Model created in COMSOL Multiphysics 6.4

Pull-In Voltage for a Biased Resonator — 2D

Introduction

Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in Ref. 1.

This model performs a pull-in analysis of the structure, to predict the point at which the biased system becomes unstable. The analysis begins from the stationary analysis performed in the accompanying model Stationary Analysis of a Biased Resonator — 2D; please review this model first.

Model Definition

The geometry, fabrication, and operation of the device are discussed for the Stationary Analysis of a Biased Resonator — 2D model.

This model computes the pull-in voltage for the resonator by solving an inverse problem. The vertical displacement of the resonator midpoint, `vmid`, relative to the gap distance is computed using a point probe. The inverse problem that COMSOL solves computes the DC voltage that must be applied to the beam in order to move the midpoint to have a given relative vertical displacement, `vrel`. This is achieved by adding a global equation for the DC voltage, `Vdc`, applied to the resonator. The equation `vrel-vmid = 0` is solved to determine the value of `Vdc`. This means that `Vdc` is adjusted until the midpoint of the resonator has a given vertical displacement. Essentially COMSOL is being asked to find the voltage that allows the beam to exist in equilibrium (stable or unstable) at a given displacement. Solving the problem in this manner avoids complications with trying to solve a problem with no solution (which is what happens if the voltage is continuously ramped up eventually exceeding the pull-in voltage). The result of the analysis is a displacement versus voltage plot, with a minimum at the pull-in voltage. Note that for a linear spring, the pull-in displacement corresponds to 1/3 of the gap distance. Although the inclusion of geometric nonlinearities in the solid mechanics solver means that the pull-in displacement changes slightly from this value, it is usually most efficient to search around this point for the pull-in voltage.

Results and Discussion

Figure 1 shows the voltage-displacement curve for the resonator at equilibrium. The pull-in voltage is 63.3 V and correspond to the displacement of around 37% of the gap distance.

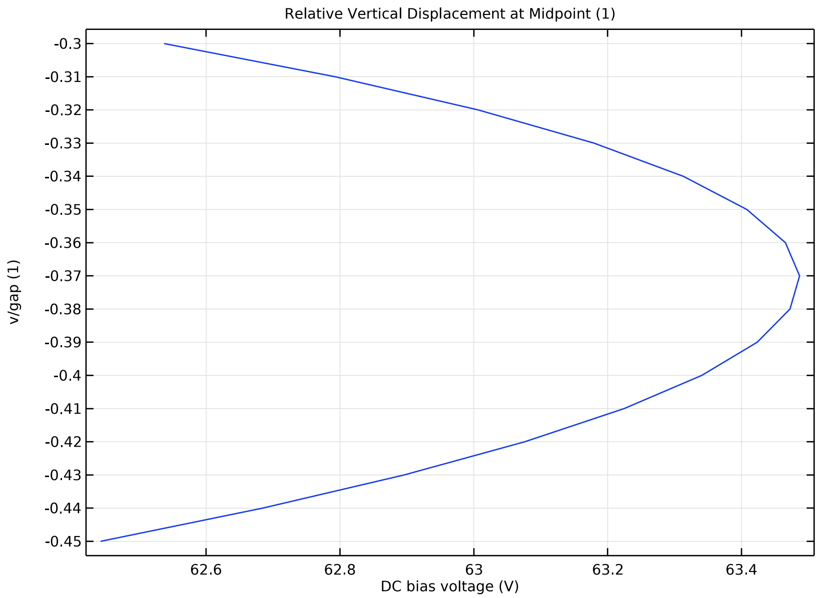

Figure 1: Voltage required to achieve a set displacement versus the target displacement. The pull-in voltage is about 63.3 V.

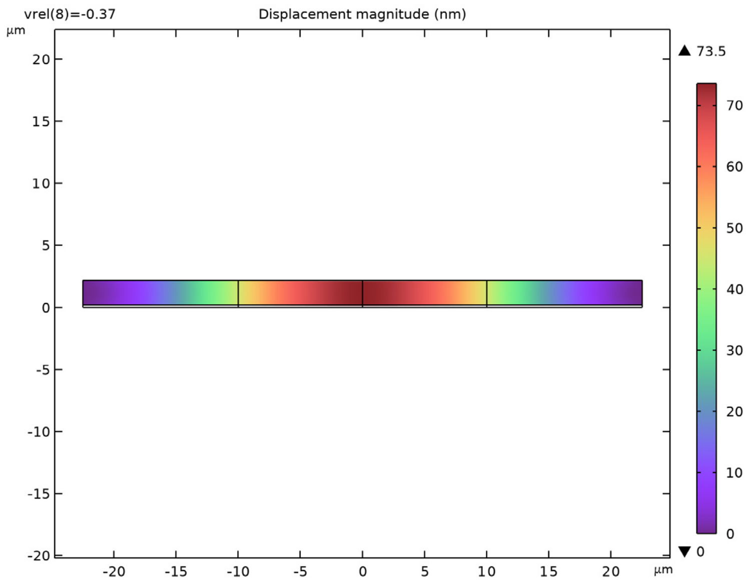

Figure 2: y-displacement of the resonator at pull-in. The displacement at pull-in is 74 nm. For a linear spring the displacement at pull in would be 66 nm.

Figure 2 shows the vertical displacement of the resonator at the pull-in voltage. The maximum displacement at pull-in is about 74 nm. This is comparable to the (approximate) linear spring value of 66 nm.

Notes About the COMSOL Implementation

To compute the voltage required to generate the desired displacement of the beam, use a global equation. A common use of global equations is for computing the value of a dependent variable based on an ordinary differential equation in the dependent variable itself. However, it is also possible to couple a global equation with the other PDEs in the model as a powerful tool to solve certain kinds of inverse problems.

Reference

1. F.D. Bannon III, J.R. Clark, and C.T.-C. Nguyen, "High-Q HF Microelectromechanical Filters," *IEEE Journal of Solid State Circuits,* vol. 35, no. 4, pp. 512–526, 2000.

Application Library path: `MEMS_Module/Actuators/biased_resonator_2d_pull_in`

Modeling Instructions

Start from the existing stationary model.

APPLICATION LIBRARIES

1 From the **File** menu, choose **Application Libraries**.

2 In the **Application Libraries** window, select **MEMS Module** > **Actuators** > **biased_resonator_2d_basic** in the tree.

3 Click **Open**.

Add a parameter and a probe variable to specify the position and displacement of the midpoint of the beam boundary.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
vrel	-0.3	-0.3	Relative vertical displacement

DEFINITIONS

Relative Vertical Displacement at Midpoint

1 In the **Definitions** toolbar, click **Probes** and choose **Point Probe**.

2 In the **Settings** window for **Point Probe**, type `Relative Vertical Displacement at Midpoint` in the **Label** text field.

3 In the **Variable name** text field, type `vmid`.

4 Select Point 8 only.

5 Locate the **Expression** section. In the **Expression** text field, type `v/gap`.

6 Locate the **Integration Settings** section. From the **Method** list, choose **Summation over nodes**.

7 From the **Frame** list, choose **Material**.

8 Click the **Show More Options** button in the **Model Builder** toolbar.

9 In the **Show More Options** dialog, in the tree, select the checkbox for the node **Physics** > **Equation Contributions**.

10 Click **OK**.

ELECTROSTATICS (ES)

Global Equations 1 (ODE1)

1 In the **Physics** toolbar, click **Global** and choose **Global Equations**.

2 In the **Settings** window for **Global Equations**, locate the **Global Equations** section.

3 In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (ut_0) (1/s)	Description
Vdc	vrel-vmid	0	0	DC bias voltage

4 Locate the **Units** section. Click **Select Dependent Variable Quantity**.

5 In the **Physical Quantity** dialog, select **Electromagnetics** > **Electric potential (V)** in the tree.

6 Click **OK**.

This equation will effectively constrain the relative displacement at the midpoint of the beam to a certain value. The DC bias voltage will be a global DOF variable that will play the role of a reaction force associated with this constraint. Thus, the electrostatic load due to the applied `Vdc` voltage will be computed automatically so that the beam midpoint will be displaced to the desired location.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Right-click and choose **Add Study**.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

Use the solution of the previous study as initial values of variables solved for.

STUDY 2

Step 1: Stationary

1 In the **Settings** window for **Stationary**, click to expand the **Values of Dependent Variables** section.

2 Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.

3 From the **Method** list, choose **Solution**.

4 From the **Study** list, choose **Stationary, Stationary**.

5 Click to expand the **Results While Solving** section. From the **Probes** list, choose **None**.

   Set up a parametric sweep over the displacement of the midpoint, `vrel`.

6 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

7 Click + **Add**.

8 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
vrel (Relative vertical displacement)	`range(-0.3,-0.01,-0.45)`	

9 In the **Study** toolbar, click **Show Default Plots**.

The problem is strongly coupled and nonlinear due to the presence of the global equation, so the solver settings need to be adjusted accordingly.

Solver Configurations

In the **Model Builder** window, expand the **Study 2** > **Solver Configurations** node.

Solution 2 (sol2)

1 In the **Model Builder** window, expand the **Study 2** > **Solver Configurations** > **Solution 2 (sol2)** node.

2 Right-click **Stationary Solver 1** and choose **Fully Coupled**.

3 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.

4 From the **Nonlinear method** list, choose **Automatic highly nonlinear (Newton)**.

5 In the **Model Builder** window, click **Study 2**.

6 In the **Settings** window for **Study**, type `Pull In` in the **Label** text field.

7 In the **Study** toolbar, click **Compute**.

RESULTS

Pull-In Plot

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Pull-In Plot` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Pull In/Solution 2 (sol2)**.

4 Locate the **Plot Settings** section.

5 Select the **y-axis label** checkbox. In the associated text field, type `v/gap (1)`.

6 Locate the **Legend** section. Clear the **Show legends** checkbox.

Global 1

1 Right-click **Pull-In Plot** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Definitions** > **vmid - Relative Vertical Displacement at Midpoint - 1**.

3 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

4 Click **Insert Expression (Ctrl+Space)** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Electrostatics** > **Vdc - DC bias voltage - V**.

5 Locate the **x-Axis Data** section. In the **Expression** text field, type `Vdc`.

6 In the **Pull-In Plot** toolbar, click **Plot**.

7 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 1.

Determine the pull-in voltage.

Pull-In Voltage

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, type `Pull-In Voltage` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Pull In/Solution 2 (sol2)**.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)** > **Electrostatics** > **Vdc - DC bias voltage - V**.

5 Locate the **Data Series Operation** section. From the **Transformation** list, choose **Maximum**.

6 Select the **Include vrel** checkbox.

7 Click **Evaluate**.

The pull-in voltage should be around 63.3 V, for which the beam middle point will be moved downward about 37% of the gap.

Pull-In Displacement

1 In the **Model Builder** window, under **Results** click **Displacement (solid)**.

2 In the **Settings** window for **2D Plot Group**, type `Pull-In Displacement` in the **Label** text field.

3 Locate the **Data** section. From the **Parameter value (vrel)** list, choose **-0.37**.

4 Locate the **Plot Settings** section. From the **Frame** list, choose **Material (X, Y, Z)**.

5 Locate the **Color Legend** section. Select the **Show maximum and minimum values** checkbox.

Surface 1

1 In the **Model Builder** window, expand the **Pull-In Displacement** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 From the **Unit** list, choose **nm**.

4 In the **Pull-In Displacement** toolbar, click **Plot**.

Compare the resulting plot with Figure 2.