



Model created in COMSOL Multiphysics 6.4

# Modeling Gyroscopic Effect

## *Introduction*

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Gyroscopes are used for measuring the orientation or maintaining the stability of airplane, spacecraft, and submarine vehicles in general. They are also used as sensors in inertial guidance systems.

This model demonstrates the modeling of a mechanical gyroscope. It analyzes the response of a spinning disc to an external torque coming on the disc due to the rotation of the frame. It is shown that the disc is able to maintain its orientation when spinning with a high speed. This fact can be explained with the principle of conservation of angular momentum.

In the second part of the model, the motion of a spinning top is analyzed. The external torque induced precession and nutation motion of the spinning top is computed.

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**Application Library path:** `Multibody_Dynamics_Module/Tutorials/gyroscope`

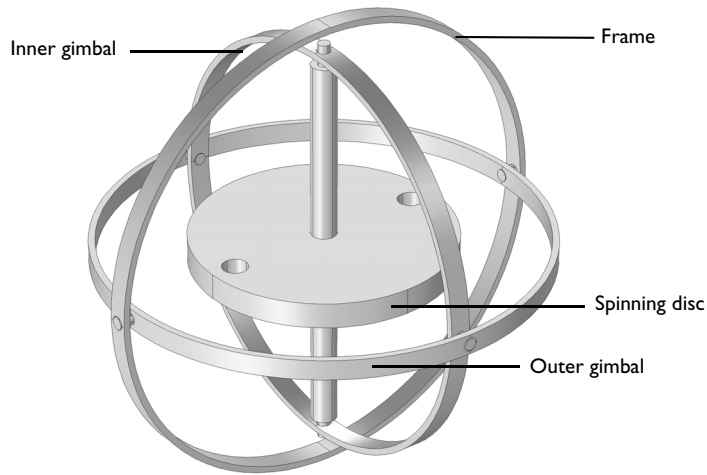
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## *Model Definition: Gyroscope*

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The geometry of a gyroscope, shown in [Figure 1](#), consists of four parts: the frame, the outer gimbal, the inner gimbal, and the spinning disc. All these parts are assumed to be rigid and they are interconnected by hinge joints.

The frame is mounted on a platform which has a rotating motion with harmonically varying magnitude. The magnitude of rotation is 2 rad and the angular frequency is  $4\pi$  Hz. The disc is spinning at 350 rad/s about its own axis. The orientation of the spinning disc is analyzed under the torques acting on it because of the frame motion.

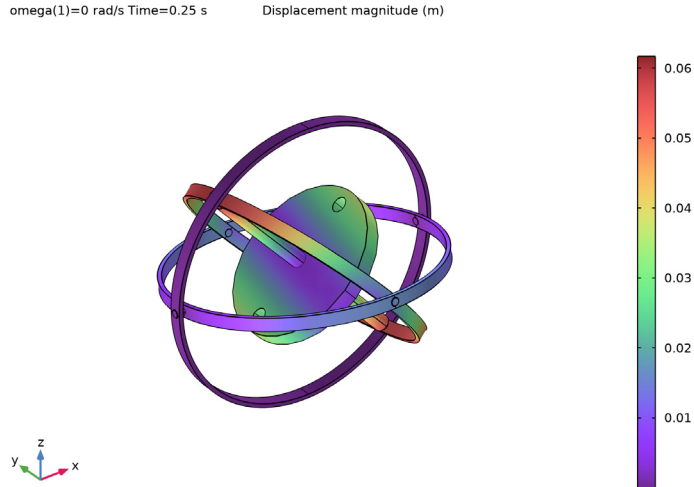


*Figure 1: Model geometry of a gyroscope.*

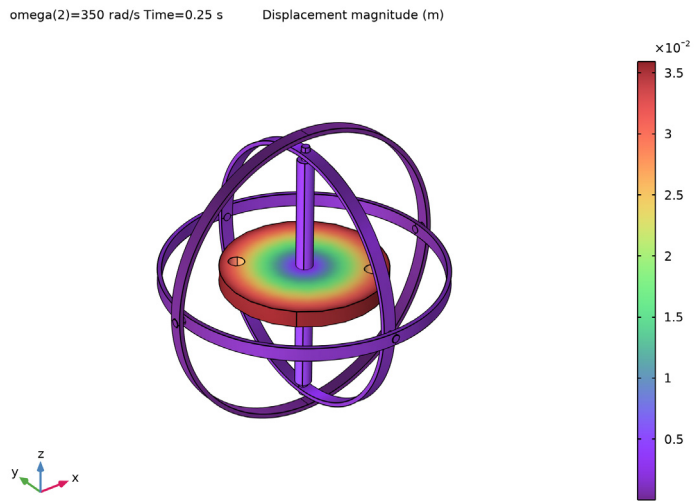
### *Results and Discussion*

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The disc orientation and the position of the gimbals for the specified frame motion, can be seen in [Figure 2](#) and [Figure 3](#). It can be seen that the disc approximately maintains its orientation when it is spinning with high speed. In the case when the disc is not spinning, it does not offer much resistance to the torques acting on it and it fails to maintain its orientation. A comparison of the inclination angle of the disc for both the cases, spinning and not spinning, can be seen in [Figure 4](#).



*Figure 2: Displacement of the gyroscope components at  $t = 0.25$  sec when the disc is not spinning.*



*Figure 3: Displacement of the gyroscope components at  $t = 0.25$  sec when the disc is spinning.*

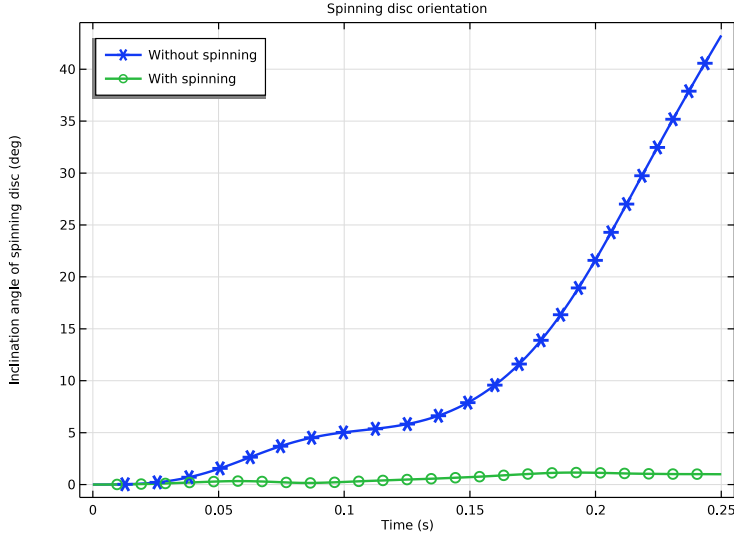


Figure 4: Comparison of the inclination angle of the spinning disc.

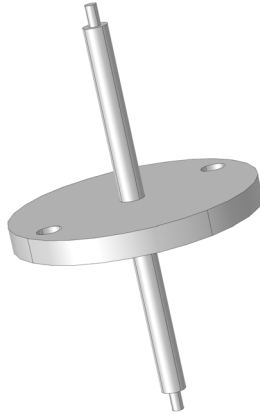
### Notes About the COMSOL Implementation

- In this model, parts are modeled as rigid elements using **Rigid Material** nodes as we are only interested in the kinematics of the mechanism. Parts can be modeled as flexible elements using the **Linear Elastic Material** node if the stresses and deformations in the parts are also of interest.
- Two different parts of the gyroscope are connected together with a pair of hinges. Both the hinges have the same axis and they are placed diagonally opposite to each other. As the parts are rigid and force computation is not of interest, it is sufficient to use only one hinge joint.
- The center of hinge joint can be chosen anywhere on the joint axis. In this model, for the convenience of modeling, the center of joint for all the joints is chosen as the geometric center of the gyroscope.

### Model Definition: Spinning Top (Building instructions given at the end)

The geometry of a spinning top is shown in Figure 5. This geometry is the spinning disc of the gyroscope assembly having an initial orientation of  $20^\circ$  from vertical.

The translational motion of the bottom point of the spinning top is constrained. The spinning top is rotating about its axis with an angular speed of 350 rad/s. The motion of the spinning top is analyzed under gravity load.



*Figure 5: Model Geometry of a spinning top.*

Instead of falling under gravity, the spinning top does have two types of motion:

#### *Precession*

If the axis of a spinning top is inclined to the vertical, the trajectory of the axis generates a vertical circular cone, so that the angle between the spinning top axis and the vertical remains constant during rotation. This kind of motion for a spinning top under an external torque is called forced or torque-induced regular precession.

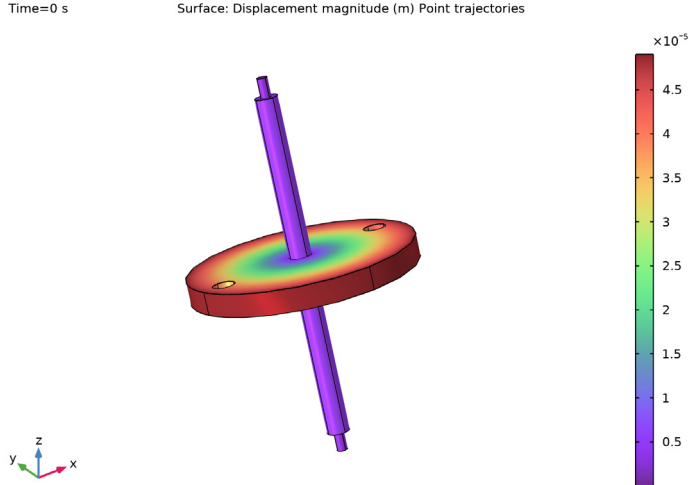
#### *Nutation*

Precession is often accompanied by nutation which can be described as a fast shivering of the precessing axis.

### *Results and Discussion*

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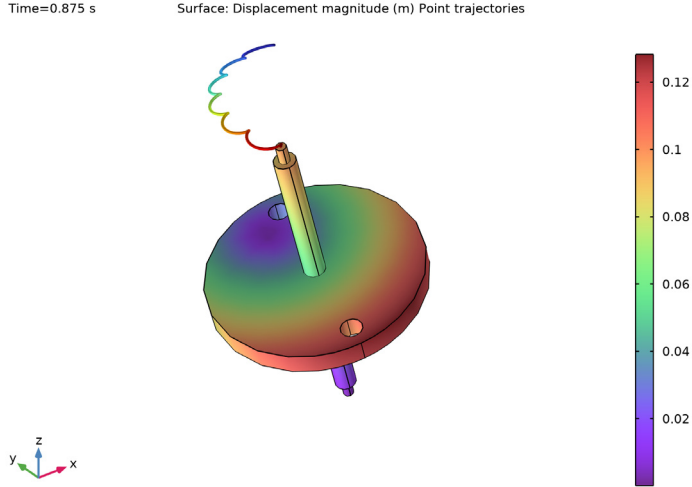
Figure 6 and Figure 7 show the total displacement of the spinning top at the beginning and at  $t = 0.875$  sec respectively. The trajectory of the topmost point of the spinning top is also shown in the latter plot.



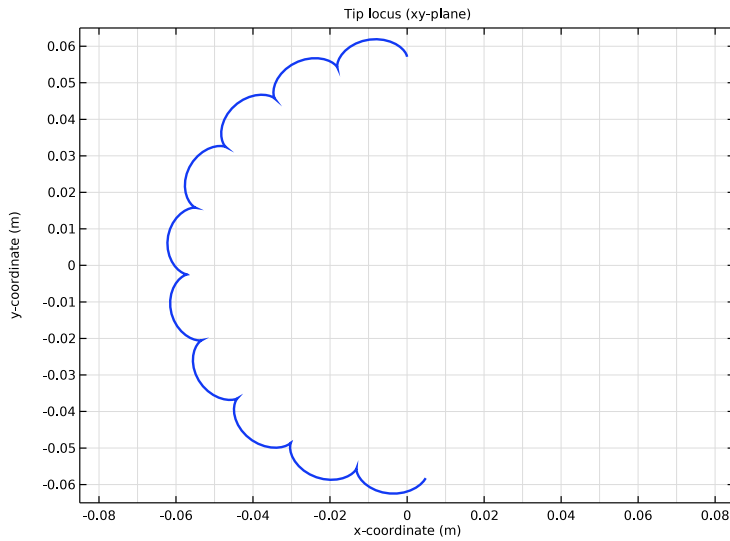
*Figure 6: Total displacement of the spinning top at the beginning.*

Figure 8 shows the locus of the tip of the spinning top in the  $xy$ -plane. The precession and nutation motion exhibited by the spinning top can be seen in this plot.

The precession and nutation velocities of the tip of the spinning top are shown in Figure 9. Both the velocity components are varying harmonically with the same frequency, but they have a phase shift of  $90^\circ$ . It can also be seen that the mean value of the nutation velocity is zero whereas the precession velocity is having a nonzero mean value with the minimum value being zero. That means the spinning top is precessing continuously in one direction, however, it is nearly stopping periodically for a small duration.



*Figure 7: Total displacement of the spinning top at  $t = 0.875$  sec. The trajectory of the topmost point can also be seen.*



*Figure 8: Locus of the tip of the spinning top in the xy-plane.*

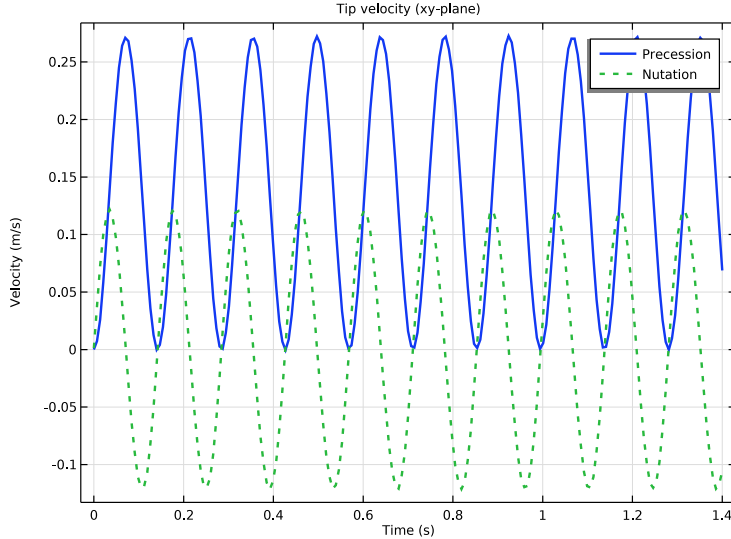


Figure 9: Precession and nutation velocity of the tip of the spinning top.


### Notes About the COMSOL Implementation

The angular velocity of spinning is an order of magnitude higher than the angular velocity of precession. Hence, care should be taken to take sufficiently small time steps to properly resolve the spinning motion in the time scale.


### Modeling Instructions (Gyroscope)

From the **File** menu, choose **New**.

#### NEW

In the **New** window, click  **Model Wizard**.

#### MODEL WIZARD

1 In the **Model Wizard** window, click  **3D**.

2 In the **Select Physics** tree, select **Structural Mechanics > Multibody Dynamics (mbd)**.

3 Click **Add**.

4 Click  **Study**.

5 In the **Select Study** tree, select **General Studies > Time Dependent**.

6 Click  **Done**.

### COMPONENT 1: GYROSCOPE

- 1 In the **Model Builder** window, click **Component 1 (comp1)**.
- 2 In the **Settings** window for **Component**, type Component 1: Gyroscope in the **Label** text field.

### GLOBAL DEFINITIONS

#### *Parameters 1*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:


Name	Expression	Value	Description
omega	350[rad/s]	350 rad/s	Angular velocity
theta0	20[deg]	0.34907 rad	Inclination angle of spinning top

### GEOMETRY 1

#### *Import 1 (imp1)*


- 1 In the **Geometry** toolbar, click  **Import**.
- 2 In the **Settings** window for **Import**, locate the **Source** section.
- 3 Click  **Browse**.
- 4 Browse to the model's Application Libraries folder and double-click the file gyroscope.mphbin.
- 5 Click  **Import**.

#### *Form Union (fin)*



- 1 In the **Model Builder** window, under **Component 1: Gyroscope (comp1) > Geometry 1** click **Form Union (fin)**.
- 2 In the **Settings** window for **Form Union/Assembly**, locate the **Form Union/Assembly** section.
- 3 From the **Action** list, choose **Form an assembly**.
- 4 Clear the **Create pairs** checkbox.
- 5 In the **Geometry** toolbar, click  **Build All**.

## DEFINITIONS

*Waveform 1 (wv1)*

- 1 In the **Definitions** toolbar, click  **More Functions** and choose **Waveform**.
- 2 In the **Settings** window for **Waveform**, locate the **Parameters** section.
- 3 In the  $T$  text field, type 0.5[s].
- 4 In the  $A$  text field, type 2.

## ADD MATERIAL

- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Built-in > Aluminum**.
- 4 Click the **Add to Component** button in the window toolbar.
- 5 In the tree, select **Built-in > Structural steel**.
- 6 Click the **Add to Component** button in the window toolbar.
- 7 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.

## MATERIALS


*Structural steel (mat2)*

Use **Structural steel** for the Spinning Disc to increase its inertia for a higher angular momentum.


- 1 Select Domain 3 only.

## MULTIBODY DYNAMICS (MBD)

*Rigid Material: Frame*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Rigid Material**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Frame in the **Label** text field.
- 3 Select Domain 2 only.

*Prescribed Displacement/Rotation 1*


- 1 In the **Physics** toolbar, click  **Attributes** and choose **Prescribed Displacement/Rotation**.
- 2 In the **Settings** window for **Prescribed Displacement/Rotation**, locate the **Prescribed Displacement at Center of Rotation** section.
- 3 Select the **Prescribed in x direction** checkbox.

- 4 Select the **Prescribed in y direction** checkbox.
- 5 Select the **Prescribed in z direction** checkbox.
- 6 Locate the **Prescribed Rotation** section. From the **By** list, choose **Prescribed rotation**.
- 7 Specify the  $\Omega$  vector as


1	x
1	y
0	z

- 8 In the  $\phi_0$  text field, type  $wv1(t)$ .


*Rigid Material: Outer Gimbal*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Rigid Material**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Outer Gimbal in the **Label** text field.
- 3 Select Domain 1 only.

*Rigid Material: Inner Gimbal*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Rigid Material**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Inner Gimbal in the **Label** text field.
- 3 Select Domain 4 only.

*Rigid Material: Spinning Disc*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Rigid Material**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Spinning Disc in the **Label** text field.
- 3 Select Domain 3 only.
- 4 Locate the **Initial Values** section. From the list, choose **Locally defined**.

*Initial Values I*

- 1 In the **Model Builder** window, click **Initial Values I**.
- 2 In the **Settings** window for **Initial Values**, locate the **Initial Values: Rotational** section.
- 3 Specify the  $\omega$  vector as

0	x
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0	y
omega	z

*Rigid Material: Frame, Rigid Material: Inner Gimbal, Rigid Material: Outer Gimbal, Rigid Material: Spinning Disc*

**1** In the **Model Builder** window, under **Component 1: Gyroscope (comp 1) > Multibody Dynamics (mbd)**, Ctrl-click to select **Rigid Material: Frame, Rigid Material: Outer Gimbal, Rigid Material: Inner Gimbal**, and **Rigid Material: Spinning Disc**.

**2** Right-click and choose **Group**.

*Rigid Materials*

In the **Settings** window for **Group**, type Rigid Materials in the **Label** text field.

*Frame-Outer Gimbal*

**1** In the **Physics** toolbar, click  **Global** and choose **Hinge Joint**.

**2** In the **Settings** window for **Hinge Joint**, type Frame-Outer Gimbal in the **Label** text field.

**3** Locate the **Attachment Selection** section. From the **Source** list, choose **Rigid Material: Frame**.

**4** From the **Destination** list, choose **Rigid Material: Outer Gimbal**.

**5** Locate the **Center of Joint** section. From the list, choose **User defined**.

*Outer Gimbal-Inner Gimbal*

**1** Right-click **Frame-Outer Gimbal** and choose **Duplicate**.

**2** In the **Settings** window for **Hinge Joint**, type Outer Gimbal-Inner Gimbal in the **Label** text field.

**3** Locate the **Attachment Selection** section. From the **Source** list, choose **Rigid Material: Outer Gimbal**.

**4** From the **Destination** list, choose **Rigid Material: Inner Gimbal**.

**5** Locate the **Axis of Joint** section. Specify the  $e_0$  vector as

0	x
1	y
0	z

*Inner Gimbal-Spinning Disc*

**1** Right-click **Outer Gimbal-Inner Gimbal** and choose **Duplicate**.

**2** In the **Settings** window for **Hinge Joint**, type Inner Gimbal-Spinning Disc in the **Label** text field.

- 3 Locate the **Attachment Selection** section. From the **Source** list, choose **Rigid Material: Inner Gimbal**.
- 4 From the **Destination** list, choose **Rigid Material: Spinning Disc**.
- 5 Locate the **Axis of Joint** section. Specify the  $e_0$  vector as

0	x
0	y
1	z


*Frame-Outer Gimbal, Inner Gimbal-Spinning Disc, Outer Gimbal-Inner Gimbal*

- 1 In the **Model Builder** window, under **Component 1: Gyroscope (comp1) > Multibody Dynamics (mbd)**, Ctrl-click to select **Frame-Outer Gimbal**, **Outer Gimbal-Inner Gimbal**, and **Inner Gimbal-Spinning Disc**.
- 2 Right-click and choose **Group**.

*Hinge Joints*

In the **Settings** window for **Group**, type **Hinge Joints** in the **Label** text field.

#### **MESH 1**

- 1 In the **Model Builder** window, under **Component 1: Gyroscope (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
- 3 From the **Element size** list, choose **Fine**.
- 4 In the table, clear the **Use** checkbox for **Geometric Analysis, Detail Size**.
- 5 Click  **Build All**.

#### **DEFINITIONS**

Define a variable for measuring the inclination angle of the spinning disc with the vertical axis. This variable can be written as a function of `mbd.hgj3.e1z`, the *joint axis, z component* variable for the **Inner Gimbal-Spinning Disc** node (listed in the **Variables** table of the node's **Equation View** subnode).

*Variables 1*

- 1 In the **Model Builder** window, under **Component 1: Gyroscope (comp1)** right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
theta	acos(mbd.hgj3.e1z)	rad	Inclination angle of spinning disc


*View 1*

- 1 In the **Model Builder** window, click **View 1**.
- 2 In the **Settings** window for **View**, locate the **View** section.
- 3 Clear the **Show grid** checkbox.

### STUDY 1: GYROSCOPE

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Study 1: Gyroscope in the **Label** text field.

*Parametric Sweep*

- 1 In the **Study** toolbar, click  **Parametric Sweep**.
- 2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
- 3 Click **+ Add**.
- 4 In the table, enter the following settings:


Parameter name	Parameter value list	Parameter unit
omega (Angular velocity)	0 350	rad/s


*Step 1: Time Dependent*

- 1 In the **Model Builder** window, click **Step 1: Time Dependent**.
- 2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 3 In the **Output times** text field, type range (0,0.0025,0.25).

In order to obtain accurate results, limit the maximum time step by following the instructions below.




*Solution 1 (sol1)*

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Time-Dependent Solver 1**.
- 3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.
- 4 From the **Maximum step constraint** list, choose **Constant**.


- 5 In the **Maximum step** text field, type  $1e-4+9e-4*(\omega=0)$ .
- 6 In the **Study** toolbar, click  **Compute**.

## RESULTS


### *Displacement (mbd)*

- 1 Click the  **Zoom Extents** button in the **Graphics** toolbar.
- 2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (omega (rad/s))** list, choose **0**.
- 4 In the **Displacement (mbd)** toolbar, click  **Plot**.
- 5 From the **Parameter value (omega (rad/s))** list, choose **350**.
- 6 In the **Displacement (mbd)** toolbar, click  **Plot**.

### *Animation: Not spinning gyro*


- 1 In the **Results** toolbar, click  **Animation** and choose **File**.
- 2 In the **Settings** window for **Animation**, type Animation: Not spinning gyro in the **Label** text field.
- 3 Locate the **Target** section. From the **Target** list, choose **Player**.
- 4 Locate the **Frames** section. In the **Number of frames** text field, type 50.

### *Animation: Spinning gyro*

- 1 In the **Results** toolbar, click  **Animation** and choose **File**.
- 2 In the **Settings** window for **Animation**, type Animation: Spinning gyro in the **Label** text field.
- 3 Locate the **Target** section. From the **Target** list, choose **Player**.
- 4 Locate the **Frames** section. In the **Number of frames** text field, type 50.
- 5 Locate the **Animation Editing** section. From the **Parameter value (omega (rad/s))** list, choose **350**.

To compare the above two cases, plot the inclination angle of the disc with time as shown in [Figure 4](#).

### *Spinning disc orientation*

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Spinning disc orientation in the **Label** text field.

- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 1: Gyroscope/ Parametric Solutions 1 (sol2)**.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **Label**.
- 5 Locate the **Legend** section. From the **Position** list, choose **Upper left**.



*Global 1*

- 1 Right-click **Spinning disc orientation** and choose **Global**.
- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1: Gyroscope (comp1) > Definitions > Variables > theta - Inclination angle of spinning disc - rad**.
- 3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
theta	deg	Inclination angle of spinning disc

- 4 Click to expand the **Coloring and Style** section. From the **Width** list, choose **2**.
- 5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
- 6 From the **Positioning** list, choose **Interpolated**.
- 7 In the **Number** text field, type 25.
- 8 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.
- 9 In the table, enter the following settings:

Legends
Without spinning
With spinning

- 10 In the **Spinning disc orientation** toolbar, click  **Plot**.
- 11 Click the  **Zoom Extents** button in the **Graphics** toolbar.

*Modeling Instructions (Spinning Top)*

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**ADD COMPONENT**

In the **Model Builder** window, right-click the root node and choose **Add Component > 3D**.

**COMPONENT 2: SPINNING TOP**

In the **Settings** window for **Component**, type Component 2: Spinning top in the **Label** text field.

## GEOMETRY 2

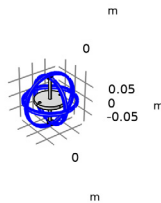
### *Import 1 (impl)*

- 1 In the **Geometry** toolbar, click  **Import**.
- 2 In the **Settings** window for **Import**, locate the **Source** section.
- 3 Click  **Browse**.
- 4 Browse to the model's Application Libraries folder and double-click the file `gyroscope.mphbin`.
- 5 Click  **Import**.

Delete the frame, inner gimbal, and outer gimbal from the gyroscope assembly.



### *Delete Entities 1 (dell)*

- 1 In the **Model Builder** window, right-click **Geometry 2** and choose **Delete Entities**.
- 2 In the **Settings** window for **Delete Entities**, locate the **Entities or Objects to Delete** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 On the object **impl(1)**, select Domain 1 only.
- 5 On the object **impl(2)**, select Domain 1 only.
- 6 On the object **impl(4)**, select Domain 1 only.



- 7 Click  **Build Selected**.


### *Rotate 1 (rot1)*

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Rotate**.
- 2 Select the object **imp1(3)** only.
- 3 In the **Settings** window for **Rotate**, locate the **Rotation** section.
- 4 In the **Angle** text field, type  $-\theta_0$ .
- 5 Locate the **Point on Axis of Rotation** section. In the **z** text field, type  $-0.0835$ .
- 6 Locate the **Rotation** section. From the **Axis type** list, choose **x-axis**.
- 7 Click  **Build All Objects**.



Define a rotated coordinate system with the same orientation as the spinning top.

### **DEFINITIONS (COMP2)**



#### *Rotated System 3 (sys3)*

- 1 In the **Definitions** toolbar, click  **Coordinate Systems** and choose **Rotated System**.
- 2 In the **Settings** window for **Rotated System**, locate the **Rotation** section.
- 3 Find the **Euler angles** subsection. In the  $\beta$  text field, type  $-\theta_0$ .

### **ADD MATERIAL**


- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Built-in > Aluminum**.
- 4 Click the **Add to Component** button in the window toolbar.
- 5 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.

### **ADD PHYSICS**

- 1 In the **Home** toolbar, click  **Add Physics** to open the **Add Physics** window.
- 2 Go to the **Add Physics** window.
- 3 In the tree, select **Structural Mechanics > Multibody Dynamics (mbd)**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Study 1: Gyroscope**.
- 5 Click the **Add to Component 2: Spinning Top** button in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Physics** to close the **Add Physics** window.

## MULTIBODY DYNAMICS 2 (MBD2)

### *Rigid Material 1*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Rigid Material**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Rigid Material**, locate the **Initial Values** section.
- 4 From the list, choose **Locally defined**.

### *Initial Values 1*

- 1 In the **Model Builder** window, click **Initial Values 1**.
- 2 In the **Settings** window for **Initial Values**, locate the **Coordinate System Selection** section.
- 3 From the **Coordinate system** list, choose **Rotated System 3 (sys3)**.
- 4 Locate the **Initial Values: Rotational** section. Specify the  $\omega$  vector as

0	x1
0	x2
omega	x3

- 5 Locate the **Center of Rotation** section. From the list, choose **Centroid of selected entities**.
- 6 From the **Entity level** list, choose **Point**.

### *Center of Rotation: Point 1*


- 1 In the **Model Builder** window, click **Center of Rotation: Point 1**.
- 2 Select Point 23 only.

It might be easier to select the correct point by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)

### *Rigid Material 1*

In the **Model Builder** window, under **Component 2: Spinning top (comp2)** > **Multibody Dynamics 2 (mbd2)** click **Rigid Material 1**.

### *Prescribed Displacement/Rotation 1*


- 1 In the **Physics** toolbar, click  **Attributes** and choose **Prescribed Displacement/Rotation**.
- 2 In the **Settings** window for **Prescribed Displacement/Rotation**, locate the **Prescribed Displacement at Center of Rotation** section.
- 3 Select the **Prescribed in x direction** checkbox.
- 4 Select the **Prescribed in y direction** checkbox.

- 5 Select the **Prescribed in z direction** checkbox.
- 6 Locate the **Center of Rotation** section. From the list, choose **Centroid of selected entities**.
- 7 From the **Entity level** list, choose **Point**.

*Center of Rotation: Point 1*

- 1 In the **Model Builder** window, click **Center of Rotation: Point 1**.
- 2 Select Point 23 only.

**MESH 2**

- 1 In the **Model Builder** window, under **Component 2: Spinning top (comp2)** click **Mesh 2**.
- 2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
- 3 In the table, clear the **Use** checkbox for **Geometric Analysis, Detail Size**.
- 4 Click  **Build All**.

**MULTIBODY DYNAMICS 2 (MBD2)**

*Gravity 1*

In the **Physics** toolbar, click  **Global** and choose **Gravity**.

**DEFINITIONS (COMP2)**

*Variables 2*



- 1 In the **Model Builder** window, under **Component 2: Spinning top (comp2)** right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 In the table, enter the following settings:

Name	Expression	Unit	Description
th	$\text{atan2}(y, x)$	rad	Precession angle
vp	$\cos(\text{th} + \pi/2) * \text{mbd2.u\_tX} + \sin(\text{th} + \pi/2) * \text{mbd2.u\_tY}$	m/s	Precession velocity
vn	$\cos(\text{th}) * \text{mbd2.u\_tX} + \sin(\text{th}) * \text{mbd2.u\_tY}$	m/s	Nutation velocity

*View 2*

- 1 In the **Model Builder** window, click **View 2**.
- 2 In the **Settings** window for **View**, locate the **View** section.
- 3 Clear the **Show grid** checkbox.

## ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies > Time Dependent**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Multibody Dynamics (mbd)**.
- 5 Click the **Add Study** button in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

## STUDY 2: SPINNING TOP



In the **Settings** window for **Study**, type Study 2: Spinning top in the **Label** text field.

### *Step 1: Time Dependent*

- 1 In the **Model Builder** window, under **Study 2: Spinning top** click **Step 1: Time Dependent**.
- 2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 3 In the **Output times** text field, type range (0,0.007,1.4).

In order to obtain accurate results, limit the maximum time step by following the instructions below.

### *Solution 5 (sol5)*

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 5 (sol5)** node, then click **Time-Dependent Solver I**.
- 3 In the **Settings** window for **Time-Dependent Solver**, locate the **Time Stepping** section.
- 4 From the **Maximum step constraint** list, choose **Constant**.
- 5 In the **Maximum step** text field, type 1e-4.
- 6 In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Displacement (mbd2)*


Add the trajectory of the topmost point of the spinning top to the default Displacement plot.

### *Point Trajectories I*

- 1 In the **Displacement (mbd2)** toolbar, click  **More Plots** and choose **Point Trajectories**.





- 2 Select Point 36 only.
- 3 In the **Settings** window for **Point Trajectories**, locate the **Coloring and Style** section.
- 4 Find the **Line style** subsection. From the **Type** list, choose **Tube**.

#### *Color Expression I*


- 1 Right-click **Point Trajectories I** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, locate the **Expression** section.
- 3 In the **Expression** text field, type **t**.
- 4 Locate the **Coloring and Style** section. Clear the **Color legend** checkbox.
- 5 In the **Displacement (mbd2)** toolbar, click  **Plot**.

#### *Displacement (mbd2)*

Follow the instructions below to plot the spinning top in various positions as shown in [Figure 6](#) and [Figure 7](#).


- 1 In the **Model Builder** window, under **Results** click **Displacement (mbd2)**.
- 2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.
- 3 From the **Time (s)** list, choose **0**.
- 4 In the **Displacement (mbd2)** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.
- 6 From the **Time (s)** list, choose **0.875**.
- 7 In the **Displacement (mbd2)** toolbar, click  **Plot**.
- 8 Click the  **Zoom Extents** button in the **Graphics** toolbar.

#### *Animation: Spinning top*

- 1 In the **Results** toolbar, click  **Animation** and choose **File**.
- 2 In the **Settings** window for **Animation**, type Animation: Spinning top in the **Label** text field.
- 3 Locate the **Target** section. From the **Target** list, choose **Player**.
- 4 Locate the **Scene** section. From the **Subject** list, choose **Displacement (mbd2)**.
- 5 Locate the **Frames** section. In the **Number of frames** text field, type 50.



Plot the locus of a point on the upper surface of the spinning top as shown in [Figure 8](#) by following the instructions below.

#### *Tip locus (xy-plane)*

- 1 In the **Results** toolbar, click  **ID Plot Group**.

- 2 In the **Settings** window for **ID Plot Group**, type Tip locus (xy-plane) in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 2: Spinning top/ Solution 5 (4) (sol5)**.
- 4 Locate the **Title** section. From the **Title type** list, choose **Label**.
- 5 Locate the **Axis** section. Select the **Preserve aspect ratio** checkbox.
- 6 Locate the **Grid** section. Select the **Manual spacing** checkbox.
- 7 In the **x spacing** text field, type 0.01.
- 8 In the **y spacing** text field, type 0.01.

#### *Point Graph I*

- 1 Right-click **Tip locus (xy-plane)** and choose **Point Graph**.
- 2 Select Point 36 only.
- 3 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.
- 4 In the **Expression** text field, type  $y$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $x$ .
- 7 Click to expand the **Coloring and Style** section. From the **Width** list, choose 2.
- 8 In the **Tip locus (xy-plane)** toolbar, click  **Plot**.
- 9 Click the  **Zoom Extents** button in the **Graphics** toolbar.

#### *Tip velocity (xy-plane)*

- 1 In the **Model Builder** window, right-click **Tip locus (xy-plane)** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type Tip velocity (xy-plane) in the **Label** text field.
- 3 Locate the **Plot Settings** section.
- 4 Select the **y-axis label** checkbox. In the associated text field, type Velocity (m/s).
- 5 Locate the **Axis** section. Clear the **Preserve aspect ratio** checkbox.
- 6 Locate the **Grid** section. Clear the **Manual spacing** checkbox.

#### *Point Graph I*

- 1 In the **Model Builder** window, expand the **Tip velocity (xy-plane)** node, then click **Point Graph I**.
- 2 In the **Settings** window for **Point Graph**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose

**Component 2: Spinning top (comp2) > Definitions > Variables > vp - Precession velocity - m/s.**

- 3 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Time**.
- 4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle**.
- 5 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 6 From the **Legends** list, choose **Manual**.
- 7 In the table, enter the following settings:

---



<b>Legends</b>
Precession

*Point Graph 2*

- 1 Right-click **Results > Tip velocity (xy-plane) > Point Graph 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Point Graph**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 2: Spinning top (comp2) > Definitions > Variables > vn - Nutation velocity - m/s**.
- 3 Locate the **Legends** section. In the table, enter the following settings:

---

<b>Legends</b>
Nutation

- 4 In the **Tip velocity (xy-plane)** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.