Model created in COMSOL Multiphysics 6.4

Transmission Line Parameters of a Coaxial Cable

Introduction

Transmission lines (TLs) are electromagnetic structures used to guide waves of alternating current and voltage at radio frequencies. Transmission lines are commonly found in electric and electronics applications, ranging from overhead lines spanning thousands of kilometers, to power cables used in motor application, and to copper traces in printed circuit board (PCB) designs, to give some examples. The first mathematical description of transmission-line structures dates back to 1876, when Oliver Heaviside proposed the well-known telegrapher's equations. One of the advantages of the telegrapher's equations is that they provide a circuital representation of a TL, which can therefore be analyzed using circuit theory rather than field theory. Another advantage of a circuital representation is that it can easily be embedded into circuit simulator tools. Typical tasks performed when analyzing transmission-line structures are: minimization of the losses, and minimization of the so-called "high-frequency effects" such as distortion, reflections, and crosstalk. Transmission line theory and its application are a cornerstone in RF and microwave engineering, and the reader is referred to the relevant literature for a comprehensive discussion about the topic; see under the section Reference. In the following, we merely provide the main theoretical background that is necessary to understand and use the simulation model.

When talking about TL models, we assume that the electromagnetic fields propagate mainly in the so-called transverse electromagnetic mode (TEM), or in the quasi-TEM mode when the losses in the conductors are small. Furthermore, defining λ as the wavelength of the line, and l the largest physical length of the line, we say that the line is electrically long when $l >> \lambda$. Transmission line theory is then necessary for electrically long lines, which require distributed models for their study, meaning, models that use the distributed parameters as computed by the Transmission Line, Parameters interface. Note that, conversely, the conductors separation in the cross section has to be electrically small for the model to be valid.

As mentioned earlier, a circuital representation of a transmission-line structure allows representing electromagnetic waves in terms of voltages and currents that can be described by resorting to circuit theory. In particular, given an section, or cell of length Δz of a generic TL that is electrically short, that is, $\Delta z << \lambda$, it can be proven that the section can be described by an equivalent circuit model as depicted in Figure 1. As a rule of thumb, an electrically short segment is such that its length is smaller than $\lambda/10$ or $\lambda/20$, depending on the application and the desired accuracy. Note that we will then need a number N of cells, such as $N\Delta z = l$, to cover the full length of the line and perform the desired analysis.

The lumped elements R, L, G, and C are the so-called distributed resistance, inductance, conductance, and capacitance, respectively, in per-unit-length, and are therefore commonly referred to as per-unit-length, or p.u.l. parameters. Note that we are considering only two-conductor TLs, namely transmission lines with one signal conductor and one reference conductor.

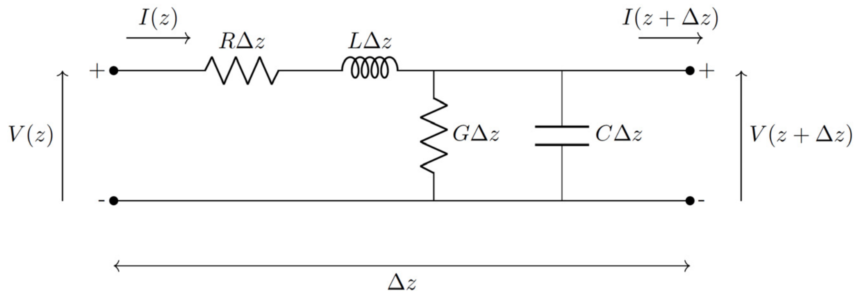


Figure 1: The equivalent-circuit representation for a section Δz of a generic TL.

Accordingly to the equivalent circuit representation provided above, the 1D, frequency-domain wave equation for the electric potential can be written as

$$\frac{\partial}{\partial z}\left(\frac{1}{R+i\omega L}\frac{\partial V}{\partial z}\right)-(G+i\omega C)V = 0$$

A similar wave equation can be deduced for the current. The solution can be expressed in terms of a forward- and a backward-propagating wave as

$$V(z) = V_+e^{-\gamma z}+V_-e^{\gamma z}$$

where γ is the complex propagation constant given by

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

The characteristic impedance Z_c of the transmission line relates the voltage and current, and can be written in terms of p.u.l. parameters as

$$Z_c = \frac{V}{I} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

The Transmission Line, Parameters interface computes the transmission line parameters R, L, G, and C, as well as γ and Z_c for two-conductor transmission lines.

Model Definition

The first step to define the p.u.l. parameters is to designate the reference conductor for the line voltage. It can be shown that, under the TEM assumption, the sum of the currents at any cross-section must be zero; this means that one of the conductors, that is, the reference conductor must serve as a "return" for the currents on the other conductors. Once the reference conductor and the signal conductor are assigned, we can proceed with the p.u.l. parameters.

Given a section Δz, the total capacitance C_{tot} of the line relates the charge Q stored on the top and bottom conductors, and the voltage between them as $Q = CV$. Then, the p.u.l. capacitance C (SI unit: F/m) is defined as

$$C = \lim_{\Delta z \to 0} \frac{C_{\mathrm{tot}}}{\Delta z}$$

where the capacitance carries the displacement current flowing in the transverse plane. Assuming that the charge is uniformly distributed around the periphery of the conductor, we can write Gauss' law to relate the surface charge density ρ integrated on the closed surface S to the charge such as

$$\oint_S (\rho \cdot ds) = Q$$

In the interface, the surface charge density ρ is computed such that the voltage between the two conductors reaches the theoretical value of the applied voltage, equal to 1 V.

The conductance G (SI unit: S/m) accounts for the bound charge losses in the dielectric, considered to be dominant with respect to conduction losses, which is generally true for dielectrics. This formulation is suitable for both homogeneous and inhomogeneous media that surround the line. Therefore, the conductance G is nonzero only for dielectric defined with dielectric losses with a complex permittivity, or via a loss tangent. Note that the conductive losses in a dielectric are neglected.

As mentioned when talking about the reference conductor, the currents are equal and oppositely directed at any cross section. Therefore, the resistance R is computed as the sum of the resistance of the reference conductor and the signal conductor. To compute the resistance on the reference conductor, we basically solve for an external electric field excitation (SI unit: V/m), such as the current approaches the theoretical value of −1 A. The resistance is then computed as the ratio between the computed external electric field and the corresponding current. Similarly, for the signal conductor we solve in the same way, but the current shall approach the theoretical value of 1 A. Note that, in a good conductor, the dominant current is the conduction current, and the displacement current is negligible. Moreover, the current decays as a function of the skin depth, defined as

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

The skin depth is a measure of the exponential drop in current density with the distance to the surface inside conductors. It can be very small and needs to be resolved by the finite element mesh. This is obtained by using a special meshing method known as boundary layer meshing automatically provided by the physics-controlled mesh.

Finally, the inductance L (SI unit: H/m) as computed by the interface accounts for both the internal and external inductances; for a deeper discussion on the topic, we refer the reader to the relevant literature, see under the section Reference. In the interface, we consider the Telegrapher's equation in the frequency domain for the voltage that reads as

$$\frac{d}{dz}V(z, \omega) = -Z(\omega)I(z, \omega)$$

where

$$Z(\omega) = R(\omega) + i\omega L_i(\omega) + i\omega L$$

with $L_i(\omega)$ is the internal impedance. Since the impedance is already computed to extract the resistance R, we can easily define the inductance L as the imaginary part of the impedance.

Results and Discussion

After completing the computation, the transmission line parameters in Table 1 are automatically evaluated by default, and they include series resistance R, series inductance L, shunt conductance G, shunt capacitance C, all calculated per unit length, as well as characteristic impedance Z_c and propagation constant γ.

TABLE 1: COMPUTED APPROXIMATE TRANSMISSION LINE PARAMETERS.

	Values
Series resistance R	4.2 Ω/m
series inductance L	2.56E-7 H/m
shunt conductance G	0 S/m
shunt capacitance C	9.8E-11 F/m

TABLE 1: COMPUTED APPROXIMATE TRANSMISSION LINE PARAMETERS.

	Values
Characteristic impedance Z_c	51-0.067j Ω
Propagation constant γ	0.041+31.5j Ω

Several default plots help to understand the electromagnetic field behavior within the transmission line structure; in particular, the electric potential is depicted in Figure 2, the electric field norm is depicted in Figure 3, and the magnetic flux norm is depicted in Figure 4.

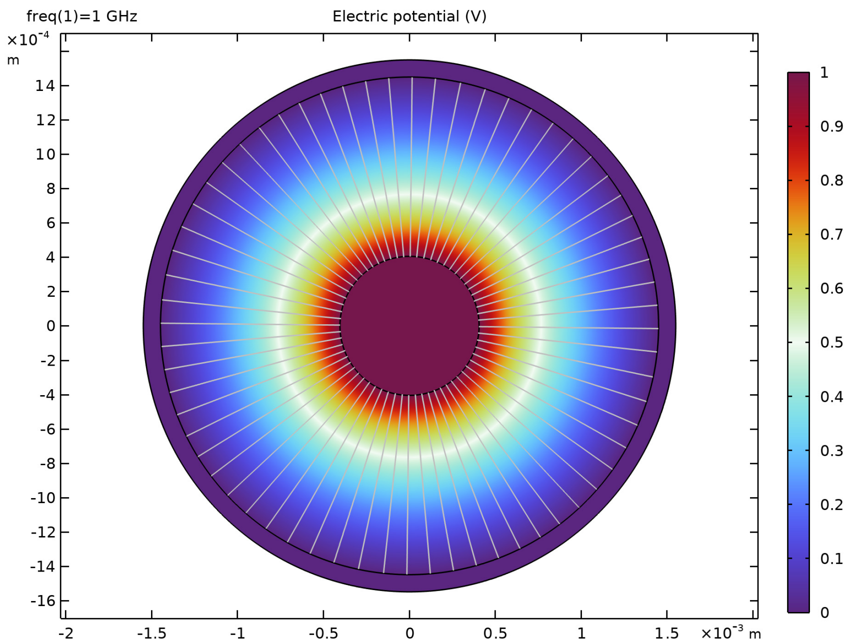


Figure 2: Surface plot of the electric potential along with a streamline plot of the electric field.

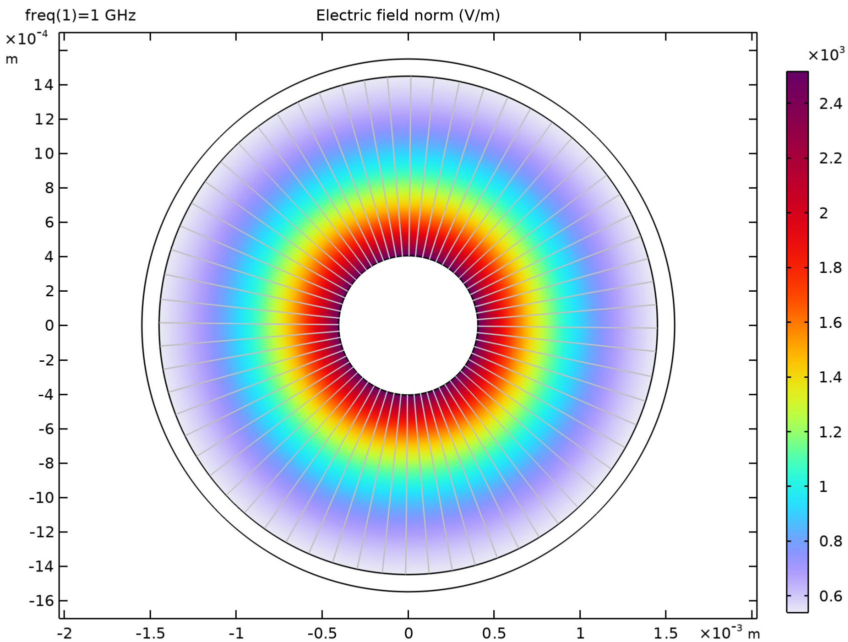


Figure 3: Surface plot of the electric field norm along with a streamline plot of the electric field.

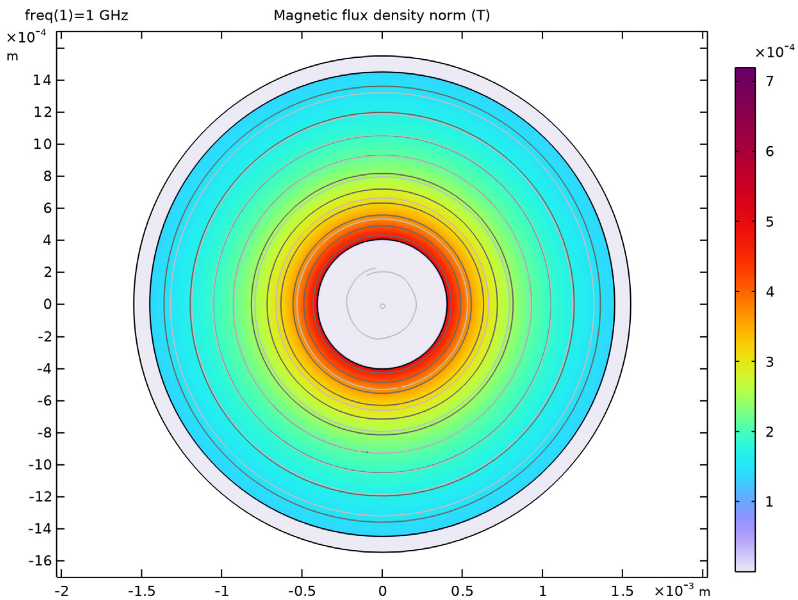


Figure 4: Surface plot of the magnetic flux density norm along with a streamline plot of the magnetic flux density and a contour plot of the z-component of the magnetic vector potential.

Notes About the COMSOL Implementation

The TEM waves assumption underlying the Transmission Line, Parameters multiphysics interface is only valid if the distance between forward (signal path) and return (ground) conductors is substantially smaller than the wavelength in the medium (<10%).

Reference

1. C.R. Paul, *Analysis of multiconductor transmission lines*, John Wiley & Sons, 2007.

Application Library path: `RF_Module/Transmission_Lines_and_Waveguides/transmission_line_coaxial`

Modeling Instructions

From the **Main Toolbar** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1. In the **Model Wizard** window, click **2D**.
2. In the **Select Physics** tree, select **Radio Frequency** > **Transmission Line, Parameters (tlpa)**.
3. Click **Add**.
4. Click **Study**.
5. In the **Select Study** tree, select **General Studies** > **Frequency Domain**.
6. Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, locate the **Parameters** section.
3. In the table, enter the following settings:

Name	Expression	Value	Description
frq_coax	1[GHz]	1E9 Hz	Frequency
Ri_coax	0.405[mm]	4.05E-4 m	Inner radius
dR_coax	1.045[mm]	0.001045 m	Dielectric thickness
d_s_coax	0.1[mm]	1E-4 m	Screen thickness
epsr_coax	2.25	2.25	Relative permittivity of dielectric
mur_coax	1	1	Relative permeability of dielectric
sigma_d_coax	1e-14[S/m]	1E-14 S/m	Conductivity of dielectric
sigma_c_coax	5.98e7[S/m]	5.98E7 S/m	Conductivity of conductors
Ro_coax	Ri_coax+dR_coax	0.00145 m	Outer radius

Name	Expression	Value	Description
w_coax	2*pi*frq_coax	6.2832E9 Hz	Angular frequency
delta_coax	sqrt(2/(w_coax* mur_coax* mu0_const* sigma_c_coax))	2.0581E-6 m	Skin depth
d_c_coax	dR_coax	0.001045 m	Conductor distance
lambda_coax	c_const/ frq_coax/ sqrt(mur_coax* epsr_coax)	0.19986 m	Wavelength in dielectric
QS_coax	d_c_coax<(0.1* lambda_coax)	1	Validity of quasistatic analysis

GEOMETRY 1

Circle 1 (c1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Geometry 1** node.

2 Right-click **Geometry 1** and choose **Circle**.

3 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

4 In the **Radius** text field, type `Ro_coax+d_s_coax`.

5 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.

6 From the **Color** list, choose **Color 16**.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Ro_coax`.

Difference 1 (dif1)

In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

Circle 1 (c1)

1 In the **Model Builder** window, click **Circle 1 (c1)**.

2 In the **Settings** window for **Circle**, click **Build Selected**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar.

Circle 2 (c2)

1 In the **Model Builder** window, click **Circle 2 (c2)**.

2 In the **Settings** window for **Circle**, click **Build Selected**.

Reference Conductor

1 In the **Model Builder** window, under **Component 1 (comp1) > Geometry 1** click **Difference 1 (dif1)**.

2 In the **Settings** window for **Difference**, type `Reference Conductor` in the **Label** text field.

3 Select the object **c1** only.

4 Locate the **Difference** section. Click to select the **Activate Selection** toggle button for **Objects to add**.

5 Select the object **c1** only.

6 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

7 Select the object **c2** only.

8 Click **Build Selected**.

Dielectric

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, type `Dielectric` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Radius** text field, type `Ro_coax`.

4 Click **Build Selected**.

Signal Conductor

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, type `Signal Conductor` in the **Label** text field.

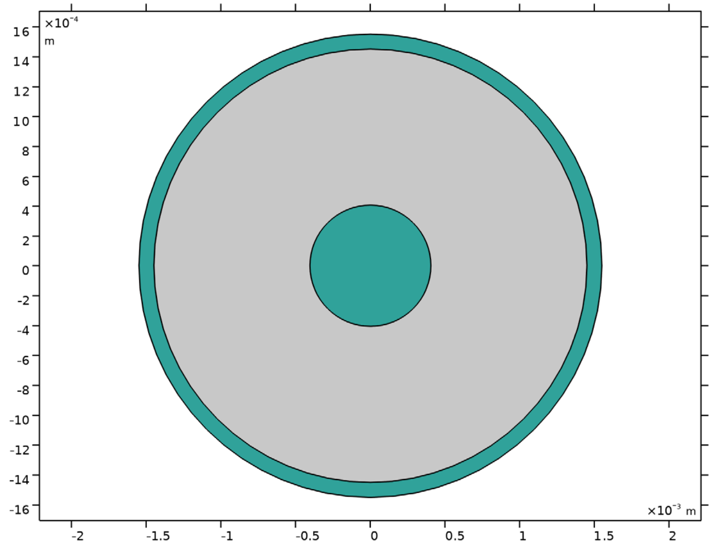
3 Locate the **Size and Shape** section. In the **Radius** text field, type `Ri_coax`.

4 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.

5 From the **Color** list, choose **Color 16**.

6 Click **Build Selected**.

7 In the **Geometry** toolbar, click **Build All**.



MATERIALS

Conductor

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Conductor` in the **Label** text field.

3 Select Domains 1 and 3 only.

4 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Materials** > **Conductor (mat1)** node, then click **Basic (def)**.

5 In the **Settings** window for **Basic**, locate the **Output Properties** section.

6 Click **Select Quantity**.

7 In the **Physical Quantity** dialog, select **Electromagnetics** > **Electric conductivity (S/m)** in the tree.

8 Click **OK**.

9 In the **Settings** window for **Basic**, locate the **Output Properties** section.

10 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Electric conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	sigma_c_coax	S/m	3x3

11 Click **Select Quantity**.

12 In the **Physical Quantity** dialog, select **Electromagnetics** > **Relative permeability (1)** in the tree.

13 Click **OK**.

14 In the **Settings** window for **Basic**, locate the **Output Properties** section.

15 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	3x3

16 Click **Select Quantity**.

17 In the **Physical Quantity** dialog, select **Electromagnetics** > **Relative permittivity (1)** in the tree.

18 Click **OK**.

19 In the **Settings** window for **Basic**, locate the **Output Properties** section.

20 In the table, enter the following settings:

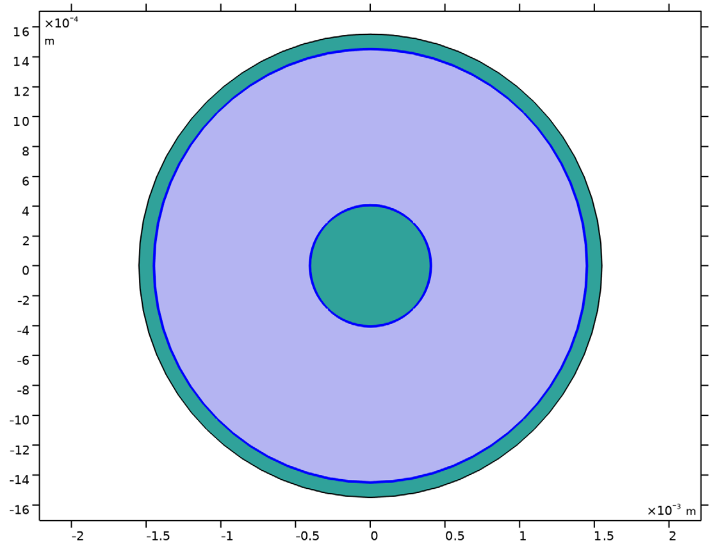
Property	Variable	Expression	Unit	Size
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	3x3

Dielectric

1 In the **Model Builder** window, right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Dielectric` in the **Label** text field.

3 Select Domain 2 only.



4 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Materials** > **Dielectric (mat2)** node, then click **Basic (def)**.

5 In the **Settings** window for **Basic**, locate the **Output Properties** section.

6 Click **Select Quantity**.

7 In the **Physical Quantity** dialog, select **Electromagnetics** > **Relative permittivity (1)** in the tree.

8 Click **OK**.

9 In the **Settings** window for **Basic**, locate the **Output Properties** section.

10 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	`epsr_coax`	1	3x3

11 Click **Select Quantity**.

12 In the **Physical Quantity** dialog, select **Electromagnetics** > **Relative permeability (1)** in the tree.

13 Click **OK**.

14 In the **Settings** window for **Basic**, locate the **Output Properties** section.

15 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	3x3

16 Click **Select Quantity**.

17 In the **Physical Quantity** dialog, select **Electromagnetics** > **Electric conductivity (S/m)** in the tree.

18 Click **OK**.

19 In the **Settings** window for **Basic**, locate the **Output Properties** section.

20 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Electric conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	1e-7	S/m	3x3

TRANSMISSION LINE, PARAMETERS (TLPA)

Reference Conductor 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Transmission Line, Parameters (tlpa)** click **Reference Conductor 1**.

2 Select Domain 1 only.

Signal Conductor 1

1 In the **Model Builder** window, click **Signal Conductor 1**.

2 Select Domain 3 only.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

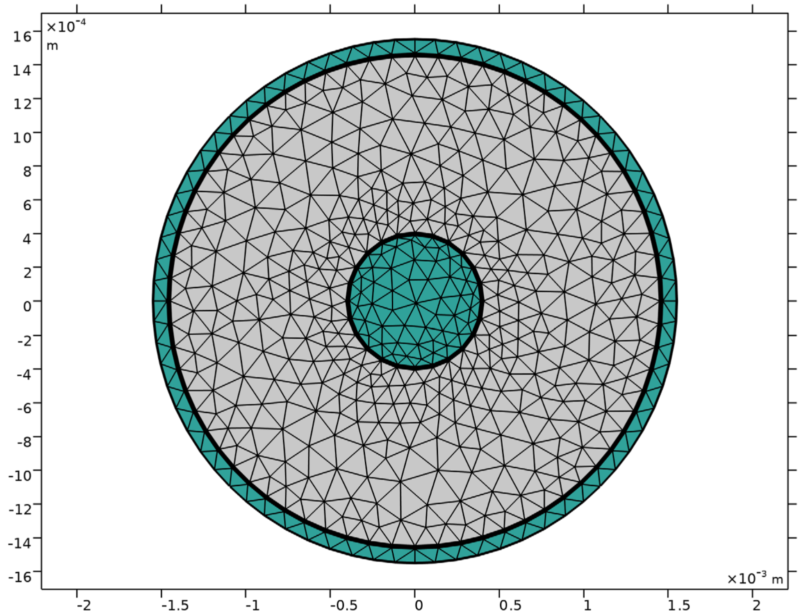
3 From the **Frequency unit** list, choose **GHz**.

4 In the **Frequencies** text field, type `frq_coax`.

MESH 1

1 In the **Model Builder** window, expand the **Results** node.

2 Right-click **Component 1 (comp1)** > **Mesh 1** and choose **Build All**.



STUDY 1

In the **Study** toolbar, click **Compute**.

RESULTS

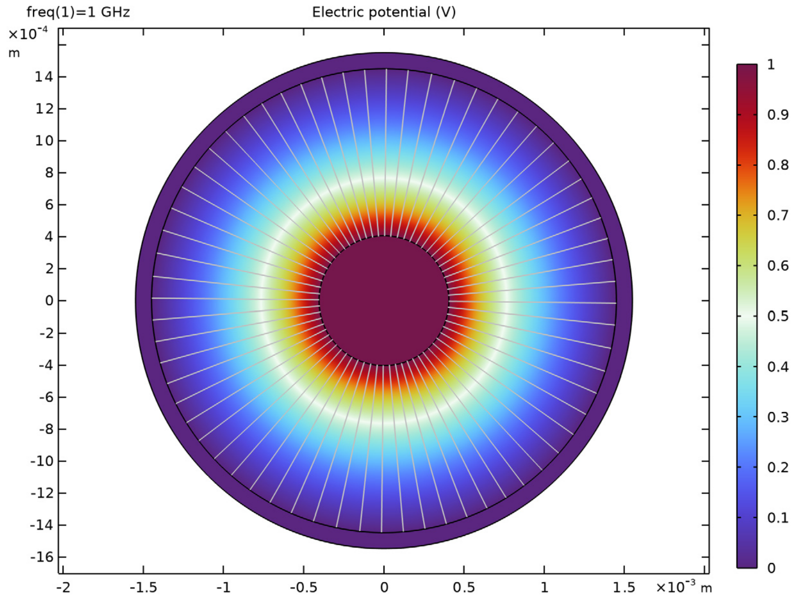
Transmission Line Parameters (tlpa)

In the **Transmission Line Parameters (tlpa)** toolbar, click **Evaluate**.

Electric Potential

1 In the **Model Builder** window, click **Electric Potential**.

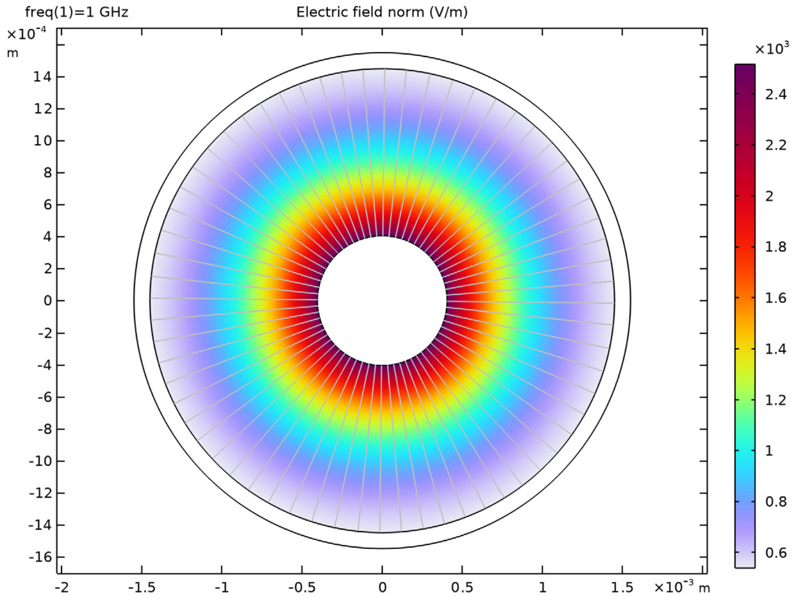
2 In the **Electric Potential** toolbar, click **Plot**.



Electric Field

1 In the **Model Builder** window, click **Electric Field**.

2 In the **Electric Field** toolbar, click **Plot**.



Magnetic Flux Density Norm

1 In the **Model Builder** window, click **Magnetic Flux Density Norm**.

2 In the **Magnetic Flux Density Norm** toolbar, click **Plot**.