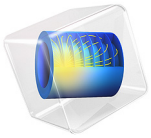


Model created in COMSOL Multiphysics 6.4

Shape Optimization of Coils

Introduction

This model demonstrates how to design a coil geometry using gradient-based shape optimization. The objective is to achieve a uniform magnetic field along the coil axis and a field minimum near the axis ends. The model assumes azimuthal symmetry, but one can use the optimization result as inspiration for a 3D design.

Model Definition

The model is set up with six inner coils and one coil at each end with the opposite current direction. The current in the two outermost coils at each end is also optimized to mimic the effect of having coils with partial turns. The result of the shape optimization is shown in Figure 2, while the initial geometry for this model is shown in Figure 1.

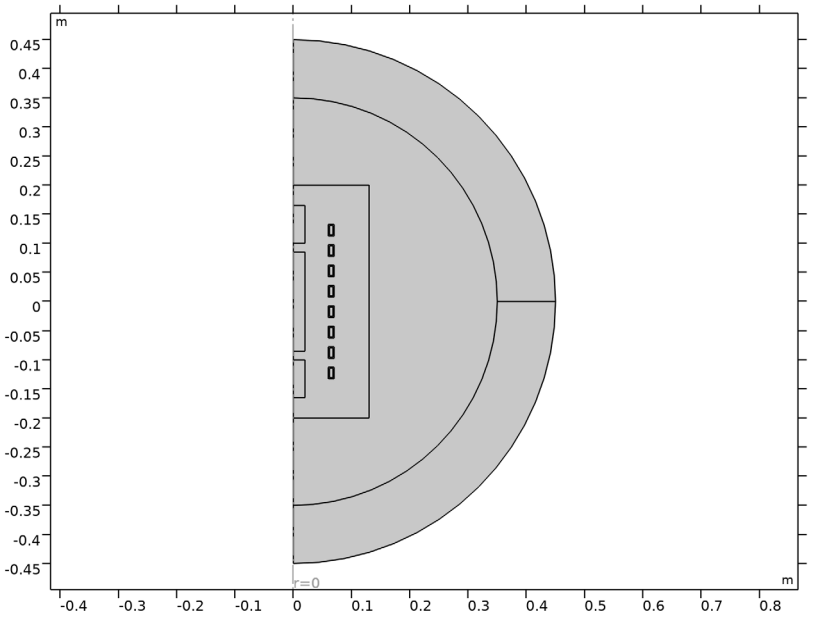


Figure 1: The initial geometry.

The **Transformation** and **Free Shape Domain** features are used to allow the coils to move in the radial direction. There are 8 control variables for the coil positions and one for the current in the outer coils. The IPOPT optimization can solve such problems fast.

The objective function, φ, consists of two terms:

$$\varphi = \varphi_1 + \varphi_2$$

$$\varphi_1 = \int_{\Omega_{\text{inner}}} (|\mathbf{B}| - \mathbf{B}_{\text{avg}})^2 d\Omega / \int_{\Omega_{\text{inner}}} d\Omega$$

$$\mathbf{B}_{\text{avg}} = \int_{\Omega_{\text{inner}}} |\mathbf{B}| d\Omega / \int_{\Omega_{\text{inner}}} d\Omega$$

$$\varphi_2 = \int_{\Omega_{\text{outer}}} |\mathbf{B}| d\Omega / \int_{\Omega_{\text{outer}}} d\Omega$$

The setup of the first objective function is simplified by the use of the **Standard Deviation** feature.

Results and Discussion

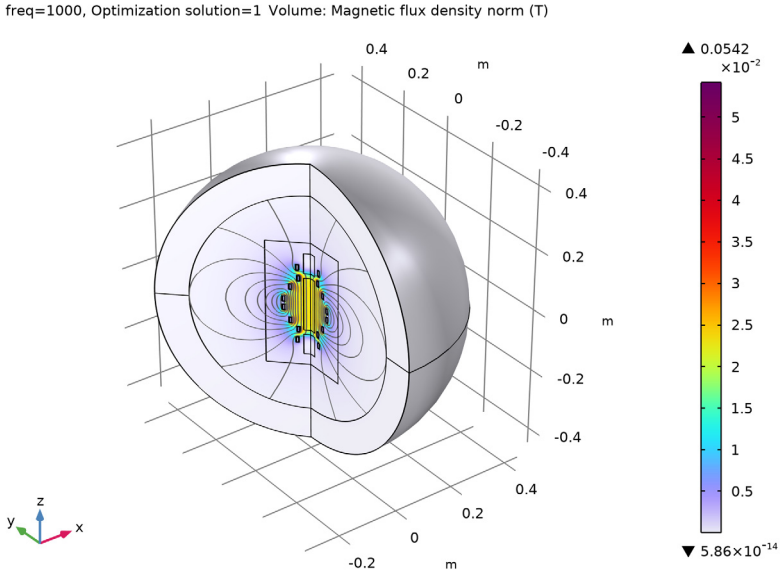


Figure 2: The optimized geometry.

Figure 2 shows the optimized design. The first and second objectives are reduced by around 90% and 50%, respectively. The strength of the magnetic field on the axis is illustrated in Figure 3.

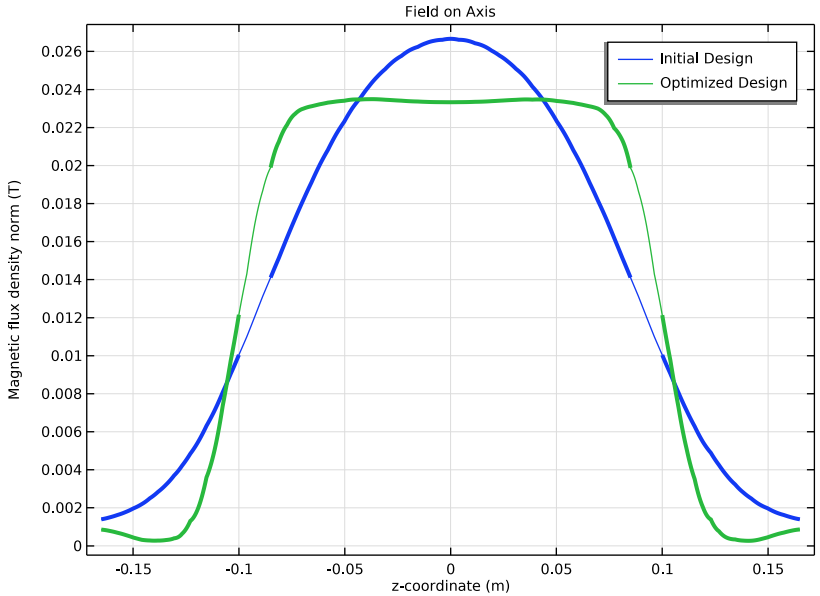


Figure 3: The optimization increases the magnetic field in the middle domain and decreases it in the outer domains.

Notes About the COMSOL Implementation

This model can be constructed in a way that exploits symmetry in the *xy*-plane, leading to a reduction in the computational time.

Application Library path: `Optimization_Module/Shape_Optimization/coil_shape_optimization`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **AC/DC** > **Electromagnetic Fields** > **Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies** > **Frequency Domain**.

6 Click **Done**.

GEOMETRY 1

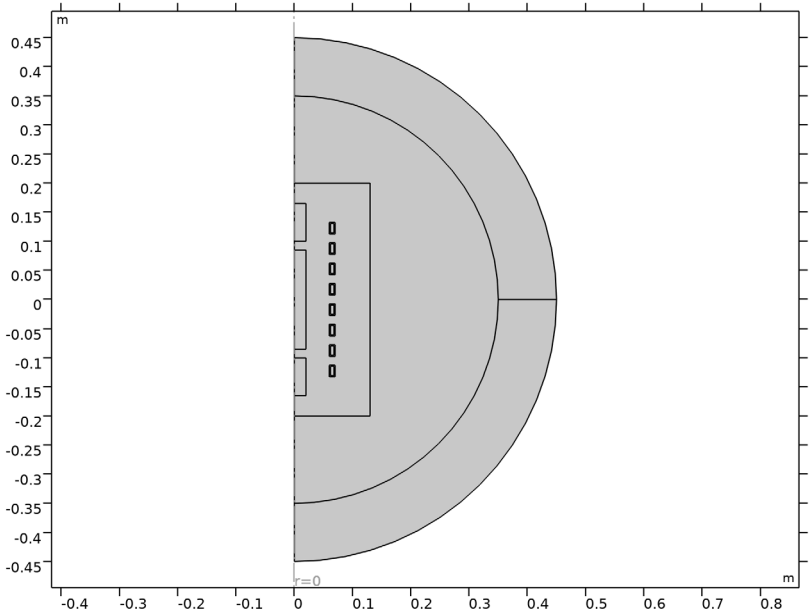
1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Geometry 1** node.

2 Right-click **Geometry 1** and choose **Insert Sequence**.

3 Browse to the model's Application Libraries folder and double-click the file `coil_shape_optimization_geom_sequence.mph`.

4 In the **Geometry** toolbar, click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



The geometry should now look like that in Figure 1.

GLOBAL DEFINITIONS

Geometry Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Geometry Parameters` in the **Label** text field.

Parameters 2

1 In the **Home** toolbar, click **Parameters** and choose **Add** > **Parameters**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	1[kHz]	1000 Hz	Frequency
lastTurns	0.5	0.5	Outer loop current factor
dmax	3[cm]	0.03 m	Maximum coil translation

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Click the **Add to Component** button in the window toolbar.

5 In the tree, select **Built-in** > **Copper**.

6 Click the **Add to Component** button in the window toolbar.

7 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Copper (mat2)

1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

2 From the **Selection** list, choose **Objects to Mirror**.

MAGNETIC FIELDS (MF)

Domain Coil 1

1 In the **Physics** toolbar, click **Domains** and choose **Domain Coil**.

2 In the **Settings** window for **Domain Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Inner Coils**.

4 Locate the **Coil** section. Select the **Coil group** checkbox.

5 In the I_{coil} text field, type `1[kA]`.

Domain Coil 2

1 Right-click **Domain Coil 1** and choose **Duplicate**.

2 In the **Settings** window for **Domain Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils 2**.

4 Locate the **Coil** section. In the I_{coil} text field, type `1[kA]*lastTurns`.

In a 2D axisymmetric model, specifying a coil with a fraction of the total current is a way to represent a coil with partial turns. Similarly, specifying a current with the opposite sign represents a coil that is wound in the opposite direction.

Domain Coil 3

1 Right-click **Domain Coil 2** and choose **Duplicate**.

2 In the **Settings** window for **Domain Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils**.

4 Locate the **Coil** section. In the I_{coil} text field, type `-1[kA]*lastTurns`.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click  **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Infinite Domains**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Infinite Domain Boundaries**.

Free Triangular 1

In the **Mesh** toolbar, click  **Free Triangular**.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Deforming Domain**.

5 Locate the **Element Size** section. From the **Predefined** list, choose **Extremely fine**.

Size 2

1 In the **Model Builder** window, right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Whole Axis**.

5 Locate the **Element Size** section. Click the **Custom** button.

6 Locate the **Element Size Parameters** section.

7 Select the **Maximum element size** checkbox. In the associated text field, type `0.004`.

8 Click  **Build All**.

COMPONENT 1 (COMP1)

Free Shape Domain 1

1 In the **Physics** toolbar, click **Optimization** and choose **Shape Optimization**.

2 In the **Settings** window for **Free Shape Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Deforming Domain**.

Transformation 1

1 In the **Shape Optimization** toolbar, click **Transformation**.

2 In the **Settings** window for **Transformation**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Objects to Mirror**.

4 Locate the **Translation** section. In the table, enter the following settings:

	Lock	Lower bound (m)	Upper bound (m)
R		`-dmax`	`dmax`
Z		`-coilSpace/4`	`coilSpace/4`

5 Locate the **Scaling** section. From the **Scaling type** list, choose **No scaling**.

DEFINITIONS

Standard Deviation 1 (std1)

1 In the **Definitions** toolbar, click **Physics Utilities** and choose **Standard Deviation**.

2 In the **Settings** window for **Standard Deviation**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Rectangle 4**.

4 Click **Replace Expression** in the upper-right corner of the **Standard Deviation** section. From the menu, choose **Component 1 (comp1)** > **Magnetic Fields** > **Magnetic** > **comp1.mf.normB - Magnetic flux density norm - T**.

5 Locate the **Quadrature Settings** section. Select the **Compute integral in revolved geometry** checkbox.

Average outer magnetic field

1 In the **Definitions** toolbar, click **Probes** and choose **Domain Probe**.

2 In the **Settings** window for **Domain Probe**, type `Average outer magnetic field` in the **Label** text field.

3 In the **Variable name** text field, type `Bouter`.

4 Locate the **Source Selection** section. From the **Selection** list, choose **Outer Objective Domain**.

5 Locate the **Expression** section. In the **Expression** text field, type `mf.normB`.

Infinite Element Domain 1 (ie1)

1 In the **Definitions** toolbar, click **Infinite Element Domain**.

2 In the **Settings** window for **Infinite Element Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Infinite Domains**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

General Optimization

1 In the **Study** toolbar, click **Optimization** and choose **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **GCMMA**.

4 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **comp1.std1 - Standard deviation**.

5 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **comp1.Bouter - Average outer magnetic field - T**.

6 Locate the **Control Variables and Parameters** section. Click **Add**.

Initialize the study to generate a plot for use while solving.

7 In the table, enter the following settings:

Parameter	Initial value	Scale	Lower bound	Upper bound	Unit
lastTurns (Outer loop current factor)	0.5	1	0	1	

8 Click to expand the **Solver Settings** section. Find the **Objective settings** subsection. From the **Objective scaling** list, choose **Initial solution based**.

9 Click to expand the **Output** section. From the **Keep solutions** list, choose **First and last**.

10 From the **Probes** list, choose **None**.

11 In the **Model Builder** window, click **Study 1**.

12 In the **Settings** window for **Study**, type `Shape Optimization` in the **Label** text field.

13 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

Shape Optimization

The plot shows the displacement. Add an **Annotation** feature to also show the value of the `lastTurns` parameter.

Annotation 1

1 In the **Model Builder** window, right-click **Shape Optimization** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `eval(lastTurns)`.

4 Locate the **Coloring and Style** section. Clear the **Show point** checkbox.

5 From the **Background color** list, choose **Gray**.

SHAPE OPTIMIZATION

General Optimization

1 In the **Model Builder** window, under **Shape Optimization** click **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Output** section.

3 Select the **Plot** checkbox.

4 In the table, enter the following settings:

Plot group	Plot window
Shape Optimization	Graphics

Enable move limits to reduce the risk of inverted elements.

Solver Configurations

In the **Model Builder** window, expand the **Shape Optimization** > **Solver Configurations** node.

Solution 1 (sol1)

1 In the **Model Builder** window, expand the **Shape Optimization** > **Solver Configurations** > **Solution 1 (sol1)** node, then click **Optimization Solver 1**.

2 In the **Settings** window for **Optimization Solver**, locate the **Optimization Solver** section.

3 Select the **Move limits** checkbox.

4 Select the **Maximum number of outer iterations** checkbox. In the associated text field, type `50`.

5 Click **Run**.

RESULTS

Magnetic Flux Density, Revolved Geometry (mf)

1 In the **Magnetic Flux Density, Revolved Geometry (mf)** toolbar, click **Plot**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

Field on Axis

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Field on Axis` in the **Label** text field.

3 Locate the **Data** section. From the **Optimization solution** list, choose **First**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 1

1 Right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Whole Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Legends** section. Select the **Show legends** checkbox.

7 From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

Legends
`Initial Design`

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Optimized Design

Line Graph 3

1 In the **Model Builder** window, right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Inner Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

7 From the **Width** list, choose **3**.

Line Graph 4

1 Right-click **Line Graph 3** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

Line Graph 3, Line Graph 4

1 In the **Model Builder** window, under **Results** > **Field on Axis**, Ctrl-click to select **Line Graph 3** and **Line Graph 4**.

2 Right-click and choose **Duplicate**.

Line Graph 5

1 In the **Settings** window for **Line Graph**, locate the **Selection** section.

2 From the **Selection** list, choose **Outer Axis**.

Line Graph 6

1 In the **Model Builder** window, click **Line Graph 6**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Outer Axis**.

4 In the **Field on Axis** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compute the value of the objective functions before and after optimization.

Evaluation Group 1

In the **Results** toolbar, click **Evaluation Group**.

Global Evaluation 1

1. Right-click **Evaluation Group 1** and choose **Global Evaluation**.
2. In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **std1 - Standard deviation - T**.
3. Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **Bouter - Average outer magnetic field - T**.
4. In the **Evaluation Group 1** toolbar, click **Evaluate**.

Compute the value of the corners, so that one can construct an interpolation function for 3D verification.

Point Displacements

1. In the **Results** toolbar, click **Evaluation Group**.
2. In the **Settings** window for **Evaluation Group**, type `Point Displacements` in the **Label** text field.
3. Locate the **Data** section. From the **Optimization solution** list, choose **Last**.

Point Evaluation 1

1. Right-click **Point Displacements** and choose **Point Evaluation**.
2. In the **Settings** window for **Point Evaluation**, locate the **Selection** section.
3. From the **Selection** list, choose **Lower left point**.
4. Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`r`	`m`	`r-coordinate`
`z`	`m`	`z-coordinate`

5. In the **Point Displacements** toolbar, click **Evaluate**.

Geometry Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **AC/DC** > **Electromagnetic Fields** > **Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies** > **Frequency Domain**.

6 Click **Done**.

GEOMETRY 1

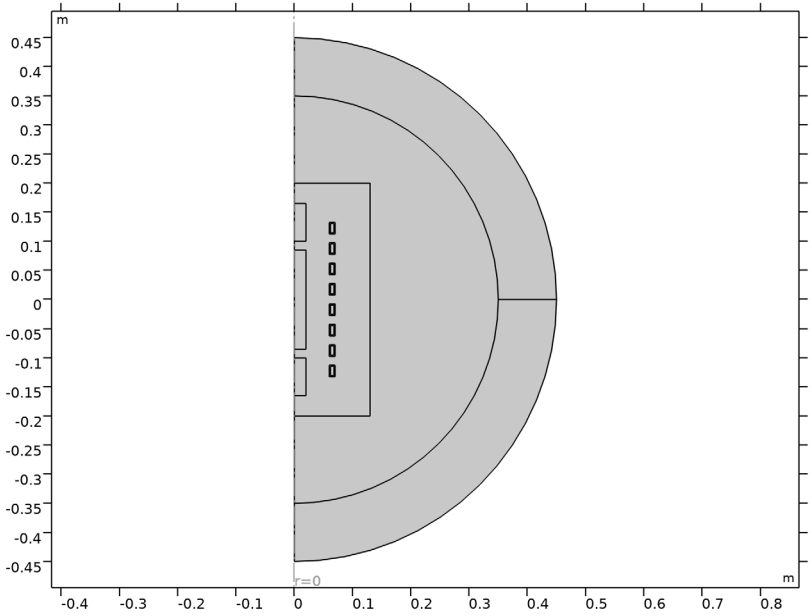
1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Geometry 1** node.

2 Right-click **Geometry 1** and choose **Insert Sequence**.

3 Browse to the model's Application Libraries folder and double-click the file `coil_shape_optimization_geom_sequence.mph`.

4 In the **Geometry** toolbar, click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



The geometry should now look like that in Figure 1.

GLOBAL DEFINITIONS

Geometry Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Geometry Parameters` in the **Label** text field.

Parameters 2

1 In the **Home** toolbar, click **Parameters** and choose **Add** > **Parameters**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	1[kHz]	1000 Hz	Frequency
lastTurns	0.5	0.5	Outer loop current factor
dmax	3[cm]	0.03 m	Maximum coil translation

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Click the **Add to Component** button in the window toolbar.

5 In the tree, select **Built-in** > **Copper**.

6 Click the **Add to Component** button in the window toolbar.

7 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Copper (mat2)

1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

2 From the **Selection** list, choose **Objects to Mirror**.

MAGNETIC FIELDS (MF)

Domain Coil 1

1 In the **Physics** toolbar, click **Domains** and choose **Domain Coil**.

2 In the **Settings** window for **Domain Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Inner Coils**.

4 Locate the **Coil** section. Select the **Coil group** checkbox.

5 In the I_{coil} text field, type `1[kA]`.

Domain Coil 2

1 Right-click **Domain Coil 1** and choose **Duplicate**.

2 In the **Settings** window for **Domain Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils 2**.

4 Locate the **Coil** section. In the I_{coil} text field, type `1[kA]*lastTurns`.

In a 2D axisymmetric model, specifying a coil with a fraction of the total current is a way to represent a coil with partial turns. Similarly, specifying a current with the opposite sign represents a coil that is wound in the opposite direction.

Domain Coil 3

1 Right-click **Domain Coil 2** and choose **Duplicate**.

2 In the **Settings** window for **Domain Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils**.

4 Locate the **Coil** section. In the I_{coil} text field, type `-1[kA]*lastTurns`.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click  **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Infinite Domains**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Infinite Domain Boundaries**.

Free Triangular 1

In the **Mesh** toolbar, click  **Free Triangular**.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Deforming Domain**.

5 Locate the **Element Size** section. From the **Predefined** list, choose **Extremely fine**.

Size 2

1 In the **Model Builder** window, right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Whole Axis**.

5 Locate the **Element Size** section. Click the **Custom** button.

6 Locate the **Element Size Parameters** section.

7 Select the **Maximum element size** checkbox. In the associated text field, type `0.004`.

8 Click  **Build All**.

COMPONENT 1 (COMP1)

Free Shape Domain 1

1 In the **Physics** toolbar, click **Optimization** and choose **Shape Optimization**.

2 In the **Settings** window for **Free Shape Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Deforming Domain**.

Transformation 1

1 In the **Shape Optimization** toolbar, click **Transformation**.

2 In the **Settings** window for **Transformation**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Objects to Mirror**.

4 Locate the **Translation** section. In the table, enter the following settings:

	Lock	Lower bound (m)	Upper bound (m)
R		`-dmax`	`dmax`
Z		`-coilSpace/4`	`coilSpace/4`

5 Locate the **Scaling** section. From the **Scaling type** list, choose **No scaling**.

DEFINITIONS

Standard Deviation 1 (std1)

1 In the **Definitions** toolbar, click **Physics Utilities** and choose **Standard Deviation**.

2 In the **Settings** window for **Standard Deviation**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Rectangle 4**.

4 Click **Replace Expression** in the upper-right corner of the **Standard Deviation** section. From the menu, choose **Component 1 (comp1)** > **Magnetic Fields** > **Magnetic** > **comp1.mf.normB - Magnetic flux density norm - T**.

5 Locate the **Quadrature Settings** section. Select the **Compute integral in revolved geometry** checkbox.

Average outer magnetic field

1 In the **Definitions** toolbar, click **Probes** and choose **Domain Probe**.

2 In the **Settings** window for **Domain Probe**, type `Average outer magnetic field` in the **Label** text field.

3 In the **Variable name** text field, type `Bouter`.

4 Locate the **Source Selection** section. From the **Selection** list, choose **Outer Objective Domain**.

5 Locate the **Expression** section. In the **Expression** text field, type `mf.normB`.

Infinite Element Domain 1 (ie1)

1 In the **Definitions** toolbar, click **Infinite Element Domain**.

2 In the **Settings** window for **Infinite Element Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Infinite Domains**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

General Optimization

1 In the **Study** toolbar, click **Optimization** and choose **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **GCMMA**.

4 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **comp1.std1 - Standard deviation**.

5 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **comp1.Bouter - Average outer magnetic field - T**.

6 Locate the **Control Variables and Parameters** section. Click **Add**.

Initialize the study to generate a plot for use while solving.

7 In the table, enter the following settings:

Parameter	Initial value	Scale	Lower bound	Upper bound	Unit
lastTurns (Outer loop current factor)	0.5	1	0	1	

8 Click to expand the **Solver Settings** section. Find the **Objective settings** subsection. From the **Objective scaling** list, choose **Initial solution based**.

9 Click to expand the **Output** section. From the **Keep solutions** list, choose **First and last**.

10 From the **Probes** list, choose **None**.

11 In the **Model Builder** window, click **Study 1**.

12 In the **Settings** window for **Study**, type `Shape Optimization` in the **Label** text field.

13 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

Shape Optimization

The plot shows the displacement. Add an **Annotation** feature to also show the value of the `lastTurns` parameter.

Annotation 1

1 In the **Model Builder** window, right-click **Shape Optimization** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `eval(lastTurns)`.

4 Locate the **Coloring and Style** section. Clear the **Show point** checkbox.

5 From the **Background color** list, choose **Gray**.

SHAPE OPTIMIZATION

General Optimization

1 In the **Model Builder** window, under **Shape Optimization** click **General Optimization**.

2 In the **Settings** window for **General Optimization**, locate the **Output** section.

3 Select the **Plot** checkbox.

4 In the table, enter the following settings:

Plot group	Plot window
Shape Optimization	Graphics

Enable move limits to reduce the risk of inverted elements.

Solver Configurations

In the **Model Builder** window, expand the **Shape Optimization** > **Solver Configurations** node.

Solution 1 (sol1)

1 In the **Model Builder** window, expand the **Shape Optimization** > **Solver Configurations** > **Solution 1 (sol1)** node, then click **Optimization Solver 1**.

2 In the **Settings** window for **Optimization Solver**, locate the **Optimization Solver** section.

3 Select the **Move limits** checkbox.

4 Select the **Maximum number of outer iterations** checkbox. In the associated text field, type `50`.

5 Click **Run**.

RESULTS

Magnetic Flux Density, Revolved Geometry (mf)

1 In the **Magnetic Flux Density, Revolved Geometry (mf)** toolbar, click **Plot**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

Field on Axis

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Field on Axis` in the **Label** text field.

3 Locate the **Data** section. From the **Optimization solution** list, choose **First**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 1

1 Right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Whole Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Legends** section. Select the **Show legends** checkbox.

7 From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

Legends
`Initial Design`

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Optimized Design

Line Graph 3

1 In the **Model Builder** window, right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Inner Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

7 From the **Width** list, choose **3**.

Line Graph 4

1 Right-click **Line Graph 3** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

Line Graph 3, Line Graph 4

1 In the **Model Builder** window, under **Results** > **Field on Axis**, Ctrl-click to select **Line Graph 3** and **Line Graph 4**.

2 Right-click and choose **Duplicate**.

Line Graph 5

1 In the **Settings** window for **Line Graph**, locate the **Selection** section.

2 From the **Selection** list, choose **Outer Axis**.

Line Graph 6

1 In the **Model Builder** window, click **Line Graph 6**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Outer Axis**.

4 In the **Field on Axis** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compute the value of the objective functions before and after optimization.

Evaluation Group 1

In the **Results** toolbar, click **Evaluation Group**.

Global Evaluation 1

1 Right-click **Evaluation Group 1** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **std1 - Standard deviation - T**.

3 Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **Bouter - Average outer magnetic field - T**.

4 In the **Evaluation Group 1** toolbar, click **Evaluate**.

Compute the value of the corners, so that one can construct an interpolation function for 3D verification.

Point Displacements

1 In the **Results** toolbar, click **Evaluation Group**.

2 In the **Settings** window for **Evaluation Group**, type `Point Displacements` in the **Label** text field.

3 Locate the **Data** section. From the **Optimization solution** list, choose **Last**.

Point Evaluation 1

1 Right-click **Point Displacements** and choose **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Selection** section.

3 From the **Selection** list, choose **Lower left point**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`r`	`m`	`r-coordinate`
`z`	`m`	`z-coordinate`

5 In the **Point Displacements** toolbar, click **Evaluate**.