



Model created in COMSOL Multiphysics 6.4

Ion Funnel

Introduction

An electrodynamic ion funnel provides an efficient means of transferring ions from regions of high pressure to high vacuum. The ion funnel can couple devices that generally operate at pressures of different orders of magnitude, such as ion mobility spectrometers and mass spectrometers, allowing mixtures of ionized gases to be separated and analyzed while minimizing losses. For this reason, ion funnels can be used to improve instrument sensitivity in a wide variety of applications, such as the analysis of complex biological molecules or the detection of explosives.

This example uses the Charged Particle Tracing interface to model the movement of ions through an electrodynamic ion funnel. The Electric Force feature is used to apply DC and RF potentials to guide ions through the funnel, while the Elastic Collision Force feature is used to model collisions with background gas molecules.

Note: This model requires the Particle Tracing Module and the AC/DC Module or the MEMS Module.

Model Definition

The ion funnel is a converging series of insulated ring-shaped electrodes, each subjected to an RF potential, with adjacent electrodes out of phase. The RF potential confines ions radially while a DC bias guides the ions toward successively narrower electrodes. The superposition of the DC and RF fields focuses the ions and offsets the effects of thermal dispersion and coulombic repulsion.

The ion funnel contains a buffer gas of neutral argon at 1 Torr, which is assumed to follow a Maxwellian velocity distribution:

$$f(v_i) = \sqrt{\frac{m_p}{2\pi k_B T_0}} \exp\left(-\frac{m_p v_i^2}{2k_B T_0}\right)$$

where

- v_i (SI unit: m/s) is the i th velocity component,
- m_p (SI unit: kg) is the molecular mass,
- T_0 (SI unit: K) is the temperature, and
- $k = 1.3806488 \times 10^{-23}$ J/K is the Boltzmann constant.

In this example, the background gas molecules have molar mass 0.04 kg/mol, for a molecular mass of 6.6422×10^{-26} kg. The temperature is assumed to be 293.15 K.

The number density of the gas can be computed using the ideal gas law,

$$p = \frac{nRT}{N_A}$$

where $R = 8.3144621$ J/(mol·K) is the universal gas constant and $N_A = 6.02214129 \times 10^{23}$ 1/mol is the Avogadro constant. For a pressure of 1 Torr this yields $n = 3.294 \times 10^{22}$ atoms/m³.

The interaction of ions with the background gas is modeled using the **Collisions** node with an **Elastic** subnode. At each time step taken by the solver, for each model particle a background gas particle is sampled at random from the Maxwellian distribution. The frequency of elastic collisions is then computed from the collision cross section, background gas number density, and the relative velocity of the model particle with respect to the randomly sampled background gas molecule:

$$\nu = N_d \sigma |\mathbf{v}_p - \mathbf{v}_g|$$

where the collision cross section σ (SI unit: m²) is usually a function of the particle kinetic energy. The collision probability is then computed as a function of the collision frequency and the time step size:

$$P = 1 - \exp(-\nu \Delta t)$$

The model uses three physics interfaces: Electrostatics, Electric Currents, and Charged Particle Tracing. The Electrostatics and Electric Currents interfaces are used to compute the DC and AC fields, respectively. These fields are then coupled to the Charged Particle Tracing interface, which models the motion of the ions due to the electric fields and interaction with neutral particles in the background gas. Interactions between the ions are neglected. In order to accurately model collisions of ions with the background gas, the average time between elastic collisions should be significantly greater than the maximum time step taken by the solver. Strict or manual time stepping is recommended.

Results and Discussion

The electric potential in the ion funnel is plotted in [Figure 1](#). The gradual DC bias guides positive ions from the larger end of the funnel to the smaller end. The AC voltage, which is out of phase between adjacent electrodes, causes the gradient of the electric potential to become very large close to the electrodes, keeping the ions confined within the funnel.

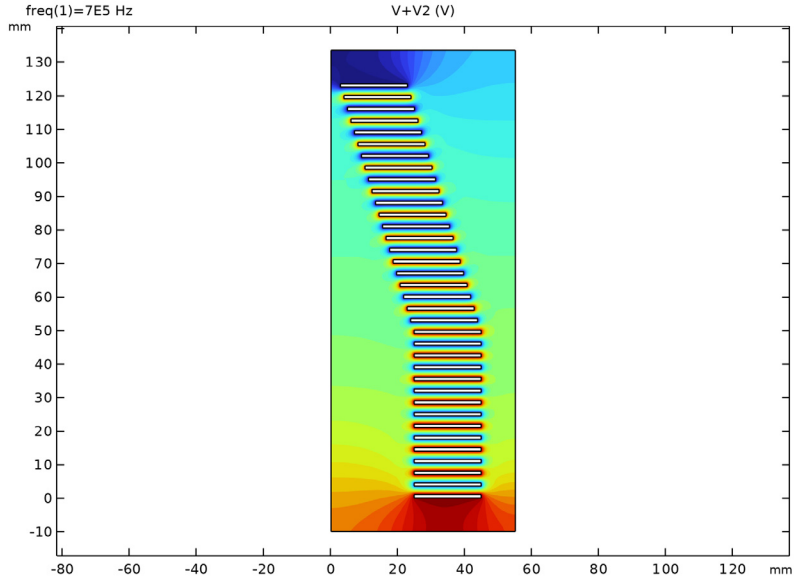


Figure 1: The combined DC and AC potential is plotted in the ion funnel at time $t=0$.

The ion trajectories are plotted in [Figure 2](#). Because the ions are confined to an area of reduced size, they can be transported to another device, such as a mass spectrometer, more efficiently.

The x - and y -coordinates of the particles at the narrow end of the funnel are plotted in [Figure 3](#). Although the ions are released along the positive x -axis, they are uniformly distributed around the z -axis by the time they reach the end of the funnel. Because the **Collisions** node uses random numbers to determine whether a collision takes place at each time step, the results may be slightly different from those shown in [Figure 2](#) and [Figure 3](#).

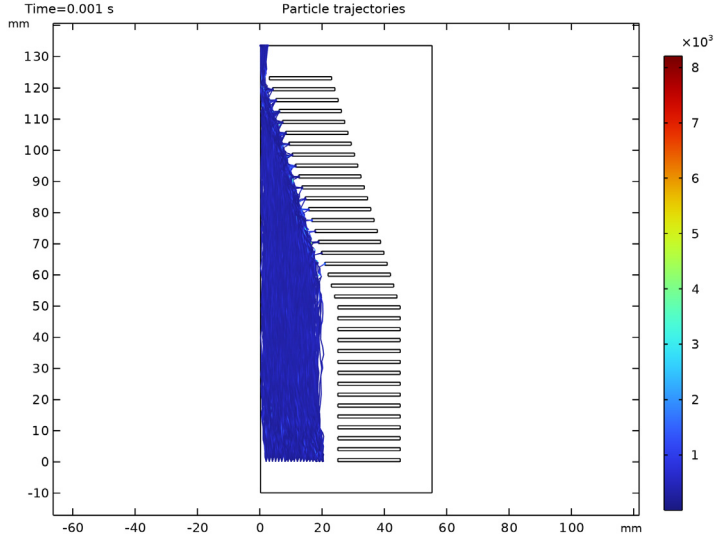


Figure 2: The trajectories of positive ions in the funnel.

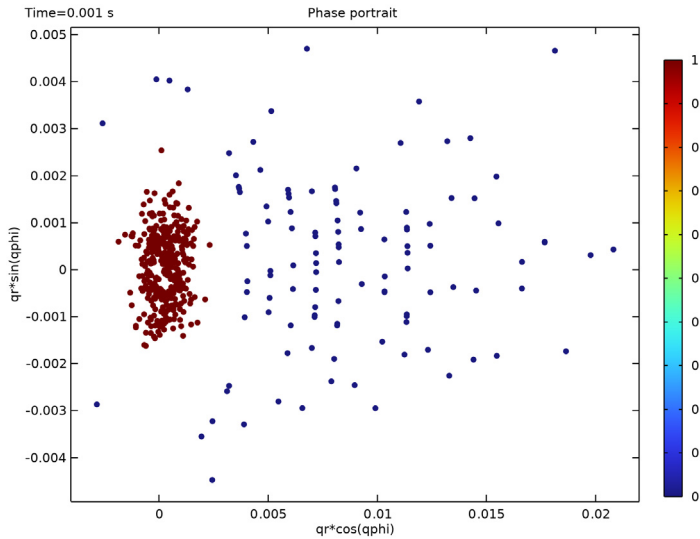


Figure 3: Phase portrait showing the x - and y -coordinates of the ions at the narrow end of the funnel. The color expression is red for particles that have exited the funnel and blue otherwise.

Reference


1. A. V. Phelps, “The application of scattering cross sections to ion flux models in discharge sheaths,” *J. Appl. Phys.* vol. 76, pp. 747-753, 1994.

Application Library path: Particle_Tracing_Module/
Charged_Particle_Tracing/ion_funnel




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.


MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D Axisymmetric**.
- 2 In the **Select Physics** tree, select **AC/DC** > **Electric Fields and Currents** > **Electrostatics (es)**.
- 3 Click **Add**.
- 4 In the **Select Physics** tree, select **AC/DC** > **Electric Fields and Currents** > **Electric Currents (ec)**.
- 5 Click **Add**.
- 6 In the **Select Physics** tree, select **AC/DC** > **Particle Tracing** > **Charged Particle Tracing (cpt)**.
- 7 Click **Add**.
- 8 Click  **Study**.
- 9 In the **Select Study** tree, select **Preset Studies for Some Physics Interfaces** > **Stationary**.
- 10 Click  **Done**.

GLOBAL DEFINITIONS

To save time, load the parameters from a file.

Parameters 1


- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click  **Load from File**.

- 4 Browse to the model's Application Libraries folder and double-click the file `ion_funnel_parameters.txt`.


GEOMETRY I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **mm**.

Rectangle 1 (r1)


- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type `welec`.
- 4 In the **Height** text field, type `telec`.
- 5 Locate the **Position** section. In the **r** text field, type `rmax`.

Rectangle 2 (r2)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type `welec`.
- 4 In the **Height** text field, type `telec`.
- 5 Locate the **Position** section. In the **r** text field, type `rmax`.
- 6 In the **z** text field, type `telec+tgap`.

Create arrays containing the electrodes which are in phase with each other. This setup allows the number of electrodes to be changed at any time by changing the corresponding parameters.

Array 1 (arr1)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r1** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.
- 5 In the **Size** text field, type $(N_{\text{straight}}+1)/2$.
- 6 Locate the **Displacement** section. In the **z** text field, type $2*(telec+tgap)$.
- 7 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.

- 8 From the **Show in physics** list, choose **Boundary selection**.
- 9 Find the **Cumulative selection** subsection. Click **New**.
- 10 In the **New Cumulative Selection** dialog, type Domains to Subtract in the **Name** text field.
- 11 Click **OK**.

1 In the **Model Builder** window, click **Array 1 (arr1)**.

2 In the **Settings** window for **Array**, click  **Build Selected**.

Array 2 (arr2)

1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.

2 Select the object **r2** only.

3 In the **Settings** window for **Array**, locate the **Size** section.

4 From the **Array type** list, choose **Linear**.

5 In the **Size** text field, type $(N_{\text{straight}} - 1) / 2$.

6 Locate the **Displacement** section. In the **z** text field, type $2 * (t_{\text{elec}} + t_{\text{gap}})$.

7 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.

8 From the **Show in physics** list, choose **Boundary selection**.

9 Find the **Cumulative selection** subsection. From the **Contribute to** list, choose **Domains to Subtract**.

10 Click  **Build Selected**.

Rectangle 3 (r3)

1 In the **Geometry** toolbar, click  **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type w_{elec} .

4 In the **Height** text field, type t_{elec} .

5 Locate the **Position** section. In the **r** text field, type r_{min} .


6 In the **z** text field, type $h_{\text{funnel}} - t_{\text{elec}}$.

Rectangle 4 (r4)


1 In the **Geometry** toolbar, click  **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.


3 In the **Width** text field, type w_{elec} .

- 4 In the **Height** text field, type `telec`.
- 5 Locate the **Position** section. In the **r** text field, type $r_{min} + (r_{max} - r_{min}) / N_{inclined}$.
- 6 In the **z** text field, type $h_{funnel} - (2 * telec + tgap)$.
- 7 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Array 3 (*arr3*)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r3** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.
- 5 In the **Size** text field, type $(N_{inclined} + 1) / 2$.
- 6 Locate the **Displacement** section. In the **r** text field, type $2 * (r_{max} - r_{min}) / N_{inclined}$.
- 7 In the **z** text field, type $-2 * (telec + tgap)$.
- 8 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.
- 9 From the **Show in physics** list, choose **Boundary selection**.
- 10 Find the **Cumulative selection** subsection. From the **Contribute to** list, choose **Domains to Subtract**.


Array 4 (*arr4*)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r4** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.
- 5 In the **Size** text field, type $(N_{inclined} - 1) / 2$.
- 6 Locate the **Displacement** section. In the **r** text field, type $2 * (r_{max} - r_{min}) / N_{inclined}$.
- 7 In the **z** text field, type $-2 * (telec + tgap)$.
- 8 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.
- 9 From the **Show in physics** list, choose **Boundary selection**.
- 10 Find the **Cumulative selection** subsection. From the **Contribute to** list, choose **Domains to Subtract**.




Add a rectangle to enclose the modeling domain. Note that the rectangle is positioned a small distance away from the axis of symmetry. This makes the particle tracing model more

robust, since the centrifugal force acting on the particles approaches infinity as the radial coordinate approaches zero.

Rectangle 5 (r5)



- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type $r_{max} + w_{elec} + 10$ [mm].
- 4 In the **Height** text field, type $h_{funnel} + 20$ [mm].
- 5 Locate the **Position** section. In the **r** text field, type 0.2 [mm].
- 6 In the **z** text field, type -10 [mm].

Difference 1 (dif1)



- 1 In the **Geometry** toolbar, click  **Booleans and Partitions** and choose **Difference**.
- 2 Select the object **r5** only.
- 3 In the **Settings** window for **Difference**, locate the **Difference** section.
- 4 From the **Objects to subtract** list, choose **Domains to Subtract**.
- 5 Click  **Build All Objects**.
- 6 Click the  **Zoom Extents** button in the **Graphics** toolbar.

DEFINITIONS



Odd electrodes

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type *Odd electrodes* in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click  **Add**.
- 5 In the **Add** dialog, in the **Selections to add** list, choose **Array 1** and **Array 4**.
- 6 Click **OK**.


Even electrodes

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type *Even electrodes* in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click  **Add**.
- 5 In the **Add** dialog, in the **Selections to add** list, choose **Array 2** and **Array 3**.
- 6 Click **OK**.



All electrodes

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type All electrodes in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click  **Add**.
- 5 In the **Add** dialog, in the **Selections to add** list, choose **Odd electrodes** and **Even electrodes**.
- 6 Click **OK**.



Exterior boundaries

- 1 In the **Definitions** toolbar, click  **Explicit**.
- 2 In the **Settings** window for **Explicit**, type Exterior boundaries in the **Label** text field.
- 3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundaries 2, 3, and 148 only.

All boundaries


- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type All boundaries in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click  **Add**.
- 5 In the **Add** dialog, in the **Selections to add** list, choose **All electrodes** and **Exterior boundaries**.
- 6 Click **OK**.

ADD MATERIAL

- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Built-in > Perfect vacuum**.
- 4 Click the **Add to Component** button in the window toolbar.
- 5 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.

ELECTROSTATICS (ES)


Electric Potential I

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Electric Potential**.
- 2 In the **Settings** window for **Electric Potential**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.


4 Locate the **Electric Potential** section. In the V_0 text field, type $EDC*z$.

ELECTRIC CURRENTS (EC)

Boundary Terminal 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Boundary Terminal**.
- 2 In the **Settings** window for **Boundary Terminal**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Odd electrodes**.
- 4 Locate the **Terminal** section. From the **Terminal type** list, choose **Voltage**.
- 5 In the V_0 text field, type Vrf .

Boundary Terminal 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Boundary Terminal**.
- 2 In the **Settings** window for **Boundary Terminal**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Even electrodes**.
- 4 Locate the **Terminal** section. From the **Terminal type** list, choose **Voltage**.
- 5 In the V_0 text field, type $-Vrf$.


CHARGED PARTICLE TRACING (CPT)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Charged Particle Tracing (cpt)**.
- 2 In the **Settings** window for **Charged Particle Tracing**, locate the **Additional Variables** section.
- 3 Select the **Include out-of-plane degrees of freedom** checkbox.



Particle Properties 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** > **Charged Particle Tracing (cpt)** click **Particle Properties 1**.
- 2 In the **Settings** window for **Particle Properties**, locate the **Particle Mass** section.
- 3 In the m_p text field, type $0.04[\text{kg/mol}]/N_A_const$.
- 4 Locate the **Charge Number** section. In the Z text field, type 1.


Wall 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Wall**.
- 2 Select Boundary 1 only.
- 3 In the **Settings** window for **Wall**, locate the **Wall Condition** section.
- 4 From the **Wall condition** list, choose **Bounce**.


Release from Grid 1

- 1 In the **Physics** toolbar, click  **Global** and choose **Release from Grid**.
- 2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.
- 3 Click  **R Range**.
- 4 In the **Range** dialog, type 2 in the **Start** text field.
- 5 In the **Step** text field, type 1.
- 6 In the **Stop** text field, type 20.
- 7 Click **Replace**.
- 8 In the **Settings** window for **Release from Grid**, locate the **Initial Velocity** section.
- 9 From the **Velocity specification** list, choose **Maxwellian**.
- 10 In the N_v text field, type 5.

Electric Force 1

- 1 In the **Physics** toolbar, click  **Domains** and choose **Electric Force**.
- 2 In the **Settings** window for **Electric Force**, locate the **Domain Selection** section.
- 3 From the **Selection** list, choose **All domains**.
- 4 Locate the **Electric Force** section. From the **E** list, choose **Electric field (es/fsp1)**.
- 5 Locate the **Advanced Settings** section. Select the **Use piecewise polynomial recovery on field** checkbox.

Electric Force 2


- 1 In the **Physics** toolbar, click  **Domains** and choose **Electric Force**.
- 2 In the **Settings** window for **Electric Force**, locate the **Domain Selection** section.
- 3 From the **Selection** list, choose **All domains**.
- 4 Locate the **Electric Force** section. From the **E** list, choose **Electric field (ec/cucns1)**.
- 5 Locate the **Advanced Settings** section. From the **Time dependence of field** list, choose **Time harmonic**.
- 6 Select the **Use piecewise polynomial recovery on field** checkbox.

Now define the cross sections used for the collision model.

DEFINITIONS

Enter the analytic approximation for momentum cross section for elastic scattering between Ar^+ ions and neutral Ar atoms from [Ref. 1](#), which depends on the kinetic energy of the particles.

Analytic 1 (an1)


- 1 In the **Definitions** toolbar, click  **Analytic**.
- 2 In the **Settings** window for **Analytic**, type Qm in the **Function name** text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type $1.15e-18*x^{(-0.1)*(1+0.015/x)^{0.6}}$.
- 4 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	eV

- 5 In the **Function** text field, type m^2 .

Enter the analytic approximation for isotropic elastic collision between Ar^+ ions and neutral Ar atoms from [Ref. 1](#), which depends on the kinetic energy of the particles.

Analytic 2 (an2)


- 1 In the **Definitions** toolbar, click  **Analytic**.
- 2 In the **Settings** window for **Analytic**, type Qi in the **Function name** text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type $2e-19/(x^{(0.5)*(1+x)})+3e-19*x/(1+x/3)^{(2.3)}$.
- 4 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	eV

- 5 In the **Function** text field, type m^2 .

CHARGED PARTICLE TRACING (CPT)

Collisions 1

- 1 In the **Physics** toolbar, click  **Domains** and choose **Collisions**.
- 2 In the **Settings** window for **Collisions**, locate the **Domain Selection** section.
- 3 From the **Selection** list, choose **All domains**.
- 4 Locate the **Fluid Properties** section. In the N_d text field, type ND.
- 5 Locate the **Collision Statistics** section. Select the **Count all collisions** checkbox.

Elastic 1


- 1 In the **Physics** toolbar, click  **Attributes** and choose **Elastic**.
- 2 In the **Settings** window for **Elastic**, locate the **Collision Frequency** section.

- 3 In the σ text field, type $Q_i(\text{cpt} \cdot \text{Ep})$.
- 4 Locate the **Collision Statistics** section. Select the **Count collisions** checkbox.

Collisions I

In the **Model Builder** window, click **Collisions I**.

Resonant Charge Exchange I



- 1 In the **Physics** toolbar, click  **Attributes** and choose **Resonant Charge Exchange**.
- 2 In the **Settings** window for **Resonant Charge Exchange**, locate the **Collision Frequency** section.
- 3 In the σ text field, type $(Q_m(\text{cpt} \cdot \text{Ep}) - Q_i(\text{cpt} \cdot \text{Ep})) / 2$.
- 4 Locate the **Collision Statistics** section. Select the **Count collisions** checkbox.

STUDY I

Step 1: Stationary

- 1 In the **Model Builder** window, under **Study I** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 3 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electric Currents (ec)**.

Step 2: Frequency Domain


- 1 In the **Study** toolbar, click  **Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type f_0 .
- 4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Electrostatics (es)**.
- 5 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 6 From the **Method** list, choose **Solution**.
- 7 From the **Study** list, choose **Study 1, Stationary**.
- 8 In the **Study** toolbar, click  **Compute**.

RESULTS



Electric Potential (es)

Create a contour plot of the electric potential when $t = 0$.



Sum of AC and DC Potentials

- 1 In the **Results** toolbar, click  **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, type Sum of AC and DC Potentials in the **Label** text field.

Contour 1


- 1 Right-click **Sum of AC and DC Potentials** and choose **Contour**.
- 2 In the **Settings** window for **Contour**, locate the **Expression** section.
- 3 In the **Expression** text field, type $V+V2$.
- 4 Locate the **Levels** section. In the **Total levels** text field, type 30.
- 5 Locate the **Coloring and Style** section. From the **Contour type** list, choose **Filled**.
- 6 Clear the **Color legend** checkbox.
- 7 In the **Sum of AC and DC Potentials** toolbar, click  **Plot**.
- 8 Click the  **Zoom Extents** button in the **Graphics** toolbar. Compare this plot to [Figure 1](#). The plot indicates that the potential gradient is steep in the area surrounding the electrodes. The large electric field magnitude in this area confines ions within the funnel.

ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies > Time Dependent**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkboxes for **Electrostatics (es)** and **Electric Currents (ec)**.
- 5 Click the **Add Study** button in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.



STUDY 2

Step 1: Time Dependent

- 1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 2 Click  **Range**.
- 3 In the **Range** dialog, type $1e-5$ in the **Step** text field.
- 4 In the **Stop** text field, type $1e-3$.
- 5 Click **Replace**.

- 6 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 7 From the **Tolerance** list, choose **User controlled**.
- 8 In the **Relative tolerance** text field, type $1e-3$.
- 9 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 10 From the **Method** list, choose **Solution**.
- 11 From the **Study** list, choose **Study 1, Frequency Domain**.


Solution 3 (sol3)

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 3 (sol3)** node, then click **Time-Dependent Solver 1**.
- 3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Absolute Tolerance** section.
- 4 From the **Tolerance method** list, choose **Manual**.
- 5 In the **Absolute tolerance** text field, type $1e-4$.
- 6 Click to expand the **Time Stepping** section. From the **Maximum step constraint** list, choose **Constant**.
- 7 In the **Maximum step** text field, type $1e-8$.
- 8 Click to expand the **Output** section. Clear the **Store reaction forces** checkbox.
- 9 Clear the **Store the first time derivative** checkbox.
- 10 In the **Model Builder** window, expand the **Study 2 > Solver Configurations > Solution 3 (sol3) > Time-Dependent Solver 1** node, then click **Advanced**.
- 11 In the **Settings** window for **Advanced**, locate the **General** section.
- 12 From the **Solver log** list, choose **Minimal**.
- 13 In the **Study** toolbar, click  **Compute**.


RESULTS

Particle Trajectories 1


- 1 In the **Model Builder** window, expand the **Particle Trajectories (cpt)** node, then click **Particle Trajectories 1**.
- 2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
- 3 Find the **Line style** subsection. From the **Type** list, choose **Line**.

- 4 In the **Particle Trajectories (cpt)** toolbar, click  **Plot**. Compare the result to [Figure 2](#).



Transverse Particle Positions

- 1 In the **Results** toolbar, click  **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, type `Transverse Particle Positions` in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Particle I**.
- 4 From the **Time (s)** list, choose **0.001**.
- 5 Locate the **Plot Settings** section. Clear the **Plot dataset edges** checkbox.

Phase Portrait I



- 1 In the **Transverse Particle Positions** toolbar, click  **More Plots** and choose **Phase Portrait**.
- 2 In the **Settings** window for **Phase Portrait**, locate the **Expression** section.
- 3 From the **x-axis** list, choose **Manual**.
- 4 In the **Expression** text field, type `qr*cos(qphi)`.
- 5 From the **y-axis** list, choose **Manual**.
- 6 In the **Expression** text field, type `qr*sin(qphi)`.

Color Expression I

- 1 Right-click **Phase Portrait I** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, locate the **Expression** section.
- 3 In the **Expression** text field, type `qz>120[mm]`.
- 4 In the **Transverse Particle Positions** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting plot to [Figure 3](#).


Create an animation showing the *x*- and *y*-components of the particle positions over time.

Animation I


- 1 In the **Results** toolbar, click  **Animation** and choose **File**.
- 2 In the **Settings** window for **Animation**, locate the **Target** section.
- 3 From the **Target** list, choose **Player**.
- 4 Locate the **Scene** section. From the **Subject** list, choose **Transverse Particle Positions**.
- 5 Click the  **Play** button in the **Graphics** toolbar.

Finally, create a 1D plot showing the average radial position over time. Set the color expression to be the total radial force, averaged over all particles.



Average Radial Particle Position

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type **Average Radial Particle Position** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Particle I**.

Particle I

- 1 In the **Average Radial Particle Position** toolbar, click  **More Plots** and choose **Particle**.
- 2 In the **Settings** window for **Particle**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp1) > Charged Particle Tracing > Particle position > qr - Particle position, r-component - m**.
- 3 Locate the **Data Series Operation** section. From the **Operation** list, choose **Average**.

Color Expression I

- 1 Right-click **Particle I** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component I (comp1) > Charged Particle Tracing > Forces > Total force - N > cpt.Ftr - Total force, r-component**.
- 3 In the **Average Radial Particle Position** toolbar, click  **Plot**.
- 4 Click the  **Zoom Extents** button in the **Graphics** toolbar.