



Model created in COMSOL Multiphysics 6.4

# Piezoelectric Energy Harvester

## *Introduction*

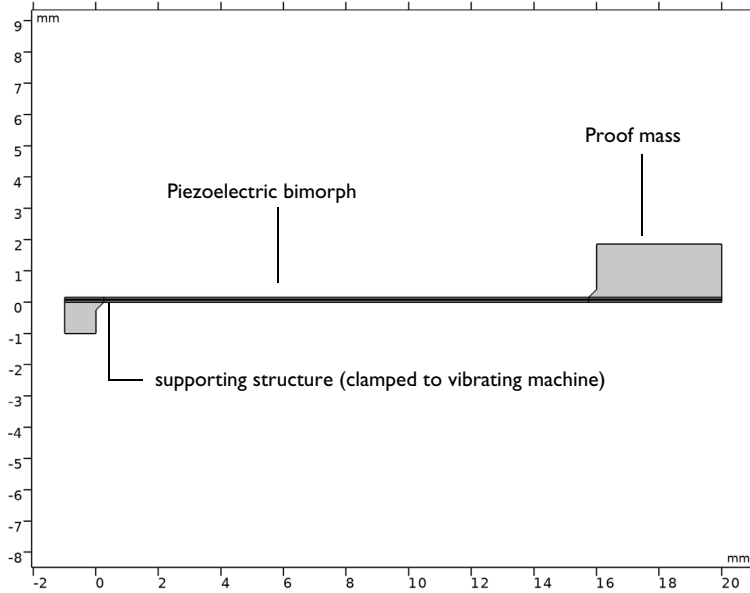
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The development of extremely low-power electronics and wireless systems has led to a strong interest in the field of energy harvesting — the development of miniature generators. Typically, these devices are used to power sensors and wireless communication systems, enabling standalone *wireless sensors* that are cheap to deploy. Wireless sensors frequently make measurements intermittently over an extended period, reporting via a wireless link to other sensors and ultimately to a base station that records readings from all the deployed sensors (creating a wireless sensor network). This model analyzes a simple “seismic” energy harvester, that is a device designed to generate electric energy from local variations in acceleration that occur, for example, when a wireless sensor is mounted on a vibrating piece of machinery. The energy harvester analyzed in this model consists of a piezoelectric bimorph clamped at one end to the vibrating machinery with a proof mass mounted at its other end. It is loosely based on the mechanical system described in detail in [Ref. 1](#).

## *Model Definition*

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[Figure 1](#) shows the device geometry. The power harvester consists of a piezoelectric bimorph clamped at one end with a proof mass mounted on the other end. The bimorph has two output electrodes embedded within it and two ground electrodes on the exterior surfaces of the cantilever beam. This configuration ensures that same voltage is induced on the output electrodes, even though the stress above and below the neutral layer is of opposite sign. Since the clamp is mounted to a piece of vibrating machinery the device is analyzed in a vibrating reference frame (modeled in COMSOL by the application of a sinusoidal body load).



*Figure 1: 2D model geometry, showing the major components of the energy harvester, including the piezoelectric bimorph, proof mass, and supporting structure.*

The model performs three analyses of the mechanical part of the energy harvester system. First, the power output is analyzed as a function of vibration frequency with a fixed electrical load. Then, the power output as a function of electrical load is explored. Finally, the DC voltage output is shown to be a linear function of the acceleration.

### *Results and Discussion*

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Figure 2 shows the input mechanical power and the power harvested (in mW) as well as the peak voltage induced across the piezoelectric bimorph (in V) as a function of frequency when the energy harvester is excited by a sinusoidal acceleration. The electrical load is 12 k $\Omega$ . The response of the system shows a peak at 71 Hz, close to the computed resonant frequency of the cantilever at 72 Hz (from a separate eigenfrequency analysis of this device).

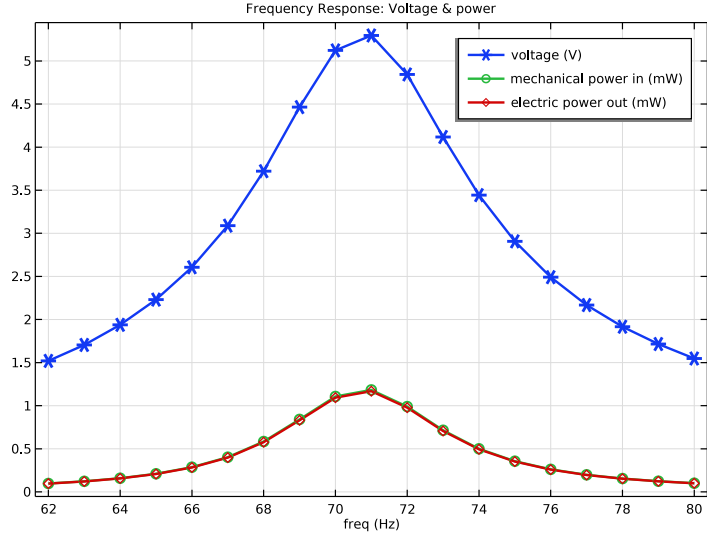


Figure 2: Energy harvester input mechanical power and the power harvested (in mW) as well as the peak voltage induced across the piezoelectric bimorph (in V) vs. excitation frequency. The load impedance is 12 kΩ and the acceleration magnitude is 1 g.

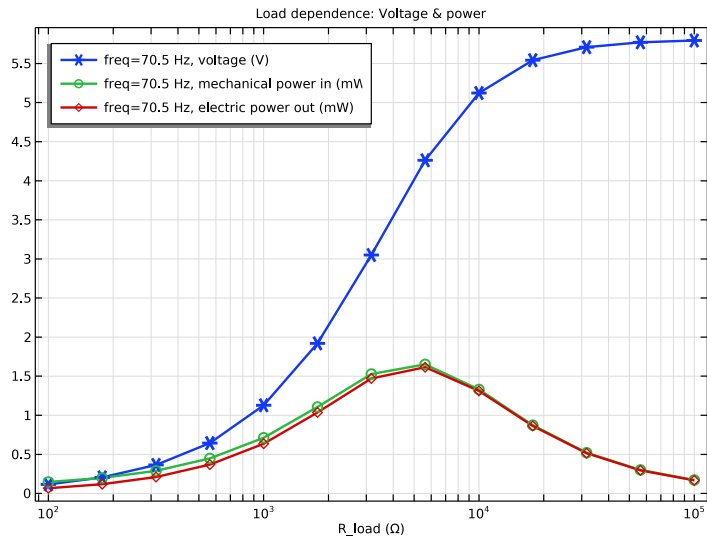


Figure 3: Power harvested from the device as a function of the electrical load resistance at an acceleration of 1 g oscillating at 70.5 Hz.

Figure 3 shows the harvested power from the device as a function of the electrical load resistance at an acceleration of 1 g oscillating at 70.5 Hz. The peak in energy harvested corresponds to an electrical load of 6 k $\Omega$ .

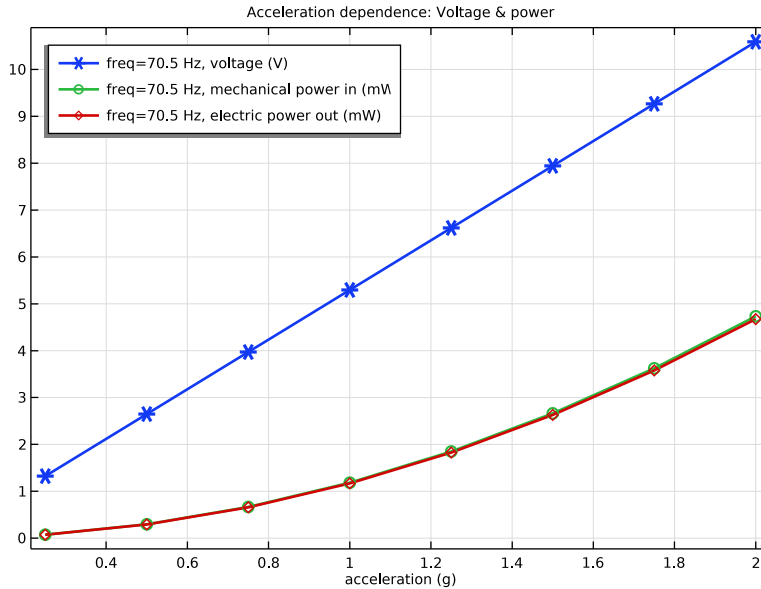


Figure 4: DC voltage and mechanical/electric power output versus the magnitude of the mechanical acceleration at a fixed frequency of 70.5 Hz with a load impedance of 12 k $\Omega$ .

Figure 4 shows the DC voltage and mechanical/electric power output versus the magnitude of the mechanical acceleration at a fixed frequency of 70.5 Hz with a load impedance of 12 k $\Omega$ . The voltage increases linearly with the load, whilst the harvested power increases quadratically, as expected from equation 4 in Ref. 1.

Note that these results are in good qualitative agreement with those presented in Ref. 1. Completely quantitative agreement would not be expected from a two dimensional model.

## Reference

1. E. Lefeuvre, D. Audiger, C. Richard, and D. Guyomar, “Buck-Boost Converter for Sensorless Power Optimization of Piezoelectric Energy Harvester,” *IEEE Transactions on Power Electronics*, vol. 22, no. 5, pp. 2018–2025, 2007.

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**Application Library path:** MEMS\_Module/Piezoelectric\_Devices/  
piezoelectric\_energy\_harvester


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### *Modeling Instructions*




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From the **File** menu, choose **New**.

#### **NEW**

In the **New** window, click  **Model Wizard**.

#### **MODEL WIZARD**

- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Structural Mechanics > Electromagnetics–Structure Interaction > Piezoelectricity > Piezoelectricity, Solid**.
- 3 Click **Add**.
- 4 In the **Select Physics** tree, select **AC/DC > Electrical Circuit (cir)**.
- 5 Click **Add**.
- 6 Click  **Study**.
- 7 In the **Select Study** tree, select **General Studies > Frequency Domain**.
- 8 Click  **Done**.

#### **GLOBAL DEFINITIONS**

##### *Parameters I*


- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

<b>Name</b>	<b>Expression</b>	<b>Value</b>	<b>Description</b>
acc	1	1	Acceleration (g)
R_load	12[kohm]	12000 Ω	Load resistance
w_plate	14[mm]	0.014 m	Out of plane dimension


## GEOMETRY I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **mm**.

### Rectangle 1 (r1)


- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Position** section.
- 3 In the **x** text field, type -1.
- 4 In the **y** text field, type -1.

### Rectangle 2 (r2)


- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type 21.
- 4 In the **Height** text field, type 0.16.
- 5 Locate the **Position** section. In the **x** text field, type -1.
- 6 Click to expand the **Layers** section. In the table, enter the following settings:


Layer name	Thickness (mm)
Layer 1	0.06
Layer 2	0.04

### Rectangle 3 (r3)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type 4.
- 4 In the **Height** text field, type 1.7.
- 5 Locate the **Position** section. In the **x** text field, type 16.
- 6 In the **y** text field, type 0.16.


### Union 1 (uni1)

- 1 In the **Geometry** toolbar, click  **Booleans and Partitions** and choose **Union**.
- 2 Select the objects **r1** and **r2** only.
- 3 In the **Model Builder** window, click **Union 1 (uni1)**.
- 4 In the **Settings** window for **Union**, locate the **Union** section.

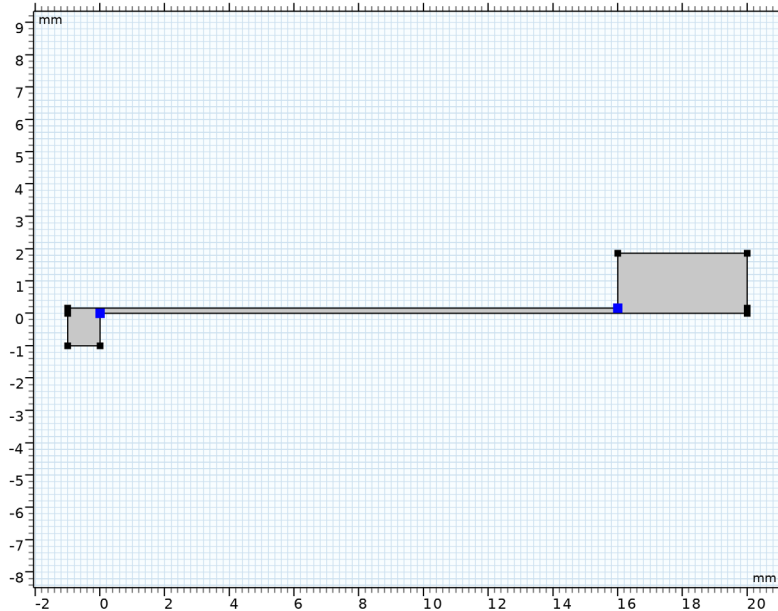
- 5 Click the  **Zoom to Selection** button for **Input objects**.
- 6 Click in the **Graphics** window and then press Ctrl+A to select all objects.
- 7 Clear the **Keep interior boundaries** checkbox.

Add chamfers to the model to avoid stress singularities at the reentrant corners.

#### *Chamfer 1 (chal)*

- 1 In the **Geometry** toolbar, click  **Chamfer**.
- 2 On the object **unil**, select Points 7 and 8 only.

It might be easier to select the correct points by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)



- 3 In the **Settings** window for **Chamfer**, locate the **Distance** section.
- 4 In the **Distance from vertex** text field, type 0.25.

#### *Rectangle 4 (r4)*



In the **Model Builder** window, under **Component 1 (comp1)** > **Geometry 1** right-click **Rectangle 2 (r2)** and choose **Duplicate**.

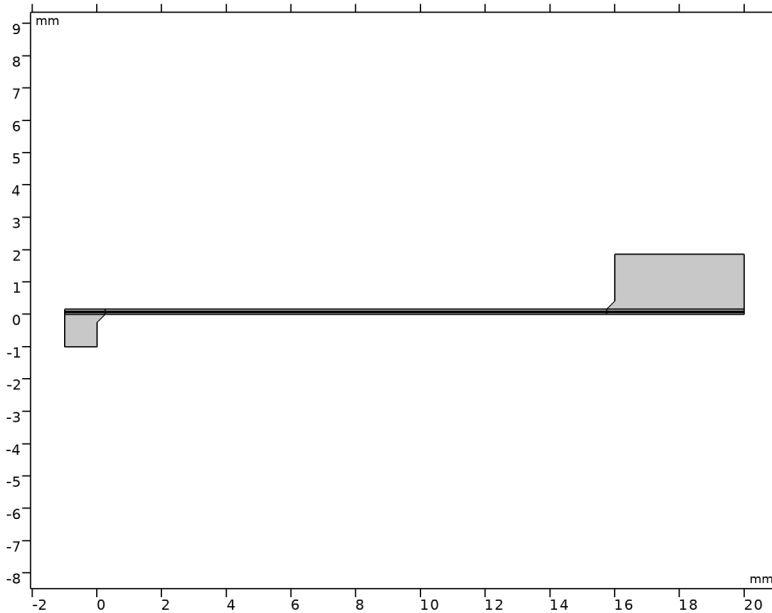
#### *Line Segment 1 (ls1)*

- 1 In the **Geometry** toolbar, click  **More Primitives** and choose **Line Segment**.


- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 Locate the **Starting Point** section. In the **x** text field, type 0.25.
- 6 Locate the **Endpoint** section. In the **x** text field, type 0.25 and **y** to 0.16.


*Line Segment 2 (ls2)*

- 1 In the **Geometry** toolbar, click  **More Primitives** and choose **Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 Locate the **Starting Point** section. In the **x** text field, type 15.75 and **y** to 0.16.
- 6 Locate the **Endpoint** section. In the **x** text field, type 15.75.
- 7 In the **Geometry** toolbar, click  **Build All**.



**ADD MATERIAL**

- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.

- 3 In the tree, select **Piezoelectric > Lead Zirconate Titanate (PZT-5A)**.
- 4 Click the **Add to Component** button in the window toolbar.
- 5 In the tree, select **Built-in > Structural steel**.
- 6 Click the **Add to Component** button in the window toolbar.
- 7 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.

## MATERIALS

### *Structural steel (mat2)*


Select Domains 1, 3, 6, 9, and 11 only.

## SOLID MECHANICS (SOLID)

### *Linear Elastic Material 1*

In the **Model Builder** window, under **Component 1 (comp1) > Solid Mechanics (solid)** click **Linear Elastic Material 1**.


### *Damping 1*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Damping**.
- 2 In the **Settings** window for **Damping**, locate the **Damping Settings** section.
- 3 From the **Damping type** list, choose **Isotropic loss factor**.
- 4 From the  $\eta_s$  list, choose **User defined**. In the associated text field, type 0.001.

### *Piezoelectric Material 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1) > Solid Mechanics (solid)** click **Piezoelectric Material 1**.
- 2 In the **Settings** window for **Piezoelectric Material**, locate the **Domain Selection** section.
- 3 From the **Selection** list, choose **Manual**.
- 4 Select Domains 2, 4, 5, 7, 8, and 10 only.

### *Mechanical Damping 1*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Mechanical Damping**.
- 2 In the **Settings** window for **Mechanical Damping**, locate the **Damping Settings** section.
- 3 From the **Damping type** list, choose **Isotropic loss factor**.
- 4 From the  $\eta_s$  list, choose **User defined**. In the associated text field, type 0.001.


### *Body Load 1*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Body Load**.

- 2 In the **Settings** window for **Body Load**, locate the **Domain Selection** section.
- 3 From the **Selection** list, choose **All domains**.
- 4 Locate the **Force** section. Specify the  $\mathbf{f}_V$  vector as

0	x
$-\text{solid.rho} * \text{g\_const} * \text{acc}$	y


*Fixed Constraint 1*

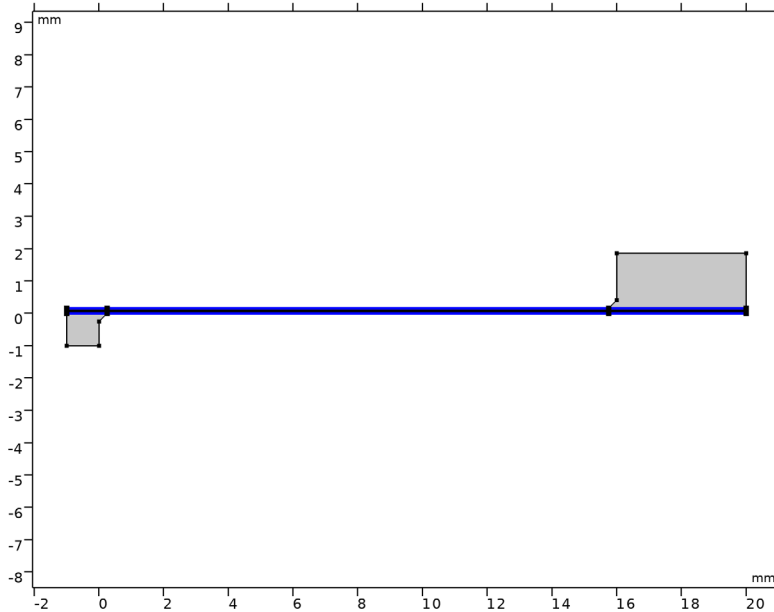
- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Fixed Constraint**.
- 2 Select Boundary 2 only.

**ELECTROSTATICS (ES)**


- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electrostatics (es)**.
- 2 Select Domains 2, 4, 5, 7, 8, and 10 only.
- 3 In the **Settings** window for **Electrostatics**, locate the **Thickness** section.
- 4 In the  $d$  text field, type  $w_{\text{plate}}$ .

*Ground 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Ground**.
- 2 Select Boundaries 4, 9, 13, 18, 20, and 26 only.




### *Boundary Terminal I*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Boundary Terminal**.
- 2 Select Boundaries 6, 8, 15, 17, 22, and 24 only.
- 3 In the **Settings** window for **Boundary Terminal**, locate the **Terminal** section.
- 4 From the **Terminal type** list, choose **Circuit**.

### **ELECTRICAL CIRCUIT (CIR)**

In the **Model Builder** window, under **Component 1 (comp1)** click **Electrical Circuit (cir)**.


### *Resistor I (RI)*

- 1 In the **Electrical Circuit** toolbar, click  **Resistor**.
- 2 In the **Settings** window for **Resistor**, locate the **Node Connections** section.
- 3 In the table, enter the following settings:

Label	Node names
n	0

- 4 Locate the **Device Parameters** section. In the  $R$  text field, type  $R_{load}$ .

### *External I-Terminal I (termI)*

- 1 In the **Electrical Circuit** toolbar, click  **External I-Terminal**.
- 2 In the **Settings** window for **External I-Terminal**, locate the **Node Connections** section.
- 3 In the **Node name** text field, type 1.
- 4 Locate the **External Terminal** section. From the  $V$  list, choose **Terminal voltage (es/termI)**.

The high aspect ratio of the modeled geometry makes this problem numerically challenging. There is only a moderate range of mesh sizes where the result is reliable within a few percent. Outside of this range, with the mesh either too coarse or too fine, the result is not reliable.

### **MESH I**

#### *Free Triangular I*

In the **Mesh** toolbar, click  **Free Triangular**.

#### *Size*

- 1 In the **Model Builder** window, click **Size**.
- 2 In the **Settings** window for **Size**, locate the **Element Size** section.

- 3 Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 0.02.
- 5 In the **Minimum element size** text field, type 0.002.

## FREQUENCY RESPONSE

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Frequency Response in the **Label** text field.

### *Step 1: Frequency Domain*

- 1 In the **Model Builder** window, under **Frequency Response** click **Step 1: Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type range (62, 1, 80).

Disable direct solver error checking which is too stringent in this case.

### *Solution 1 (sol1)*

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node.

## DEFINITIONS

Define a nonlocal integration coupling to calculate mechanical power input later when plotting results.


### *Integration 1 (intop1)*

- 1 In the **Model Builder** window, expand the **Frequency Response > Solver Configurations > Solution 1 (sol1) > Stationary Solver 1** node.
- 2 Right-click **Component 1 (comp1) > Definitions** and choose **Nonlocal Couplings > Integration**.
- 3 In the **Settings** window for **Integration**, locate the **Source Selection** section.
- 4 From the **Selection** list, choose **All domains**.

## FREQUENCY RESPONSE


### *Solution 1 (sol1)*

- 1 In the **Model Builder** window, under **Frequency Response > Solver Configurations > Solution 1 (sol1) > Stationary Solver 1** click **Direct**.
- 2 In the **Settings** window for **Direct**, click to expand the **Error** section.
- 3 From the **Check error estimate** list, choose **No**.

4 In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Frequency Response: Voltage & Power*

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Frequency Response: Voltage & Power in the **Label** text field.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
- 4 In the **Title** text area, type Frequency Response: Voltage & power.

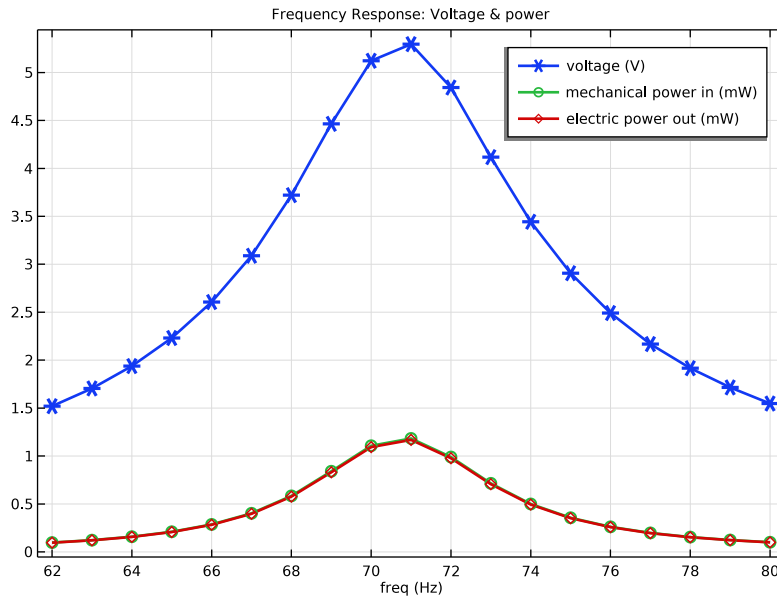
### *Global 1*

- 1 Right-click **Frequency Response: Voltage & Power** and choose **Global**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:



Expression	Unit	Description
$\text{abs}(\text{cir.R1.v})$	V	voltage (V)
$0.5 * \text{intop1}(\text{realdot}(-\text{solid.rho} * \text{g\_const} * \text{acc}, \text{solid.u\_tY})) * \text{w\_plate}$	mW	mechanical power in (mW)
$0.5 * \text{realdot}(\text{cir.R1.i}, \text{cir.R1.v})$	mW	electric power out (mW)

- 4 Click to expand the **Coloring and Style** section. From the **Width** list, choose **2**.
- 5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 In the **Frequency Response: Voltage & Power** toolbar, click  **Plot**.




## ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Frequency Domain**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

## STUDY 2

### Step 1: Frequency Domain

- 1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 2 In the **Frequencies** text field, type 70.5.
- 3 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 4 Click  **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
R_load (Load resistance)	10^range(2, 0.25, 5)	$\Omega$

6 In the **Model Builder** window, click **Study 2**.

7 In the **Settings** window for **Study**, type Load Dependence in the **Label** text field.

8 Locate the **Study Settings** section. Clear the **Generate default plots** checkbox.

#### *Solution 2 (sol2)*

1 In the **Study** toolbar, click  **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node.

3 In the **Model Builder** window, expand the **Load Dependence > Solver Configurations > Solution 2 (sol2) > Stationary Solver 1** node, then click **Direct**.

4 In the **Settings** window for **Direct**, locate the **Error** section.

5 From the **Check error estimate** list, choose **No**.

6 In the **Study** toolbar, click  **Compute**.

## **RESULTS**

### *Load Dependence: Voltage & Power*

1 In the **Model Builder** window, right-click **Frequency Response: Voltage & Power** and choose **Duplicate**.


2 In the **Settings** window for **ID Plot Group**, type Load Dependence: Voltage & Power in the **Label** text field.

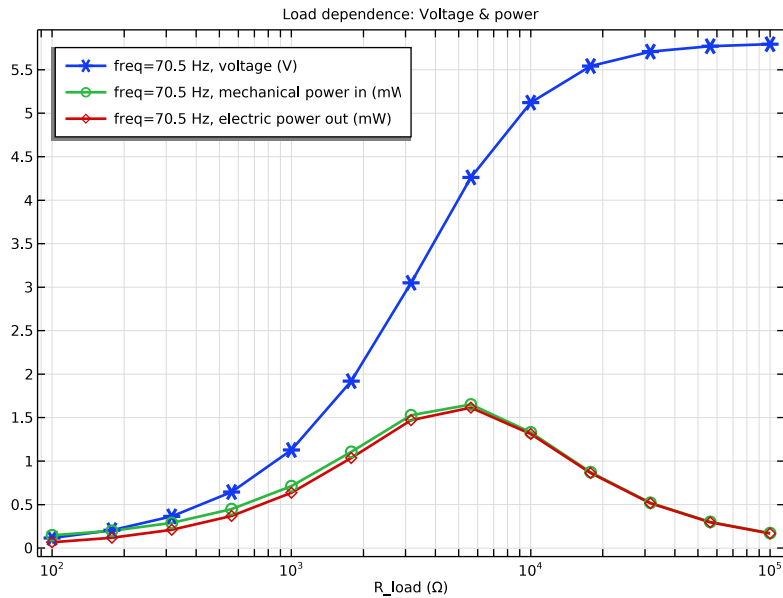
3 Locate the **Data** section. From the **Dataset** list, choose **Load Dependence/Solution 2 (sol2)**.

4 Locate the **Title** section. In the **Title** text area, type Load dependence: Voltage & power.

5 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

6 Click the  **x-Axis Log Scale** button in the **Graphics** toolbar.

7 In the **Load Dependence: Voltage & Power** toolbar, click  **Plot**.



## ROOT

From the **Home** menu, choose **Add Study**.

## ADD STUDY

- 1 Go to the **Add Study** window.
- 2 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Frequency Domain**.
- 3 Click the **Add Study** button in the window toolbar.
- 4 From the **Home** menu, choose **Add Study**.

## STUDY 3

*Step 1: Frequency Domain*



- 1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 2 In the **Frequencies** text field, type 70.5.
- 3 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.
- 4 Click **+ Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
acc (Acceleration (g))	range(0.25,0.25,2)	

- 6 In the **Model Builder** window, expand the **Load Dependence: Voltage & Power** node, then click **Study 3**.
- 7 In the **Settings** window for **Study**, type Acceleration Dependence in the **Label** text field.
- 8 Locate the **Study Settings** section. Clear the **Generate default plots** checkbox.

#### *Solution 3 (sol3)*

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 3 (sol3)** node.
- 3 In the **Model Builder** window, expand the **Acceleration Dependence > Solver Configurations > Solution 3 (sol3) > Stationary Solver I** node, then click **Direct**.
- 4 In the **Settings** window for **Direct**, locate the **Error** section.
- 5 From the **Check error estimate** list, choose **No**.
- 6 In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Acceleration Dependence: Voltage & Power*

- 1 In the **Model Builder** window, right-click **Load Dependence: Voltage & Power** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type Acceleration Dependence: Voltage & Power in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Acceleration Dependence/ Solution 3 (sol3)**.
- 4 Locate the **Plot Settings** section.
- 5 Select the **x-axis label** checkbox. In the associated text field, type acceleration (g).
- 6 Locate the **Axis** section. Clear the **x-axis log scale** checkbox.
- 7 Locate the **Title** section. In the **Title** text area, type Acceleration dependence: Voltage & power.

8 In the **Acceleration Dependence: Voltage & Power** toolbar, click  **Plot**.

