



Model created in COMSOL Multiphysics 6.4

Wideband RCS Calculation Using Time-Domain Simulation and FFT

Introduction

This model shows how to calculate the radar cross section (RCS) of a scatterer in a wide frequency range with the Electromagnetic Waves, Time Explicit physics interface. The problem is solved in the scattered-field formulation, where the background field is a temporally modulated Gaussian pulse. The simulated results present the scattered field in the frequency domain and time domain, and the RCS per unit length of a circle in the frequency domain.

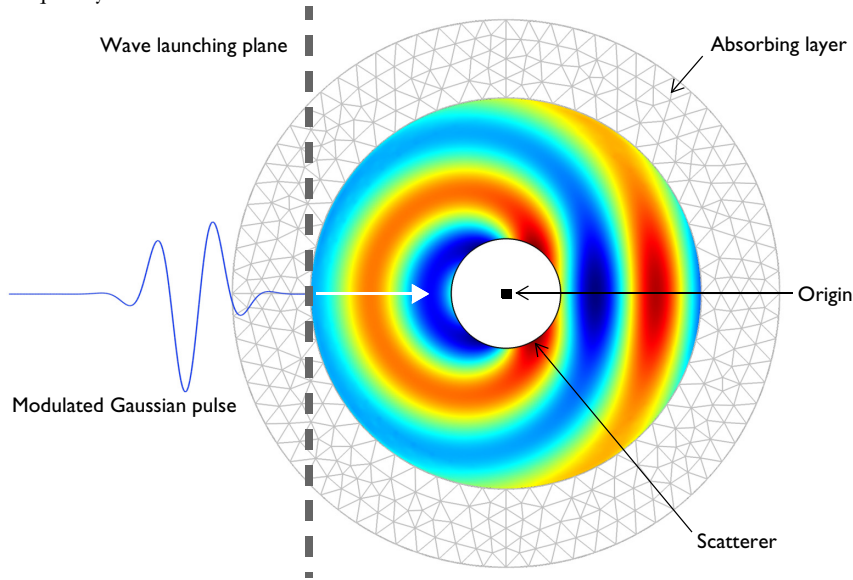


Figure 1: A PEC circle is enclosed by a circular vacuum domain. The outermost layer is finished with an absorbing layer to absorb outgoing waves from the modulated Gaussian background pulse and the scattered field from the circular scatterer.

Model Definition

The model is solved using the scattered field formulation, that is automatically activated by adding a background field domain. The built-in Gaussian pulse, modulated with a sinusoidal function at 200 MHz, is used for background field. The wave vector of the background field is pointing toward the $+x$ direction with z -directional polarization and $120\pi \Omega$ wave impedance. The distance from the coordinate system origin to the wave launching plane is set to 1.5 m (See Figure 1). So, the wave is launched at the boundary of the vacuum domain.

The metallic circle is modeled using the perfect electric conductor boundary condition and its inner part is removed from the model domain. This is done because the skin depth is much smaller than the size of the circle. The circle's scattered field is computed on the boundaries configured by the Far-Field Domain node and its Far-Field Calculation subfeature, by performing a near-field to far-field transformation in the frequency domain after a fast Fourier transform (FFT). The PEC circle is surrounded by vacuum. The simulations are performed with two different absorbing features:

- 1 A scattering boundary condition (first order absorbing boundary condition) applied on the exterior boundary of the vacuum domain.
- 2 An absorbing layer domain on the outside of the surrounding vacuum domain acting as an absorber of the scattered field. The absorbing layer is placed at more than one wavelength away from the scatterer. The effect of the absorbing layer is to make the outermost layer appear as much larger for the outgoing waves. At the outermost boundary, the waves will reach the boundary propagating in the normal direction. Thus, the scattering boundary condition, added to the outermost boundary, will effectively absorb the outgoing waves.

The entire simulation consists of two study steps and one study extension step. The first step is a Time Dependent study for computing a temporal solution. The second step is a Time to Frequency FFT study, where an FFT is performed to obtain a frequency spectrum of the temporal solution. The last step is a Combine Solutions study extension step. This study extension step removes the unwanted parts of the frequency spectrum after the FFT (the first and the last 5% of the frequency spectrum), using a user-defined expression such as `freq<0.1*fb0 || freq>2*fb0-0.1*fb0`.

Results and Discussion

In [Figure 2](#), the dB-scaled RCS per unit length is plotted for 100 MHz, 200 MHz, and 300 MHz when using either an absorbing layer or a scattering boundary condition to absorb the outgoing wave from the background field and scattered field from the PEC circle. The RCS difference between two cases is less than 0.2 dB at the backward scattering direction and around 0.5 dB at the forward scattering direction.

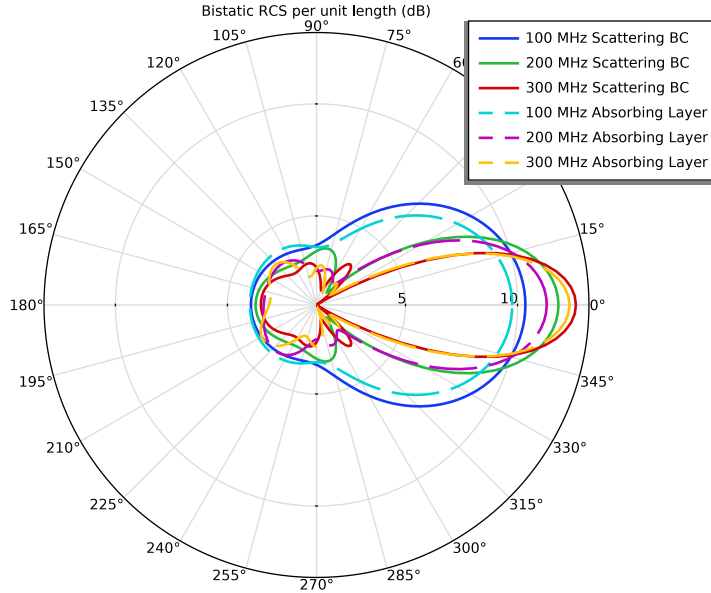


Figure 2: The dB-scaled RCS per unit length with the absorbing layer (solid) and scattering boundary condition (dashed).

Notes About the COMSOL Implementation

In the scattered field formulation, the total field is defined as $\mathbf{E}_{\text{total}} = \mathbf{E}_{\text{rel}} + \mathbf{E}_{\text{b}}$ where \mathbf{E}_{rel} is the relative field and \mathbf{E}_{b} is the background field. The relative field is the difference between the total field caused by the presence of the scatterer and the background field. After performing the time dependent study and the FFT, only the relative field and postprocessing variables related to far-field analysis are available in the frequency domain, since the FFT takes only dependent variables. Other postprocessing variables are valid only in the time domain and can be accessed via stored solutions.

TABLE I: AVAILABILITY OF VARIABLES FOR POSTPROCESSING.


VARIABLE	DESCRIPTION	AVAILABILITY
comp1.Ez or Ez	Dependent variable, Relative (scattered) field	Frequency domain
ewte.Ez	Total field	Time domain
ewte.Ebz	Background field	Time domain
ewte.bRCS2D	RCS per unit length	Frequency domain

Application Library path: RF_Module/Scattering_and_RCS/rcs_time_explicit




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Radio Frequency > Electromagnetic Waves, Time Explicit (ewte)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Time Dependent with FFT**.
- 6 Click  **Done**.

GLOBAL DEFINITIONS

Parameters 1


- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
fb0	200[MHz]	2E8 Hz	Modulated Gaussian center frequency
lda0	c_const/fb0	1.499 m	Wavelength
k0	2*pi/lda0	4.1917 1/m	Wave number
T0	1/fb0	5E-9 s	Modulation period


Here, c_const used in the free space wavelength is a predefined COMSOL constant for the speed of light in vacuum.

GEOMETRY 1




Circle 1 (c1)

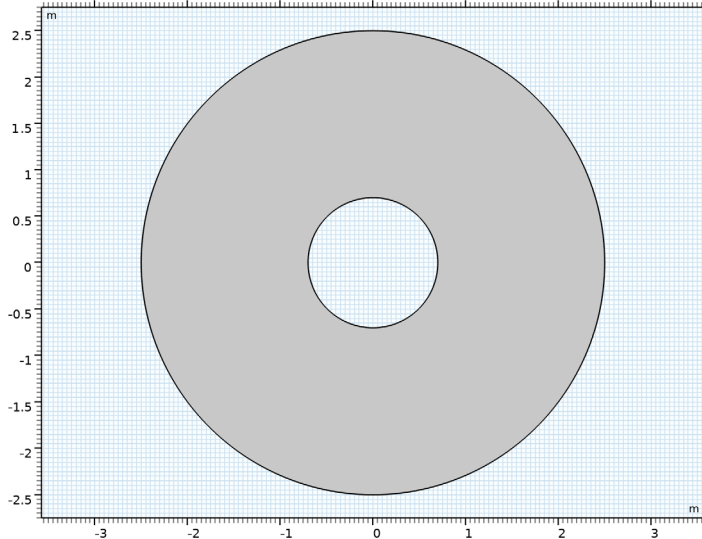
- 1 In the **Geometry** toolbar, click  **Circle**.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type 2.5.

Circle 2 (c2)

- 1 In the **Geometry** toolbar, click  **Circle**.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type 0.7.

Difference 1 (dif1)


- 1 In the **Geometry** toolbar, click  **Booleans and Partitions** and choose **Difference**.
- 2 Select the object **c1** only.
- 3 In the **Settings** window for **Difference**, locate the **Difference** section.
- 4 Click to select the  **Activate Selection** toggle button for **Objects to subtract**.
- 5 Select the object **c2** only.
- 6 Click  **Build All Objects**.




ELECTROMAGNETIC WAVES, TIME EXPLICIT (EWTE)

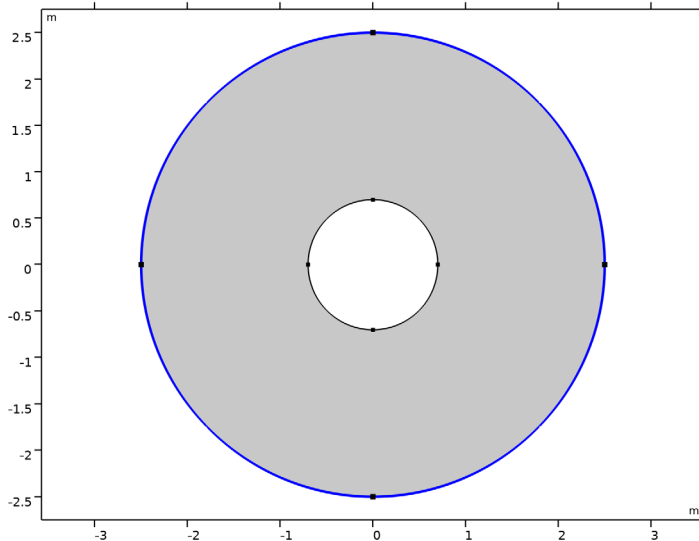
- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Time Explicit (ewte)**.
- 2 In the **Settings** window for **Electromagnetic Waves, Time Explicit**, locate the **Components** section.
- 3 From the **Field components solved for** list, choose **H in plane (TE wave)**.

Background Field 1

- 1 In the **Physics** toolbar, click  **Domains** and choose **Background Field**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Background Field**, locate the **Settings** section.
- 4 From the **Polarization direction** list, choose **z**.
- 5 In the f_0 text field, type fb0.
- 6 In the d_{offset} text field, type 2.5.

Scattering Boundary Condition 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Scattering Boundary Condition**.
- 2 Select Boundaries 1, 2, 5, and 8 only.

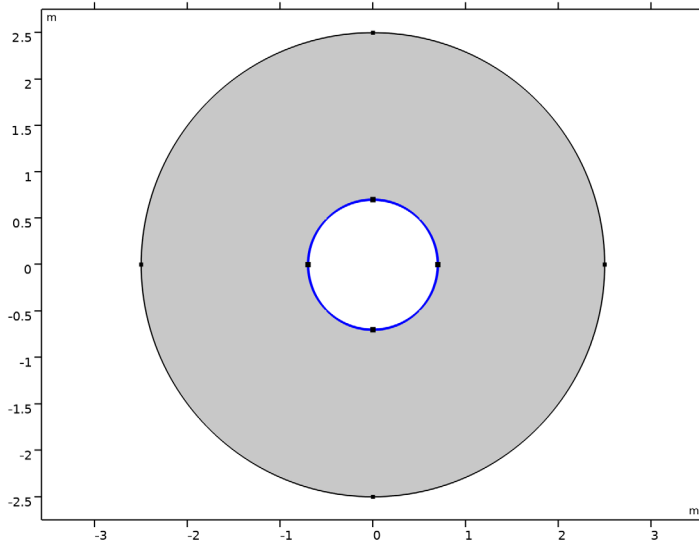


Far-Field Domain 1

- In the **Physics** toolbar, click  **Domains** and choose **Far-Field Domain**.

Far-Field Calculation 1

- 1 In the **Model Builder** window, expand the **Far-Field Domain 1** node, then click **Far-Field Calculation 1**.
- 2 Select Boundaries 3, 4, 6, and 7 only.



MATERIALS


Material 1 (mat1)

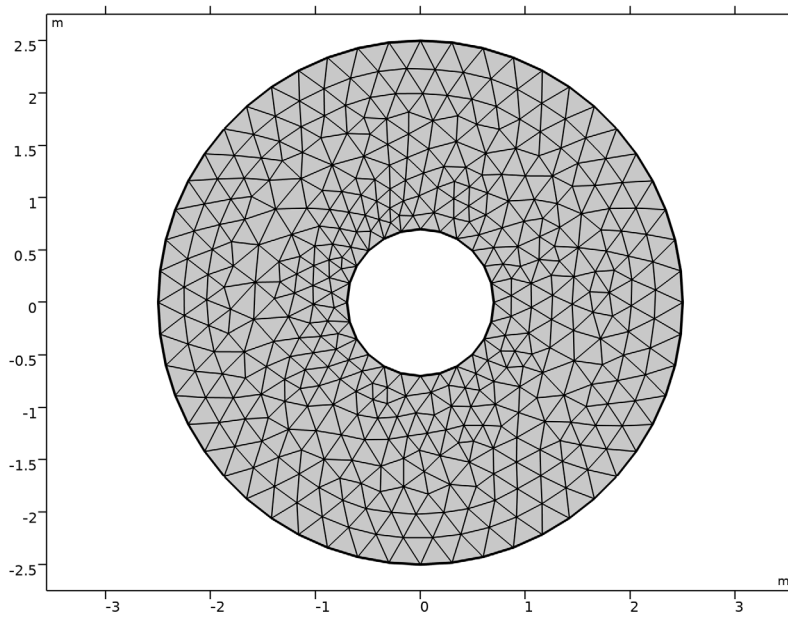
- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, locate the **Material Contents** section.
- 3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilon _{n_} iso ; epsilon _n rii = epsilon _n iso, epsilon _n rij = 0	1		Basic

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; muri = mur_iso , $\text{murij} = 0$	1		Basic
Electric conductivity	sigma_iso ; sigma_{ii} = sigma_iso , $\text{sigma}_{ij} = 0$	0	S/m	Basic

MESH 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Time Explicit (ewte)** section.
- 3 In the **Maximum element size in free space** text field, type $1\text{da}0/5$.
This sets the maximum mesh element size to 0.2 wavelengths.
- 4 Click  **Build All**.



STUDY 1


Step 1: Time Dependent

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.
- 2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 3 In the **Output times** text field, type range $(0, 1 / (4 * f_{b0}), 10 * T_0)$. The Sampling rate $4 * f_{b0}$ satisfies the Nyquist condition for the time to frequency fast Fourier transform (FFT) where its bandwidth is $2 * f_{b0}$ excluding negative frequencies.

Step 2: Time to Frequency FFT

- 1 In the **Model Builder** window, click **Step 2: Time to Frequency FFT**.
- 2 In the **Settings** window for **Time to Frequency FFT**, locate the **Study Settings** section.
- 3 In the **End time** text field, type $20 * T_0$. This makes sure that the FFT end time is longer than the simulation time so zero-padding can be applied during the time to frequency FFT. This will generate a finer frequency resolution in the resulting frequency response.
- 4 In the **Maximum output frequency** text field, type $2 * f_{b0}$.

Step 3: Combine Solutions

- 1 In the **Model Builder** window, click **Step 3: Combine Solutions**.
- 2 In the **Settings** window for **Combine Solutions**, locate the **Combine Solutions Settings** section.
- 3 In the **Excluded if** text field, type $\text{freq} < 0.1 * f_{b0} \ || \ \text{freq} > 2 * f_{b0} - 0.1 * f_{b0}$. This excludes the first 5% and last 5% of the frequency response after FFT.
- 4 In the **Study** toolbar, click  **Compute**.


RESULTS

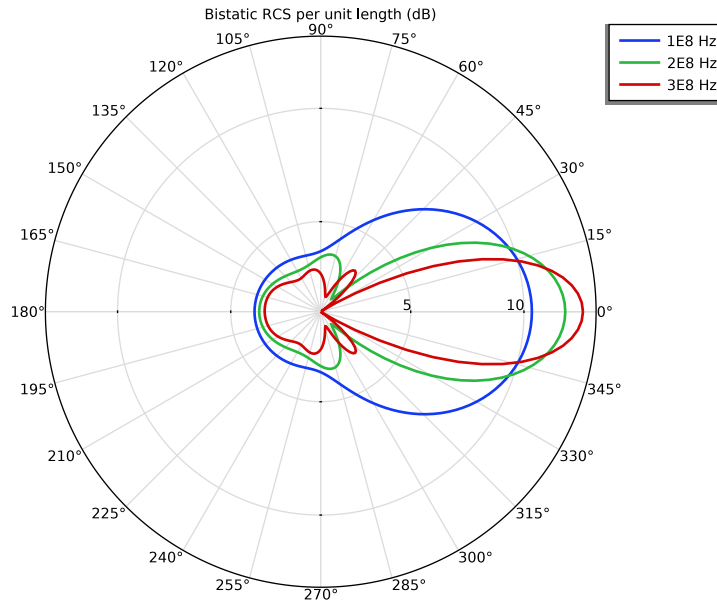
2D Far Field (ewte)

- 1 In the **Settings** window for **Polar Plot Group**, locate the **Data** section.
- 2 From the **Parameter selection (freq)** list, choose **From list**.
- 3 In the **Parameter values (freq (Hz))** list, choose **1E8**, **2E8**, and **3E8**.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
- 5 In the **Title** text area, type Bistatic RCS per unit length (dB).

Radiation Pattern 1


- 1 In the **Model Builder** window, expand the **2D Far Field (ewte)** node, then click **Radiation Pattern 1**.
- 2 In the **Settings** window for **Radiation Pattern**, locate the **Expression** section.

- 3 In the **Expression** text field, type $10 \cdot \log_{10}(\text{ewte}.\text{bRCS2D})$.
- 4 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 5 Click to expand the **Coloring and Style** section. From the **Width** list, choose **2**.
- 6 In the **2D Far Field (ewte)** toolbar, click  **Plot**.




The RCS per unit length of the circle is plotted.

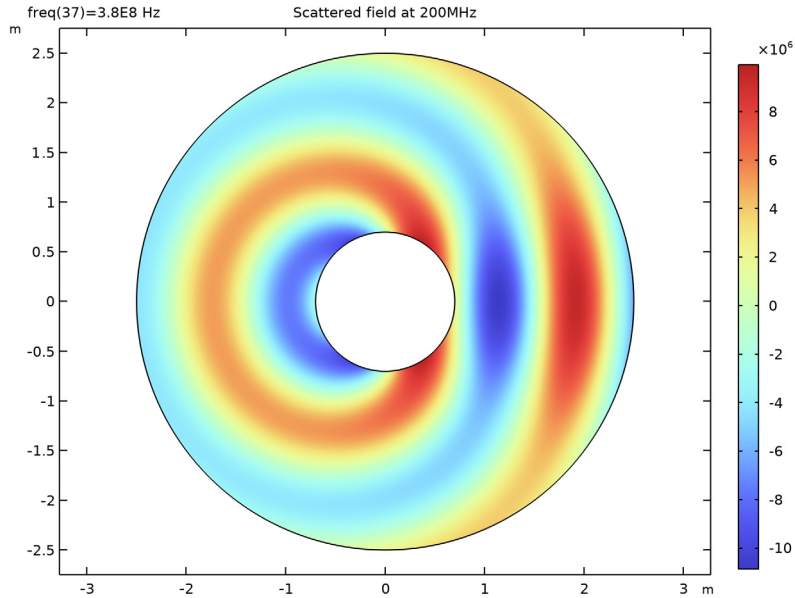
2D Plot Group 2

- 1 In the **Results** toolbar, click  **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (freq (Hz))** list, choose **2E8**.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
- 5 In the **Title** text area, type Scattered field at 200MHz.


Surface 1

- 1 Right-click **2D Plot Group 2** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type E_z . This dependent variable is the z-component of the scattered field.

4 In the **2D Plot Group 2** toolbar, click  **Plot**.



2D Plot Group 3

1 In the **Results** toolbar, click  **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution Store 1 (sol2)**.

4 From the **Time (s)** list, choose **1E-8**.

5 Locate the **Title** section. From the **Title type** list, choose **Manual**.

6 In the **Title** text area, type **Scattered field at 10ns**.


Surface 1

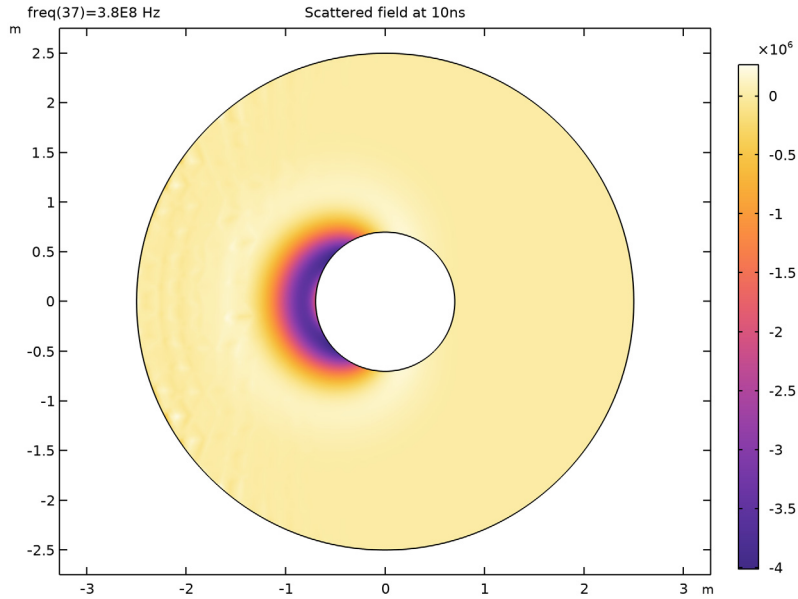
1 Right-click **2D Plot Group 3** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type $ewt_e.Ez - ewt_e.Ebz$. This is the difference in z-components between the total field and background field.

4 Locate the **Coloring and Style** section. From the **Color table** list, choose **HeatCameraLight**.

5 In the **2D Plot Group 3** toolbar, click  **Plot**.



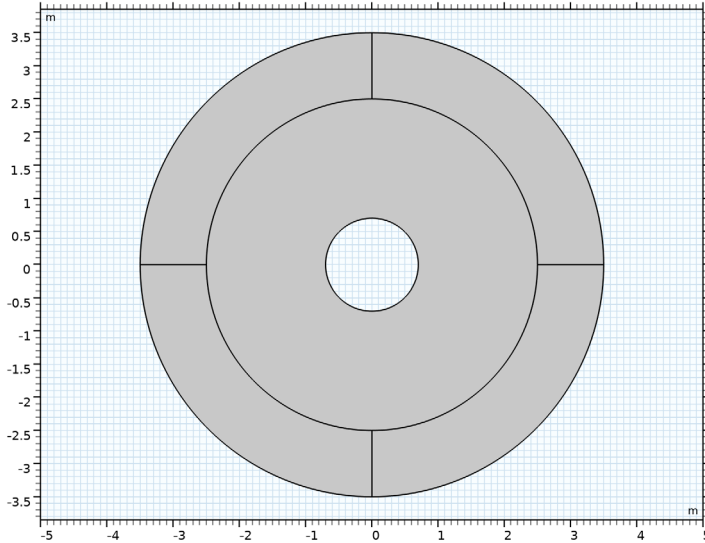
GEOMETRY 1


Circle 1 (c1)

- 1 In the **Model Builder** window, under **Component 1 (comp1) > Geometry 1** click **Circle 1 (c1)**.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type **3.5**.
- 4 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	1


5 Click  **Build All Objects**.



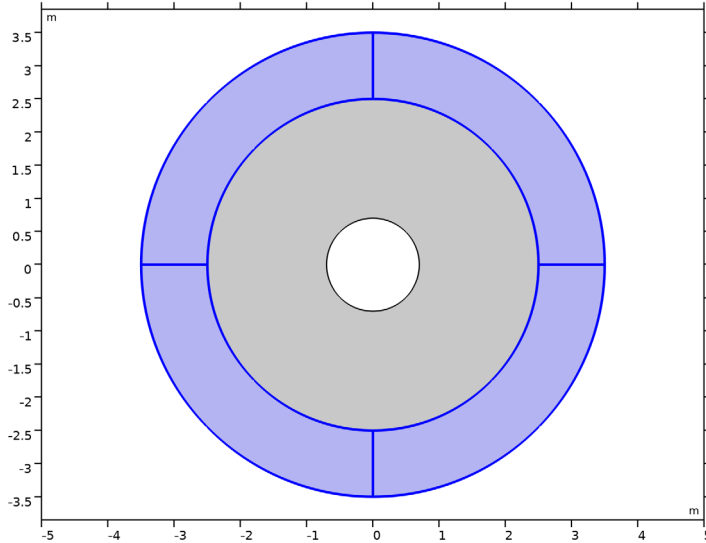
6 Click the  **Zoom Extents** button in the **Graphics** toolbar.

DEFINITIONS

Absorbing Layer 1 (abl)

1 In the **Definitions** toolbar, click  **Absorbing Layer**.

2 Select Domains 1–4 only.



3 In the **Settings** window for **Absorbing Layer**, locate the **Geometry** section.

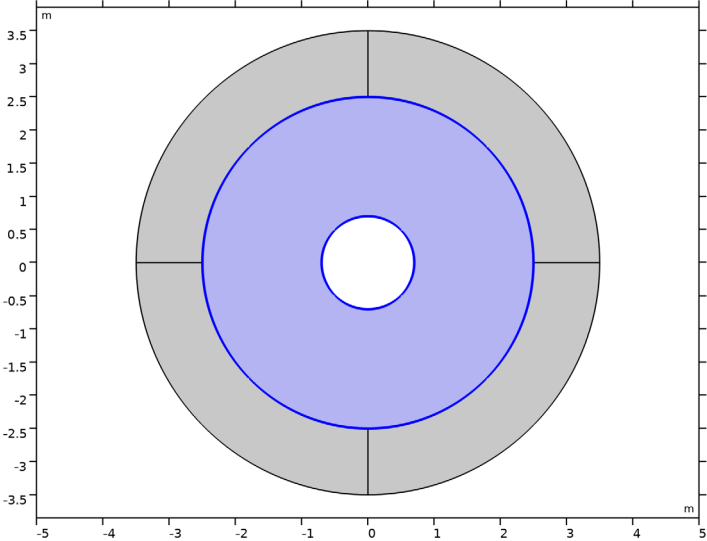
4 From the **Type** list, choose **Cylindrical**.

ELECTROMAGNETIC WAVES, TIME EXPLICIT (EWTE)

Background Field 1

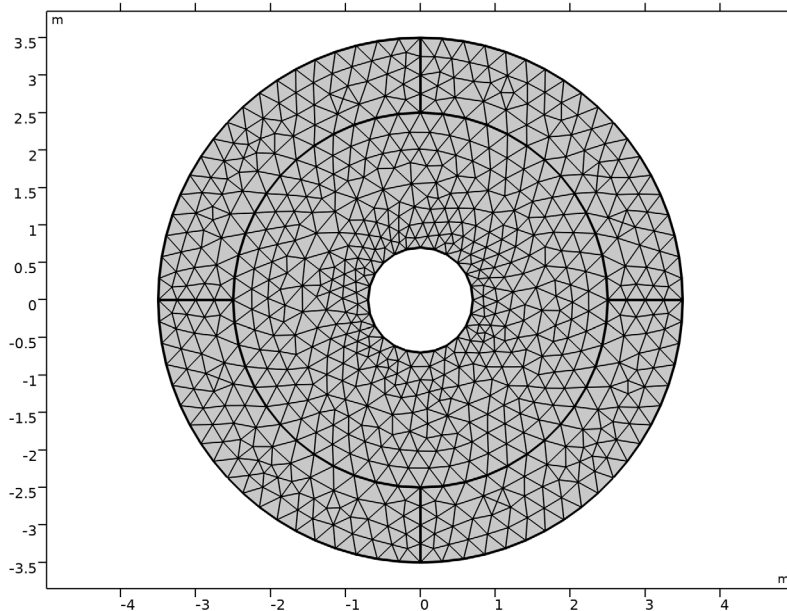
1 In the **Model Builder** window, under **Component 1 (comp1) > Electromagnetic Waves, Time Explicit (ewte)** click **Background Field 1**.

2 Select Domain 5 only.





MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.



ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Time Dependent with FFT**.
- 4 Right-click and choose **Add Study**.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Time Dependent


- 1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 2 In the **Output times** text field, type range $(0, 1 / (4 * f_{b0}), 10 * T_0)$.

Step 2: Time to Frequency FFT

- 1 In the **Model Builder** window, click **Step 2: Time to Frequency FFT**.

- 2 In the **Settings** window for **Time to Frequency FFT**, locate the **Study Settings** section.
- 3 In the **End time** text field, type $20 \cdot T_0$.
- 4 In the **Maximum output frequency** text field, type $2 \cdot f_{b0}$.

Step 3: Combine Solutions

- 1 In the **Model Builder** window, click **Step 3: Combine Solutions**.
- 2 In the **Settings** window for **Combine Solutions**, locate the **Combine Solutions Settings** section.
- 3 In the **Excluded if** text field, type $\text{freq} < 0.1 \cdot f_{b0} \ || \ \text{freq} > 2 \cdot f_{b0} - 0.1 \cdot f_{b0}$.
- 4 In the **Study** toolbar, click  **Compute**.

RESULTS

Radiation Pattern 2

- 1 In the **Model Builder** window, under **Results > 2D Far Field (ewte)** right-click **Radiation Pattern 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Radiation Pattern**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2/Solution 4 (sol4)**.
- 4 From the **Parameter selection (freq)** list, choose **From list**.
- 5 In the **Parameter values (freq (Hz))** list, choose **1E8**, **2E8**, and **3E8**.

Radiation Pattern 1

- 1 In the **Model Builder** window, click **Radiation Pattern 1**.
- 2 In the **Settings** window for **Radiation Pattern**, locate the **Legends** section.
- 3 From the **Legends** list, choose **Evaluated**.
- 4 In the **Legend** text field, type $\text{eval}(\text{ewte}.\text{freq}/1\text{e}6)$ MHz Scattering BC.

Radiation Pattern 2

- 1 In the **Model Builder** window, click **Radiation Pattern 2**.
- 2 In the **Settings** window for **Radiation Pattern**, locate the **Legends** section.
- 3 From the **Legends** list, choose **Evaluated**.
- 4 In the **Legend** text field, type $\text{eval}(\text{ewte}.\text{freq}/1\text{e}6)$ MHz Absorbing Layer.
- 5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

The RCS per unit length when using the scattering boundary condition and absorbing layer is shown in [Figure 2](#).

