



Model created in COMSOL Multiphysics 6.4

Implementing a Point Source

Introduction

Consider Poisson's equation on the unit circle with a point source at the origin. Its formal expression is:

$$\begin{cases} -\nabla \cdot (\nabla u) = \delta & \Omega \\ u = 0 & \partial\Omega \end{cases}$$


where δ is the Dirac δ distribution located at the origin. The exact solution to this boundary value problem is $-(1/2\pi)\log(r)$, which has a singularity at the origin. You can model the point source by adding a **Point Source** node to your COMSOL Multiphysics model.

Application Library path: COMSOL_Multiphysics/Equation_Based/point_source




Modeling Instructions

From the **File** menu, choose **New**.

NEW


In the **New** window, click  **Model Wizard**.

MODEL WIZARD


- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Mathematics** > **Classical PDEs** > **Laplace's Equation (lpeq)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies** > **Stationary**.
- 6 Click  **Done**.

GEOMETRY

Circle 1 (c1)

In the **Geometry** toolbar, click  **Circle**.


Point 1 (pt1)

1 In the **Geometry** toolbar, click  **Point**.


- 2 In the **Settings** window for **Point**, click  **Build All Objects**.

LAPLACE'S EQUATION (LPEQ)

Dirichlet Boundary Condition 1


- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Dirichlet Boundary Condition**.
- 2 In the **Settings** window for **Dirichlet Boundary Condition**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.

Point Source 1

- 1 In the **Physics** toolbar, click  **Points** and choose **Point Source**.
- 2 Select Point 3 only.
- 3 In the **Settings** window for **Point Source**, locate the **Source Term** section.
- 4 In the f text field, type 1.

STUDY 1

Step 1: Stationary

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, click to expand the **Adaptation and Error Estimates** section.
- 3 From the **Adaptation and error estimates** list, choose **Adaptation and error estimates**.
- 4 In the **Study** toolbar, click  **Compute**.

RESULTS

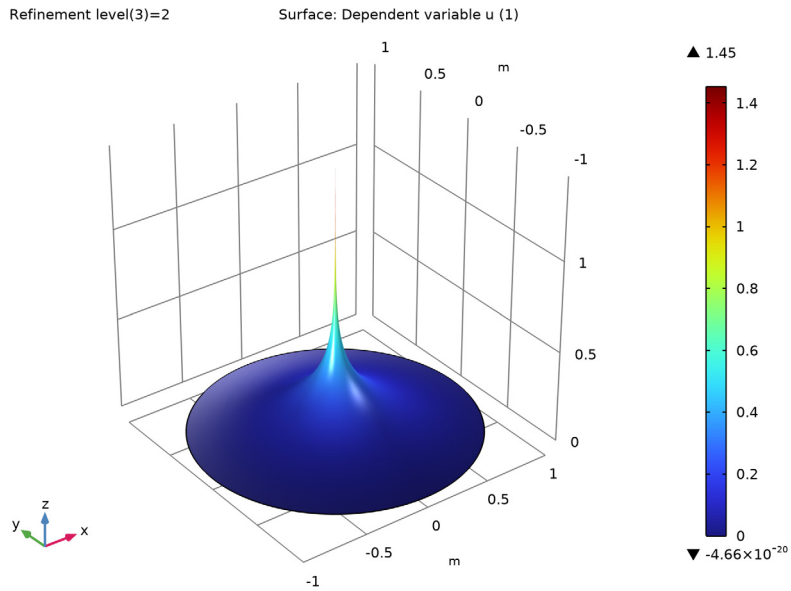
- 1 In the **Settings** window for **2D Plot Group**, locate the **Color Legend** section.
- 2 Select the **Show maximum and minimum values** checkbox.

Height Expression 1


- 1 In the **Model Builder** window, expand the **Laplace's Equation** node.

2 Right-click **Surface 1** and choose **Height Expression**.

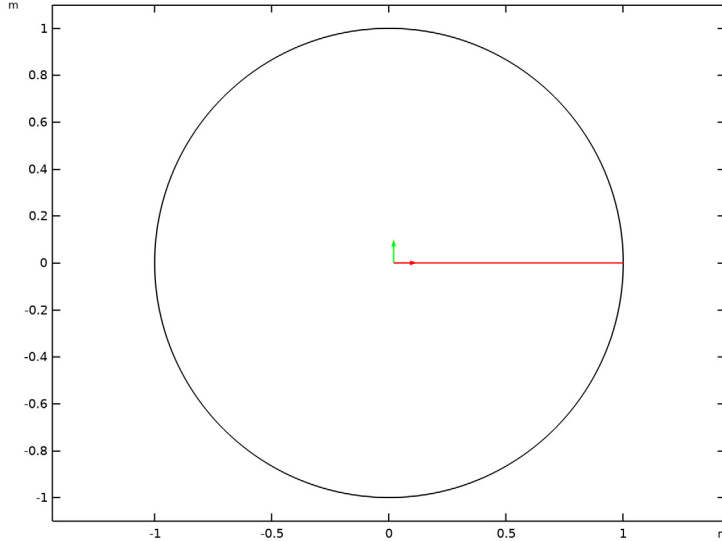
The height plot appears directly.




Cut Line 2D 1

- 1 In the **Results** toolbar, click  **Cut Line 2D**.
- 2 In the **Settings** window for **Cut Line 2D**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 1/Adaptive Mesh Refinement Solutions 1 (sol2)**.
- 4 Locate the **Line Data** section. In row **Point 1**, set **X** to 0.02.

5 Click  **Plot**.




ID Plot Group 2

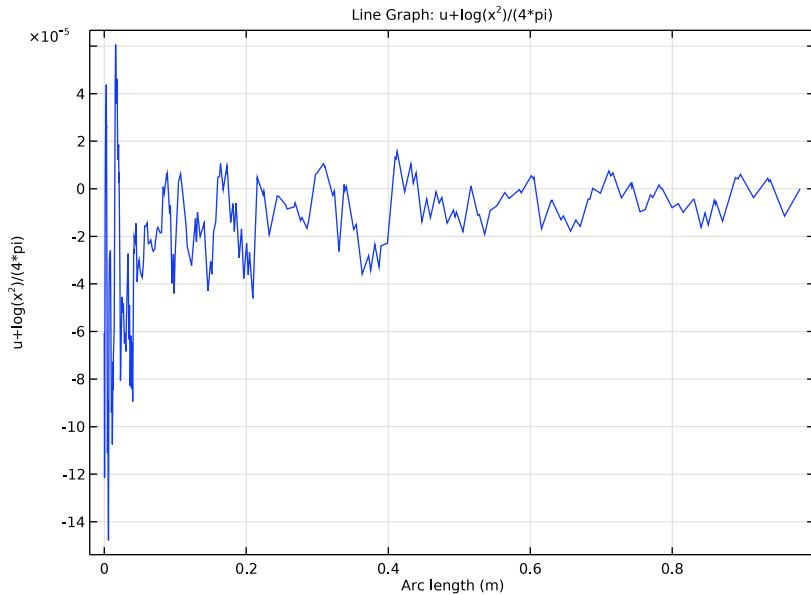
- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Cut Line 2D 1**.
- 4 From the **Parameter selection (Refinement level)** list, choose **Last**.

Line Graph 1

- 1 Right-click **ID Plot Group 2** and choose **Line Graph**.
- 2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 3 In the **Expression** text field, type $u + \log(x^2) / (4 \cdot \pi)$.

4 In the **ID Plot Group 2** toolbar, click  **Plot**.

The resulting plot shows the error in the solution.



Surface Integration 1

1 In the **Results** toolbar, click 8.85×10^{-12} **More Derived Values** and choose **Integration > Surface Integration**.

2 In the **Settings** window for **Surface Integration**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Adaptive Mesh Refinement Solutions 1 (sol2)**.

4 From the **Parameter selection (Refinement level)** list, choose **Last**.

5 Select Domain 1 only.

6 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
$\text{abs}(u + \log(\sqrt{x^2 + y^2})) / (2 * \pi)$		

7 Click  **Evaluate**.

TABLE 1

1 Go to the **Table 1** window.

The result of this integration shows that the error is small.

