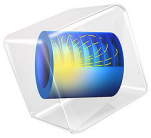

Model created in COMSOL Multiphysics 6.4

Radio Frequency Quadrupole Ion Trap

Introduction

A radio frequency quadrupole ion trap utilizes a radio frequency quadrupole (RFQ) in order to trap an ion beam. A continuous high energy ion beam is first cooled and converted into a spatially confined bunch, which can then be released with minimal heating of the beam (Ref. 1). The device is also commonly referred to as a Paul trap where an oscillating AC voltage is used to contain the beam radially. The axial confinement of the particles is achieved by using a DC voltage along the length of the quadrupole designed to create a potential well that can contain the beam.

The RFQ is often used to guide particle beams with kinetic energies in the order of 10–100 eV. In order to efficiently trap these particles, the kinetic energies are reduced through a process called buffer-gas cooling. The high energy charged beam is made to pass through a buffer gas, often consisting of a low molecular mass inert gas such as He or Ar. The highly energetic ions undergo elastic collisions with the light and slow moving buffer gas molecules, thus reducing the overall kinetic energy of the particle beam. The RF electric fields help contain the collisional scattering in the radial direction.

The device can be operated in two configurations: trapping and extraction. In the trapping configuration, the axial DC voltage contains a minima which helps in trapping the cooled beam. The cooled beam is therefore collected near the minima as a bunch of particles. This bunch can then be extracted by altering the DC voltage to help guide the beam out of the quadrupole. This two-stage operation thus allows for a continuous beam to be converted to a pulsed beam.

Model Definition

The geometry consists of four parallel cylindrical rods placed along a fixed radius as shown in Figure 1. An outer case is present to bound the particle trajectories. The particle beam enters the quadrupole from one end of the device and can exit through any surface of the case.

A positive potential of magnitude V_{AC} is applied to the north and south rods, while a negative potential of the same magnitude is applied to the east and west rods. The motion of a charged particle in a RFQ is determined by the Mathieu equation whose solutions outline the stability region for the particle motion. In this model, the geometry and voltage parameters are chosen to ensure the stable motion of the charged particles (ions).

Once the ions enter the RFQ, they undergo elastic collisions with the buffer gas which helps in reducing the kinetic energy of the ions, thus cooling the beam. The ions in this model are composed of Cs^+ ions which undergo collisions with He atoms. The He atoms

are modeled as a background gas and thus their motion is not explicitly modeled. The collisional interactions are modeled using the **Collisions** node with an **Elastic** subnode. This approach utilizes a Monte-Carlo approach for modeling the collisions whereby at each time step taken by the solver, a He atom is sampled from a Maxwell–Boltzmann distribution. The probability of collision is then computed for each ion and if a collision is deemed to occur, an elastic collision is modeled where the energy of the collision is conserved. The collision frequency depends on the number density of the gas, the collision cross section, and the relative velocity between the ion and the background gas atom. This model uses the Hard Sphere collision model where the collision cross section is constant and independent of the relative velocity. The gas pressure chosen in this model is 6 Pa.

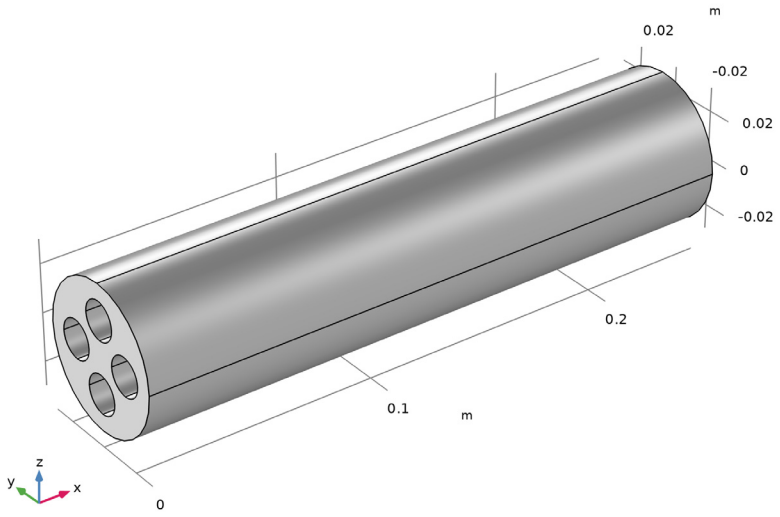

Figure 1: Model geometry of a radio frequency quadrupole ion trap.

This model demonstrates the two-stage operation of the RFQ ion trap. In the first stage, a trapping DC potential is used in addition to the buffer gas cooling in order to trap and bunch the ions spatially. Once the ion beam is cooled and bunched, the DC potential is altered to the extraction mode which allows the cooled beam to exit the RFQ. Thus a

continuous ion beam can be converted to a pulsed beam. The two potentials are shown in Figure 2.

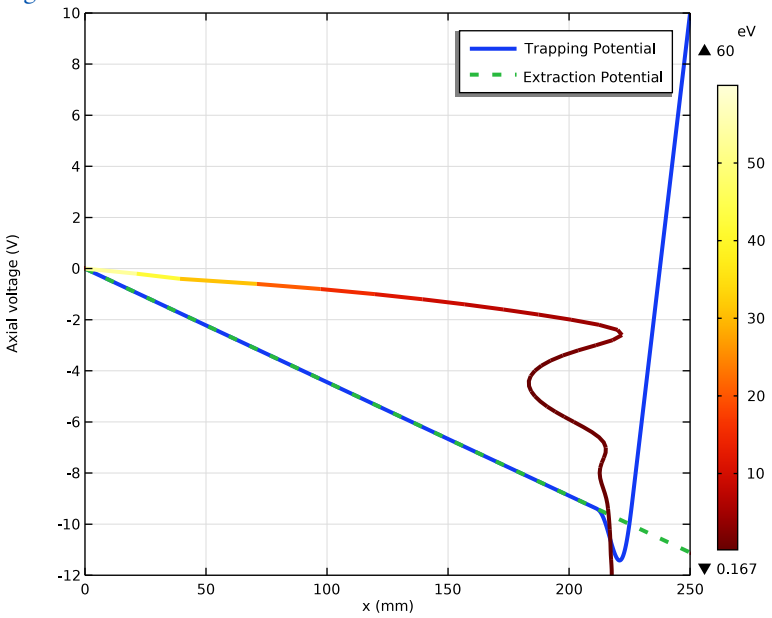

Figure 2: Trapping and extraction configuration of the axial DC potentials. The unlabeled line represents the beam trajectory as it is cooled and trapped.

The ion beam is released into the domain using the **Particle Beam** feature. In order to release a continuous beam, 20 ions are released at a time interval equal to 1/20 of the RF cycle. Ions are released up to five cycles such that a total of 2000 ions are released. The ions are released with an initial longitudinal kinetic energy of 60 eV. Two **Electric Force** features are used in this model to account for the electric forces resulting from the AC and DC voltages.

Notes on COMSOL Implementation

The model uses three physics interfaces: Electrostatics, Electric Currents and Charged Particle Tracing. The model is solved in four steps.

First the Electrostatics (using the trapping potential), Electric Currents interfaces are used to compute the DC and AC electric fields respectively. These fields are then coupled to the Charged Particle Tracing interface to compute the particle motion under the influence of the electric forces and collisions with the buffer gas.

Once the ion beams are cooled and bunched near the potential minima, the DC voltage in the Electrostatics interface is altered to the extraction potential. The Electrostatics and the Electric Currents interfaces are again used to compute the new DC and AC electric fields. Finally, the Charged Particle Tracing interface utilizes the particle positions and velocities from the previous study as the initial conditions to compute the particle trajectories under the influence of the newly computed electric fields and buffer gas collisions. In order to enable this initialization of the particle properties from a previous study, the **Store particle status data** checkbox must be selected in the **Additional Variables** section in the settings for the Charged Particle Tracing interface.

Results and Discussion

The particle positions at the end of the trapping stage (300 µs) are plotted in Figure 3. The particles are colored by their overall kinetic energy (in eV). The ion beam entered the domain with a longitudinal kinetic energy of 60 eV. Therefore, it is clear that the beam has been significantly cooled. Furthermore, a continuous beam of ions have been collected and bunched as can be seen by the narrow longitudinal spread of the ions.

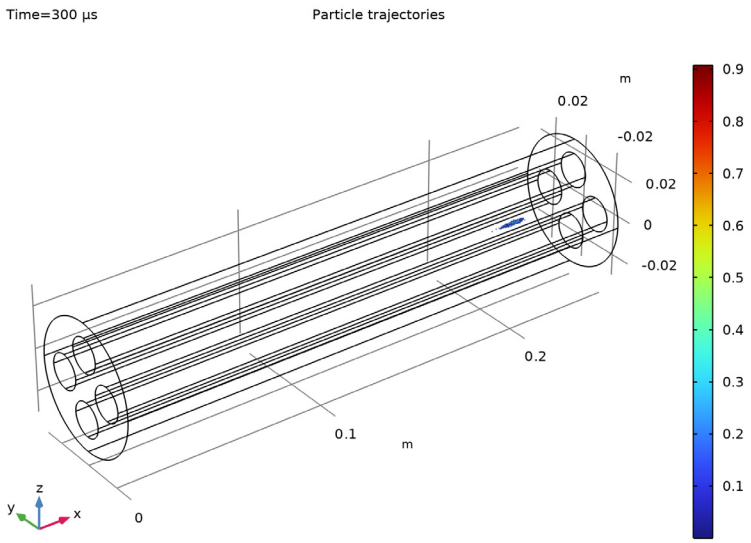

Figure 3: Positions of the cooled and bunched ion beam.

As the beam cools and encounters the potential barrier, it is repulsed and thus the beam oscillates axially and asymptotically settles near the potential minima (Ref. 1). This is schematically shown in Figure 2 where the unlabeled line represents the beam's average axial position (x-axis) as a function of time (y-axis). The time is scaled so that the plot superimposes onto the trapping potential. This line is colored by its average kinetic energy.

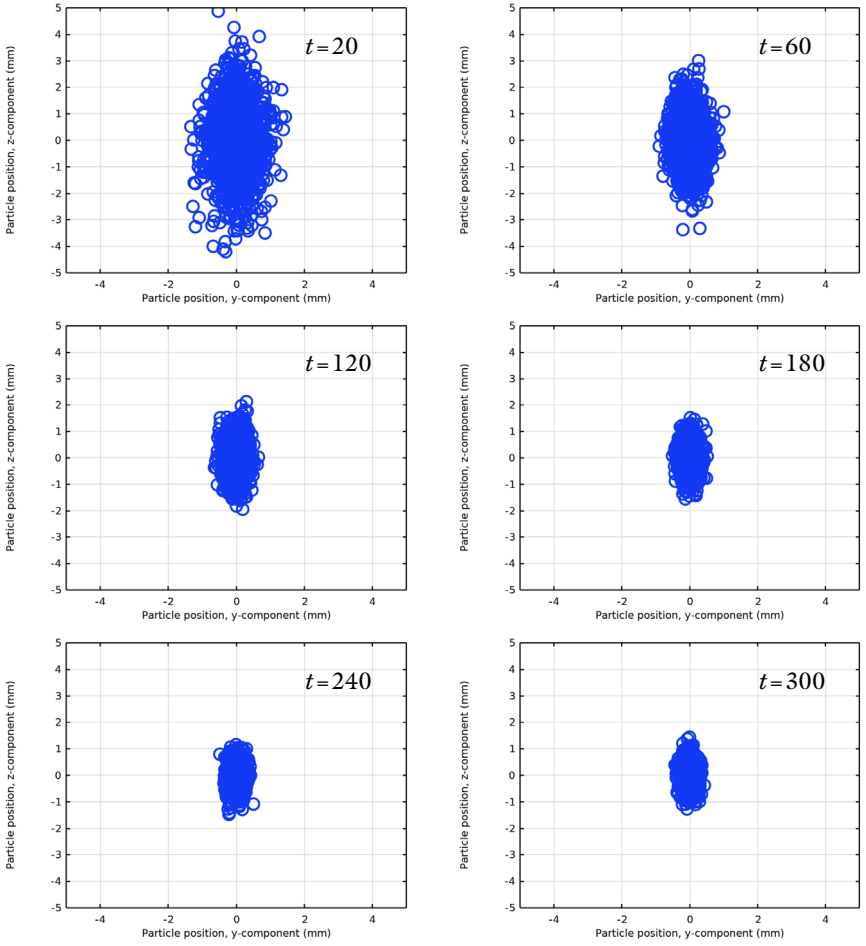

Figure 4: Transverse beam profiles at various solution times (annotated in μs).

The repeated ion-gas collisions reduces the kinetic energy of the ions and the RF voltages constrain the radial diffusion of the particles. As a result, the radial spread of the particles is greatly reduced. This is visualized in Figure 4 where the transverse beam profiles are

plotted at different solution times. The radial spread of the ion beam clearly diminishes as a function of time. A similar effect is also observed in the transverse phase portrait plots of the position-velocity phase space as shown in Figure 5.

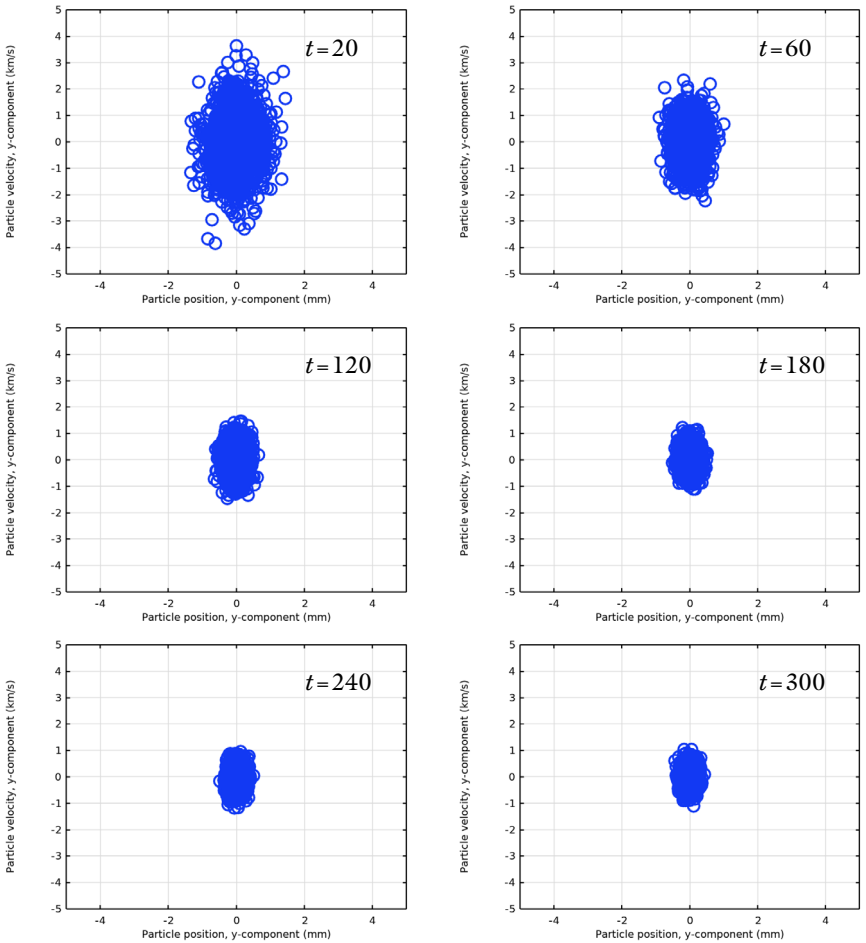

Figure 5: Transverse phase portraits showing the position-velocity phase space of the beam at various solution times (annotated in μs).

Finally, the cooled and bunched beam is released by altering the DC voltage to the extraction mode and the particle trajectories of this ion bunch are plotted in Figure 6. It can be seen that the particles promptly exit the domain and the heating of the beam is minimal.

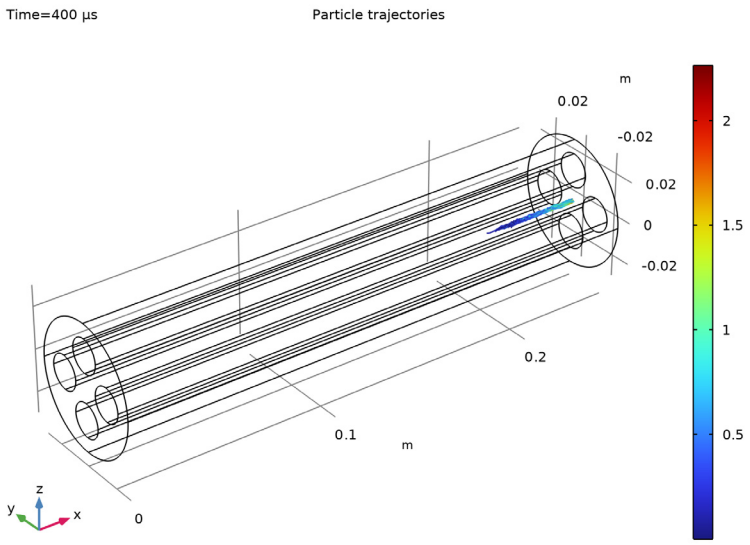

Figure 6: Particle trajectories under the influence of the extraction potential and buffer-gas cooling.

Reference

1. T. Brunner, M.J. Smith, M. Brodeur, S. Ettenauer, A.T. Gallant, V.V. Simon, A. Chaudhuri, and others, "TITAN's digital RFQ ion beam cooler and buncher, operation and performance," *Nucl. Instrum. Methods Phys. Res. Sect. A.*, vol. 676, pp. 32–43, 2012.

Application Library path: `Particle_Tracing_Module/Charged_Particle_Tracing/rfq_ion_trap`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **AC/DC** > **Electric Fields and Currents** > **Electrostatics (es)**.

3 Click **Add**.

4 In the **Select Physics** tree, select **AC/DC** > **Electric Fields and Currents** > **Electric Currents (ec)**.

5 Click **Add**.

6 In the **Select Physics** tree, select **AC/DC** > **Particle Tracing** > **Charged Particle Tracing (cpt)**.

7 Click **Add**.

8 Click **Study**.

9 In the **Select Study** tree, select **Empty Study**.

10 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

Load the model parameters from a file.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `rfq_ion_trap_parameters.txt`.

DEFINITIONS

Piecewise 1 (pw1)

1 In the **Definitions** toolbar, click **Piecewise**.

2 In the **Settings** window for **Piecewise**, type `trap_pot` in the **Function name** text field.

3 Locate the **Definition** section. From the **Smoothing** list, choose **Continuous second derivative**.

4 In the **Size of transition zone** text field, type `L_rod/4`.

5 Find the **Intervals** subsection. In the table, enter the following settings:

Start	End	Function
0	x_min	V1/x_min*x
x_min	L_rod	V1+((V2-V1)/(L_rod-x_min))*(x-x_min)

6 Locate the **Units** section. In the **Arguments** text field, type m.

7 In the **Function** text field, type V.

8 Click **Create Plot**.

Analytic 1 (an1)

1 In the **Definitions** toolbar, click **Analytic**.

2 In the **Settings** window for **Analytic**, type ext_pot in the **Function name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type V1/x_min*x.

4 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	m

5 Locate the **Plot Parameters** section. In the table, enter the following settings:

Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
√	x	0	L_rod	0	m

Now plot the two functions to visualize the trapping and extraction potentials.

RESULTS

Axial DC voltages

1 In the **Model Builder** window, under **Results** click **1D Plot Group 1**.

2 In the **Settings** window for **1D Plot Group**, type Axial DC voltages in the **Label** text field.

Function 1

1 In the **Model Builder** window, expand the **Axial DC voltages** node, then click **Function 1**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Description** text field, type Trapping Potential.

4 Locate the **x-Axis Data** section. From the **Unit** list, choose **mm**.

5 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle**.

6 From the **Width** list, choose **3**.

7 Click to expand the **Legends** section. Select the **Show legends** checkbox.

8 In the **Axial DC voltages** toolbar, click **Plot**.

Grid 1D 1

1 In the **Model Builder** window, expand the **Results** > **Datasets** node, then click **Grid 1D 1**.

2 In the **Settings** window for **Grid 1D**, locate the **Data** section.

3 From the **Function** list, choose **All**.

Function 2

1 In the **Model Builder** window, under **Results** > **Axial DC voltages** right-click **Function 1** and choose **Duplicate**.

2 In the **Settings** window for **Function**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `comp1.ext_pot(x)`.

4 In the **Description** text field, type `Extraction Potential`.

Axial DC voltages

1 In the **Model Builder** window, click **Axial DC voltages**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **x-axis label** checkbox.

4 Select the **y-axis label** checkbox. In the associated text field, type `Axial voltage (V)`.

5 Click to expand the **Title** section. From the **Title type** list, choose **None**.

6 In the **Axial DC voltages** toolbar, click **Plot**. The trapping and extraction potentials should resemble Figure 2.

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Perfect vacuum**.

4 Click the **Add to Component** button in the window toolbar.

5 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

GEOMETRY 1

Work Plane 1 (wp1)

1 In the **Geometry** toolbar, click **Work Plane**.

2 In the **Settings** window for **Work Plane**, locate the **Plane Definition** section.

3 From the **Plane** list, choose **yz-plane**.

Work Plane 1 (wp1) > Plane Geometry

In the **Model Builder** window, click **Plane Geometry**.

Work Plane 1 (wp1) > Circle 1 (c1)

1 In the **Work Plane** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r_rod`.

4 Locate the **Position** section. In the **xw** text field, type `r_rod+r0`.

5 Click **Build Selected**.

Work Plane 1 (wp1) > Circle 2 (c2)

1 Right-click **Component 1 (comp1)** > **Geometry 1** > **Work Plane 1 (wp1)** > **Plane Geometry** > **Circle 1 (c1)** and choose **Duplicate**.

2 In the **Settings** window for **Circle**, locate the **Position** section.

3 In the **xw** text field, type `-(r_rod+r0)`.

Work Plane 1 (wp1) > Circle 3 (c3)

1 Right-click **Component 1 (comp1)** > **Geometry 1** > **Work Plane 1 (wp1)** > **Plane Geometry** > **Circle 2 (c2)** and choose **Duplicate**.

2 In the **Settings** window for **Circle**, locate the **Position** section.

3 In the **xw** text field, type `0`.

4 In the **yw** text field, type `-(r_rod+r0)`.

Work Plane 1 (wp1) > Circle 4 (c4)

1 Right-click **Component 1 (comp1)** > **Geometry 1** > **Work Plane 1 (wp1)** > **Plane Geometry** > **Circle 3 (c3)** and choose **Duplicate**.

2 In the **Settings** window for **Circle**, locate the **Position** section.

3 In the **yw** text field, type `r_rod+r0`.

4 In the **Home** toolbar, click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Extrude 1 (ext1)

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Geometry 1** right-click **Work Plane 1 (wp1)** and choose **Extrude**.

2 In the **Settings** window for **Extrude**, locate the **Distances** section.

3 In the table, enter the following settings:

Distances (m)
L_rod

4 Click **Build Selected**.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r_case`.

4 In the **Height** text field, type `L_rod`.

5 Locate the **Axis** section. From the **Axis type** list, choose **x-axis**.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **cyl1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Click to select the **Activate Selection** toggle button for **Objects to subtract**.

5 Select the object **ext1** only.

6 Click **Build Selected**.

Form Union (fin)

1 In the **Geometry** toolbar, click **Build All**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

DEFINITIONS

All rods

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `All rods` in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select the **Group by continuous tangent** checkbox.

5 Select Boundaries 4–11, 13–16, and 18–21 only.

y-pair

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `y-pair` in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select the **Group by continuous tangent** checkbox.

5 Select Boundaries 4–7 and 18–21 only.

z-pair

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `z-pair` in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select the **Group by continuous tangent** checkbox.

5 Select Boundaries 8–11 and 13–16 only.

ELECTROSTATICS (ES)

Electric Potential 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.

2 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

3 In the V_0 text field, type `trap_pot(x)`.

4 Locate the **Boundary Selection** section. From the **Selection** list, choose **All rods**.

ELECTRIC CURRENTS (EC)

In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.

1 In the **Settings** window for **Electric Potential**, locate the **Boundary Selection** section.

2 From the **Selection** list, choose **z-pair**.

3 Locate the **Electric Potential** section. In the V_0 text field, type `V_AC`.

Electric Potential 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.

2 In the **Settings** window for **Electric Potential**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **y-pair**.

4 Locate the **Electric Potential** section. In the V_0 text field, type `-V_AC`.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Extra fine**.

4 Click **Build All**.

First solve for the electric fields. The **Electrostatics (es)** physics interface will be solved using a **Stationary** study step, while the **Electric Currents (ec)** physics interface will be solved using a **Time Dependent** study step

STUDY 1: ELECTRIC FIELDS

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Study 1: Electric Fields` in the **Label** text field.

Step 1: Stationary

1 In the **Study** toolbar, click **Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electric Currents (ec)**.

Step 2: Frequency Domain

1 In the **Study** toolbar, click **Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f`.

4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electrostatics (es)**.

5 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

6 From the **Method** list, choose **Solution**.

7 From the **Study** list, choose **Study 1: Electric Fields, Stationary**.

8 In the **Study** toolbar, click **Compute**.

Now that the electric fields are solved for, set up the **Charged Particle Tracing (cpt)** physics interface to compute the particle trajectories.

CHARGED PARTICLE TRACING (CPT)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Charged Particle Tracing (cpt)**.

2 In the **Settings** window for **Charged Particle Tracing**, locate the **Additional Variables** section.

3 Select the **Store particle status data** checkbox.

Selecting this checkbox is necessary for utilizing the particle trajectories of one study step as the initial conditions for another study step.

Wall 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Charged Particle Tracing (cpt)** click **Wall 1**.

2 In the **Settings** window for **Wall**, locate the **Wall Condition** section.

3 From the **Wall condition** list, choose **Disappear**.

Particle Properties 1

1 In the **Model Builder** window, click **Particle Properties 1**.

2 In the **Settings** window for **Particle Properties**, locate the **Particle Mass** section.

3 In the m_p text field, type `mass_ion`.

4 Locate the **Charge Number** section. In the Z text field, type `1`.

Use the particle beam release feature to release a continuous beam of charged particles during the first five cycles of the oscillating voltage.

Particle Beam 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Particle Beam**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Particle Beam**, locate the **Release Times** section.

4 In the **Release times** text field, type `range(0,(5*T_osc-0)/99,5*T_osc)`.

5 Locate the **Initial Position** section. In the N text field, type `20`.

6 Locate the **Initial Transverse Velocity** section. From the **Transverse velocity distribution specification** list, choose **Specify phase space ellipse dimensions**.

7 In the x_m text field, type `r_in`.

8 Locate the **Initial Longitudinal Velocity** section. In the E text field, type `E_in`.

Electric Force 1

1 In the **Physics** toolbar, click **Domains** and choose **Electric Force**.

2 Select Domain 1 only.

3 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.

4 From the **E** list, choose **Electric field (es/fsp1)**.

Electric Force 2

1 In the **Physics** toolbar, click **Domains** and choose **Electric Force**.

2 Select Domain 1 only.

3 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.

4 From the **E** list, choose **Electric field (ec/cucns1)**.

5 Locate the **Advanced Settings** section. From the **Time dependence of field** list, choose **Time harmonic**.

Collisions 1

1 In the **Physics** toolbar, click **Domains** and choose **Collisions**.

2 Select Domain 1 only.

3 In the **Settings** window for **Collisions**, locate the **Fluid Properties** section.

4 In the N_d text field, type `nd`.

5 In the M_g text field, type `2[amu]*N_A_const`.

6 In the T text field, type `Tgas`.

Elastic 1

In the **Physics** toolbar, click **Attributes** and choose **Elastic**.

Outlet 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Outlet**.

2 Select Boundary 22 only.

Now add another study to compute the particle trajectories utilizing the previously computed electric fields.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Time Dependent**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkboxes for **Electrostatics (es)** and **Electric Currents (ec)**.

5 Click the **Add Study** button in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2: PARTICLE TRACING - TRAPPING POTENTIAL

In the **Settings** window for **Study**, type `Study 2: Particle Tracing - Trapping Potential` in the **Label** text field.

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 2: Particle Tracing - Trapping Potential** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **µs**.

4 In the **Output times** text field, type `range(0,5.0,300)`.

5 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

6 From the **Method** list, choose **Solution**.

7 From the **Study** list, choose **Study 1: Electric Fields, Frequency Domain**.

The modeling of the ion-gas collisions requires that the time steps taken by the solver be less than the mean collision time. Therefore, a fixed time step of 1e-8 s is utilized in this study.

Solution 3 (sol3)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 3 (sol3)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.

4 From the **Steps taken by solver** list, choose **Manual**.

5 In the **Time step** text field, type `1e-8`.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Particle Trajectories - Trapping Potential

In the **Settings** window for **3D Plot Group**, type `Particle Trajectories - Trapping Potential` in the **Label** text field.

Particle Trajectories 1

1. In the **Model Builder** window, expand the **Particle Trajectories - Trapping Potential** node, then click **Particle Trajectories 1**.
2. In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
3. Find the **Point style** subsection.
4. Select the **Radius scale factor** checkbox. In the associated text field, type `0.25`.
5. In the **Particle Trajectories - Trapping Potential** toolbar, click **Plot**.

Color Expression 1

1. In the **Model Builder** window, expand the **Particle Trajectories 1** node, then click **Color Expression 1**.
2. In the **Settings** window for **Color Expression**, locate the **Expression** section.
3. In the **Expression** text field, type `cpt.Ep`.
4. From the **Unit** list, choose **eV**.
5. In the **Particle Trajectories - Trapping Potential** toolbar, click **Plot**.

   The particle trajectories at the last time step should resemble Figure 3.

Animation 1

1. In the **Particle Trajectories - Trapping Potential** toolbar, click **Animation** and choose **Player**.
2. In the **Settings** window for **Animation**, locate the **Frames** section.
3. From the **Frame selection** list, choose **All**.
4. Click the **Play** button in the **Graphics** toolbar.

It is clear that the collisions with the Ar gas diminish the beam energy, and the shape of the axial potential causes the cooled beam to be trapped near the well. This can also be visualized by superimposing the average axial position of the beam as a function of time over the trapping potential.

Axial DC voltages

In the **Model Builder** window, under **Results** click **Axial DC voltages**.

Global 1

1 In the **Axial DC voltages** toolbar, click **Global**.

2 In the **Settings** window for **Global**, locate the **Data** section.

3 From the **Dataset** list, choose **Particle 1**.

4 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
`-t/300[us]*12[V]`	`us`	`Scaled time`

The time parameter is scaled so as to fit neatly over the trapping potential.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `cpt.ave(qx)`.

7 From the **Unit** list, choose **mm**.

8 Click to expand the **Coloring and Style** section. From the **Width** list, choose **3**.

9 Click to expand the **Legends** section. Clear the **Show legends** checkbox.

10 In the **Axial DC voltages** toolbar, click **Plot**.

Color Expression 1

1 In the **Axial DC voltages** toolbar, click **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `cpt.Epave`.

4 From the **Unit** list, choose **eV**.

5 Locate the **Coloring and Style** section. From the **Color table** list, choose **Thermal**.

Axial DC voltages

1 In the **Model Builder** window, under **Results** click **Axial DC voltages**.

2 In the **Settings** window for **1D Plot Group**, locate the **Axis** section.

3 Select the **Manual axis limits** checkbox.

4 In the **x minimum** text field, type `0`.

5 In the **x maximum** text field, type `250`.

6 In the **y minimum** text field, type `-12`.

7 In the **y maximum** text field, type `10`.

8 Locate the **Color Legend** section. Select the **Show maximum and minimum values** checkbox.

9 Select the **Show units** checkbox. The plot demonstrates the trapping of the beam and should resemble Figure 2.

Beam Profile

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Beam Profile` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Particle 1**.

4 From the **Time selection** list, choose **Interpolated**.

5 In the **Times (μs)** text field, type `20`.

Particle 1

1 In the **Beam Profile** toolbar, click **More Plots** and choose **Particle**.

2 In the **Settings** window for **Particle**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `qz`.

4 From the **Unit** list, choose **mm**.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `qy`.

7 From the **Unit** list, choose **mm**.

8 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

9 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

Beam Profile

1 In the **Model Builder** window, click **Beam Profile**.

2 In the **Settings** window for **1D Plot Group**, locate the **Axis** section.

3 Select the **Manual axis limits** checkbox.

4 In the **x minimum** text field, type `-5`.

5 In the **x maximum** text field, type `5`.

6 In the **y minimum** text field, type `-5`.

7 In the **y maximum** text field, type `5`.

8 Locate the **Title** section. From the **Title type** list, choose **None**.

9 In the **Beam Profile** toolbar, click **Plot**.

Plot the beam profile at different solution times by selecting values from the **Time (s)** list. The beam profiles at different solution times are shown in Figure 4.

Transverse Phase Space

1 Right-click **Beam Profile** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Transverse Phase Space` in the **Label** text field.

3 Locate the **Data** section. In the **Times (µs)** text field, type `20`.

Particle 1

1 In the **Model Builder** window, expand the **Transverse Phase Space** node, then click **Particle 1**.

2 In the **Settings** window for **Particle**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `cpt.vy`.

4 From the **Unit** list, choose **km/s**. km/s is equivalent to the convenient scale of mm/µs.

5 In the **Transverse Phase Space** toolbar, click **Plot**.

Plot the transverse phase portraits at different solution times by selecting values from the **Time (s)** list. The transverse phase portraits at different solution times are shown in Figure 5.

Transverse Phase Space

Now that the particles have been cooled and bunched together, alter the DC voltage to the extraction mode in order to release the cooled ions.

ELECTROSTATICS (ES)

Electric Potential 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Electrostatics (es)** click **Electric Potential 1**.

2 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

3 In the V_0 text field, type `ext_pot(x)`.

STUDY 1: ELECTRIC FIELDS

In the **Study** toolbar, click **Compute**.

Add a new study to compute the particle trajectories under the influence of the extraction potential. This new study step will utilize the solution from **Study 2** to initialize the particle position and velocities.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Time Dependent**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkboxes for **Electrostatics (es)** and **Electric Currents (ec)**.

5 Click the **Add Study** button in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3: PARTICLE TRACING - EXTRACTION POTENTIAL

In the **Settings** window for **Study**, type `Study 3: Particle Tracing - Extraction Potential` in the **Label** text field.

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 3: Particle Tracing - Extraction Potential** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **µs**.

4 In the **Output times** text field, type `range(300,10,400)`.

5 Locate the **Values of Dependent Variables** section. Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.

6 From the **Method** list, choose **Solution**.

7 From the **Study** list, choose **Study 2: Particle Tracing - Trapping Potential, Time Dependent**.

8 From the **Time (µs)** list, choose **Last**.

9 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

10 From the **Method** list, choose **Solution**.

11 From the **Study** list, choose **Study 1: Electric Fields, Frequency Domain**.

Solution 4 (sol4)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 4 (sol4)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, locate the **Time Stepping** section.

4 From the **Steps taken by solver** list, choose **Manual**.

5 In the **Time step** text field, type `1e-8`.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Particle Trajectories - Extraction potential

In the **Settings** window for **3D Plot Group**, type `Particle Trajectories - Extraction potential` in the **Label** text field.

Particle Trajectories 1

1 In the **Model Builder** window, expand the **Particle Trajectories - Extraction potential** node, then click **Particle Trajectories 1**.

2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.

3 Find the **Line style** subsection. From the **Type** list, choose **Line**.

4 Find the **Point style** subsection.

5 Select the **Radius scale factor** checkbox. In the associated text field, type `0.25`.

Color Expression 1

1 In the **Model Builder** window, expand the **Particle Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `cpt.Ep`.

4 From the **Unit** list, choose **eV**.

5 In the **Particle Trajectories - Extraction potential** toolbar, click **Plot**.

The particle trajectories at the last time step should resemble Figure 6.

Animation 2

1 In the **Model Builder** window, under **Results** > **Export** right-click **Animation 1** and choose **Duplicate**.

2 In the **Settings** window for **Animation**, locate the **Scene** section.

3 From the **Subject** list, choose **Particle Trajectories - Extraction potential**.

4 Click the **Play** button in the **Graphics** toolbar.