



Model created in COMSOL Multiphysics 6.4

# Lossy Circular Waveguide

## Introduction

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In mode analysis it is usually the primary goal to find a *propagation constant*. This quantity is often, but not always, real valued; if the analysis involves some lossy part, such as a nonzero conductivity or an open boundary, the eigenvalue is complex. In such situations, the real and imaginary parts have separate interpretations:

- The real part is the propagation constant
- The imaginary part is the *attenuation constant*, measuring the damping in space

## Model Definition

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The mode analysis study for electromagnetic waves solves the eigenvalue problem

$$\nabla \times (\mu^{-1} \nabla \times \mathbf{E}) - \lambda \mathbf{E} = 0$$

where

$$\lambda = k_0^2 \left( \epsilon_r - \frac{j\sigma}{\omega} \right)$$

is the eigenvalue. For time-harmonic problems, the electric field for out-of-plane propagation can be written as

$$\mathbf{E}(\mathbf{r}, t) = \text{Re}(\tilde{\mathbf{E}}(\mathbf{r})e^{j\omega t - \alpha z})$$

where  $z$  is the known out-of-plane direction.

The spatial parameter,  $\alpha = \delta_z + j\beta = -\lambda$ , can have a real part and an imaginary part. The propagation constant is equal to the imaginary part, and the real part,  $\delta_z$ , represents the damping along the propagation direction.

### VARIABLES INFLUENCED BY MODE ANALYSIS

The following table lists the variables that are influenced by the mode analysis in terms of the eigenvalue lambda:

NAME	EXPRESSION	CAN BE COMPLEX	DESCRIPTION
beta	imag(-lambda)	No	Propagation constant
dampz	real(-lambda)	No	Attenuation constant

NAME	EXPRESSION	CAN BE COMPLEX	DESCRIPTION
dampzdB	$20 \cdot \log_{10}(\exp(1)) \cdot \text{dampz}$	No	Attenuation constant per meter, dB
neff	$j \cdot \lambda / k_0$	Yes	Effective mode index

This two-dimensional model finds the modes of a circular waveguide with walls made of a nonperfect conductor, which is copper in this case. The losses in the walls lead to attenuation of the propagating wave. The propagation constant  $\beta$  is obtained as the imaginary part of  $\alpha = -\lambda$  and the damping  $\delta_z$  is obtained as the real part. Since the wave in the waveguide is attenuated in the  $z$  direction as  $e^{-\delta_z z}$ , the attenuation in dB scale is calculated using the formula

$$\Delta_{\text{dB}} = 20 \delta_z \log_e$$

### *Results and Discussion*

The eigenvalue solver returns six eigenvalues. Table 1 shows the six effective mode indices,  $n_{\text{eff}}$ , closest to 1, where

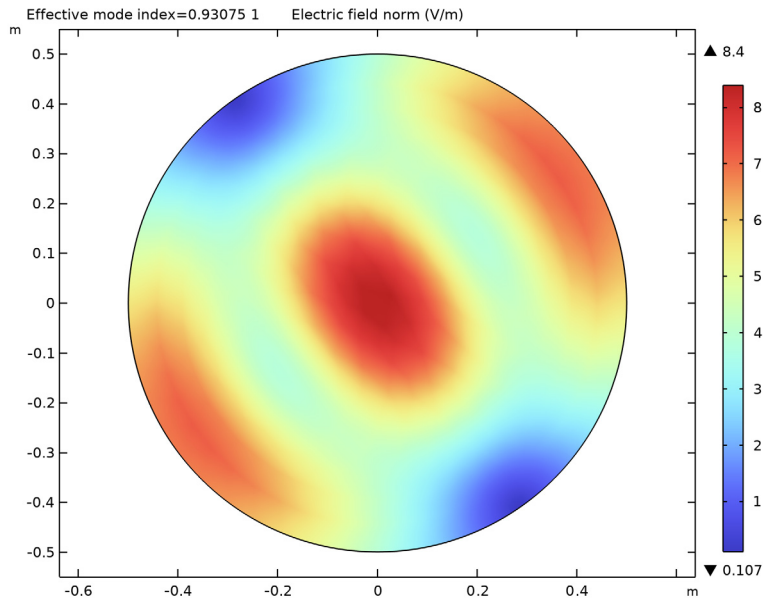
$$n_{\text{eff}} = j \frac{\lambda}{k_0}$$

and  $k_0$  is the wave number in vacuum. The table also lists the propagation constant and damping in dB/m for each eigenmode.

TABLE 1: EFFECTIVE MODE INDICES, PROPAGATION CONSTANTS, AND ATTENUATION.

Effective mode index	Propagation constant (1/m)	Attenuation constant per meter (dB/m)
$0.9308 - 2.2082 \cdot 10^{-6}i$	19.5071	$4.0199 \cdot 10^{-4}$
$0.9733 - 2.1116 \cdot 10^{-6}i$	20.3992	$3.844 \cdot 10^{-4}$
$0.9566 - 1.7954 \cdot 10^{-6}i$	20.0486	$3.2684 \cdot 10^{-4}$
$0.9566 - 1.7954 \cdot 10^{-6}i$	20.0486	$3.2684 \cdot 10^{-4}$
$0.9844 - 9.38 \cdot 10^{-7}i$	20.6324	$1.7076 \cdot 10^{-4}$
$0.9844 - 9.38 \cdot 10^{-7}i$	20.6324	$1.7076 \cdot 10^{-4}$

The default surface plot shows the norm of the electric field for the effective mode index  $0.9308 - 2.208 \cdot 10^{-6}j$ . This plot is shown in [Figure 1](#).



*Figure 1: The surface plot visualizes the norm of the electric field for the effective mode index  $0.9308 - 2.208 \cdot 10^{-6}j$ .*

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**Application Library path:** RF\_Module/Transmission\_Lines\_and\_Waveguides/lossy\_circular\_waveguide


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### *Modeling Instructions*


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From the **File** menu, choose **New**.

#### **NEW**

In the **New** window, click  **Model Wizard**.



#### **MODEL WIZARD**

I In the **Model Wizard** window, click  **2D**.


- 2 In the **Select Physics** tree, select **Radio Frequency > Electromagnetic Waves, Frequency Domain (emw)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Mode Analysis**.
- 6 Click  **Done**.

## GEOMETRY I

*Circle 1 (c1)*

- 1 In the **Geometry** toolbar, click  **Circle**.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type 0.5.
- 4 Click  **Build All Objects**.

## ADD MATERIAL

- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Built-in > Air**.
- 4 Click the **Add to Component** button in the window toolbar.


## MATERIALS

*Air (mat1)*

By default the first material you add apply for all domains.

Next, specify copper as the material on the boundaries.

## ADD MATERIAL

- 1 Go to the **Add Material** window.
- 2 In the tree, select **Built-in > Copper**.
- 3 Click the **Add to Component** button in the window toolbar.
- 4 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.


## MATERIALS

*Copper (mat2)*

- 1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.
- 2 From the **Geometric entity level** list, choose **Boundary**.
- 3 From the **Selection** list, choose **All boundaries**.

## ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

*Impedance Boundary Condition 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Impedance Boundary Condition**.
- 2 In the **Settings** window for **Impedance Boundary Condition**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.


## MESH 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
- 3 In the table, clear the **Use** checkbox for **Electromagnetic Waves, Frequency Domain (emw)**.

Solve for the 6 effective mode indices closest to 1.

## STUDY 1

*Step 1: Mode Analysis*

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Mode Analysis**.
- 2 In the **Settings** window for **Mode Analysis**, locate the **Study Settings** section.
- 3 Select the **Desired number of modes** checkbox.
- 4 In the **Study** toolbar, click  **Compute**.




## RESULTS

*Electric Field (emw)*

The default plot shows the electric field norm for the lowest mode found; compare with [Figure 1](#).

Calculate the propagation constant and the attenuation constant (in dB) for each effective mode index.

*Global Evaluation 1*

- 1 In the **Results** toolbar, click  **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Frequency Domain > Global > emw.beta - Propagation constant - rad/m**.
- 3 Click  **Evaluate**.  
Compare the results with those in the second column of [Table 1](#).
- 4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1) > Electromagnetic Waves, Frequency Domain > Global > emw.dampzdB - Attenuation constant per meter, dB - dB/m**.
- 5 Click  **Evaluate**.  
Compare with the third column of [Table 1](#).