacements and load. In order to avoid ill conditioning of the method, it is advisable to scale to load appropriately. In this example, a scale factor of  $10^5$  is used for the load.

### *Results*

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In Figure 3 you can see the applied load as a function of the panel center displacement. The figure shows clearly a nonunique solution for a given applied load (between  $-400$  N

to 600 N) or a given displacement (between 14.4 mm and 17 mm). The result matches the reference results computed using the global equation approach.

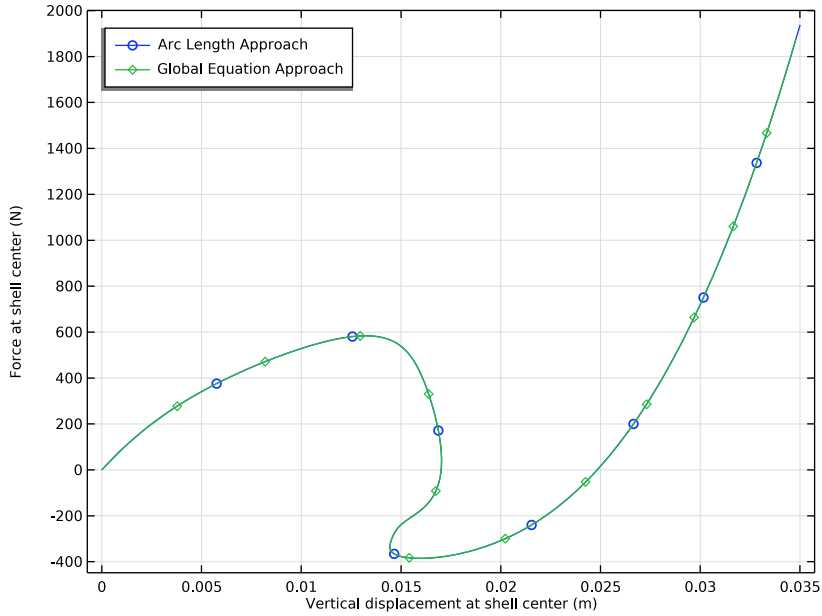


Figure 3: Applied load versus panel center displacement.

As shown in Table 1, the results agree well with the target data from Ref. 1.

TABLE 1: COMPARISON BETWEEN TARGET AND COMPUTED DATA.

Applied Load (N)	Displacement target (mm)	Displacement computed (mm)
155.1	1.846	1.8531
574.2	11.904	12.051
485.1	15.501	15.563
24.9	17.008	17.029
-300.3	14.520	14.536
-381.3	16.961	16.785
-1.8	24.824	24.818
1469.4	33.388	33.345

## *Notes About the COMSOL Implementation*

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The main feature of this model is that a limit point instability occurs at the buckling load. Neither a load control, nor a point displacement control, would be able to track the jump between the stable solution paths (see [Figure 1](#)). To solve this type of problem one needs either of the approaches:

- Find a proper parameter that increases monotonically. Then, use a **Global Equation** node where the load is made a function of this monotonically increasing parameter. The Application Library example [Postbuckling Analysis of a Hinged Cylindrical Shell](#) demonstrates this approach.
- Use an arc length method. The current example demonstrate this approach, which is useful if you cannot find a suitable monotonically increasing control parameter.

The arc length method is not built-in, so you have to implement yourself, using a **Model Method**. The code used in this example is fairly general, so it should be possible to copy it into other models, and reuse it with no or small modifications.

## *Reference*

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1. K.Y. Sze, X.H. Liua, and S.H. Lob, “Popular Benchmark Problems for Geometric Nonlinear Analysis of Shells,” *Finite Element in Analysis and Design*, vol. 40, issue 11, pp. 1551–1569, 2004.
2. Y.K. Cheung, and S.H. Chen, “Application of the Incremental Harmonic Balance Method to Cubic Non-linearity Systems,” *Journal of Sound and Vibration*, vol. 140, pp. 273-286, 1990.

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**Application Library path:** Structural\_Mechanics\_Module/  
Verification\_Examples/postbuckling\_shell\_arclength

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## *Modeling Instructions*


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### **ROOT**

In this example, you will start from an existing model in the Structural Mechanics Module.

### **APPLICATION LIBRARIES**

- 1 From the **File** menu, choose **Application Libraries**.

- 2 In the **Application Libraries** window, select **Structural Mechanics Module** > **Verification Examples** > **postbuckling\_shell** in the tree.
- 3 Click  **Open**.

### COMPONENT 1 (COMP1)


Remove the **Postbuckling Study**. Start by copying the evaluation group results to a table for future comparison.

### RESULTS

*Evaluation Group: Force vs. Displacement*

In the **Model Builder** window, expand the **Component 1 (comp1)** node, then click **Results** > **Evaluation Group: Force vs. Displacement**.

*Table: Reference Result*

- 1 In the **Evaluation Group: Force vs. Displacement** toolbar, click  **Copy to Table**.
- 2 In the **Settings** window for **Table**, type **Table: Reference Result** in the **Label** text field.

### POSTBUCKLING STUDY

In the **Model Builder** window, right-click **Postbuckling Study** and choose **Delete**.

Modify the parameter list.

### GLOBAL DEFINITIONS

*Parameters 1*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
p	0	0	Dimensionless load
p_scaleFactor	1E5[N]	1E5 N	Load scale factor
P	p*p_scaleFactor	0 N	Total load

### DEFINITIONS

*Average 1 (aveop1)*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Definitions** node.

2 Right-click **Component 1 (comp1)** > **Definitions** > **Average 1 (aveop1)** and choose **Delete**.

Delete the **Global Equation** node from the physics.

## **SHELL (SHELL)**


### *Global Equations 1 (ODE1)*

1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Shell (shell)** node.

2 Right-click **Component 1 (comp1)** > **Shell (shell)** > **Global Equations 1 (ODE1)** and choose **Delete**.

Create model methods to run the postbuckling study with the arc length method. Note that the method editor is only available in the Windows® version of the COMSOL Desktop.

## **APPLICATION BUILDER**

In the **Home** toolbar, click  **Application Builder**.

## **METHODS**

### *codeUtil*

1 In the **Home** toolbar, click  **More Libraries** and choose **Utility Class**.

2 Right-click **util1** and choose **Rename**.

3 In the **Rename Utility Class** dialog, type **codeUtil** in the **New name** text field.

4 Click **OK**.

5 Right-click **codeUtil** and choose **Edit**.

6 Copy the following code into the **codeUtil** window:

```
//This utility creates the requested methods

//Method to remove autocreated model nodes
public static void removeAutoCreatedNodes() {
    if (contains(model.study().tags(), "std_in"))
        model.study().remove("std_in");
    if (contains(model.study().tags(), "std_c"))
        model.study().remove("std_c");
    if (contains(model.study().tags(), "std_p"))
        model.study().remove("std_p");
    if (contains(model.sol().tags(), "sol_1"))
        model.sol().remove("sol_1");
    if (contains(model.sol().tags(), "sol_2"))
        model.sol().remove("sol_2");
    if (contains(model.sol().tags(), "sol_3"))
        model.sol().remove("sol_3");
    if (contains(model.sol().tags(), "sol_4"))
```

```

        model.sol().remove("sol_4");
        if (contains(model.result().dataset().tags(), "dset_c"))
            model.result().dataset().remove("dset_c");
        if (contains(model.result().evaluationGroup().tags(), "eg_c"))
            model.result().evaluationGroup().remove("eg_c");
    }

    //Method to create a placeholder study for the initial guess
    public static void createPlaceholderStudyForInitialGuess() {
        Study study = model.study().create("std_in");
        String stdLabel = "Placeholder Study for Initial Guess";
        boolean isLabelExists = false;
        for (Study std : model.study()) {
            if (std.label().equals(stdLabel))
                isLabelExists = true;
        }
        if (!isLabelExists)
            study.label(stdLabel);
        study.create("stat_in", "Stationary");
        study.feature("stat_in").set("geometricNonlinearity", true);
        SolverSequence solin = model.sol().create("sol_in");
        solin.createAutoSequence("std_in");
        solin.attach("std_in");
    }

    //Method to create a study for the postbuckling analysis
    public static void createPostbucklingStudy() {
        Study study = model.study().create("std_c");
        String stdLabel = "Postbuckling Study";
        boolean isLabelExists = false;
        for (Study std : model.study()) {
            if (std.label().equals(stdLabel))
                isLabelExists = true;
        }
        if (!isLabelExists)
            study.label(stdLabel);
        study.create("stat_c", "Stationary");
        study.feature("stat_c").set("geometricNonlinearity", true);
        SolverSequence solc = model.sol().create("sol_c");
        solc.createAutoSequence("std_c");
        solc.attach("std_c");
    }

    //Method to create a placeholder study for the parametric solution
    public static void createPlaceholderStudyForParametricSolution(String p) {
        Study study = model.study().create("std_p");
        String stdLabel = "Placeholder Study for Parametric Solution";
        boolean isLabelExists = false;
        for (Study std : model.study()) {
            if (std.label().equals(stdLabel))
                isLabelExists = true;
        }
        if (!isLabelExists)
            study.label(stdLabel);
        study.create("stat_p", "Stationary");
    }

```

```

SolverSequence solp = model.sol().create("sol_p");
solp.createAutoSequence("std_p");
solp.attach("std_p");
StudyFeature studyFeature = model.study("std_p").feature("stat_p");
studyFeature.set("geometricNonlinearity", true);
studyFeature.set("useparam", true);
studyFeature.setIndex("pname", p, 0);
studyFeature.setIndex("plistarr", "1 2", 0);
studyFeature.setIndex("punit", "N", 0);
}

//Method to create an evaluation group
public static void createEvaluationGroup(String globalVariable) {
    DatasetFeature dsetc = model.result().dataset().create("dset_c", "Solution");
    dsetc.set("solution", "sol_c");
    EvaluationGroupFeature eg = model.result().evaluationGroup().create("eg_c",
"EvaluationGroup");
    eg.set("data", "dset_c");
    eg.create("gev1", "EvalGlobal");
    eg.feature("gev1").setIndex("expr", globalVariable, 0);
    eg.set("includeparameters", false);
}

//Method for creating a stop condition in the solver
public static boolean stopCondition(double globalVariableMin, double
globalVariableMax) {
    boolean stop = false;
    EvaluationGroupFeature eg = model.result().evaluationGroup("eg_c");
    eg.run();
    double[][] globalVariable = eg.getReal();
    if (globalVariable[0][0] > globalVariableMax || globalVariable[0][0] <
globalVariableMin)
        stop = true;
    return stop;
}

```

## NEW METHOD

- 1 In the **Application Builder** window, right-click **Methods** and choose **New Method**.
- 2 In the **New Method** dialog, type `postBucklingWithArclength` in the **Name** text field.
- 3 Click **OK**.

### *postBucklingWithArclength*

- 1 In the **Application Builder** window, under **Methods** click **postBucklingWithArclength**.
- 2 Copy the following code into the **postBucklingWithArclength** window:

```

clearDebugLog();
long startTime = timeStamp();
if (!contains(model.param().varnames(), p))
    error("The load parameter with the given name does not exist in the parameter
list of the model, check the model.");

```

```

if (p.equals("") || deltap == 0 || numberOfIter == 0 || (isStopConditionGiven
&& globalVariable.equals("")))
    error("The inputs assigned to the model method are either empty or zero, check
the model.");

if (isStopConditionGiven && globalVariableMin == globalVariableMax)
    error("The lower and upper limits of the global variable in stop condition
should not be the same, check the model.");

model.param().set(p, "0");
//Remove the autocreated model nodes
codeUtil.removeAutoCreatedNodes();
//Create a placeholder study to store the intial guess
codeUtil.createPlaceholderStudyForInitialGuess();
//Create a postbuckling study
codeUtil.createPostbucklingStudy();
//Create a placeholder study to store the parametric solutions
codeUtil.createPlaceholderStudyForParametricSolution(p);
//Create an evaluation group for the evaluation of the stop condition
if (isStopConditionGiven)
    codeUtil.createEvaluationGroup(globalVariable);

//First iteration
model.sol("sol_c").runAll();
model.sol("sol_p").setU(1, model.sol("sol_c").getU());
model.sol("sol_c").copySolution("sol_1");
//Second iteration
model.param().set(p, toString(deltap));
model.sol("sol_c").runAll();
model.sol("sol_p").setU(2, model.sol("sol_c").getU());
model.sol("sol_c").copySolution("sol_2");
//Third iteration
model.param().set(p, toString(2*deltap));
model.sol("sol_c").runAll();
model.sol("sol_p").setU(3, model.sol("sol_c").getU());
model.sol("sol_c").copySolution("sol_3");
//Fourth iteration
model.param().set(p, toString(3*deltap));
model.sol("sol_c").runAll();
model.sol("sol_p").setU(4, model.sol("sol_c").getU());
model.sol("sol_c").copySolution("sol_4");

//Next iterations by the arclength method
double[] parametricFull = new double[4+numberOfIter];
parametricFull[0] = 0;
parametricFull[1] = 1*deltap;
parametricFull[2] = 2*deltap;
parametricFull[3] = 3*deltap;
int pfinal = 4+numberOfIter;
model.study("std_c").feature("stat_c").set("useinitsol", true);
model.study("std_c").feature("stat_c").set("initmethod", "sol");
model.study("std_c").feature("stat_c").set("initstudy", "std_in");
model.study("std_c").feature("stat_c").set("initsol", "sol_in");
int sizeOfSolution = pfinal;

```

```

//Arclength method
for (int ii = 4; ii < pfinal; ii++) {
    double[] w_3 = remove(model.sol("sol_4").getU(), 0);
    double[] w_2 = remove(model.sol("sol_3").getU(), 0);
    double[] w_1 = remove(model.sol("sol_2").getU(), 0);
    double[] w_0 = remove(model.sol("sol_1").getU(), 0);

    double p_3 = parametricFull[ii-1];
    double p_2 = parametricFull[ii-2];
    double p_1 = parametricFull[ii-3];
    double p_0 = parametricFull[ii-4];

    double[] W_3 = append(w_3, p_3);
    double[] W_2 = append(w_2, p_2);
    double[] W_1 = append(w_1, p_1);
    double[] W_0 = append(w_0, p_0);

    double As3 = 0;
    double As2 = 0;
    double As1 = 0;
    for (int i = 0; i < W_3.length; i++) {
        As3 = As3+(W_3[i]-W_2[i])*(W_3[i]-W_2[i]);
        As2 = As2+(W_2[i]-W_1[i])*(W_2[i]-W_1[i]);
        As1 = As1+(W_1[i]-W_0[i])*(W_1[i]-W_0[i]);
    }
    double A3 = Math.sqrt(As3);
    double A2 = Math.sqrt(As2);
    double A1 = Math.sqrt(As1);
    double deltaA = A3;
    double yw0 = 0.0;
    double yw1 = yw0+A1;
    double yw2 = yw1+A2;
    double yw3 = yw2+A3;
    double yw4 = yw3+deltaA;
    double t0 = ((yw4-yw1)/(yw0-yw1))*((yw4-yw2)/(yw0-yw2))*((yw4-yw3)/(yw0-
yw3));
    double t1 = ((yw4-yw0)/(yw1-yw0))*((yw4-yw2)/(yw1-yw2))*((yw4-yw3)/(yw1-
yw3));
    double t2 = ((yw4-yw0)/(yw2-yw0))*((yw4-yw1)/(yw2-yw1))*((yw4-yw3)/(yw2-
yw3));
    double t3 = ((yw4-yw0)/(yw3-yw0))*((yw4-yw1)/(yw3-yw1))*((yw4-yw2)/(yw3-
yw2));
    double[] w_ini = new double[W_3.length];
    for (int i = 0; i < W_3.length; i++) {
        w_ini[i] = t0*W_0[i]+t1*W_1[i]+t2*W_2[i]+t3*W_3[i];
    }
    //Initial values
    double[] W_ini = new double[w_ini.length-1];
    for (int i = 0; i < w_ini.length-1; i++) {
        W_ini[i] = w_ini[i];
    }
    //Current value for the load
    double p_ini = w_ini[w_ini.length-1];

    parametricFull[ii] = p_ini;
}

```

```

double[] W_ini_up = insert(W_ini, 1, 0);

//Setting up the initial guess
model.sol("sol_in").setU(W_ini_up);
model.sol("sol_in").createSolution();

//Load update
model.param().set(p, p_ini);

//Computing the current solution
model.sol("sol_c").runAll();

//Creating a parametric solution based on the current solution
model.sol("sol_p").setU(ii+1, model.sol("sol_c").getU());

//Updating the four last converged iterations
model.sol("sol_2").copySolution("sol_1");
model.sol("sol_3").copySolution("sol_2");
model.sol("sol_4").copySolution("sol_3");
model.sol("sol_c").copySolution("sol_4");

//Debug log
debugLog("Increment "+toString(ii));
debugLog("The load is "+toString(p_ini));

//Evaluating the stop condition
if (isStopConditionGiven) {
    if (codeUtil.stopCondition(globalVariableMin, globalVariableMax)) {
        debugLog("Stop condition fulfilled for load "+toString(p_ini));
        pfinal = ii;
        sizeOfSolution = ii+1;
    }
}
}

double[] parametric = new double[sizeOfSolution];
for (int ii = 0; ii < sizeOfSolution; ii++) {
    parametric[ii] = parametricFull[ii];
}
model.sol("sol_p").setPVals(parametric);
model.sol("sol_p").createSolution();
if (contains(model.result().dataset().tags(), "dset_c"))
    model.result().dataset().remove("dset_c");
long endTime = timeStamp();
long totalTime = (endTime-startTime);
String timeString = formattedTime(totalTime, "hr:min:sec");
debugLog("The solution time is "+timeString);

```

- 3 In the **Settings** window for **Method**, locate the **Inputs and Output** section.
- 4 Find the **Inputs** subsection. Click **+** **Add** seven times.

5 In the table, enter the following settings:


Name	Type	Default	Description	Unit
p	String		Dimensionless Load Parameter	
deltap	Double	0	Dimensionless Load Step size	
numberOfIter	Integer	0	Number of Iterations	
isStopConditionGiven	Boolean		Activate Stop Condition	
globalVariable	String		Global Variable in Stop Condition	
globalVariableMin	Double	0	Lower Limit of Global Variable	
globalVariableMax	Double	0	Upper Limit of Global Variable	

## METHODS

In the **Home** toolbar, click  **Model Builder** to switch to the main desktop.

Call the method **postBucklingWithArclength** in order to run it.

## MODEL BUILDER

1 In the **Home** toolbar, click  **Model Builder**.

2 In the **Developer** toolbar, click  **Method Call** and choose **postBucklingWithArclength**.

## GLOBAL DEFINITIONS

*Run Postbuckling Study with Arclength Method*

1 In the **Model Builder** window, under **Global Definitions** click **PostBucklingWithArclength I**.


2 In the **Settings** window for **Method Call**, type Run Postbuckling Study with Arclength Method in the **Label** text field.

3 In the **Tag** text field, type postBucklingWithArclength.

Before running the **Run Postbuckling Study with Arclength Method**, assign correct inputs to the method.


4 Locate the **Inputs** section. In the **Dimensionless Load Parameter** text field, type p.

5 In the **Dimensionless Load Step size** text field, type 0.0003.

- 6 In the **Number of Iterations** text field, type 200.
- 7 Select the **Activate Stop Condition** checkbox.
- 8 In the **Global Variable in Stop Condition** text field, type `w_center`.
- 9 In the **Upper Limit of Global Variable** text field, type 0.035.
- 10 Click  **Run**.


## METHODS

*postBucklingWithArclength*

In the **Home** toolbar, click  **Model Builder**.

## RESULTS

*Stress*

- 1 In the **Results** toolbar, click  **3D Plot Group**.
- 2 In the **Settings** window for **3D Plot Group**, type **Stress** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Placeholder Study for Parametric Solution/Solution 3 (sol\_p)**.


*Surface 1*

- 1 Right-click **Stress** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type `shell.misesGp`.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **Prism**.

*Deformation 1*

- 1 Right-click **Surface 1** and choose **Deformation**.
- 2 In the **Settings** window for **Deformation**, locate the **Scale** section.
- 3 Select the **Scale factor** checkbox. In the associated text field, type 1.

*Evaluation Group: Force vs. Displacement*


- 1 In the **Results** toolbar, click  **Evaluation Group**.
- 2 In the **Settings** window for **Evaluation Group**, type **Evaluation Group: Force vs. Displacement** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Placeholder Study for Parametric Solution/Solution 3 (sol\_p)**.

### *Point Evaluation 1*


- 1 Right-click **Evaluation Group: Force vs. Displacement** and choose **Point Evaluation**.
- 2 Select Point 4 only.
- 3 In the **Settings** window for **Point Evaluation**, locate the **Expressions** section.
- 4 In the table, enter the following settings:

Expression	Unit	Description
w_center	m	Vertical displacement at shell center
P	N	Total load

### *Evaluation Group: Force vs. Displacement*

- 1 In the **Model Builder** window, click **Evaluation Group: Force vs. Displacement**.
- 2 In the **Settings** window for **Evaluation Group**, click to expand the **Format** section.
- 3 From the **Include parameters** list, choose **Off**.
- 4 In the **Evaluation Group: Force vs. Displacement** toolbar, click  **Evaluate**.

### *Force vs. Displacement*

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Force vs. Displacement in the **Label** text field.
- 3 Locate the **Plot Settings** section.
- 4 Select the **x-axis label** checkbox. In the associated text field, type Vertical displacement at shell center (m).
- 5 Select the **y-axis label** checkbox. In the associated text field, type Force at shell center (N).
- 6 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

### *Table Graph 1*

- 1 Right-click **Force vs. Displacement** and choose **Table Graph**.
- 2 In the **Settings** window for **Table Graph**, locate the **Data** section.
- 3 From the **Source** list, choose **Evaluation group**.
- 4 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.
- 5 From the **Positioning** list, choose **Interpolated**.
- 6 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 7 From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

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**Legends**

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Arc Length Approach

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*Table Graph 2*

- 1 Right-click **Table Graph 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Table Graph**, locate the **Data** section.
- 3 From the **Source** list, choose **Table**.
- 4 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Diamond**.
- 5 In the **Number** text field, type 12.
- 6 Locate the **Legends** section. In the table, enter the following settings:

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**Legends**

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Global Equation Approach

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*Force vs. Displacement*

- 1 In the **Model Builder** window, click **Force vs. Displacement**.
- 2 In the **Force vs. Displacement** toolbar, click  **Plot**.