Model created in COMSOL Multiphysics 6.4

Alvarez Lens

Introduction

Freeform optics has generated renewed interest due to advances in fabrication and manufacturing technology. This model studies a specific freeform design called the Alvarez lens, where two complementary cubic surfaces are shifted laterally with respect to each other to achieve variable optical power. Departure from conventional (mostly spherical) optical surfaces provides additional degrees of freedom, can open up many opportunities for systems design, and achieve better optical performance. For example, in Ref. 1 a modified Alvarez lens is used to design a miniature multimodal microscope with variable depth focusing in a small form factor.

Model Definition

The model uses a design described in Ref. 2, in which the surface sags are described as

$$t_1(X, Y) = 6\sqrt{8}a\left(X^2Y + \frac{Y^3}{3}\right) - 3\sqrt{8}aY$$

$$t_1(X, Y) = -t_2(X, Y)$$

where $t_1(X, Y)$ and $t_2(X, Y)$ are the surface sags of the first and the second elements; a is a surface coefficient; X and Y are the normalized coordinates

$$X = \frac{x}{r_n}, Y = \frac{y}{r_n}$$

and r_n is the normalizing radius. For transverse shifts of ΔX, ΔY and zero longitudinal shift, the total contribution to phase for both surfaces is given by

$$t(X, Y) = t_1(X \pm \Delta X, Y \pm \Delta Y) + t_2(X \mp \Delta X, Y \mp \Delta Y)$$

Note that for $\Delta X = 0$, the total profile becomes quadratic and proportional to ΔY:

$$t \propto 24\sqrt{2}a\Delta Y\left(\frac{x^2 + y^2}{r_n^{\ 3}}\right)$$

A longitudinal shift of zero is not physical and therefore the optical power and the aberrations introduced in the model deviate slightly from the expressions above. Geometry module can be used to easily parameterize and model these effects. The parameters shown in Table 1 are used for the initial values and a parametric sweep varies the relative shift in the y direction to observe the change in the optical power. The geometry shown in Figure 1 is generated using the parameters in Table 1.

TABLE 1: PARAMETER DEFINITIONS.

Parameter	**Value**	**Description**
r_n	20 mm	Normalizing radius
thickness	5 mm	Central thickness
a	0.43 mm	Surface coefficient
n	1.505	Refractive index
Δx	0	Relative shift, x-coordinate
Δy	-4 mm	Relative shift, y-coordinate
Δz	2 mm	Relative shift, z-coordinate

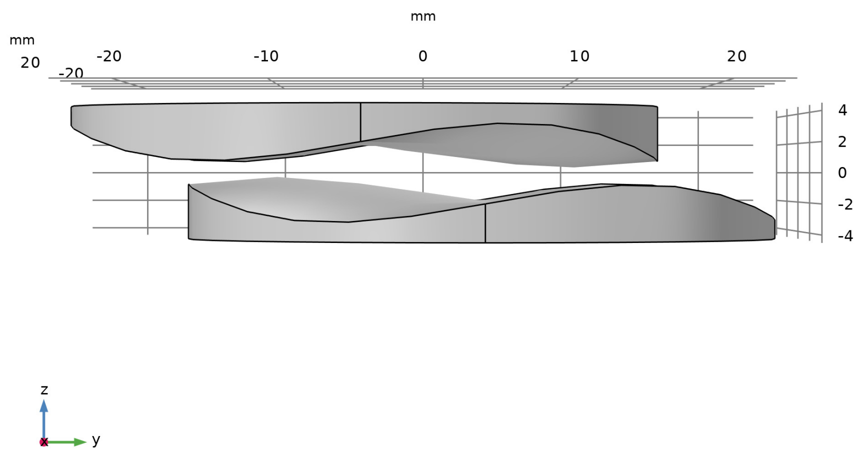

Figure 1: Alvarez lens where the cubic surfaces are laterally shifted to achieve variable optical power.

In this model, collimated rays are released with a hexapolar distribution and a parametric sweep is performed to observe the change in the focal point as a function of the lateral shift. Spot diagrams are generated to study the aberrations introduced.

Figure 2 shows that a lateral shift in y causes a change in optical power. The corresponding spot diagrams shown in Figure 3 indicate that the aberrations change at every configuration. Ref. 1 discusses strategies for reducing aberrations for the entire range using two Alvarez lens pairs.

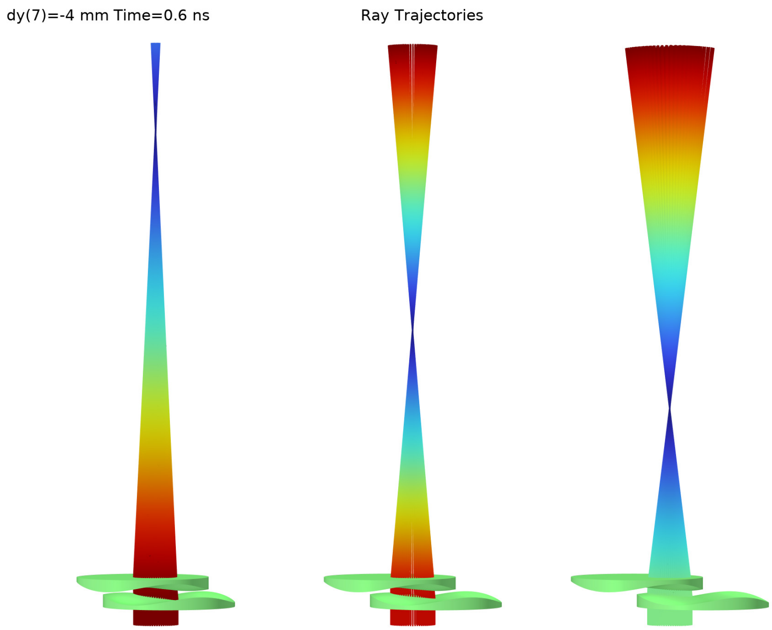

Figure 2: Ray trajectories for different lateral shifts demonstrate variable optical power.

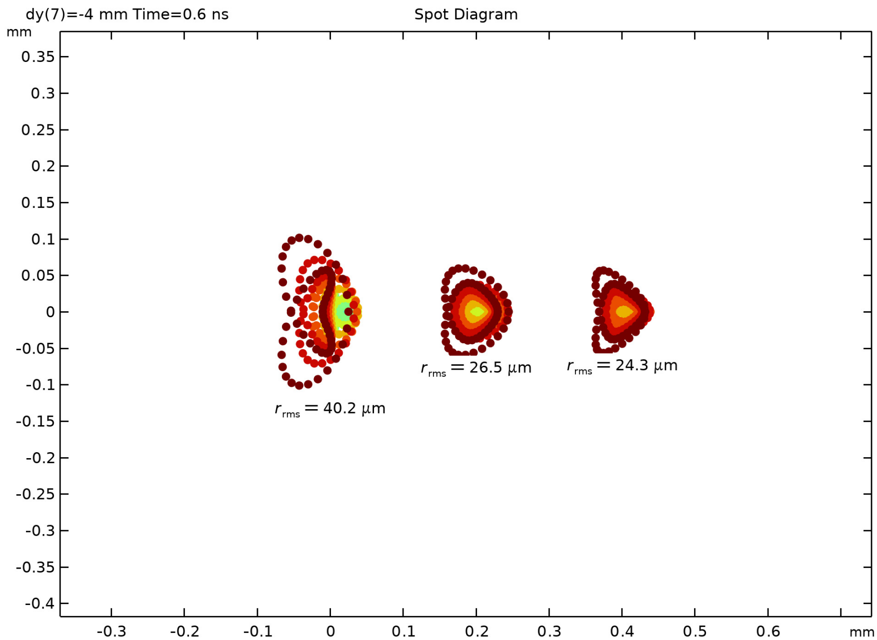

Figure 3: Spot diagrams for different lateral shifts, showing that performance varies between configurations.

References

1. S.S. Rege and others, “Application of the Alvarez-Humphrey Concept to the Design of a Miniaturized Scanning Microscope,” *Optics Express*, vol. 12, no. 12, pp. 2574–2588, 2004.

2. D. Gonzalez-Utrera and others, “Modeling, Fabrication, and Metrology of 3D Printed Alvarez Lenses Prototypes,” *Optics Express*, vol. 32, no. 3, pp. 3512–3527, 2024.

Application Library path: `Ray_Optics_Module/Lenses_Cameras_and_Telescopes/alvarez_lens`

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics** > **Ray Optics** > **Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
rn	20[mm]	0.02 m	Normalizing radius
thickness	5[mm]	0.005 m	Central thickness
a	0.43[mm]	4.3E-4 m	Surface coefficient
nref	1.505	1.505	Refractive index
dx	0[mm]	0 m	Displacement in x
dy	-4[mm]	-0.004 m	Displacement in y
dz	2[mm]	0.002 m	Displacement in z

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `rn`.

4 In the **Height** text field, type `thickness`.

5 Locate the **Position** section. In the **z** text field, type `-thickness/2`.

Parametric Surface 1 (ps1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Parametric Surface**.

2 In the **Settings** window for **Parametric Surface**, locate the **Parameters** section.

3 Find the **First parameter** subsection. In the **Minimum** text field, type `-1`.

4 Find the **Second parameter** subsection. In the **Minimum** text field, type `-1`.

5 Locate the **Expressions** section. In the **x** text field, type `s1*rn`.

6 In the **y** text field, type `s2*rn`.

7 In the **z** text field, type `6*sqrt(8)*a*(s1^2*s2+s2^3/3) - 3*sqrt(8)*a*s2`.

Partition Objects 1 (par1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Partition Objects**.

2 Select the object **cyl1** only.

3 In the **Settings** window for **Partition Objects**, locate the **Partition Objects** section.

4 Click to select the **Activate Selection** toggle button for **Tool objects**.

5 Select the object **ps1** only.

Extract 1 (extract1)

1 In the **Geometry** toolbar, click **Extract**.

2 In the **Settings** window for **Extract**, locate the **Entities or Objects to Extract** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 On the object **par1**, select Domain 2 only.

5 From the **Input object handling** list, choose **Create remainder object**.

Move 1 (mov1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Move**.

2 Select the object **extract1(1)** only.

3 In the **Settings** window for **Move**, locate the **Displacement** section.

4 In the **x** text field, type `dx`.

5 In the **y** text field, type `dy`.

6 In the **z** text field, type `dz`.

Move 2 (mov2)

1 In the **Geometry** toolbar, click **Transforms** and choose **Move**.

2 Select the object **extract1(2)** only.

3 In the **Settings** window for **Move**, locate the **Displacement** section.

4 In the **x** text field, type `-dx`.

5 In the **y** text field, type `-dy`.

6 In the **z** text field, type `-dz`.

7 Click **Build All Objects**.

8 Click the **Go to YZ View** button in the **Graphics** toolbar. The geometry should look like Figure 1.

MATERIALS

Material 1 (mat1)

1 In the **Materials** toolbar, click **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	`nref`	1	Refractive index

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Ray Release and Propagation** section.

3 In the **Maximum number of secondary rays** text field, type `0`.

Material Discontinuity 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Geometrical Optics (gop)** click **Material Discontinuity 1**.

2 In the **Settings** window for **Material Discontinuity**, locate the **Rays to Release** section.

3 From the **Release reflected rays** list, choose **Never**.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 From the **Grid type** list, choose **Hexapolar**.

4 Specify the $\mathbf{q}_c$ vector as

-2*thickness	z

5 Specify the $\mathbf{r}_c$ vector as

0	x
0	y
1	z

6 In the R_c text field, type `rn/3`.

7 In the N_c text field, type `10`.

8 Locate the **Ray Direction Vector** section. Specify the $\mathbf{L}_0$ vector as

0	x
0	y
1	z

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Coarse**.

4 Click **Build All**.

STUDY 1

Step 1: Ray Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 In the **Output times** text field, type `range(0,0.01,0.6)`.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
dy (Displacement in y)	`range(-10, 1, -4)`	`mm`

5 In the **Study** toolbar, click **Compute**.

RESULTS

Ray Trajectories (gop)

1 In the **Settings** window for **3D Plot Group**, click to expand the **Title** section.

2 From the **Title type** list, choose **Manual**.

3 In the **Title** text area, type `Ray Trajectories`.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

5 Click next to **cycle_plot_level**, then choose **Parameter value (dy (mm))**.

6 Click **Plot First**. Scroll through different values of dy and note the change in the optical power.

7 In the **Model Builder** window, click **Ray Trajectories (gop)**.

8 Locate the **Data** section. From the **Dataset** list, choose **None**.

9 Locate the **Color Legend** section. Clear the **Show legends** checkbox.

10 Click to expand the **Plot Array** section. From the **Array type** list, choose **Linear**.

11 From the **Array axis** list, choose **y**.

Ray Trajectories 1

1 In the **Model Builder** window, click **Ray Trajectories 1**.

2 In the **Settings** window for **Ray Trajectories**, locate the **Data** section.

3 From the **Dataset** list, choose **Ray 1**.

Color Expression 1

1 In the **Model Builder** window, click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `gop.rrms`.

4 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

Ray Trajectories (gop)

In the **Ray Trajectories (gop)** toolbar, click **Volume**.

Volume 1

1 In the **Settings** window for **Volume**, locate the **Data** section.
2 From the **Dataset** list, choose **Ray 1**.
3 Click to expand the **Plot Array** section. Select the **Manual indexing** checkbox.
4 Click the **Go to YZ View** button in the **Graphics** toolbar.

Ray Trajectories 2

1 In the **Model Builder** window, under **Results** > **Ray Trajectories (gop)** right-click **Ray Trajectories 1** and choose **Duplicate**.
2 In the **Settings** window for **Ray Trajectories**, locate the **Data** section.
3 From the **Parameter value (dy (mm))** list, choose **-7**.
4 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

Volume 2

1 In the **Model Builder** window, under **Results** > **Ray Trajectories (gop)** right-click **Volume 1** and choose **Duplicate**.
2 In the **Settings** window for **Volume**, locate the **Data** section.
3 From the **Parameter value (dy (mm))** list, choose **-7**.
4 Locate the **Plot Array** section. In the **Index** text field, type 1.
5 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

Ray Trajectories 3

1 In the **Model Builder** window, under **Results** > **Ray Trajectories (gop)** right-click **Ray Trajectories 2** and choose **Duplicate**.
2 In the **Settings** window for **Ray Trajectories**, locate the **Data** section.
3 From the **Parameter value (dy (mm))** list, choose **-10**.
4 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

Volume 3

1 In the **Model Builder** window, under **Results** > **Ray Trajectories (gop)** right-click **Volume 2** and choose **Duplicate**.
2 In the **Settings** window for **Volume**, locate the **Data** section.
3 From the **Parameter value (dy (mm))** list, choose **-10**.
4 Locate the **Plot Array** section. In the **Index** text field, type 2.
5 In the **Ray Trajectories (gop)** toolbar, click **Plot**.
6 Click the **Show Axis Orientation** button in the **Graphics** toolbar.

7 Click the **Show Grid** button in the **Graphics** toolbar.

8 Click the **Zoom Extents** button in the **Graphics** toolbar. The plot should look like Figure 2.

Intersection Point 3D 1

1 In the **Results** toolbar, click **More Datasets** and choose **Intersection Point 3D**.

2 In the **Settings** window for **Intersection Point 3D**, locate the **Data** section.

3 From the **Parameter selection (dy)** list, choose **From list**.

4 In the **Parameter values (dy (mm))** list box, select **-4**.

5 Locate the **Surface** section. Find the **Point** subsection. In the **z** text field, type `139.63`.

Intersection Point 3D 2

1 Right-click **Intersection Point 3D 1** and choose **Duplicate**.

2 In the **Settings** window for **Intersection Point 3D**, locate the **Data** section.

3 In the **Parameter values (dy (mm))** list box, select **-7**.

4 Locate the **Surface** section. Find the **Point** subsection. In the **z** text field, type `79.32`.

Intersection Point 3D 3

1 Right-click **Intersection Point 3D 2** and choose **Duplicate**.

2 In the **Settings** window for **Intersection Point 3D**, locate the **Data** section.

3 In the **Parameter values (dy (mm))** list box, select **-10**.

4 Locate the **Surface** section. Find the **Point** subsection. In the **z** text field, type `55.67`.

Spot Diagram

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Spot Diagram` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **None**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type `Spot Diagram`.

6 Locate the **Color Legend** section. Clear the **Show legends** checkbox.

Spot Diagram 1

1 In the **Spot Diagram** toolbar, click **More Plots** and choose **Spot Diagram**.

2 In the **Settings** window for **Spot Diagram**, locate the **Data** section.

3 From the **Image surface** list, choose **Intersection Point 3D 1**.

4 In the **Spot Diagram** toolbar, click **Plot**.

Color Expression 1

1 In the **Spot Diagram** toolbar, click **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `at(0, gop.rrel)`.

4 In the **Spot Diagram** toolbar, click **Plot**.

Spot Diagram 2

1 In the **Model Builder** window, under **Results** > **Spot Diagram** right-click **Spot Diagram 1** and choose **Duplicate**.

2 In the **Settings** window for **Spot Diagram**, locate the **Data** section.

3 From the **Image surface** list, choose **Intersection Point 3D 2**.

4 Click to expand the **Position** section. In the **x** text field, type `0.2`.

5 In the **Spot Diagram** toolbar, click **Plot**.

Spot Diagram 3

1 Right-click **Spot Diagram 2** and choose **Duplicate**.

2 In the **Settings** window for **Spot Diagram**, locate the **Data** section.

3 From the **Image surface** list, choose **Intersection Point 3D 3**.

4 Locate the **Position** section. In the **x** text field, type `0.4`.

5 In the **Spot Diagram** toolbar, click **Plot**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar. The plot should look like Figure 3.