



Model created in COMSOL Multiphysics 6.4

# GEC ICP Reactor Coupled with the Two-Term Boltzmann Equation

## Introduction

The GEC cell was introduced by NIST (National Institute of Standards and Technology) in order to provide a standardized platform for experimental and modeling studies of discharges in different laboratories. The plasma is sustained via inductive heating. The Reference Cell operates as an inductively-coupled plasma in this model.

This tutorial models the GEC ICP reactor by solving plasma fluid equations fully coupled with the homogeneous and time-independent electron Boltzmann equation in the classical two-term approximation. The approximated Boltzmann equation is solved for in each position of space and is coupled with the fluid equations by way of the electron mean energy. The rate constants of electron impact reactions and the electron transport parameters are obtained by suitable integration of the computed electron energy distribution function over electron scattering cross sections. Simulation results from this model are compared with results from the model [GEC ICP Reactor, Argon Chemistry](#) where the electron energy distribution function (EEDF) is assumed Maxwellian.



*Figure 1: GEC ICP reactor geometry consisting of a 5 turn copper coil, plasma volume, dielectrics, and wafer with pedestal.*

---

**Note:** This application requires the Plasma Module and the AC/DC Module.

---

## *Model Definition*

---

### **FLUID MODEL**

Electron transport is modeled by solving the continuity equation, the momentum equation under the drift-diffusion approximation, and the mean electron energy equation (for detailed information on electron transport, see *Theory for the Drift Diffusion Interface* in the *Plasma Module User's Guide*)

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\mu_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e] = R_e$$

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\mu_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e] + \mathbf{E} \cdot \Gamma_e = S_{en}$$

The source coefficients in the above equations are determined by the plasma chemistry. The electron rate expression is defined as

$$R_e = N_A \sum_j \nu_{e,j} r_j$$

where  $\nu_{e,j}$  is the stoichiometric coefficient, and the reaction rate is defined as

$$r_j = k_j^f \prod_{i \in \text{react}} c_i^{-\nu_{i,j}} + k_j^r \prod_{i \in \text{prod}} c_i^{\nu_{i,j}}$$

where  $k_j^f$  is the forward rate constant and  $k_j^r$  is the reversed rate constant. Both the Electron Impact Reaction feature and Reaction feature can contribute to the electron rate expression. However, when using the Reaction feature it is important to note that the associated electron energy gain or loss is not included in the source term of the electron mean energy equation.

When *Townsend coefficients* are used, the reaction rate is defined as

$$r_j = \frac{\alpha_j}{N_n} |\Gamma_e| \prod_{i \neq e \in \text{react}} c_i^{-\nu_{i,j}}$$

where  $\alpha_j/N_n$  is the reduced Townsend coefficient for reaction  $j$  (SI unit:  $m^2$ ) and  $\Gamma_e$  is the electron flux as defined above (SI unit:  $1/(m^2 \cdot s)$ ). Townsend coefficients can increase the stability of the numerical scheme when the electron flux is field driven as is the case with DC discharges.

The total electron energy loss or gained is calculated by summing the collisional energy changes from all reactions defined with the Electron Impact Reaction feature as

$$S_{\text{en}} = -\sum_j r_j \Delta \epsilon_j F$$

where  $\Delta \epsilon_j$  is the energy loss from reaction  $j$  (SI unit: V) and  $F$  is the Faraday constant (SI unit: C/mol). For excitation and ionization collisions  $\Delta \epsilon_j$  corresponds to the energy of the excited state being excited/deexcited or ionized, for attachment  $\Delta \epsilon_j$  is set to zero, and for elastic collisions

$$\Delta \epsilon = 2 \frac{m_e}{m_k} \frac{3}{2} \left[ T_e (\text{eV}) - \frac{k_B}{e} T_{\text{gas}} (\text{K}) \right]$$

where  $m_e$  and  $m_k$  are the electron and heavy species mass in kg,  $T_e$  is the electron temperature in eV, and  $T_{\text{gas}}$  is the gas temperature in K. For heavy species, the following equation is solved for the mass fraction of each species (for detailed information on the transport of the nonelectron species, see *Theory for the Heavy Species Transport Interface* in the *Plasma Module User's Guide*):

$$\rho \frac{\partial}{\partial t} (w_k) + \rho (\mathbf{u} \cdot \nabla) w_k = \nabla \cdot \mathbf{j}_k + R_k$$

The electrostatic field is computed using the following equation:

$$-\nabla \cdot \epsilon_0 \epsilon_r \nabla V = \rho$$

The space charge density  $\rho$  is automatically computed based on the plasma chemistry specified in the model using the formula:

$$\rho = q \left( \sum_{k=1}^N Z_k n_k - n_e \right)$$

For detailed information about electrostatics see *Theory for the Electrostatics Interface* in the *Plasma Module User's Guide*.

For a nonmagnetized, nonpolarized plasma, the induction currents are computed in the frequency domain using the following equation:

$$(j\omega\sigma - \omega^2 \epsilon_0) \mathbf{A} + \nabla \times (\mu_0^{-1} \nabla \times \mathbf{A}) = \mathbf{J}^e$$

The plasma conductivity is specified using the cold plasma approximation

$$\sigma = \frac{q^2 n_e}{m_e (v_{\text{eff}} + j\omega)} \phi$$

$$v_{\text{eff}} = -\frac{\mu_r}{\mu_i} \omega$$

$$\phi = -\frac{\mu_r^2 + \mu_i^2 m_e \omega}{\mu_i} \frac{e}{e}$$

where  $n_e$  is the electron density,  $q$  is the electron charge,  $m_e$  is the electron mass,  $\omega$  is the angular frequency  $\mu_r$  and  $\mu_i$  are the real and imaginary parts of the HF mobility. The effective collision frequency  $v_{\text{eff}}$  and the factor  $\phi$  are obtained in coherence with the Boltzmann equation approach here used.

### **BOLTZMANN EQUATION IN THE TWO-TERM APPROXIMATION**

The EEDF used in this model is obtained from the solution of the homogeneous and time-independent electron Boltzmann equation in the classical two-term approximation

$$\frac{\partial}{\partial \epsilon} \left( W F_0 - D \frac{\partial F_0}{\partial \epsilon} \right) = S$$

where  $F_0$  is the isotropic part of an EEDF constant in time and space that satisfies the following normalization

$$\int_0^\infty \epsilon^{1/2} F_0 d\epsilon = 1.$$

In this model the Boltzmann equation in the two-term approximation is solved in the high-frequency limit including the effects of electron–electron collisions. The different terms are presented below:

$$W = -\gamma \epsilon^2 \sigma_\epsilon - 3\alpha \left( \frac{n_e}{N_n} \right) A_1$$

$$D = \frac{\gamma}{3} \left( \frac{E}{N_n} \right)^2 \left( \frac{\varepsilon}{Q} \right) + \frac{\gamma k_b T}{q} \varepsilon^2 \sigma_\varepsilon + 2a \left( \frac{n_e}{N_n} \right) (A_2 + \varepsilon^{3/2} A_3)$$

The following definitions apply:

$$a = \frac{q^2 \gamma}{24 \pi \varepsilon_0} \ln \Lambda$$

$$\Lambda = \frac{12 \pi (2 \varepsilon_0 e \bar{\varepsilon} / 3)^{3/2}}{q^3 n_e^{1/2}}$$

$$\sigma_m = \sum_{k = \text{all}} x_k \sigma_k$$

$$\sigma_\varepsilon = \sum_{k = \text{elastic}} 2 \left( \frac{m_e}{M} \right) x_k \sigma_k$$

$$\tilde{\sigma}_m = \sigma_m + \frac{\lambda}{\varepsilon^{1/2}}$$

$$A_1 = \int_0^\varepsilon u^{1/2} F_0(u) du$$

$$A_2 = \int_0^\varepsilon u^{3/2} F_0(u) du$$

$$A_3 = \int_\varepsilon^\infty F_0(u) du$$

$$Q = 2 \frac{\tilde{\sigma}_m^2 + q_w^2}{\tilde{\sigma}_m^2}$$

$$q_w = \frac{\omega}{N_n} \frac{1}{\gamma \varepsilon^{1/2}}$$

Here:

- $\gamma = (2q/m_e)^{1/2}$  (SI unit:  $C^{1/2}/kg^{1/2}$ )

- $m_e$  is the electron mass (SI unit: kg)
- $\varepsilon = (v/\gamma)^2$  is energy (SI unit: V)
- $\sigma_\varepsilon$  is the total elastic collision cross section (SI unit: m<sup>2</sup>)
- $\sigma_m$  is the total momentum collision cross section (SI unit: m<sup>2</sup>)
- $\tilde{\sigma}_m$  is the normalized total momentum collision cross section (SI unit: m<sup>2</sup>)
- $q$  is the electron charge (SI unit: C)
- $\varepsilon_0$  is the permittivity of free space (SI unit: F/m)
- $T$  is the temperature of the background gas (SI unit: K)
- $k_b$  is the Boltzmann constant (SI unit: J/K)
- $n_e$  is the electron density (SI unit: 1/m<sup>3</sup>)
- $N_n$  is the background gas density (SI unit: 1/m<sup>3</sup>)
- $\Lambda$  is the Coulomb logarithm, and
- $\lambda$  is a scalar-valued renormalization factor that ensures that the EEDF is normalized to 1 as explained above. An ODE is implemented to solve for the value of  $\lambda$ .
- $M$  is the mass of the target species (SI unit: kg).

The source term,  $S$  represents energy loss due to inelastic collisions. Because the energy loss due to an inelastic collision is quantized, the source term is nonlocal in the energy space. The source term can be decomposed into four parts where the following definitions apply:

$$S = \sum_{k = \text{inelastic}} C_k - \gamma \lambda \varepsilon^{1/2} F_0$$

$$C_{k = \text{excitation}} = -\gamma x_k [\varepsilon \sigma_k(\varepsilon) F_0(\varepsilon) - (\varepsilon + \Delta \varepsilon_k) \sigma_k(\varepsilon + \Delta \varepsilon_k) F_0(\varepsilon + \Delta \varepsilon_k)]$$

$$C_{k = \text{att}} = -\gamma x_k \varepsilon \sigma_k(\varepsilon) F_0(\varepsilon)$$

$$C_{\text{ion}} = -\gamma x_k [\varepsilon \sigma_k(\varepsilon) F_0(\varepsilon) - 4(2\varepsilon + \Delta \varepsilon_k) \sigma_k(2\varepsilon + \Delta \varepsilon_k) F_0(2\varepsilon + \Delta \varepsilon_k)]$$

where  $x_k$  is the mole fraction of the target species for reaction  $k$ ,  $\sigma_k$  is the collision cross section for reaction  $k$ ,  $\Delta \varepsilon_k$  is the energy loss from collision  $k$ , and  $\delta$  is the delta function at  $\varepsilon = 0$ .

The mean electron energy is defined by the integral

$$\bar{\varepsilon} = \int_0^{\infty} F_0 \varepsilon^{3/2} d\varepsilon.$$

The external excitation in the Boltzmann equation comes from an electric field. If the Boltzmann equation is to be solved for a given mean electron energy (as in the present model) a Lagrange multiplier is introduced to solve for the reduced electric field, such that the following equation, presented in the weak form, is satisfied:

$$\left( \int_0^{\infty} F_0 \varepsilon^{3/2} d\varepsilon - \bar{\varepsilon} \right) \left( \frac{\tilde{E}}{N_n} \right) = 0$$

where tilde denotes test function. For detailed information about electrostatics see *Theory for the Boltzmann Equation, Two-Term Approximation Interface* in the *Plasma Module User's Guide*.

### **COUPLING BETWEEN THE FLUID MODEL AND THE BOLTZMANN EQUATION**

The fluid model equations and the Boltzmann equation are solved fully coupled. The Boltzmann equation is solved at every point in which the fluid model is solved. This is the space where the mean electron energy, mole fraction of the different heavy species, and electron density (the gas density is assumed constant and as such it does not intervene in the computation of the ionization degree and the reduced angular frequency) are computed. The EEDFs obtained from the solution of the Boltzmann equation are used to compute macroscopic rate constants and transport coefficients intervening in the fluid model equations thus closing the loop.

### **ELECTRON RATE COEFFICIENTS AND TRANSPORT PARAMETERS**

Rate coefficients for electron impact reactions are computed using cross section data and the computed EEDF by the following integral

$$k^f = N_A \gamma \int_0^{\infty} \varepsilon \sigma_k(\varepsilon) F_0(\varepsilon) d\varepsilon.$$

The reduced electron mobility associated with the DC transport is computed using

$$\mu_e N_n = - \left( \frac{\gamma}{3} \right) \int_0^{\infty} \frac{\varepsilon}{\sigma_m} \left( \frac{\partial F_0}{\partial \varepsilon} \right) d\varepsilon$$

and the real and imaginary part of the mobility, used to computed the plasma conductivity, are computed with

$$\mu_r N_n = -\left(\frac{\gamma}{3}\right) \int_0^\infty \frac{\tilde{\sigma}_m \varepsilon}{\tilde{\sigma}_m + q} \left(\frac{\partial F_0}{\partial \varepsilon}\right) d\varepsilon$$

$$\mu_i N_n = \left(\frac{\gamma}{3}\right) \int_0^\infty \frac{q \varepsilon}{\tilde{\sigma}_m + q} \left(\frac{\partial F_0}{\partial \varepsilon}\right) d\varepsilon.$$

### BOUNDARY CONDITIONS

Electrons are lost to the wall due to random motion within a few mean free paths of the wall and gained due to secondary emission effects, resulting in the following boundary condition for the electron flux:

$$\mathbf{n} \cdot \Gamma_e = \left(\frac{1}{2} v_{e, \text{th}} n_e\right)$$

and the electron energy flux:

$$\mathbf{n} \cdot \Gamma_\varepsilon = \left(\frac{5}{6} v_{e, \text{th}} n_\varepsilon\right)$$

For the heavy species, ions are lost to the wall due to surface reactions and the fact that the electric field is directed toward the wall:

$$\mathbf{n} \cdot \mathbf{j}_k = M_w R_k + M_w c_k Z \mu_k (\mathbf{E} \cdot \mathbf{n}) [Z_k \mu_k (\mathbf{E} \cdot \mathbf{n}) > 0]$$

The walls of the reactor are grounded.

### PLASMA CHEMISTRY

Because the physics occurring in an inductively coupled plasma is rather complex, it is always best to start a modeling project with a simple chemical mechanism. Argon is one of the simplest mechanisms to implement at low pressures. The electronically excited states can be lumped into a single species, which results in a chemical mechanism consisting of only 3 species and 7 reactions (electron impact cross sections are obtained from [Ref. 3](#)):

TABLE 1: TABLE OF COLLISIONS AND REACTIONS MODELED.

REACTION	FORMULA	TYPE	$\Delta\varepsilon$ (eV)
1	$e + \text{Ar} \Rightarrow e + \text{Ar}$	Elastic	0
2	$e + \text{Ar} \Rightarrow e + \text{Ar}_s$	Excitation	11.5
3	$e + \text{Ar}_s \Rightarrow e + \text{Ar}$	Superelastic	-11.5
4	$e + \text{Ar} \Rightarrow 2e + \text{Ar}^+$	Ionization	15.8

TABLE 1: TABLE OF COLLISIONS AND REACTIONS MODELED.

REACTION	FORMULA	TYPE	$\Delta\epsilon(\text{eV})$
5	$e+\text{Ar}_s \Rightarrow 2e+\text{Ar}^+$	Ionization	4.24
6	$\text{Ar}_s+\text{Ar}_s \Rightarrow e+\text{Ar}+\text{Ar}^+$	Penning ionization	-
7	$\text{Ar}_s+\text{Ar} \Rightarrow \text{Ar}+\text{Ar}$	Metastable quenching	-

Stepwise ionization (reaction 5) can play an important role in sustaining low pressure argon discharges. Excited argon atoms are consumed via superelastic collisions with electrons, quenching with neutral argon atoms, ionization or Penning ionization where two metastable argon atoms react to form a neutral argon atom, an argon ion and an electron. In addition to volumetric reactions, the following surface reactions are implemented:

TABLE 2: TABLE OF SURFACE REACTIONS.

REACTION	FORMULA	STICKING COEFFICIENT
1	$\text{Ar}_s \Rightarrow \text{Ar}$	1
2	$\text{Ar}^+ \Rightarrow \text{Ar}$	1

When a metastable argon atom makes contact with the wall, it reverts to the ground state argon atom with some probability (the sticking coefficient).

### ELECTRICAL EXCITATION

From an electrical point of view, the GEC reactor behaves as a transformer. A current is applied to the driving coil (the primary) and this induces a current in the plasma (the secondary). The plasma then induces an opposing current back in the coil, increasing its resistance. The current flowing in the plasma depends on the current applied to the coil and the reaction kinetics. The total plasma current can vary from no current (plasma not sustained) to the same current as the primary, which corresponds to perfect coupling between the coil and the plasma.

In this example a fixed power of 1500 W is applied to the coil. Some of this power is dissipated in the coil, some is deposited into the plasma.

### *Results and Discussion*

In this section, results are presented from the model. For reference, results of the fluid model when assuming a Maxwellian EEDF are also presented.

The macroscopic quantities that change the most are the electron temperature and the number density of the argon excited state. When comparing the simulation results of the models using an assumed and computed EEDFs one observes that the model with the

computed EEDF: have an *electron temperature* 1 eV higher in most of the plasma bulk; the maximum value for the *number density of the argon excited* state is only about 15% larger, however the spatial distribution is considerable different; the *electron density* is about 15% lower and the profiles are similar; the *power absorbed by the plasma* maintains almost the same absolute value but there is a deeper penetration of the absorbed power into the plasma bulk. This is because the lower plasma density implies a larger skin depth.

Figure 10 presents computed EEDFs for several axial positions between the wafer and the dielectric window. When comparing with Maxwellian EEDFs with the same mean energy (not shown in the figure), the computed EEDFs tend to be less populated in the regions of high energy above 16 eV and low energy below 5 eV, and tend to be more populated in the mid energies between 5 and 16 eV. The electron–electron collisions cause the EEDF to tend toward a Maxwellian distribution function, populating the high energy tail that otherwise would present a much steeper cutoff around the ionization energy region of 16 eV.

When answering the question “which EEDF should I use?” one can conclude that if the goal of a work is to study the electric properties of the ICP discharge the use of a Maxwellian EEDF (or a generalized EEDF with a less populated tail) could be enough since the electron density and the power absorbed by the plasma are fairly similar. However, if the goal is to study a complex chemistry that strongly depends on the electron temperature the model with computed EEDF might be necessary.

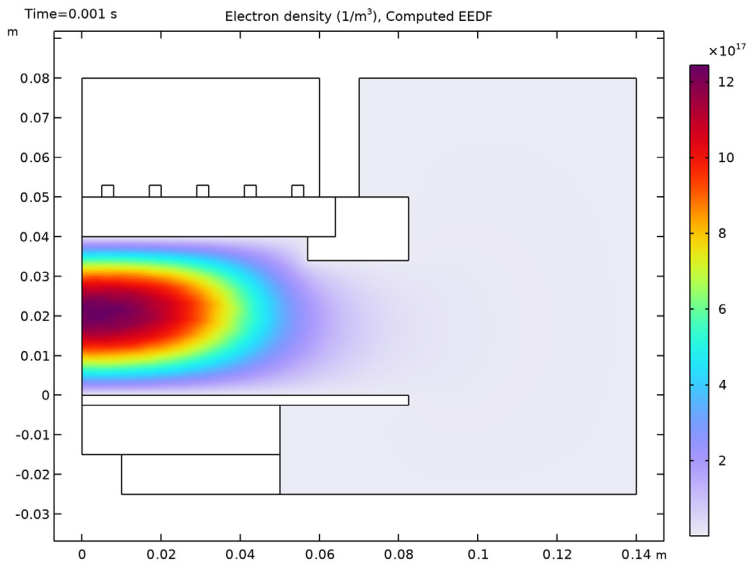


Figure 2: Electron density obtained with the computed EEDF.

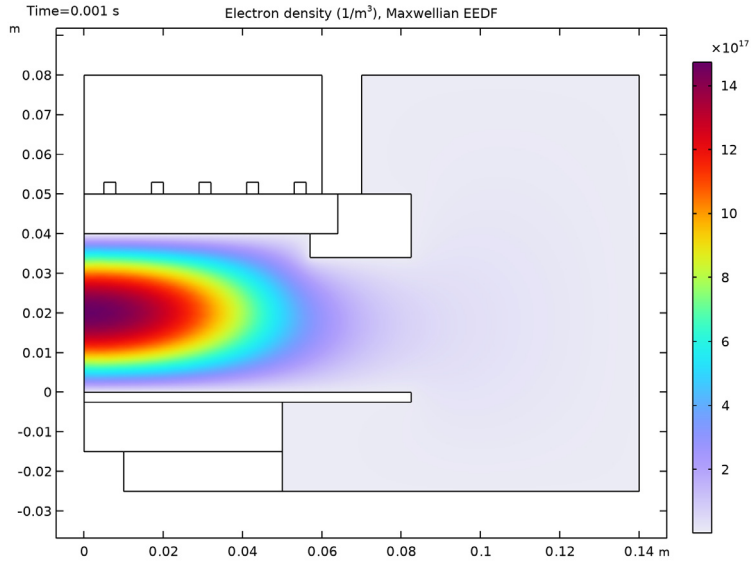


Figure 3: Electron density obtained with a Maxwellian EEDF.

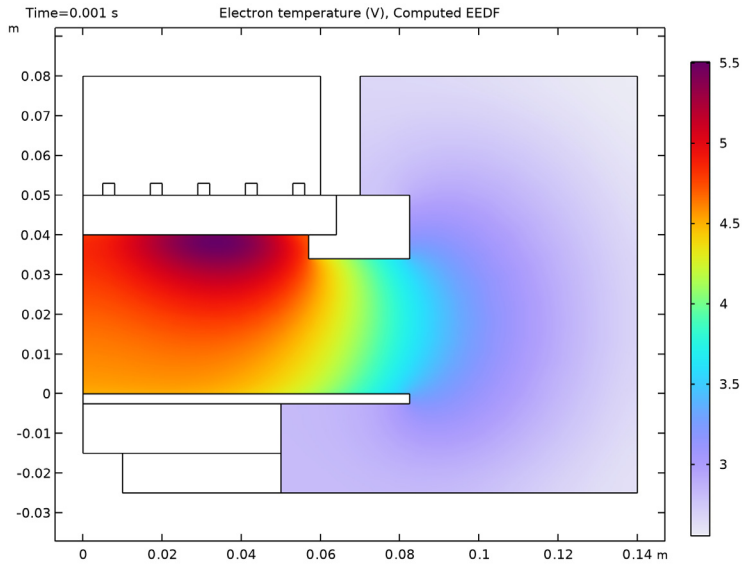


Figure 4: Electron temperature obtained with the computed EEDF.

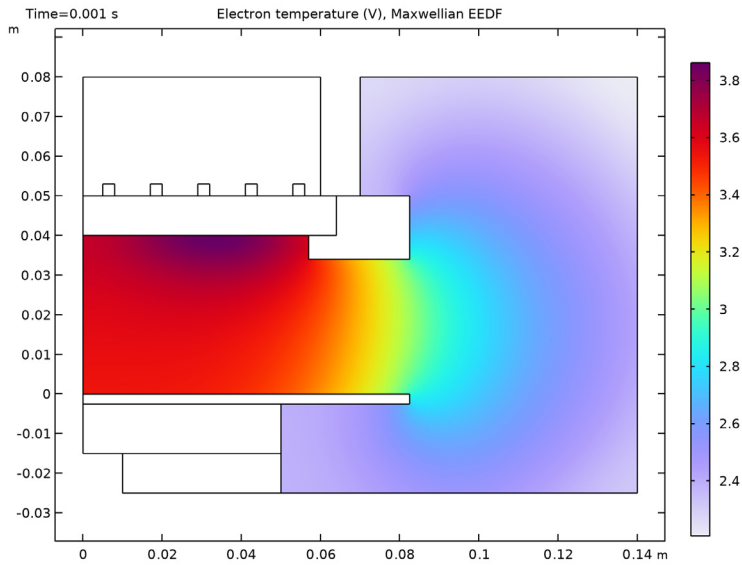


Figure 5: Electron temperature obtained with a Maxwellian EEDF.

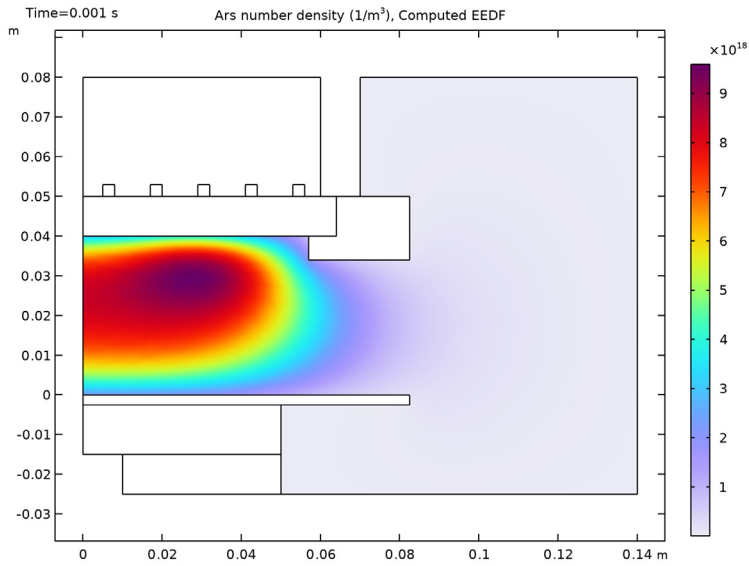


Figure 6: Number density of excited argon atoms obtained with the computed EEDF.

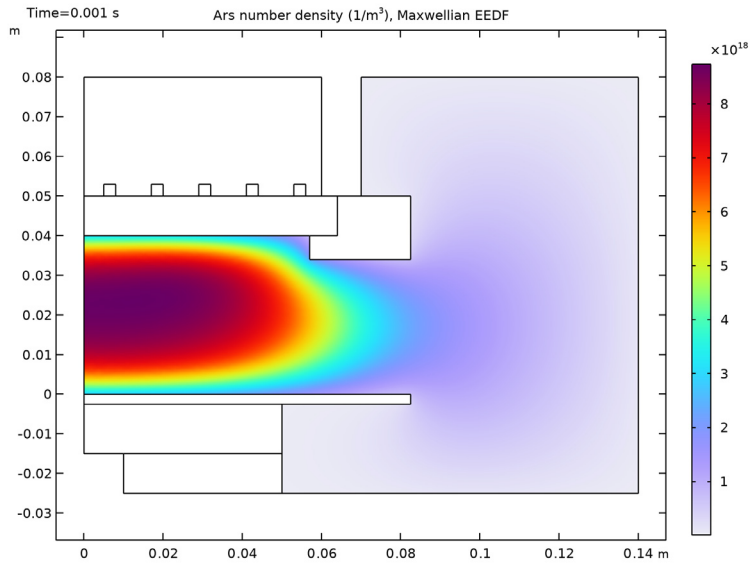


Figure 7: Number density of excited argon atoms obtained with a Maxwellian EEDF.

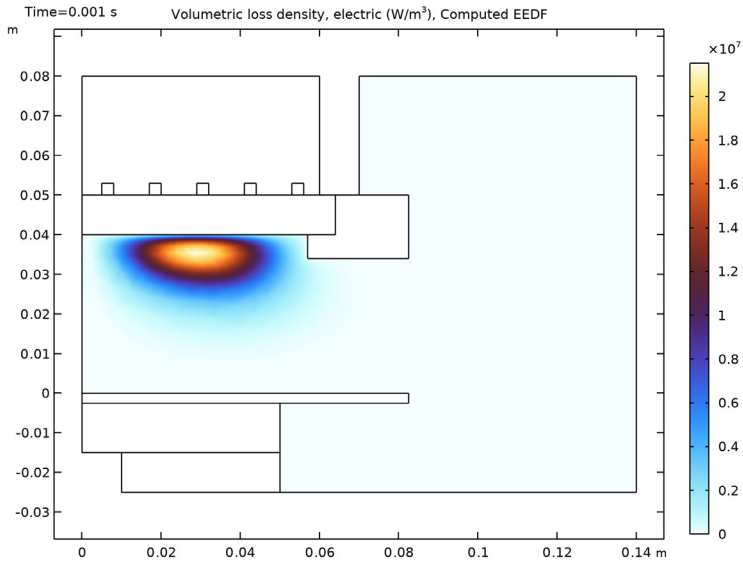


Figure 8: Power deposition into the plasma obtained with the computed EEDF.

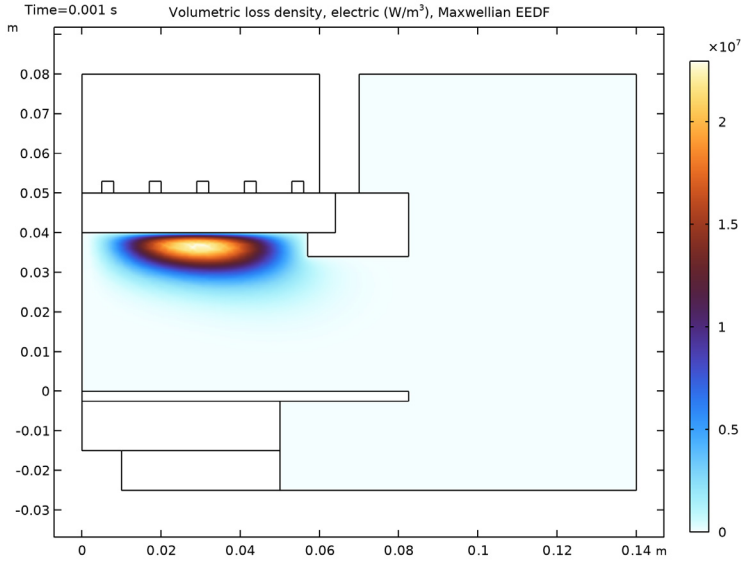


Figure 9: Power deposition into the plasma obtained with a Maxwellian EEDF.

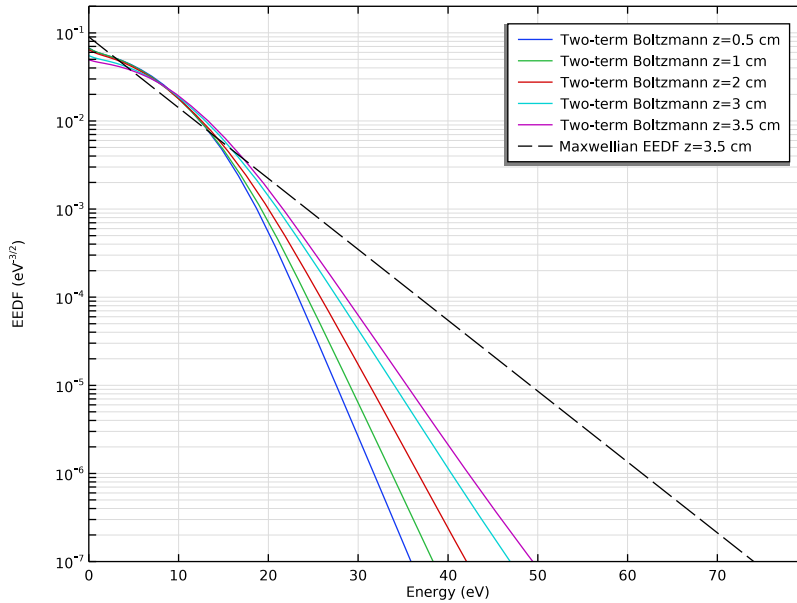


Figure 10: Computed EEDFs at a radial position of 3 cm and for several axial positions. For reference, it is presented a Maxwellian EEDF with the mean energy of 5.4 eV, which is the value found at  $z = 3.5$  cm for the model with the computed EEDF.

## References

1. G.J.M. Hagelaar and L.C. Pitchford, “Solving the Boltzmann Equation to Obtain Electron Transport Coefficients and Rate Coefficients for Fluid Models,” *Plasma Sources Sci. Technol.*, vol. 14, pp. 722–733, 2005.
2. D.P. Lymberopoulos and D.J. Economou, “Two-Dimensional Self-Consistent Radio Frequency Plasma Simulations Relevant to the Gaseous Electronics Conference RF Reference Cell,” *J. Res. Natl. Inst. Stand. Technol.*, vol. 100, pp. 473–494, 1995.
3. Phelps database, [www.lxcat.net](http://www.lxcat.net), retrieved 2017.

---

**Application Library path:** Plasma\_Module/Space-Dependent\_EEDF\_Modeling/  
argon\_gec\_icp\_boltzmann

---

### ROOT


The following instructions show how to create a model for an ICP reactor that solves plasma fluid type equations fully coupled with the Boltzmann equation in the two-term approximation.

Go to the **Application Libraries** and open the `argon_gec_icp` model.

Select to solve the Boltzmann equation in the two-term approximation, add the contribution from electron-electron collisions and the oscillating electric field.

Set the number of elements in the extra dimension to 50, refined the mesh in the extra dimension in the low energies region, and choose to compute the maximum energy automatically.

### APPLICATION LIBRARIES

- 1 From the **File** menu, choose **Application Libraries**.
- 2 In the **Application Libraries** window, select **Plasma Module > Inductively Coupled Plasmas > argon\_gec\_icp** in the tree.
- 3 Click  **Open**.

### PLASMA (PLAS)

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)** node, then click **Plasma (plas)**.
- 2 In the **Settings** window for **Plasma**, locate the **Electron Energy Distribution Function Settings** section.
- 3 From the **Electron energy distribution function** list, choose **Boltzmann equation, two-term approximation (linear)**.
- 4 Select the **Electron–electron collisions** checkbox.
- 5 Select the **Oscillating field** checkbox.
- 6 In the  $\omega/N$  text field, type  $2.11E-14[m^3/s]$ .
- 7 In the  $N$  text field, type 50.
- 8 In the  $R$  text field, type 30.
- 9 Select the **Compute maximum energy** checkbox.

Choose to define the transport parameters using the **Mobility from electron energy distribution function**. This way, the electron mobility is obtained from suitable

integration of the EEDF over the cross section for momentum transfer. The remaining transport parameters are computed using the electron mobility.

#### *Plasma Model I*

- 1 In the **Model Builder** window, expand the **Plasma (plas)** node, then click **Plasma Model I**.
- 2 In the **Settings** window for **Plasma Model**, locate the **Electron Density and Energy** section.
- 3 From the **Electron transport properties** list, choose **Mobility from electron energy distribution function**.

#### **MULTIPHYSICS**

Choose to define the plasma conductivity using an effective collision frequency for momentum transfer computed from the EEDF.

#### *Plasma Conductivity Coupling I (pccI)*


- 1 In the **Model Builder** window, expand the **Component I (compI) > Multiphysics** node, then click **Plasma Conductivity Coupling I (pccI)**.
- 2 In the **Settings** window for **Plasma Conductivity Coupling**, locate the **Collision Frequency Specification** section.
- 3 From the **Specify collision frequency using** list, choose **Electron energy distribution function**.

Since the computation time of this type of problems are very long the mesh is coarsened.

#### **MESH I**

In the **Model Builder** window, expand the **Component I (compI) > Mesh I** node.

#### *Size I*

- 1 In the **Model Builder** window, expand the **Component I (compI) > Mesh I > Free Triangular I** node, then click **Size I**.
- 2 In the **Settings** window for **Size**, locate the **Element Size** section.
- 3 From the **Predefined** list, choose **Normal**.
- 4 Click  **Build All**.

#### **MESH I**

In the **Model Builder** window, collapse the **Component I (compI) > Mesh I** node.

#### **MULTIPHYSICS**

In the **Model Builder** window, collapse the **Component I (compI) > Multiphysics** node.



## PLASMA (PLAS)

- 1 In the **Model Builder** window, collapse the **Component 1 (comp1) > Plasma (plas)** node.  
Label the study and group the plots so that it is easy to identify that they refer the model that uses a Maxwellian EEDF.


## MAXWELLIAN EEDF

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Maxwellian EEDF in the **Label** text field.  
Add a **EEDF Initialization** study to solve for the EEDF only. The solution of this study is going to be used as the initial condition for the study that solves the fluid type equations coupled with the Boltzmann equation.

## ADD STUDY

- 1 In the **Study** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Multiphysics couplings in study** subsection. In the table, clear the **Solve** checkboxes for **Plasma Conductivity Coupling 1 (pcc1)** and **Electron Heat Source 1 (ehs1)**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Magnetic Fields (mf)**.
- 5 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > EEDF Initialization**.
- 6 Click the **Add Study** button in the window toolbar.
- 7 In the **Study** toolbar, click  **Add Study** to close the **Add Study** window.

## EEDF INITIALIZATION



- 1 In the **Settings** window for **Study**, type EEDF Initialization in the **Label** text field.
- 2 In the **Home** toolbar, click  **Compute**.

## RESULTS

### *Electron Energy Distribution Function, Initialization*


- 1 In the **Settings** window for **ID Plot Group**, type Electron Energy Distribution Function, Initialization in the **Label** text field.  
Add a new study to compute the plasma fluid type equations fully coupled with the Boltzmann equation in the two term approximation.

## ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Multiphysics > Frequency–Transient**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

## STUDY 3

### *Step 1: Frequency–Transient*

- 1 In the **Settings** window for **Frequency–Transient**, locate the **Study Settings** section.
- 2 In the **Output times** text field, type  $0 \cdot 10^{\{\text{range}(-8, 5/20, -3)\}}$ .
- 3 In the **Frequency** text field, type  $13.56\text{e}6$ .
- 4 Click to expand the **Values of Dependent Variables** section. Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.
- 5 From the **Method** list, choose **Solution**.
- 6 From the **Study** list, choose **EEDF Initialization, EEDF Initialization**.
- 7 In the **Model Builder** window, click **Study 3**.
- 8 In the **Settings** window for **Study**, type **Computed EEDF** in the **Label** text field.  
Choose to **Get the initial value** to organize the output plots and to set the electron temperature to be plotted while solving.
- 9 In the **Study** toolbar, click  **Get Initial Value**.

### *Solver Configurations*

In the **Model Builder** window, expand the **Computed EEDF > Solver Configurations** node.


### *Solution 3 (sol3)*

- 1 In the **Model Builder** window, expand the **Computed EEDF > Solver Configurations > Solution 3 (sol3) > Time-Dependent Solver 1** node, then click **Segregated 1**.
- 2 In the **Settings** window for **Segregated**, click to expand the **Results While Solving** section.
- 3 Select the **Plot** checkbox.

4 In the table, enter the following settings:

Plot group	Plot window
Electron Temperature (plas) 1	Graphics

### COMPUTED EEDF

1 In the **Study** toolbar, click  **Compute**.

Prepare a plot of the computed EEDF at several heights for  $r=3\text{ cm}$  and compare with a Maxwellian EEDF.

### RESULTS

*Computed EEDF  $r=3\text{ cm}$*

- 1 In the **Settings** window for **ID Plot Group**, type Computed EEDF  $r=3\text{ cm}$  in the **Label** text field.
- 2 Locate the **Data** section. From the **Time selection** list, choose **Last**.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 4 Locate the **Plot Settings** section. In the **x-axis label** text field, type Energy (eV).
- 5 In the **y-axis label** text field, type EEDF (eV<sup>-3/2</sup>).
- 6 Locate the **Axis** section. Select the **Manual axis limits** checkbox.
- 7 In the **x minimum** text field, type 0.
- 8 In the **x maximum** text field, type 80.
- 9 In the **y minimum** text field, type  $1\text{e-}7$ .
- 10 In the **y maximum** text field, type 0.2.

*Two-term Boltzmann  $z=0.5\text{ cm}$*

- 1 In the **Model Builder** window, expand the **Computed EEDF  $r=3\text{ cm}$**  node, then click **Two-term Boltzmann**.
- 2 In the **Settings** window for **Line Graph**, type Two-term Boltzmann  $z=0.5\text{ cm}$  in the **Label** text field.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.005, plas.fcaps)`.
- 4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.005, plas.xeedf)`.
- 5 Click to expand the **Legends** section.

*Two-term Boltzmann z=1 cm*

- 1 Right-click **Two-term Boltzmann z=0.5 cm** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, type Two-term Boltzmann z=1 cm in the **Label** text field.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.01, plas.fcap)`.
- 4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.01, plas.xeedf)`.

*Two-term Boltzmann z=2 cm*

- 1 Right-click **Two-term Boltzmann z=1 cm** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, type Two-term Boltzmann z=2 cm in the **Label** text field.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.02, plas.fcap)`.
- 4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.02, plas.xeedf)`.

*Two-term Boltzmann z=3 cm*

- 1 Right-click **Two-term Boltzmann z=2 cm** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, type Two-term Boltzmann z=3 cm in the **Label** text field.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.03, plas.fcap)`.
- 4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.03, plas.xeedf)`.

*Two-term Boltzmann z=3.5 cm*

- 1 Right-click **Two-term Boltzmann z=3 cm** and choose **Duplicate**.
- 2 In the **Settings** window for **Line Graph**, type Two-term Boltzmann z=3.5 cm in the **Label** text field.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.035, plas.fcap)`.
- 4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.035, plas.xeedf)`.

### *Druyvesteyn*

In the **Model Builder** window, right-click **Druyvesteyn** and choose **Disable**.

### *Generalized (g=3)*

In the **Model Builder** window, right-click **Generalized (g=3)** and choose **Disable**.

### *Maxwellian EEDF z=3.5 cm*

- 1 In the **Model Builder** window, right-click **Maxwellian** and choose **Move Down** six times.
- 2 In the **Settings** window for **Line Graph**, type Maxwellian EEDF z=3.5 cm in the **Label** text field.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.035, plas.fmax)`.
- 4 Locate the **x-Axis Data** section. In the **Expression** text field, type `atxd2(0.03,0.035, plas.xeedf)`.
- 5 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.
- 6 From the **Color** list, choose **Black**.

## **RESULTS**


### *Computed EEDF r=3 cm*

- 1 In the **Model Builder** window, collapse the **Results > Computed EEDF > Computed EEDF r=3 cm** node.  
Plot the excited argon number density and the power deposition.

### *Excited Argon Number Density 1*

- 1 In the **Model Builder** window, right-click **Electron Density (plas) 1** and choose **Duplicate**.
- 2 In the **Settings** window for **2D Plot Group**, type Excited Argon Number Density 1 in the **Label** text field.

### *Surface 1*

- 1 In the **Model Builder** window, expand the **Excited Argon Number Density 1** node, then click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type `plas.n_wArs`.
- 4 In the **Excited Argon Number Density 1** toolbar, click  **Plot**.

## RESULTS

### *Excited Argon Number Density 1*

In the **Model Builder** window, collapse the **Results > Computed EEDF > Excited Argon Number Density 1** node.


### *Power Deposition 1*

- 1 In the **Model Builder** window, right-click **Excited Argon Number Density 1** and choose **Duplicate**.
- 2 In the **Settings** window for **2D Plot Group**, type Power Deposition 1 in the **Label** text field.

### *Surface 1*

- 1 In the **Model Builder** window, expand the **Power Deposition 1** node, then click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type  $m_f \cdot Q_{rh}$ .
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **ThermalWave**.

### *Selection 1*

- 1 Right-click **Surface 1** and choose **Selection**.
- 2 Select Domain 3 only.
- 3 In the **Power Deposition 1** toolbar, click  **Plot**.

## RESULTS

### *Power Deposition 1*

In the **Model Builder** window, collapse the **Results > Computed EEDF > Power Deposition 1** node.