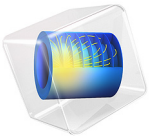

Model created in COMSOL Multiphysics 6.4

DC Glow Discharge Coupled with the Two-Term Boltzmann Equation

Introduction

DC glow discharges in the low pressure regime have long been used for gas lasers and fluorescent lamps. DC discharges are attractive to study because the solution is time independent. This model shows how to use the Plasma interface to set up an analysis of a positive column. The discharge is sustained by emission of secondary electrons at the cathode.

This tutorial models a DC discharge by solving plasma fluid equations fully coupled with the homogeneous and time-independent electron Boltzmann equation in the classical two-term approximation. The approximated Boltzmann equation is solved for each position of space and is coupled with the fluid equations by way of the electron mean energy. The rate constants of electron impact reactions and the electron transport parameters are obtained by suitable integration of the computed electron energy distribution function over electron scattering cross sections. Simulation results from this model are compared with results from the model DC Glow Discharge, 1D where the electron energy distribution function (EEDF) is assumed to be Maxwellian.

Model Definition

The DC discharge consists of two electrodes, one powered (the anode) and one grounded (the cathode):

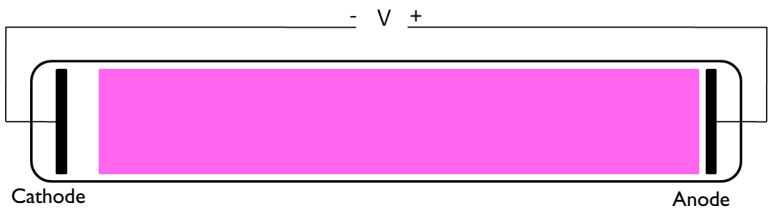

Figure 1: Schematic of the DC discharge. The voltage applied across the electrodes leads to formation of a plasma.

DOMAIN EQUATIONS

Electron transport is modeled by solving the continuity equation, the momentum equation under the drift-diffusion approximation, and the mean electron energy equation (for detailed information on electron transport, see *Theory for the Drift Diffusion Interface* in the *Plasma Module User's Guide*)

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\mu_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e] = R_e$$

$$\frac{\partial}{\partial t}(n_\varepsilon) + \nabla \cdot [-n_\varepsilon(\mu_\varepsilon \bullet \mathbf{E}) - \mathbf{D}_\varepsilon \bullet \nabla n_\varepsilon] + \mathbf{E} \cdot \Gamma_e = S_{\text{en}}$$

The source coefficients in the above equations are determined by the plasma chemistry. The electron rate expression is defined as

$$R_e = N_A \sum_j \nu_{e,j} r_j$$

where $\nu_{e,j}$ is the stoichiometric coefficient, and the reaction rate is defined as

$$r_j = k_j^f \prod_{i \in \text{ react}} c_i^{-\nu_{i,j}} + k_j^r \prod_{i \in \text{ prod}} c_k^{\nu_{i,j}}$$

where k_j^f is the forward rate constant and k_j^r is the reversed rate constant. Both the Electron Impact Reaction feature and Reaction feature can contribute to the electron rate expression. However, when using the Reaction feature it is important to note that the associated electron energy gain or loss is not included in the source term of the electron mean energy equation.

When *Townsend coefficients* are used, the reaction rate is defined as

$$r_j = \frac{\alpha_j}{N_n} |\Gamma_e| \prod_{i \neq e \in \text{ react}} c_i^{-\nu_{i,j}}$$

where α_j/N_n is the reduced Townsend coefficient for reaction j (SI unit: m^2) and Γ_e is the electron flux as defined above (SI unit: $1/(\text{m}^2 \cdot \text{s})$). Townsend coefficients can increase the stability of the numerical scheme when the electron flux is field driven as is the case with DC discharges.

The total electron energy loss or gained is calculated by summing the collisional energy changes from all reactions defined with the Electron Impact Reaction feature as

$$S_{\text{en}} = -\sum_j r_j \Delta\varepsilon_j F$$

where $\Delta\varepsilon_j$ is the energy loss from reaction j (SI unit: V) and F is the Faraday constant (SI unit: C/mol). For excitation and ionization collisions $\Delta\varepsilon_j$ corresponds to the energy of the excited state being excited/deexcited or ionized, for attachment $\Delta\varepsilon_j$ is set to zero, and for elastic collisions

$$\Delta\varepsilon = 2\frac{m_e}{m_k}\frac{3}{2}\left[T_e(eV) - \frac{k_B}{e}T_{gas}(K)\right]$$

where m_e and m_k are the electron and heavy species mass in kg, T_e is the electron temperature in eV, and T_{gas} is the gas temperature in K. For heavy species, the following equation is solved for the mass fraction of each species (for detailed information on the transport of the nonelectron species, see *Theory for the Heavy Species Transport Interface* in the *Plasma Module User's Guide*):

$$\rho\frac{\partial}{\partial t}(w_k) + \rho(\mathbf{u}\cdot\nabla)w_k = \nabla\cdot\mathbf{j}_k + R_k$$

The electrostatic field is computed using the following equation:

$$-\nabla\cdot\varepsilon_0\varepsilon_r\nabla V = \rho$$

The space charge density ρ is automatically computed based on the plasma chemistry specified in the model using the formula:

$$\rho = q\left(\sum_{k=1}^{N} Z_k n_k - n_e\right)$$

For detailed information about electrostatics see *Theory for the Electrostatics Interface* in the *Plasma Module User's Guide*.

BOLTZMANN EQUATION IN THE TWO-TERM APPROXIMATION

The EEDF used in this model is obtained from the solution of the homogeneous and time-independent electron Boltzmann equation in the classical two-term approximation

$$\frac{\partial}{\partial\varepsilon}\left(WF_0 - D\frac{\partial F_0}{\partial\varepsilon}\right) = S$$

where F_0 is the isotropic part of an EEDF constant in time and space that verifies the following normalization:

$$\int_0^\infty \varepsilon^{1/2}F_0 d\varepsilon = 1\,.$$

The different terms are presented below:

$$W = -\gamma\varepsilon^2\sigma_\varepsilon$$

$$D = \frac{\gamma}{3}\left(\frac{E}{N_n}\right)^2\left(\frac{\varepsilon}{Q}\right) + \frac{\gamma k_b T}{q}\varepsilon^2\sigma_\varepsilon$$

The following definitions apply:

$$\sigma_m = \sum_{k = \text{all}} x_k\sigma_k$$

$$\sigma_\varepsilon = \sum_{k = \text{elastic}} 2\left(\frac{m_e}{M}\right)x_k\sigma_k$$

$$\tilde{\sigma}_m = \sigma_m + \frac{\lambda}{\varepsilon^{1/2}}$$

$$Q = \tilde{\sigma}_m$$

Here:

- $\gamma = (2q/m_e)^{1/2}$ (SI unit: $C^{1/2}/kg^{1/2}$)
- m_e is the electron mass (SI unit: kg)
- $\varepsilon = (v/\gamma)^2$ is energy (SI unit: V)
- σ_ε is the total elastic collision cross section (SI unit: m^2)
- σ_m is the total momentum collision cross section (SI unit: m^2)
- $\tilde{\sigma}_m$ is the normalized total momentum collision cross section (SI unit: m^2)
- q is the electron charge (SI unit: C)
- ε_0 is the permittivity of free space (SI unit: F/m)
- T is the temperature of the background gas (SI unit: K)
- k_b is the Boltzmann constant (SI unit: J/K)
- n_e is the electron density (SI unit: $1/m^3$)
- N_n is the background gas density (SI unit: $1/m^3$)
- λ is a scalar-valued renormalization factor that ensures that the EEDF is normalized to 1as explained above. An ODE is implemented to solve for the value of λ.
- M is the mass of the target species (SI unit: kg).

The source term, S represents energy loss due to inelastic collisions. Because the energy loss due to an inelastic collision is quantized, the source term is nonlocal in the energy

space. The source term can be decomposed into four parts where the following definitions apply:

$$S = \sum_{k = \text{inelastic}} C_k - \gamma\lambda\varepsilon^{1/2}F_0$$

$$C_{k = \text{excitation}} = -\gamma x_k[\varepsilon\sigma_k(\varepsilon)F_0(\varepsilon) - (\varepsilon + \Delta\varepsilon_k)\sigma_k(\varepsilon + \Delta\varepsilon_k)F_0(\varepsilon + \Delta\varepsilon_k)]$$

$$C_{k = \text{all}} = -\gamma x_k\varepsilon\sigma_k(\varepsilon)F_0(\varepsilon)$$

$$C_{\text{ion}} = -\gamma x_k[\varepsilon\sigma_k(\varepsilon)F_0(\varepsilon) - 4(2\varepsilon + \Delta\varepsilon_k)\sigma_k(2\varepsilon + \Delta\varepsilon_k)F_0(2\varepsilon + \Delta\varepsilon_k)]$$

where x_k is the mole fraction of the target species for reaction k, σ_k is the collision cross section for reaction k, $\Delta\varepsilon_k$ is the energy loss from collision k, and δ is the delta function at $\varepsilon = 0$.

The mean electron energy is defined by the integral

$$\bar{\varepsilon} = \int_0^\infty F_0\varepsilon^{3/2}d\varepsilon\,.$$

The external excitation in the Boltzmann equation comes from an electric field. If the Boltzmann equation is to be solved for a given mean electron energy (as in the present model) a Lagrange multiplier is introduced to solve for the reduced electric field, such that the following equation, presented in the weak form of the constraint, is satisfied:

$$\left(\int_0^\infty F_0\varepsilon^{3/2}d\varepsilon - \bar{\varepsilon}\right)\left(\frac{\tilde{E}}{N_n}\right) = 0$$

where the tilde denotes a test function. For detailed information about electrostatics see *Theory for the Boltzmann Equation, Two-Term Approximation Interface* in the *Plasma Module User's Guide*.

COUPLING BETWEEN THE FLUID MODEL AND THE BOLTZMANN EQUATION

The fluid model and the Boltzmann equation are solved fully coupled. The Boltzmann equation is solved at every point in space using as input the space dependent mean electron energy and the mole fraction of the different heavy species obtained from the fluid model. The EEDFs obtained from the solution of the Boltzmann equation are used to compute macroscopic rate constants and transport coefficients intervening in the fluid model equations thus closing the loop.

ELECTRON RATE COEFFICIENTS, TOWNSEND COEFFICIENTS, AND TRANSPORT PARAMETERS

Rate coefficient for electron impact reaction are computed using cross section data and computed EEDFs by the integral

$$k_k = \gamma \int_0^\infty \varepsilon \sigma_k(\varepsilon) F_0(\varepsilon) d\varepsilon \,.$$

The Townsend coefficients are computed using

$$\alpha_k / N_n = \frac{(E/N_n)(\nu_k/N_n)}{(P/N_n)}$$

where ν_k is the frequency associated with the reaction rate k_k

$$\nu_k = k_k N_n$$

and P is the power absorbed by the electrons from the DC electric field. The reduced electron mobility is computed using

$$\mu_e N_n = -\left(\frac{\gamma}{3}\right) \int_0^\infty \frac{\varepsilon}{\sigma_m} \left(\frac{\partial F_0}{\partial \varepsilon}\right) d\varepsilon \,.$$

BOUNDARY CONDITIONS

Unlike RF discharges, the mechanism for sustaining the discharge is emission of secondary electrons from the cathode. An electron is emitted from the cathode surface with a specified probability when struck by an ion. These electrons are then accelerated by the strong electric field close to the cathode where they acquire enough energy to initiate ionization. The net result is a rapid increase in the electron density close to the cathode in a region often known as the *cathode fall* or *Crookes dark space*.

Electrons are lost to the wall due to random motion within a few mean free paths of the wall and gained due to secondary emission effects, resulting in the following boundary condition for the electron flux:

$$\mathbf{n} \cdot \Gamma_e = \left(\frac{1}{2} \nu_{e,\,\mathrm{th}} n_e\right) - \sum_p \gamma_p (\Gamma_p \cdot \mathbf{n}) \tag{1}$$

and the electron energy flux:

$$\mathbf{n} \cdot \Gamma_\varepsilon = \left(\frac{5}{6} \nu_{e,\,\mathrm{th}} n_\varepsilon\right) - \sum_p \varepsilon_p \gamma_p (\Gamma_p \cdot \mathbf{n}) \tag{2}$$

The second term on the right-hand side of Equation 1 is the gain of electrons due to secondary emission effects, γ_p being the secondary emission coefficient. The second term in Equation 2 is the secondary emission energy flux, ε_p being the mean energy of the secondary electrons. For the heavy species, ions are lost to the wall due to surface reactions and the fact that the electric field is directed toward the wall:

$$\mathbf{n} \cdot \mathbf{j}_k = M_w R_k + M_w c_k Z \mu_k (\mathbf{E} \cdot \mathbf{n}) [Z_k \mu_k (\mathbf{E} \cdot \mathbf{n}) > 0]$$

PLASMA CHEMISTRY

Argon is one of the simplest mechanisms to implement at low pressures. The electronically excited states can be lumped into a single species which results in a chemical mechanism consisting of only 3 species and 7 reactions (electron impact cross sections are obtained from Ref. 3):

TABLE 1: TABLE OF COLLISIONS AND REACTIONS MODELED.

Reaction	Formula	Type	$\Delta\varepsilon$(eV)
1	e+Ar=>e+Ar	Elastic	0
2	e+Ar=>e+Ars	Excitation	11.5
3	e+Ars=>e+Ar	Superelastic	-11.5
4	e+Ar=>2e+Ar+	Ionization	15.8
5	e+Ars=>2e+Ar+	Ionization	4.24
6	Ars+Ars=>e+Ar+Ar+	Penning ionization	-
7	Ars+Ar=>Ar+Ar	Metastable quenching	-

In this discharge, the electron density and density of excited species is relatively low so stepwise ionization is not as important as in high density discharges. In addition to volumetric reactions, the following surface reactions are implemented:

TABLE 2: TABLE OF SURFACE REACTIONS.

Reaction	Formula	Sticking coefficient
1	Ars=>Ar	1
2	Ar+=>Ar	1

When a metastable argon atom makes contact with the wall, it reverts to the ground state argon atom with some probability (the sticking coefficient).

Results and Discussion

In this section, results are presented from the model. For reference, results of the fluid model when assuming a Maxwellian EEDF are also presented.

Figure 2 through Figure 7 present the electron density, the electron temperature, the potential, the mass fraction of the argon excited state, the current densities of the charged species, and the electron transport parameters along the discharge. All quantities presented change significantly when the EEDF is computed. In summary: the *electron density* is more than 50% smaller and the maximum deviates toward the cathode; the *electron temperature* profile is similar but it is 1 eV higher corresponding to an increase of 50%; the *mass fraction of the argon excited* is considerable different with a much more pronounced maximum near the cathode and a much smaller population in the positive column; the different *current densities* maintain roughly the same profile and magnitude with a difference of only 10%.

Perhaps the most important difference between the two models is the fact that the creation of excited species and ions in the positive column is much smaller for the case when the EEDF is computed. This can be verified from the plots of the source terms of these species (not presented). For the ions, the source term can be verified to be close to zero since the ion current above 20 cm is practically constant meaning that the source term is zero (the divergence of the ion flux is zero).

At a first look this might seem incoherent with the fact that the electron temperature increases for the case of the computed EEDF. However, this can be understandable by looking at the shape of the computed EEDFs in the region of the positive column presented in Figure 8. The EEDFs deviates considerable from a Maxwellian with the high energy region above the excitation and ionization thresholds being strongly unpopulated. EEDFs with these shapes when integrated over the excitation or ionization cross section give small rate coefficients.

When modeling a DC discharge one might find that using a computed EEDF gives better agreement with experiments. By inspecting the EEDFs the use of an analytic EEDF with a depleted tail, representative of the EEDFs in positive column, might not be adequate since the EEDF in the cathode region has a different shape closer to a Maxwellian. For this case, a good option to avoid large computational times is to give to the fluid model a

previously computed EEDF in the form of a lookup table as a function of the mean electron energy.

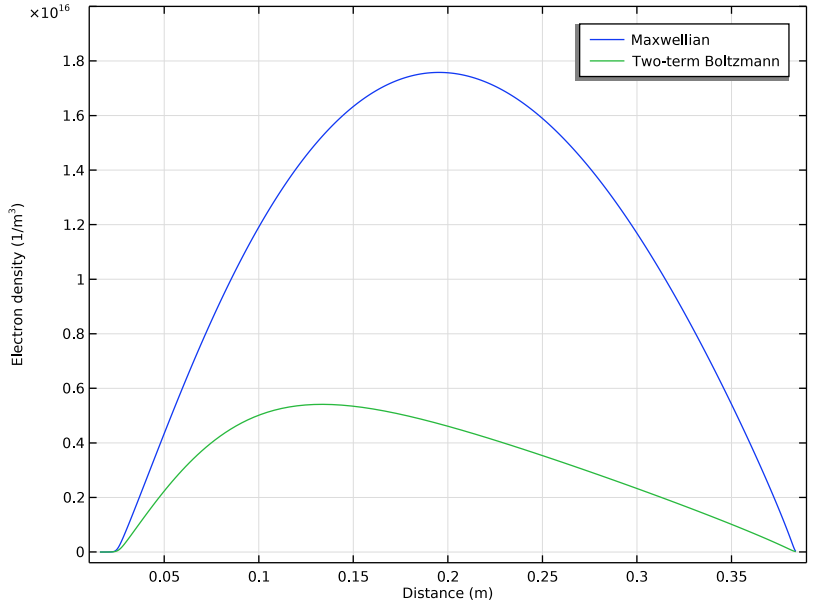

Figure 2: Electron density obtained with the present model that computes the EEDF and from the same model where the EEDF was assumed to be Maxwellian.

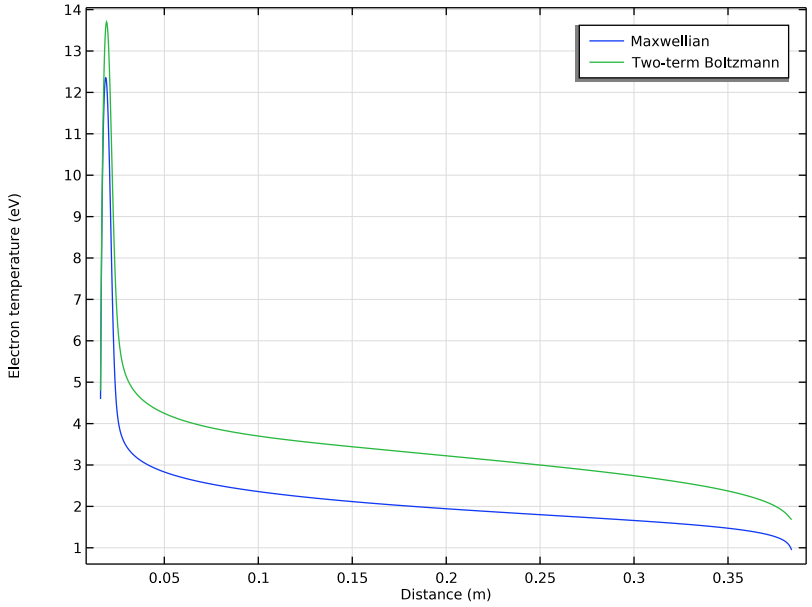

Figure 3: Electron temperature obtained with the present model that computes the EEDF and from the same model where the EEDF was assumed to be Maxwellian.

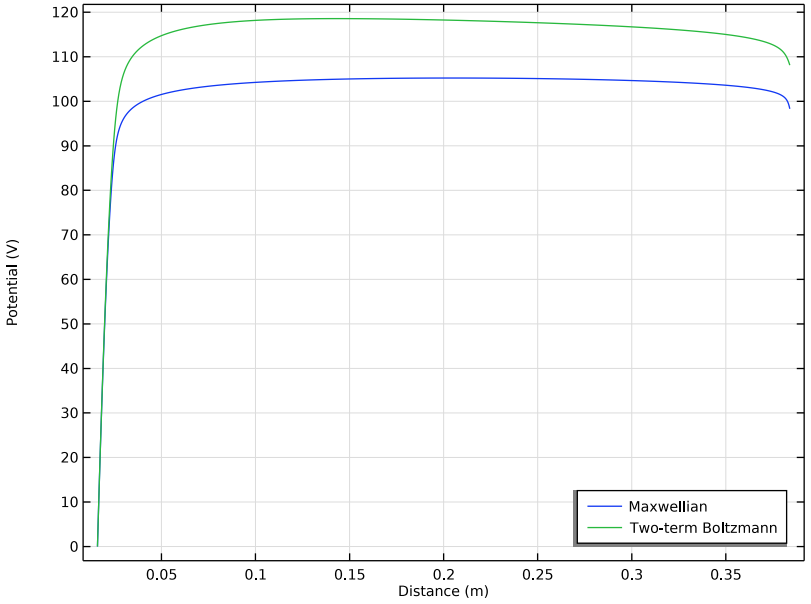

Figure 4: Potential obtained with the present model that computes the EEDF and from the same model where the EEDF was assumed to be Maxwellian.

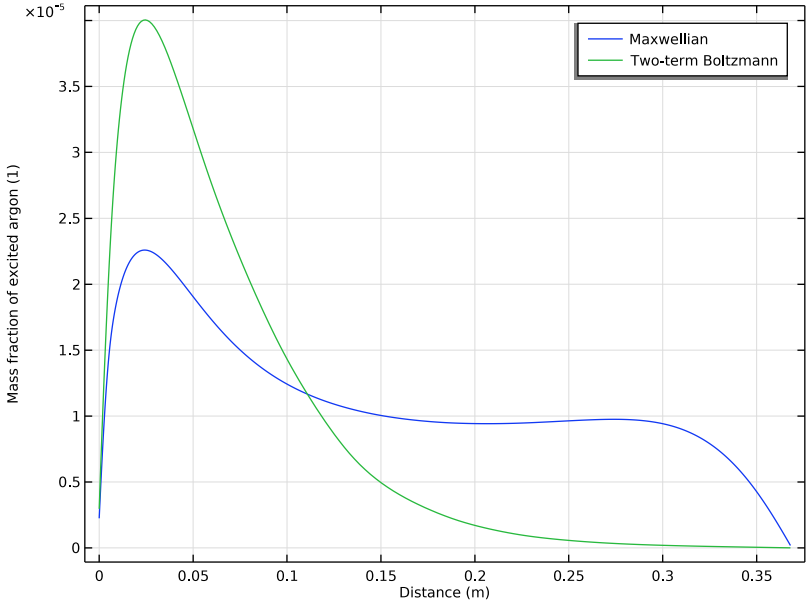

Figure 5: Mass fraction of the excited argon obtained with the present model that computes the EEDF and from the same model where the EEDF was assumed to be Maxwellian.

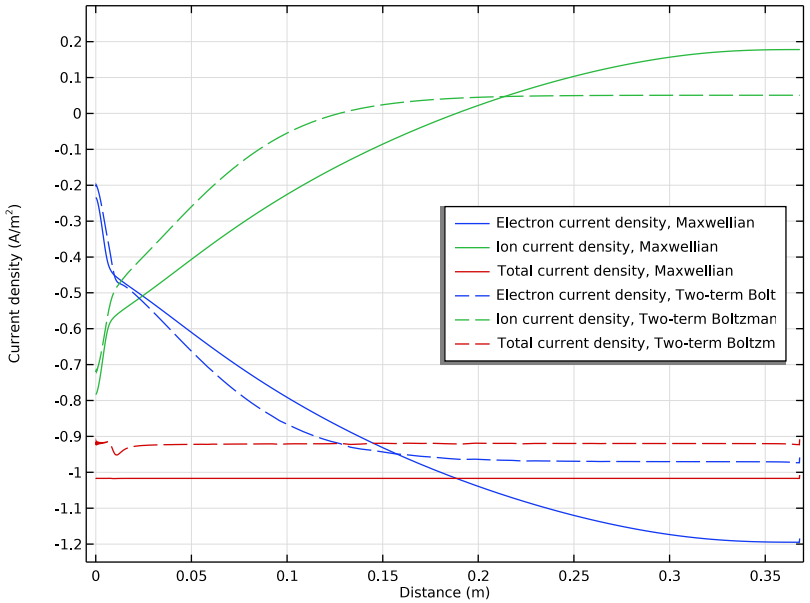

Figure 6: Electron current density (blue), the ion current density (green), and the total current density (red) obtained with the present model that computes the EEDF and from the same model where the EEDF was assumed to be Maxwellian.

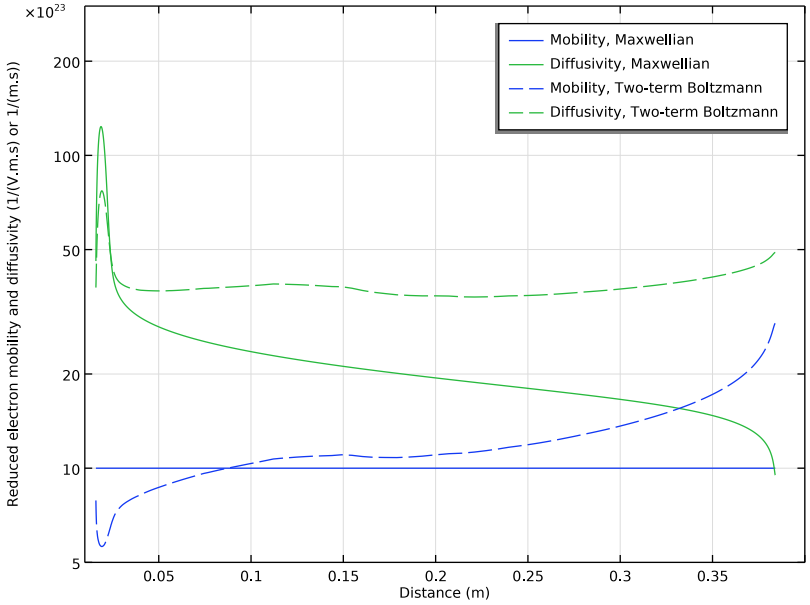

Figure 7: Reduced electron mobility (blue) and diffusivity (green) electron current density (blue), the ion current density (green) and the total current density (red) obtained with the present model that computes the EEDF and from the same model where the EEDF was assumed to be Maxwellian.

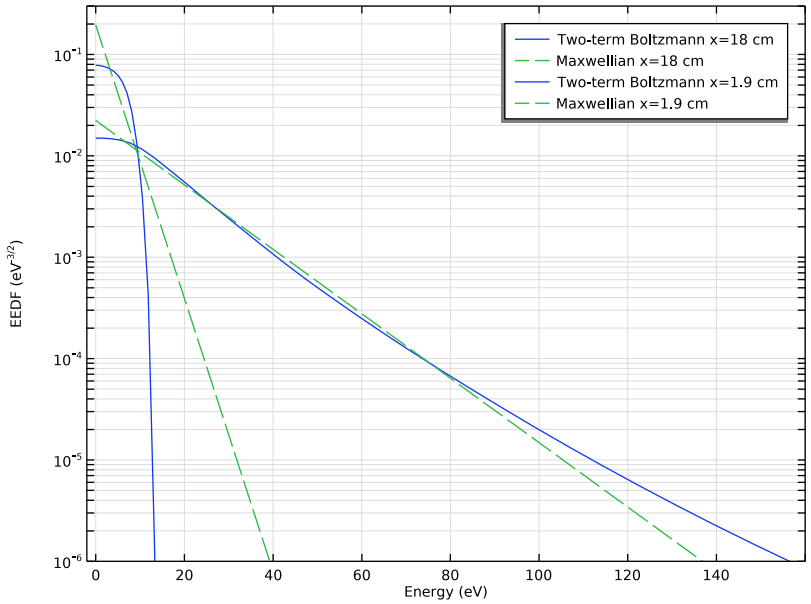

Figure 8: Computed EEDFs in the positive column region (1.9 cm) and in the cathode fall region (18 cm). For reference, it is presented a Maxwellian EEDF with the mean energy obtained from the model with the computed EEDF.

References

1. M.A. Lieberman and A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, John Wiley & Sons, 2005.

2. G.J.M. Hagelaar and L.C. Pitchford, "Solving the Boltzmann Equation to Obtain Electron Transport Coefficients and Rate Coefficients for Fluid Models," *Plasma Sources Science and Technology*, vol. 14, pp. 722–733, 2005.

3. Phelps database, www.lxcat.net, retrieved 2017.

Application Library path: `Plasma_Module/Space-Dependent_EEDF_Modeling/positive_column_1d_boltzmann`

ROOT

The following instructions show how to create a model that solves plasma–fluid-type equations fully coupled with the Boltzmann equation in the two-term approximation.

APPLICATION LIBRARIES

The DC positive column application-library model is used as an example

1. From the **File** menu, choose **Application Libraries**.
2. In the **Application Libraries** window, select **Plasma Module** > **Direct Current Discharges** > **positive_column_1d** in the tree.
3. Click **Open**.

Choose to solve the Boltzmann equation in the two-term approximation, set the number of elements in the extra dimension to 50, and choose to compute the maximum energy automatically.

COMPONENT 1 (COMP1)

In the **Model Builder** window, expand the **Component 1 (comp1)** node.

PLASMA (PLAS)

1. In the **Model Builder** window, expand the **Component 1 (comp1)** > **Plasma (plas)** node, then click **Plasma (plas)**.
2. In the **Settings** window for **Plasma**, locate the **Electron Energy Distribution Function Settings** section.
3. From the **Electron energy distribution function** list, choose **Boltzmann equation, two-term approximation (linear)**.
4. In the *N* text field, type `50`.
5. Select the **Compute maximum energy** checkbox.
6. Click to expand the **Discretization** section. From the **Formulation** list, choose **Finite element, log formulation (linear shape function)**.

Change the two reactions that use Townsend coefficients from lookup tables to use cross-section data and select **Use Townsend coefficient**. This way, the Townsend coefficients are computed using the electron impact cross section and the computed EEDF.

2: e+Ar=>e+Ars

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Plasma (plas)** click **2: e+Ar=>e+Ars**.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Collision** section.

3 From the **Specify reaction using** list, choose **Cross-section data**.

4 Locate the **Reaction Parameters** section. Select the **Use Townsend coefficient** checkbox.

4: e+Ar=>2e+Ar+

1 In the **Model Builder** window, click **4: e+Ar=>2e+Ar+**.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Collision** section.

3 From the **Specify reaction using** list, choose **Cross-section data**.

4 Locate the **Reaction Parameters** section. Select the **Use Townsend coefficient** checkbox.

Plasma Model 1

1 In the **Model Builder** window, click **Plasma Model 1**.

2 In the **Settings** window for **Plasma Model**, locate the **Electron Density and Energy** section.

3 From the **Electron transport properties** list, choose **Mobility from electron energy distribution function**. This way, the electron mobility is obtained from suitable integration of the EEDF over the cross section for momentum transfer. The remaining transport parameters are computed using the electron mobility.

MAXWELLIAN EEDF

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Maxwellian EEDF` in the **Label** text field.

ADD STUDY

Add an **EEDF Initialization** study to solve for the EEDF only. Later, the solution of this study will serve as the initial condition for the study that solves the fluid-type equations coupled with the Boltzmann equation.

1 In the **Study** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **EEDF Initialization**.

4 Click the **Add Study** button in the window toolbar.

5 In the **Study** toolbar, click **Add Study** to close the **Add Study** window.

EEDF INITIALIZATION

1 In the **Settings** window for **Study**, type `EEDF Initialization` in the **Label** text field.

2 In the **Home** toolbar, click **Compute**.

RESULTS

Electron Energy Distribution Function, EEDF Initialization

1 In the **Settings** window for **1D Plot Group**, type `Electron Energy Distribution Function, EEDF Initialization` in the **Label** text field.

2 Locate the **Axis** section. Select the **Manual axis limits** checkbox.

3 Locate the **Plot Settings** section. In the **x-axis label** text field, type `Energy (eV)`.

4 In the **y-axis label** text field, type `EEDF (eV<sup>-3/2</sup>)`.

5 Locate the **Axis** section. In the **x minimum** text field, type `0`.

6 In the **x maximum** text field, type `50`.

7 In the **y minimum** text field, type `1e-8`.

8 In the **y maximum** text field, type `1`.

9 In the **Electron Energy Distribution Function, EEDF Initialization** toolbar, click **Plot**.

ADD STUDY

Add another study to solve for the plasma–fluid-type equations fully coupled with the Boltzmann equation in the two-term approximation.

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Time Dependent**.

4 Click the **Add Study** button in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3

Step 1: Time Dependent

1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

2 In the **Output times** text field, type `0 10^{range(-8,8/100,0)}`.

3 Click to expand the **Values of Dependent Variables** section. Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **EEDF Initialization, EEDF Initialization**.

6 In the **Model Builder** window, click **Study 3**.

7 In the **Settings** window for **Study**, locate the **Study Settings** section.

8 Clear the **Generate default plots** checkbox.

9 In the **Label** text field, type `Computed EEDF`.

10 In the **Study** toolbar, click **Compute**.

RESULTS

Compare the results of the model that used a Maxwellian EEDF with those of the one that computes the EEDF.

Electron Density (plas)

1 In the **Model Builder** window, under **Results** click **Electron Density (plas)**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Axis** section. Select the **Manual axis limits** checkbox.

5 In the **y maximum** text field, type `2E16`.

Line Graph 1

1 In the **Model Builder** window, expand the **Electron Density (plas)** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, click to expand the **Legends** section.

3 Select the **Show legends** checkbox.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
Maxwellian

Line Graph 2

1 Right-click **Results** > **Electron Density (plas)** > **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Two-term Boltzmann

6 In the **Electron Density (plas)** toolbar, click **Plot**.

RESULTS

Electron Density (plas)

In the **Model Builder** window, collapse the **Results** > **Electron Density (plas)** node.

Electron Temperature (plas)

1 In the **Model Builder** window, click **Electron Temperature (plas)**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

Line Graph 1

1 In the **Model Builder** window, expand the **Electron Temperature (plas)** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** checkbox.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
Maxwellian

Line Graph 2

1 Right-click **Results** > **Electron Temperature (plas)** > **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Two-term Boltzmann

6 In the **Electron Temperature (plas)** toolbar, click **Plot**.

RESULTS

Electron Temperature (plas)

In the **Model Builder** window, collapse the **Results** > **Electron Temperature (plas)** node.

Electric Potential (plas)

1 In the **Model Builder** window, click **Electric Potential (plas)**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Legend** section. From the **Position** list, choose **Lower right**.

Line Graph 1

1 In the **Model Builder** window, expand the **Electric Potential (plas)** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** checkbox.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
Maxwellian

Line Graph 2

1 Right-click **Results** > **Electric Potential (plas)** > **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Two-term Boltzmann

6 In the **Electric Potential (plas)** toolbar, click **Plot**.

RESULTS

Electric Potential (plas)

In the **Model Builder** window, collapse the **Results** > **Electric Potential (plas)** node.

Excited Argon Mass Fraction

1 In the **Model Builder** window, click **Excited Argon Mass Fraction**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

Line Graph 1

1 In the **Model Builder** window, expand the **Excited Argon Mass Fraction** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** checkbox.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
Maxwellian

Line Graph 2

1 Right-click **Results** > **Excited Argon Mass Fraction** > **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Two-term Boltzmann

6 In the **Excited Argon Mass Fraction** toolbar, click **Plot**.

RESULTS

Excited Argon Mass Fraction

In the **Model Builder** window, collapse the **Results** > **Excited Argon Mass Fraction** node.

Argon Ion Number Density

1 In the **Model Builder** window, click **Argon Ion Number Density**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

Line Graph 1

1 In the **Model Builder** window, expand the **Argon Ion Number Density** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** checkbox.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
Maxwellian

Line Graph 2

1 Right-click **Results** > **Argon Ion Number Density** > **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Two-term Boltzmann

6 In the **Argon Ion Number Density** toolbar, click **Plot**.

RESULTS

Argon Ion Number Density

In the **Model Builder** window, collapse the **Results** > **Argon Ion Number Density** node.

Line Graph 1

1 In the **Model Builder** window, under **Results** > **Current Density** click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 In the table, enter the following settings:

Legends
Electron current density, Maxwellian

Line Graph 4

1 Right-click **Results** > **Current Density** > **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

6 Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

7 Locate the **Legends** section. In the table, enter the following settings:

Legends
Electron current density, Two-term Boltzmann

Line Graph 2

1 In the **Model Builder** window, click **Line Graph 2**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 In the table, enter the following settings:

Legends
Ion current density, Maxwellian

Line Graph 5

1 Right-click **Results** > **Current Density** > **Line Graph 2** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
Ion current density, Two-term Boltzmann

Line Graph 3

1 In the **Model Builder** window, click **Line Graph 3**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 In the table, enter the following settings:

Legends
Total current density, Maxwellian

Line Graph 6

1 Right-click **Results** > **Current Density** > **Line Graph 3** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Total current density, Two-term Boltzmann`

Current Density

1 In the **Model Builder** window, click **Current Density**.

2 In the **Settings** window for **1D Plot Group**, locate the **Axis** section.

3 Select the **Manual axis limits** checkbox.

4 In the **x minimum** text field, type `-0.005`.

5 In the **x maximum** text field, type `0.37`.

6 In the **y minimum** text field, type `-1.25`.

7 In the **y maximum** text field, type `0.3`.

8 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

9 In the **Current Density** toolbar, click **Plot**.

Electron Mobility and Diffusivity

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electron Mobility and Diffusivity` in the **Label** text field.

3 Locate the **Data** section. From the **Time selection** list, choose **Last**.

4 Locate the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** checkbox. In the associated text field, type `Distance (m)`.

7 Select the **y-axis label** checkbox. In the associated text field, type `Reduced electron mobility and diffusivity (1/(V.m.s) or 1/(m.s))`.

8 Locate the **Axis** section. Select the **y-axis log scale** checkbox.

9 Select the **Manual axis limits** checkbox.

10 In the **x minimum** text field, type `0.01`.

11 In the **x maximum** text field, type `0.4`.

12 In the **y minimum** text field, type `5e23`.

13 In the **y maximum** text field, type `3e25`.

Line Graph 1

1 Right-click **Electron Mobility and Diffusivity** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **All domains**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.muexx*plas.Nn`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `x`.

7 Locate the **Legends** section. Select the **Show legends** checkbox.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Mobility, Maxwellian`

10 In the **Electron Mobility and Diffusivity** toolbar, click **Plot**.

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `plas.Dexx*plas.Nn`.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Diffusivity, Maxwellian`

Line Graph 3

1 In the **Model Builder** window, under **Results** > **Electron Mobility and Diffusivity** right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Mobility, Two-term Boltzmann

6 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

7 Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

Line Graph 4

1 In the **Model Builder** window, under **Results** > **Electron Mobility and Diffusivity** right-click **Line Graph 2** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Computed EEDF/Solution 3 (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
Diffusivity, Two-term Boltzmann

7 In the **Electron Mobility and Diffusivity** toolbar, click **Plot**.

RESULTS

Electron Mobility and Diffusivity

In the **Model Builder** window, collapse the **Results** > **Electron Mobility and Diffusivity** node.

Computed EEDF/Solution 3 (5) (sol3)

1 In the **Model Builder** window, expand the **Results** > **Datasets** node.

2 Right-click **Results** > **Datasets** > **Computed EEDF/Solution 3 (sol3)** and choose **Duplicate**.

3 In the **Settings** window for **Solution**, locate the **Solution** section.

4 From the **Component** list, choose **Extra Dimension from Plasma (plas_eedf_xdim)**.

RESULTS

In the **Model Builder** window, collapse the **Results** > **Datasets** node.

Electron Energy Distribution Function

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electron Energy Distribution Function` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Computed EEDF/ Solution 3 (5) (sol3)**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Title** section. From the **Title type** list, choose **None**.

6 Locate the **Plot Settings** section.

7 Select the **x-axis label** checkbox. In the associated text field, type `Energy (eV)`.

8 Select the **y-axis label** checkbox. In the associated text field, type `EEDF (eV<sup>-3/2</sup>)`.

9 Locate the **Axis** section. Select the **y-axis log scale** checkbox.

10 Select the **Manual axis limits** checkbox.

11 In the **x minimum** text field, type `-2`.

12 In the **x maximum** text field, type `160`.

13 In the **y minimum** text field, type `1e-6`.

14 In the **y maximum** text field, type `0.3`.

Line Graph 1

1 Right-click **Electron Energy Distribution Function** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **All domains**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd1(plas.rcx, plas.fcap)`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `atxd1(plas.rcx,plas.xeedf)`.

7 Locate the **Legends** section. Select the **Show legends** checkbox.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Two-term Boltzmann x=18 cm`

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `atxd1(plas.rcx,plas.fmax)`.

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Maxwellian x=18 cm

6 In the **Electron Energy Distribution Function** toolbar, click **Plot**.

Line Graph 3

1 In the **Model Builder** window, under **Results** > **Electron Energy Distribution Function** right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `atxd1(0.019,plas.fcap)`.

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd1(0.019, plas.xeedf)`.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
Two-term Boltzmann x=1.9 cm

Line Graph 4

1 In the **Model Builder** window, under **Results** > **Electron Energy Distribution Function** right-click **Line Graph 2** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `atxd1(0.019,plas.fmax)`.

4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd1(0.019, plas.xeedf)`.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Maxwellian x=1.9 cm

RESULTS

Electron Energy Distribution Function

1 In the **Model Builder** window, collapse the **Results** > **Electron Energy Distribution Function** node.

2 In the **Model Builder** window, click **Electron Energy Distribution Function**.

3 In the **Electron Energy Distribution Function** toolbar, click **Plot**.