



Model created in COMSOL Multiphysics 6.4

# P-N Junction 1D

This model sets up a simple 1D benchmark of a p–n junction, or semiconductor diode, based on an example given in [Ref. 1](#).

## Introduction

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A semiconductor diode consists of two regions with different doping: a p-type region with a dominant concentration of holes, and an n-type region with a dominant concentration of electrons. The anode contact is adjacent to the p-type region, and the cathode connects to the n-type region. Impurities that the manufacturing process adds to the semiconductor material determine each region’s doping type. Donor impurities add additional electrons to the material conduction band and create n-type material. Acceptor impurities remove electrons from the valence band and create p-type material.

It is possible to derive a semiconductor model from Maxwell’s equations and the Boltzmann equation for carrier transport. The three basic semiconductor equations (for a stationary problem) are

$$\begin{aligned}\nabla \cdot (\epsilon \nabla V) &= -q(p - n + N_D^+ - N_A^-) \\ \nabla \cdot \mathbf{J}_n &= -qR_{\text{SRH}} \\ \nabla \cdot \mathbf{J}_p &= qR_{\text{SRH}}\end{aligned}$$

where  $\epsilon$  is the dielectric permittivity of the semiconductor;  $V$  is the electric potential;  $p$  and  $n$  are the electron and hole concentrations, respectively;  $N_D^+$  and  $N_A^-$  are the concentrations of ionized donors and acceptors, respectively;  $\mathbf{J}_n$  and  $\mathbf{J}_p$  are the electron and hole currents, respectively; and  $R_{\text{SRH}}$  is the Shockley–Read–Hall recombination rate (see below for a description; in this case this term is assumed to be the only source of electron and hole recombination).

The electron and hole currents can be expressed in terms of  $V$ ,  $n$ , and  $p$  in the following manner (assuming an isothermal, nondegenerate semiconductor with a constant band structure):

$$\begin{aligned}\mathbf{J}_n &= -nq\mu_n \nabla V + \mu_n k_B T \nabla n \\ \mathbf{J}_p &= -pq\mu_p \nabla V - \mu_p k_B T \nabla p\end{aligned}$$

where  $\mu_n$  and  $\mu_p$  are the carrier mobilities,  $k_B$  is Boltzmann’s constant, and  $T$  is the absolute temperature. In this model, the mobility is a complex-valued function of the temperature and the donor and acceptor concentrations, as defined in [Ref. 1](#).

The term  $R_{\text{SRH}}$  gives the rate at which electrons recombine (by a mechanism known as Shockley–Read–Hall recombination, which is often dominant in silicon). This recombination process involves traps in the forbidden band gap of the semiconductor. The recombination rate due to this mechanism is given by

$$R_{\text{SRH}} = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

where  $n_i$  is the intrinsic carrier concentration (that is, the carrier concentration in an undoped semiconductor),  $\tau_n$  and  $\tau_p$  are the carrier lifetimes, and  $n_1$  and  $p_1$  are parameters related to the trap energy level. If the trap level is located at the middle of the band gap (which is assumed in this model), then  $n_1$  and  $p_1$  equal  $n_i$ .

### *Model Definition*

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This model simulates the behavior of a p–n junction under reverse, equilibrium, and forward bias. The modeled junction has a length of 5  $\mu\text{m}$  and a net doping concentration of  $1 \times 10^{15} \text{ cm}^{-3}$  for both the p- and n-doped side. A Shockley–Read–Hall recombination feature is also added to the model in order to simulate recombination usually observed in indirect band-gap semiconductor such as silicon, which is the material used in this model. The model uses the material parameters used in [Ref. 1](#) and compares the carrier concentration profiles obtained from the computation with those obtained in the reference under different biasing conditions ( $-4 \text{ V}$ ,  $0 \text{ V}$ , and  $0.5 \text{ V}$ ). Two different discretization methods are used to solve the model: the Finite Element Log Formation discretization and the Finite Volume discretization. The results of these two computation methods are found to be in good agreement.

### *Results and Discussion*

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[Figure 1](#) shows the electric potential in the junction for different biasing conditions. A good agreement is shown between the finite element, finite volume, and reference for all biasing conditions. [Figure 2](#), [Figure 4](#), and [Figure 6](#) show the energy diagrams for each biasing conditions. These figures display a perfect agreement between the finite element (using streamline-diffusion stabilization) and finite volume computations. Comparing the carrier concentrations also shows a very good agreement between the reference, finite element, and finite volume computations — see [Figure 3](#), [Figure 5](#), and [Figure 7](#).

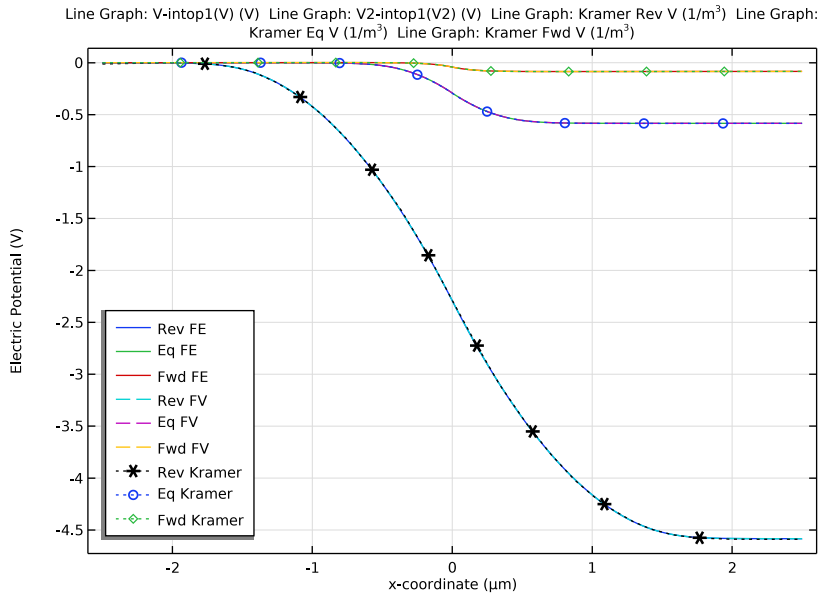


Figure 1: Electric potential obtained using the finite element (FE) and finite volume (FV) methods under different bias, that is, reverse (Rev), equilibrium (Eq), and forward (Fwd) bias. The results are compared to those calculated in Ref. 1 (Kramer)

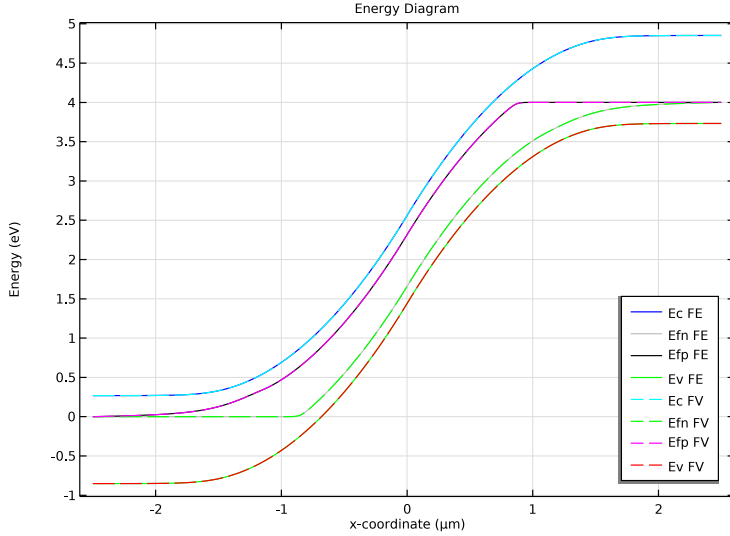


Figure 2: Energy diagram obtained using the finite element (FE) and finite volume (FV) methods under reverse bias (-4 V).

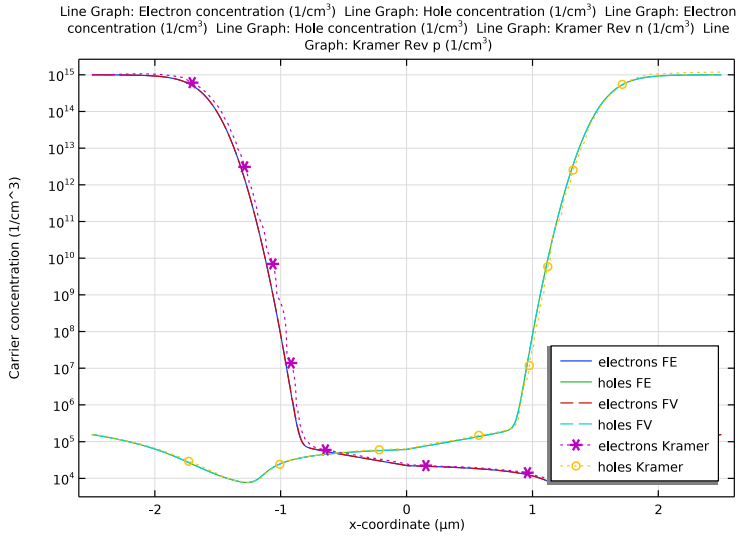


Figure 3: Electron and hole concentrations obtained using the finite element (FE) and finite volume (FV) methods under reverse bias (-4 V). The profiles are compared to those calculated in Ref. 1 (Kramer).

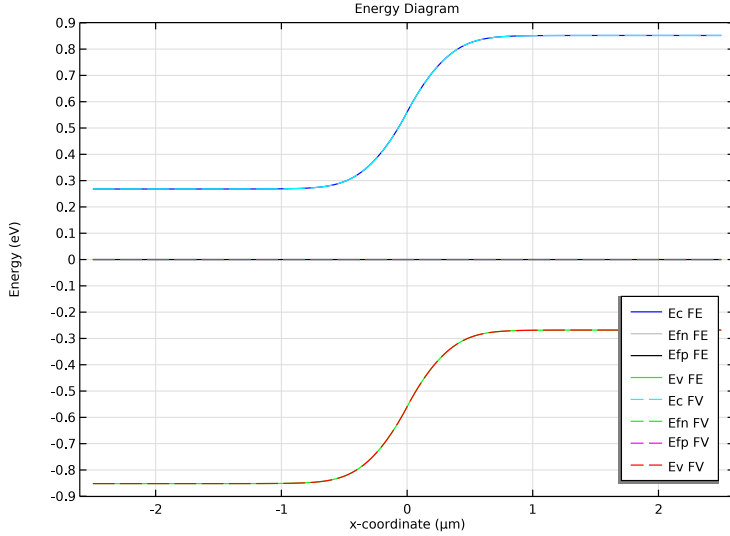


Figure 4: Energy diagram obtained using the finite element (FE) and finite volume (FV) methods at equilibrium (0 V).

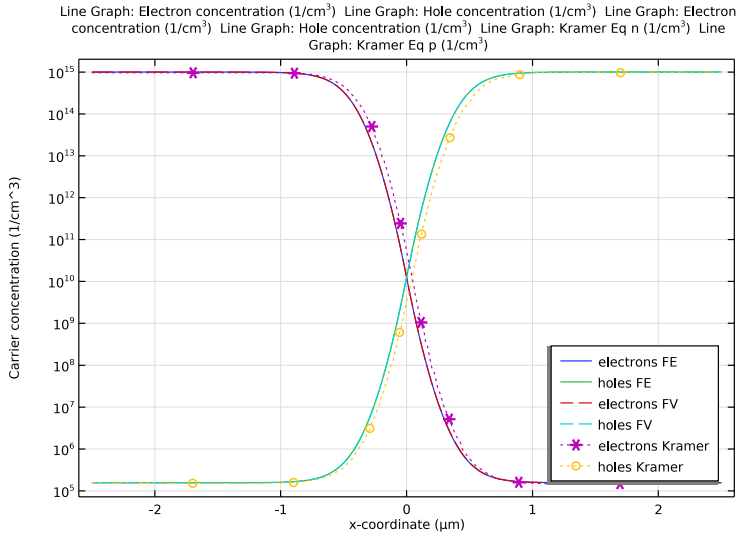


Figure 5: Electron and hole concentrations obtained using the finite element (FE) and finite volume (FV) methods at equilibrium (0 V). The profiles are compared to those calculated in Ref. 1 (Kramer).

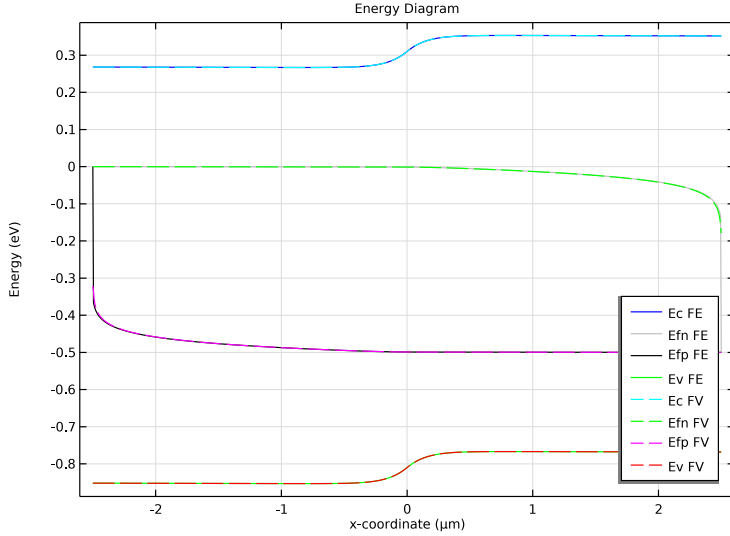


Figure 6: Energy diagram obtained using the finite element (FE) and finite volume (FV) methods under forward bias (0.5 V).

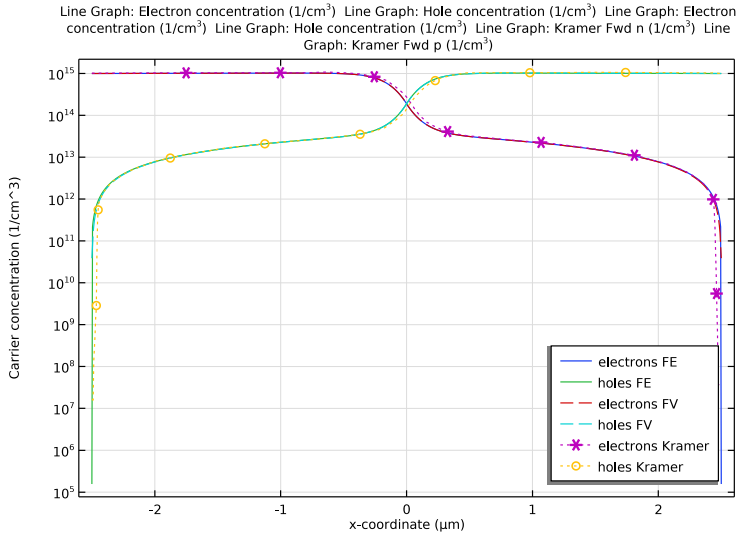


Figure 7: Electron and hole concentrations obtained using the finite element (FE) and finite volume (FV) methods under forward bias (0.5 V). The profiles are compared to those calculated in Ref. 1 (Kramer).

## Reference

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1. K.M. Kramer and W.N.G. Hitchon, *Semiconductor Devices a Simulation Approach*, Prentice Hall, Upper Saddle River, NJ, 1997.

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**Application Library path:** Semiconductor\_Module/Verification\_Examples/  
pn\_junction\_1d


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## Modeling Instructions



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From the **File** menu, choose **New**.

### NEW

In the **New** window, click  **Model Wizard**.

### MODEL WIZARD

- 1 In the **Model Wizard** window, click **ID**.
- 2 In the **Select Physics** tree, select **Semiconductor** > **Semiconductor (semi)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies** > **Stationary**.
- 6 Click  **Done**.

### SEMICONDUCTOR (SEMI)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Semiconductor (semi)**.
- 2 In the **Settings** window for **Semiconductor**, click to expand the **Discretization** section.
- 3 From the **Formulation** list, choose **Finite element, log formulation (linear shape function)**.

### GEOMETRY 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose  $\mu\text{m}$ .


### Interval 1 (i1)

- 1 Right-click **Component 1 (comp1)** > **Geometry 1** and choose **Interval**.

- 2 In the **Settings** window for **Interval**, locate the **Interval** section.
- 3 In the table, enter the following settings:


<b>Coordinates (<math>\mu\text{m}</math>)</b>
-2.5
2.5

#### *Point 1 (pt1)*

- 1 In the **Model Builder** window, right-click **Geometry 1** and choose **Point**.
- 2 In the **Settings** window for **Point**, click  **Build All Objects**.



### **GLOBAL DEFINITIONS**

#### *Parameters 1*



- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_parameters.txt.

### **DEFINITIONS**

#### *Variables 1*

- 1 In the **Definitions** toolbar, click  **Local Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_variables.txt.

#### *Kramer Eq V*

- 1 In the **Definitions** toolbar, click  **Interpolation**.
- 2 In the **Settings** window for **Interpolation**, locate the **Definition** section.
- 3 In the **Function name** text field, type kramer\_eq\_V.
- 4 Click  **Load from File**.
- 5 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_eq\_V.txt.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:



Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_eq_V	1/m <sup>3</sup>

8 In the **Label** text field, type Kramer Eq V.

*Kramer Fwd V*

- 1 In the **Definitions** toolbar, click  **Interpolation**.
- 2 In the **Settings** window for **Interpolation**, locate the **Definition** section.
- 3 In the **Function name** text field, type kramer\_fwd\_V.
- 4 Click  **Load from File**.
- 5 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_fwd\_V.txt.
- 6 Locate the **Units** section. In the **Argument** table, enter the following settings:



Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_fwd_V	1/m <sup>3</sup>

8 In the **Label** text field, type Kramer Fwd V.

*Kramer Rev V*

- 1 In the **Definitions** toolbar, click  **Interpolation**.
- 2 In the **Settings** window for **Interpolation**, locate the **Definition** section.
- 3 In the **Function name** text field, type kramer\_rev\_V.
- 4 Click  **Load from File**.
- 5 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_rev\_V.txt.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:

Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_rev_V	1/m^3

8 In the **Label** text field, type Kramer Rev V.

*Kramer Eq n*

1 In the **Definitions** toolbar, click  **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 Click  **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_eq\_n.txt.

5 In the **Function name** text field, type kramer\_eq\_n.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:

Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_eq_n	1/m^3

8 In the **Label** text field, type Kramer Eq n.

*Kramer Eq p*

1 In the **Definitions** toolbar, click  **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 Click  **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_eq\_p.txt.

5 In the **Function name** text field, type kramer\_eq\_p.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:

Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_eq_p	1/m <sup>3</sup>

8 In the **Label** text field, type Kramer Eq p.

*Kramer Fwd n*

1 In the **Definitions** toolbar, click  **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 In the **Function name** text field, type kramer\_fwd\_n.

4 Click  **Load from File**.

5 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_fwd\_n.txt.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:

Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_fwd_n	1/m <sup>3</sup>

8 In the **Label** text field, type Kramer Fwd n.

*Kramer Fwd p*

1 In the **Definitions** toolbar, click  **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 In the **Function name** text field, type kramer\_fwd\_p.

4 Click  **Load from File**.

5 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_fwd\_p.txt.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:

Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_fwd_p	1/m^3

8 In the **Label** text field, type Kramer Fwd p.

*Kramer Rev n*

1 In the **Definitions** toolbar, click  **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 In the **Function name** text field, type kramer\_rev\_n.

4 Click  **Load from File**.

5 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_rev\_n.txt.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:

Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_rev_n	1/m^3

8 In the **Label** text field, type Kramer Rev n.

*Kramer Rev p*

1 In the **Definitions** toolbar, click  **Interpolation**.

2 In the **Settings** window for **Interpolation**, locate the **Definition** section.

3 Click  **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file pn\_junction\_1d\_Kramer\_rev\_p.txt.

5 In the **Function name** text field, type kramer\_rev\_p.

6 Locate the **Units** section. In the **Argument** table, enter the following settings:


Argument	Unit
t	m

7 In the **Function** table, enter the following settings:

Function	Unit
kramer_rev_p	1/m <sup>3</sup>

8 In the **Label** text field, type Kramer Rev p.

*Integration 1 (intop1)*


- 1 In the **Definitions** toolbar, click  **Nonlocal Couplings** and choose **Integration**.
- 2 In the **Settings** window for **Integration**, locate the **Source Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundary 1 only.

## SEMICONDUCTOR (SEMI)


*Semiconductor Material Model 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1) > Semiconductor (semi)** click **Semiconductor Material Model 1**.
- 2 In the **Settings** window for **Semiconductor Material Model**, locate the **Model Input** section.
- 3 In the *T* text field, type T1.
- 4 Locate the **Mobility Model** section. From the  $\mu_n$  list, choose **User defined**. In the associated text field, type mu\_n.
- 5 From the  $\mu_p$  list, choose **User defined**. In the associated text field, type mu\_p.

*Trap-Assisted Recombination 1*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Trap-Assisted Recombination**.
- 2 In the **Settings** window for **Trap-Assisted Recombination**, locate the **Domain Selection** section.
- 3 From the **Selection** list, choose **All domains**.

*Analytic Doping Model 1*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Analytic Doping Model**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Analytic Doping Model**, locate the **Impurity** section.

4 From the **Impurity type** list, choose **Donor doping (n-type)**.

5 In the  $N_{D0}$  text field, type Nd.

#### *Analytic Doping Model 2*

1 In the **Physics** toolbar, click **Domains** and choose **Analytic Doping Model**.

2 Select Domain 2 only.

3 In the **Settings** window for **Analytic Doping Model**, locate the **Impurity** section.

4 In the  $N_{A0}$  text field, type Na.

#### *Metal Contact 1*

1 In the **Physics** toolbar, click **Boundaries** and choose **Metal Contact**.

2 Select Boundary 1 only.

#### *Metal Contact 2*

1 In the **Physics** toolbar, click **Boundaries** and choose **Metal Contact**.

2 In the **Settings** window for **Metal Contact**, locate the **Terminal** section.

3 In the  $V_0$  text field, type Va.

4 Select Boundary 3 only.

### **MATERIALS**

#### *Material 1 (mat1)*

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilon <sub>nr_iso</sub> ; epsilon <sub>nr<sub>ii</sub></sub> = epsilon <sub>nr_iso</sub> , epsilon <sub>nr<sub>ij</sub></sub> = 0	epsilon <sub>nr_param</sub>	I	Basic
Band gap	Eg0	Eg0_param	V	Semiconductor material
Electron affinity	chi0	chi0_param	V	Semiconductor material
Effective density of states, conduction band	Nc	Nc_param	I/m <sup>3</sup>	Semiconductor material

Property	Variable	Value	Unit	Property group
Effective density of states, valence band	Nv	Nv_param	1/m <sup>3</sup>	Semiconductor material
Electron lifetime, SRH	taun	taun_param	s	Shockley-Read-Hall recombination
Hole lifetime, SRH	taup	taup_param	s	Shockley-Read-Hall recombination

### SEMICONDUCTOR (SEMI)


In the **Model Builder** window, under **Component 1 (comp1)** right-click **Semiconductor (semi)** and choose **Copy**.

### SEMICONDUCTOR 2 (SEMI2)

- 1 In the **Model Builder** window, right-click **Component 1 (comp1)** and choose **Paste Semiconductor**.
- 2 In the **Messages from Paste** dialog, click **OK**.
- 3 In the **Settings** window for **Semiconductor**, locate the **Discretization** section.
- 4 From the **Formulation** list, choose **Finite volume (constant shape function)**.

### MESH 1

#### Edge 1

In the **Mesh** toolbar, click  **Edge**.

#### Distribution 1

- 1 Right-click **Edge 1** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 In the **Number of elements** text field, type 500.

Configure the first study. The finite element log formulation discretization requires the voltage applied to the device to be ramped on. This is achieved using the 'sweep' variable, which is ramped from 0 to 1 using the solver's continuation functionality. The voltage applied to the device,  $V_a$ , is calculated using  $V_a = \text{bias} * \text{sweep}$ , as can be seen in the parameters table within the Definitions node. The first study will be configured to apply three different biases, with each bias being ramped from 0 V to the intended value as 'sweep' increases from 0 to 1.

## STUDY 1 - FINITE ELEMENT LOG FORMULATION

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Study 1 - Finite Element Log Formulation in the **Label** text field.

### Step 1: Stationary

- 1 In the **Model Builder** window, under **Study 1 - Finite Element Log Formulation** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 3 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Semiconductor 2 (semi2)**.
- 4 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 5 From the **Sweep type** list, choose **All combinations**.
- 6 Click **+ Add**.
- 7 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
bias (Device bias)	-4 0 0.5	V

- 8 Click **+ Add**.
- 9 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
sweep (Sweep parameter for gradually turning on bias)	0 1	

- 10 In the **Study** toolbar, click **= Compute**.

## RESULTS

### Energy Levels (semi)

- 1 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 2 From the **Parameter selection (bias)** list, choose **First**.
- 3 From the **Parameter selection (sweep)** list, choose **Last**.

### Conduction Band Energy Level

- 1 In the **Model Builder** window, expand the **Energy Levels (semi)** node, then click **Conduction Band Energy Level**.

- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.
- 3 From the **Parameter** list, choose **Expression**.
- 4 In the **Expression** text field, type  $x$ .
- 5 Click to expand the **Legends** section. In the table, enter the following settings:

---

**Legends**

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$E_c$  FE

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*Electron Quasi-Fermi Energy Level*

- 1 In the **Model Builder** window, click **Electron Quasi-Fermi Energy Level**.
- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.
- 3 From the **Parameter** list, choose **Expression**.
- 4 In the **Expression** text field, type  $x$ .
- 5 Locate the **Legends** section. In the table, enter the following settings:

---

**Legends**

---

$E_{fn}$  FE

---

- 6 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Solid**.
- 7 From the **Color** list, choose **Gray**.

*Hole Quasi-Fermi Energy Level*

- 1 In the **Model Builder** window, click **Hole Quasi-Fermi Energy Level**.
- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.
- 3 From the **Parameter** list, choose **Expression**.
- 4 In the **Expression** text field, type  $x$ .
- 5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Solid**.
- 6 Locate the **Legends** section. In the table, enter the following settings:

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**Legends**

---

$E_{fp}$  FE

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*Valence Band Energy Level*

- 1 In the **Model Builder** window, click **Valence Band Energy Level**.
- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.

- 3 From the **Parameter** list, choose **Expression**.
- 4 In the **Expression** text field, type x.
- 5 Locate the **Legends** section. In the table, enter the following settings:

<b>Legends</b>	
EV	FE

#### *Energy Levels Reverse Bias*

- 1 In the **Model Builder** window, click **Energy Levels (semi)**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Legend** section.
- 3 From the **Position** list, choose **Lower right**.
- 4 In the **Label** text field, type Energy Levels Reverse Bias.

#### *Carrier Concentrations Reverse Bias*

- 1 In the **Model Builder** window, click **Carrier Concentrations (semi)**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Legend** section.
- 3 From the **Position** list, choose **Lower right**.
- 4 In the **Label** text field, type Carrier Concentrations Reverse Bias.
- 5 Locate the **Data** section. From the **Parameter selection (bias)** list, choose **First**.
- 6 From the **Parameter selection (sweep)** list, choose **Last**.

#### *Electron Concentration*

- 1 In the **Model Builder** window, expand the **Carrier Concentrations Reverse Bias** node, then click **Electron Concentration**.
- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.
- 3 From the **Parameter** list, choose **Expression**.
- 4 In the **Expression** text field, type x.
- 5 Locate the **Legends** section. In the table, enter the following settings:

<b>Legends</b>	
electrons	FE

#### *Hole Concentration*

- 1 In the **Model Builder** window, click **Hole Concentration**.
- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.
- 3 From the **Parameter** list, choose **Expression**.

- 4 In the **Expression** text field, type  $x$ .
- 5 Locate the **Legends** section. In the table, enter the following settings:

<b>Legends</b>
holes FE

*Line Graph 1*

- 1 In the **Model Builder** window, expand the **Results > Electric Potential (semi)** node, then click **Line Graph 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type  $V - \text{intop1}(V)$ .
- 4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 5 In the **Expression** text field, type  $x$ .
- 6 Locate the **Legends** section. Select the **Show legends** checkbox.
- 7 From the **Legends** list, choose **Manual**.
- 8 In the table, enter the following settings:

<b>Legends</b>
Rev FE
Eq FE
Fwd FE

*Electric Potential (semi)*

- 1 In the **Model Builder** window, click **Electric Potential (semi)**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Legend** section.
- 3 From the **Position** list, choose **Lower left**.
- 4 Locate the **Data** section. From the **Parameter selection (sweep)** list, choose **Last**.



*Net Dopant Concentration (semi)*

- 1 In the **Model Builder** window, click **Net Dopant Concentration (semi)**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 From the **Parameter selection (bias)** list, choose **First**.
- 4 From the **Parameter selection (sweep)** list, choose **First**.

- 5 In the **Net Dopant Concentration (semi)** toolbar, click  **Plot**.

In order to compare the finite element log formulation discretization with finite volume discretization, change the discretization and add a second study to re-solve the model with the new selection.

#### ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies > Stationary**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.


#### STUDY 2 - FINITE VOLUME

- 1 In the **Settings** window for **Study**, type Study 2 - Finite Volume in the **Label** text field.
- 2 Locate the **Study Settings** section. Clear the **Generate default plots** checkbox.  
Unlike in the previous study, the finite volume discretization does not require that the voltage be ramped up gradually. Therefore the desired values of 'Va' can be set directly in the auxiliary sweep.

##### *Step 1: Stationary*

- 1 In the **Model Builder** window, under **Study 2 - Finite Volume** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 3 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Semiconductor (semi)**.
- 4 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 5 Click **+ Add**.
- 6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Va (Applied voltage)	-4 0 0.5	V

- 7 In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Line Graph 2*

- 1 In the **Model Builder** window, under **Results > Electric Potential (semi)** right-click **Line Graph 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 4 Locate the **y-Axis Data** section. In the **Expression** text field, type  $V2 - \text{intop1}(V2)$ .
- 5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 6 Locate the **Legends** section. In the table, enter the following settings:

---

<b>Legends</b>
Rev FV
Eq FV
Fwd FV

---

### *Line Graph 3*

- 1 Right-click **Line Graph 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type  $\text{kramer\_rev\_V}(x)$ .
- 4 Locate the **Data** section. From the **Dataset** list, choose **Study 1 - Finite Element Log Formulation/Solution 1 (sol1)**.
- 5 From the **Parameter selection (bias)** list, choose **First**.
- 6 From the **Parameter selection (sweep)** list, choose **Last**.
- 7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dotted**.
- 8 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
- 9 Locate the **Legends** section. In the table, enter the following settings:

---

<b>Legends</b>
Rev Kramer

---

### *Line Graph 4*

- 1 Right-click **Line Graph 3** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.


- 3 In the **Expression** text field, type `kramer_eq_V(x)`.
- 4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Eq Kramer

*Line Graph 5*

- 1 Right-click **Line Graph 4** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type `kramer_fwd_V(x)`.
- 4 Locate the **Legends** section. In the table, enter the following settings:

Legends
Fwd Kramer

- 5 In the **Electric Potential (semi)** toolbar, click  **Plot**.

*Conduction Band Energy Level, Electron Quasi-Fermi Energy Level, Hole Quasi-Fermi Energy Level, Valence Band Energy Level*

- 1 In the **Model Builder** window, under **Results > Energy Levels Reverse Bias**, Ctrl-click to select **Conduction Band Energy Level**, **Electron Quasi-Fermi Energy Level**, **Hole Quasi-Fermi Energy Level**, and **Valence Band Energy Level**.
- 2 Right-click and choose **Duplicate**.

*Conduction Band Energy Level 1*

- 1 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 2 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 3 From the **Parameter selection (Va)** list, choose **First**.
- 4 Locate the **y-Axis Data** section. In the **Expression** text field, type `semi2.Ec_e`.
- 5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 6 From the **Color** list, choose **Cyan**.
- 7 Locate the **Legends** section. In the table, enter the following settings:

Legends
Ec FV

### *Electron Quasi-Fermi Energy Level 1*

- 1 In the **Model Builder** window, click **Electron Quasi-Fermi Energy Level 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 4 From the **Parameter selection (Va)** list, choose **First**.
- 5 Locate the **y-Axis Data** section. In the **Expression** text field, type `semi2.Efn_e`.
- 6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 7 From the **Color** list, choose **Green**.
- 8 Locate the **Legends** section. In the table, enter the following settings:

---

**Legends**

---

Efn FV

---

### *Hole Quasi-Fermi Energy Level 1*

- 1 In the **Model Builder** window, click **Hole Quasi-Fermi Energy Level 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 4 From the **Parameter selection (Va)** list, choose **First**.
- 5 Locate the **y-Axis Data** section. In the **Expression** text field, type `semi2.Efp_e`.
- 6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 7 From the **Color** list, choose **Magenta**.
- 8 Locate the **Legends** section. In the table, enter the following settings:

---

**Legends**

---

Efp FV

---

### *Valence Band Energy Level 1*

- 1 In the **Model Builder** window, click **Valence Band Energy Level 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 4 From the **Parameter selection (Va)** list, choose **First**.
- 5 Locate the **y-Axis Data** section. In the **Expression** text field, type `semi2.Ev_e`.

- 6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 7 From the **Color** list, choose **Red**.
- 8 Locate the **Legends** section. In the table, enter the following settings:

<b>Legends</b>	
Ev	FV

- 9 In the **Energy Levels Reverse Bias** toolbar, click  **Plot**.

*Electron Concentration, Hole Concentration*

- 1 In the **Model Builder** window, under **Results** > **Carrier Concentrations Reverse Bias**, Ctrl-click to select **Electron Concentration** and **Hole Concentration**.
- 2 Right-click and choose **Duplicate**.

*Electron Concentration 1*

- 1 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 2 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 3 From the **Parameter selection (Va)** list, choose **First**.
- 4 Locate the **y-Axis Data** section. In the **Expression** text field, type `semi2.N`.
- 5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 6 Locate the **Legends** section. In the table, enter the following settings:

<b>Legends</b>	
electrons	FV

*Hole Concentration 1*

- 1 In the **Model Builder** window, click **Hole Concentration 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2 - Finite Volume/Solution 2 (sol2)**.
- 4 From the **Parameter selection (Va)** list, choose **First**.
- 5 Locate the **y-Axis Data** section. In the **Expression** text field, type `semi2.P`.
- 6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

7 Locate the **Legends** section. In the table, enter the following settings:

---

**Legends**

---

holes FV

---

*Electron Concentration I, Hole Concentration I*

1 In the **Model Builder** window, under **Results** > **Carrier Concentrations Reverse Bias**, Ctrl-click to select **Electron Concentration I** and **Hole Concentration I**.

2 Right-click and choose **Duplicate**.

*Electron Concentration I.1*

1 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

2 In the **Expression** text field, type `kramer_rev_n(x)`.

3 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dotted**.

4 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

5 Locate the **Legends** section. In the table, enter the following settings:

---

**Legends**

---

electrons Kramer

---

*Hole Concentration I.1*

1 In the **Model Builder** window, click **Hole Concentration I.1**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `kramer_rev_p(x)`.

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dotted**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 Locate the **Legends** section. In the table, enter the following settings:

---

**Legends**

---

holes Kramer

---

7 In the **Carrier Concentrations Reverse Bias** toolbar, click  **Plot**.

*Carrier Concentrations Reverse Bias, Energy Levels Reverse Bias*

1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Energy Levels Reverse Bias** and **Carrier Concentrations Reverse Bias**.

- 2 Right-click and choose **Duplicate**.

#### *Carrier Concentrations Reverse Bias I, Energy Levels Reverse Bias I*

- Right-click and choose **Duplicate**.

#### *Energy Levels Equilibrium*

- 1 In the **Settings** window for **ID Plot Group**, type Energy Levels Equilibrium in the **Label** text field.
- 2 Locate the **Data** section. From the **Parameter selection (sweep)** list, choose **From list**.
- 3 In the **Parameter values (sweep)** list box, select **0**.

#### *Conduction Band Energy Level I*

- 1 In the **Model Builder** window, expand the **Energy Levels Equilibrium** node, then click **Conduction Band Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **From list**.
- 4 In the **Parameter values (Va (V))** list box, select **0**.


#### *Electron Quasi-Fermi Energy Level I*

- 1 In the **Model Builder** window, click **Electron Quasi-Fermi Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **From list**.
- 4 In the **Parameter values (Va (V))** list box, select **0**.

#### *Hole Quasi-Fermi Energy Level I*

- 1 In the **Model Builder** window, click **Hole Quasi-Fermi Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **From list**.
- 4 In the **Parameter values (Va (V))** list box, select **0**.

#### *Valence Band Energy Level I*

- 1 In the **Model Builder** window, click **Valence Band Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **From list**.
- 4 In the **Parameter values (Va (V))** list box, select **0**.
- 5 In the **Energy Levels Equilibrium** toolbar, click  **Plot**.

### *Carrier Concentrations Reverse Bias I*

- 1 In the **Model Builder** window, under **Results** click **Carrier Concentrations Reverse Bias I**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 From the **Parameter selection (sweep)** list, choose **From list**.
- 4 In the **Parameter values (sweep)** list box, select **0**.

### *Electron Concentration I*

- 1 In the **Model Builder** window, expand the **Carrier Concentrations Reverse Bias I** node, then click **Electron Concentration I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **From list**.
- 4 In the **Parameter values (Va (V))** list box, select **0**.


### *Hole Concentration I*

- 1 In the **Model Builder** window, click **Hole Concentration I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **From list**.
- 4 In the **Parameter values (Va (V))** list box, select **0**.

### *Electron Concentration I.I*

- 1 In the **Model Builder** window, click **Electron Concentration I.I**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type `kramer_eq_n(x)`.

### *Hole Concentration I.I*

- 1 In the **Model Builder** window, click **Hole Concentration I.I**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type `kramer_eq_p(x)`.
- 4 In the **Carrier Concentrations Reverse Bias I** toolbar, click  **Plot**.

### *Carrier Concentrations Equilibrium*

- 1 In the **Model Builder** window, right-click **Carrier Concentrations Reverse Bias I** and choose **Rename**.
- 2 In the **Rename ID Plot Group** dialog, type **Carrier Concentrations Equilibrium** in the **New label** text field.
- 3 Click **OK**.

#### *Energy Levels Reverse Bias I.I*

- 1 In the **Model Builder** window, click **Energy Levels Reverse Bias I.I**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 From the **Parameter selection (bias)** list, choose **Last**.

#### *Conduction Band Energy Level I*

- 1 In the **Model Builder** window, expand the **Energy Levels Reverse Bias I.I** node, then click **Conduction Band Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **Last**.


#### *Electron Quasi-Fermi Energy Level I*

- 1 In the **Model Builder** window, click **Electron Quasi-Fermi Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **Last**.

#### *Hole Quasi-Fermi Energy Level I*

- 1 In the **Model Builder** window, click **Hole Quasi-Fermi Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **Last**.

#### *Valence Band Energy Level I*

- 1 In the **Model Builder** window, click **Valence Band Energy Level I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **Last**.
- 4 In the **Energy Levels Reverse Bias I.I** toolbar, click  **Plot**.

#### *Energy Levels Forward Bias*

- 1 In the **Model Builder** window, right-click **Energy Levels Reverse Bias I.I** and choose **Rename**.
- 2 In the **Rename ID Plot Group** dialog, type Energy Levels Forward Bias in the **New label** text field.
- 3 Click **OK**.

#### *Carrier Concentrations Forward Bias*

- 1 In the **Model Builder** window, under **Results** click **Carrier Concentrations Reverse Bias I.I**.
- 2 In the **Settings** window for **ID Plot Group**, type Carrier Concentrations Forward Bias in the **Label** text field.

- 3 Locate the **Data** section. From the **Parameter selection (bias)** list, choose **Last**.

#### *Electron Concentration I*

- 1 In the **Model Builder** window, expand the **Carrier Concentrations Forward Bias** node, then click **Electron Concentration I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **Last**.


#### *Hole Concentration I*

- 1 In the **Model Builder** window, click **Hole Concentration I**.
- 2 In the **Settings** window for **Line Graph**, locate the **Data** section.
- 3 From the **Parameter selection (Va)** list, choose **Last**.

#### *Electron Concentration I.I*

- 1 In the **Model Builder** window, click **Electron Concentration I.I**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type `kramer_fwd_n(x)`.

#### *Hole Concentration I.I*

- 1 In the **Model Builder** window, click **Hole Concentration I.I**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type `kramer_fwd_p(x)`.
- 4 In the **Carrier Concentrations Forward Bias** toolbar, click  **Plot**.