Model created in COMSOL Multiphysics 6.4

Radiative Heat Transfer in Finite Cylindrical Media — P1 Method

This application uses the P1 approximation to solve a 3D radiative transfer problem in an emitting, absorbing, and linearly anisotropic-scattering finite cylindrical medium. The calculated incident radiation and heat fluxes are compared to the results obtained with the highly accurate S6 discrete ordinates method (see Radiative Heat Transfer in Finite Cylindrical Media). The results obtained with this method show fairly good agreement with published results obtained by transformed integral methods (see Ref. 1) for optically thick media.

Introduction

There are numerous engineering applications of radiative transfer in absorbing, emitting, and anisotropically scattering media with variable radiation properties. Examples include, among others, coal-fired combustion systems, light-weight fibrous insulations, and heat transfer systems containing small scattering particles. Furthermore, the efficiency of radiative transfer depends on the boundary conditions, for example, the temperature and the emissivity of the surrounding walls, and the target where heat transfer is desired. Studies have shown that radiative transfer is highly sensitive to the wall emissivity. In this study you build a validation model representing a cylinder with homogeneous walls. Then you go on to consider walls with space-dependent emissivity and investigate the effects of albedo and scattering.

Model Definition

In this tutorial you validate the P1 formulation in COMSOL Multiphysics by examining three benchmark cases. You investigate the method's efficiency through parametric analyses by changing the single-scattering albedo, wall emissivity, and linear function. In particular, in the final case, you approximate the scattering phase function by a linear function reflecting highly backward, isotropic, and highly forward scattering. The results are compared to the results obtained with the DOM formulation as shown in Radiative Heat Transfer in Finite Cylindrical Media.

The model geometry, shown in Figure 1, is a cylinder of radius $R = 0.5$ m and height $L = 1$ m.

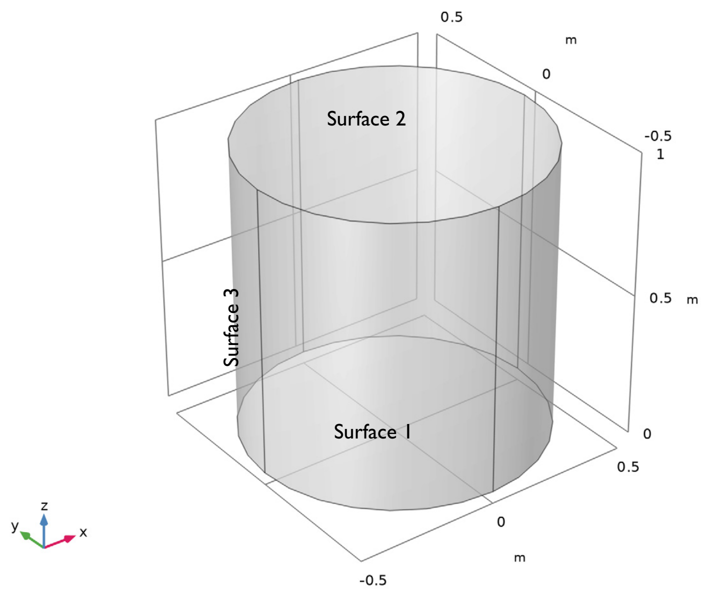


Figure 1: Schematic diagram of the physical model.

These examples use the P1 approximation method for predicting the heat flux on the enclosure side walls and the incident radiation distribution inside the domain.

VERIFICATION CASE

For the initial study, assume cold boundaries, that is, $T = 0\text{K}$. Furthermore, assume that the walls diffusively reflect radiation, that is, $\varepsilon = 0.5$. The medium is at a uniform temperature T such that the blackbody radiation intensity $I_b(T) = \sigma T^4/\pi$ is equal to 1 W/m^2.

WALLS WITH VARIABLE EMISSIVITY

Two cases are computed for comparison purposes. These 3D cases represent opaque partial side wall diffuse emission and/or reflection. In both cases, the radiosity on the side walls (Surface 3) varies with the angular coordinate along the full height of the finite cylinder according to $\varepsilon_3 = (1 - y/R)/2$. Both cases also have cold walls at the cylinder top (Surface 1) and bottom (Surface 2).

Case 1 has isotropic scattering function and compares results for different albedos. Case 2 has constant albedo and compares results with highly forward, isotropic, and highly backward scattering function parameterized by the Legendre coefficient a_1.

TABLE 1: NONSTANDARD TEST CASES.

CASE	MEDIUM PROPERTIES)
1	albedo = 0.1, 0.5, 0.9 $a_1 = 0$
2	albedo = 0.5 $a_1 = -0.99, 0, 0.99$

THERMAL ANALYSIS

The P1 approximation method is the simplest form of the method of spherical harmonics (PN) to describe radiative transport in participating media. The radiation transport equation (RTE) for this type of configuration can be written as

$$\Omega \cdot \nabla I(\Omega, s) = \kappa I_b(T) - \beta I(\Omega, s) + \frac{\sigma_s}{4\pi}\int_0^{4\pi} I(\Omega, \Omega')\phi(\Omega, \Omega')\partial\Omega'$$

where

- $I(\Omega, s)$ is the radiation intensity at a given position s in the direction Ω
- T is the temperature
- κ, β, and σ_s are absorption, extinction, and scattering coefficients, respectively
- $I_b(T)$ is the blackbody radiation intensity
- $\phi(\Omega, \Omega') = 1 + a_1\mu_0$ where $\mu_0 = \Omega \cdot \Omega'$ is the cosine of the scattering angle.

The RTE (an integro-partial differential equation) is transformed into a set of partial differential equations. The remaining task is to solve an additional equation for

$$G = \int_0^{4\pi} I(\Omega)d\Omega$$

which adds a heat source/sink to account for radiative heat transfer.

$$-\nabla \cdot (D_{P1}\nabla G) = Q_r$$

where D_{P1} is a diffusion coefficient and Q_r is the radiative heat source. The boundary condition for opaque surfaces then is

$$-\mathbf{n} \cdot D_{P1}\nabla G = -q_{r,\,net}$$

Results and Discussion

The results demonstrate that the P1 method provides a fast alternative to the DOM method. The solution time is only a few seconds whereas it is about 4 hours for the DOM method on the same mesh. The results represent the radiation characteristics well. The numerical values show good agreement for small scattering coefficients (`omega`) and high absorption coefficients (`kappa`). The relation between both values is $\kappa = \omega/(\omega - 1)$.

VERIFICATION CASE

This case examines the effects of the scattering albedo on the incident radiation and heat fluxes. Figure 2 shows the distribution of the net heat flux $q_{r,\,net}(R, 0, z)$ versus axial optical thickness. There is good agreement with the results obtained from the DOM model. The results for the net radiative heat flux (Figure 2) show good agreement with distance to the top and bottom boundary. The relative error for small scattering albedo is below 10% and increases with increasing scattering albedo.

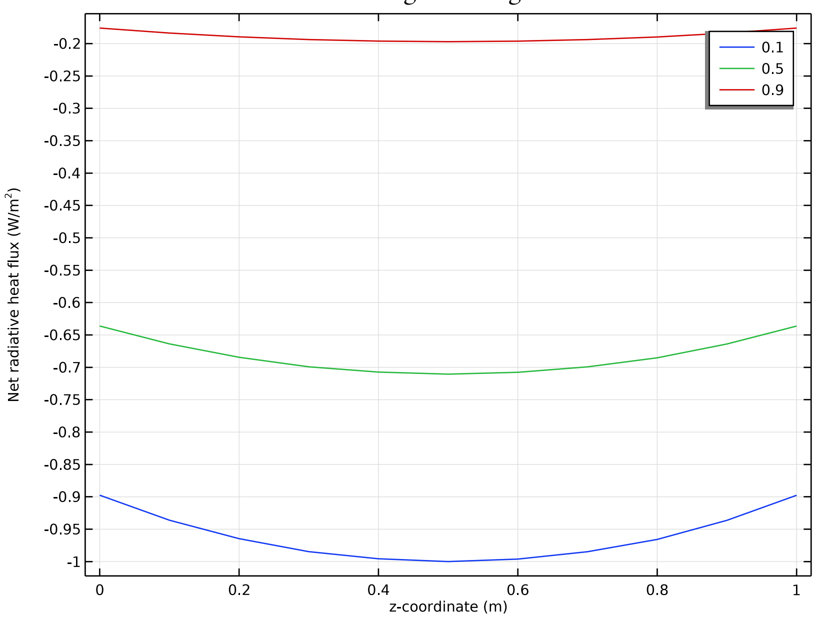


Figure 2: The effects of the scattering albedo on the radial heat flux $q_{r,\,net}(R, 0, z)$*; for a hot cylindrical medium enclosed by cold walls,* $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$.

The effects of albedo on the distribution of centerline incident radiation in axial direction are shown in Figure 3. The incident radiation is symmetric with respect to $z = L/2$ plane and decrease with increasing scattering albedo. Furthermore, results become more uniform with larger scattering albedo.

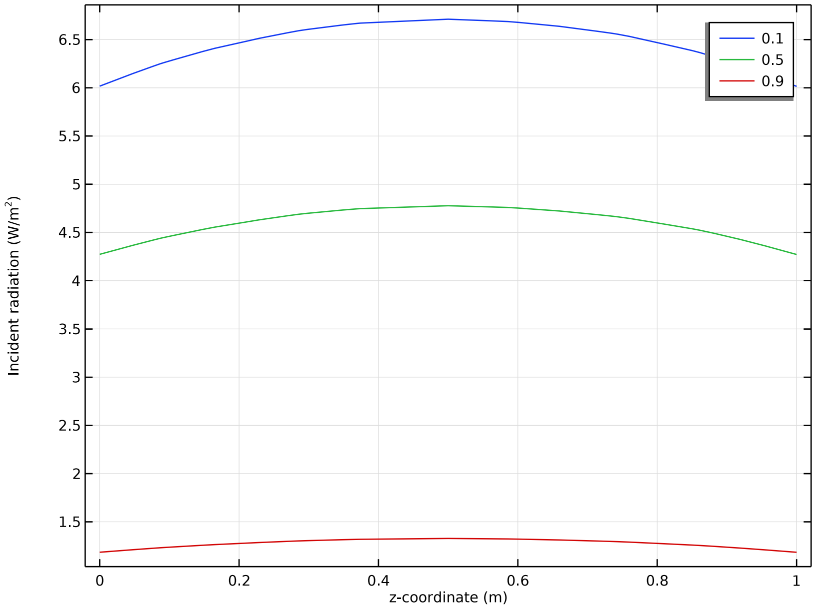


Figure 3: The effects of the scattering albedo on the distribution of centerline incident radiation $G(0, 0, z)$ *for a hot cylindrical medium enclosed by cold walls,* $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$.

Figure 4 displays the distributions of the incident radiation across the midplane radius $G(x, 0, L/2)$ with respect to normalized optical thickness x/R.

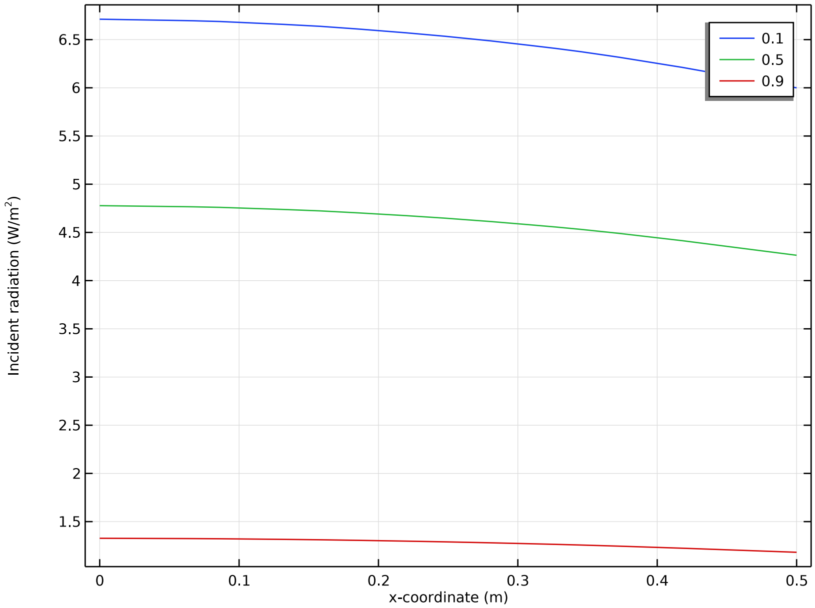


Figure 4: The effects of the scattering albedo on the distributions of the incident radiation $G(x, 0, L/2)$ *with respect to normalized optical thickness* x/R *for a hot cylindrical medium enclosed by cold walls,* $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$.

For the validation case the relative error compared to the DOM model is below 20%.

Walls with Variable Emissivity

The incident radiation at the midplane $z = L/2$ at the radial position $R/2$ is shown in Figure 5. At the side surface $R/2$ distance from the cylinder axis, $G(R, 0, L/2)$ changes with the azimuthal angle. The changes in scattering albedo are also illustrated. The smallest albedo makes the biggest change around the azimuthal angle. The numerical error for small scattering albedo again is low (around 10%).

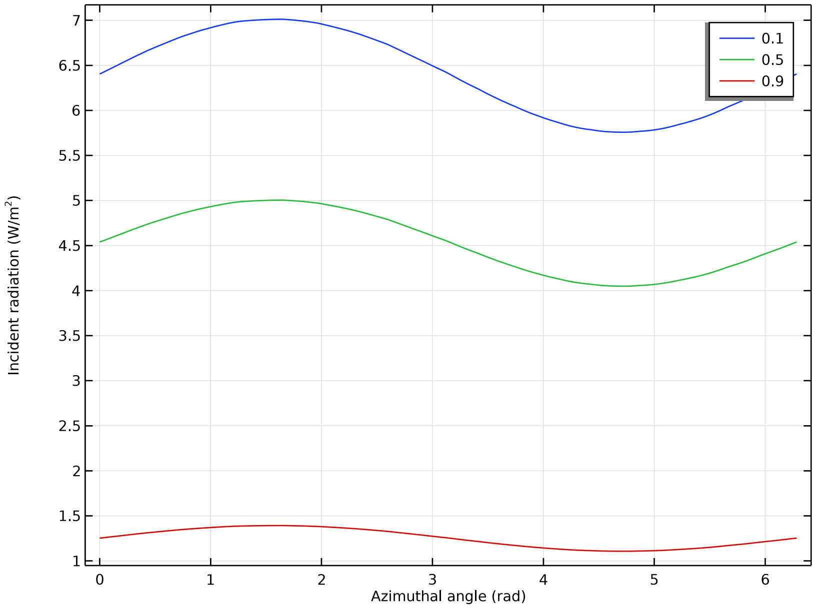


Figure 5: The effects of scattering albedo on the distribution of incident radiation at midplane $z = L/2$ at $R/2$ distance from the cylinder axis with respect to an azimuthal coordinate for Case 1.

[Figure 6](#) shows the effect of a nonzero linear anisotropic scattering coefficient a_1. As expected, the differences between isotropic scattering, forward scattering, and backward scattering are most accentuated far from the boundary.

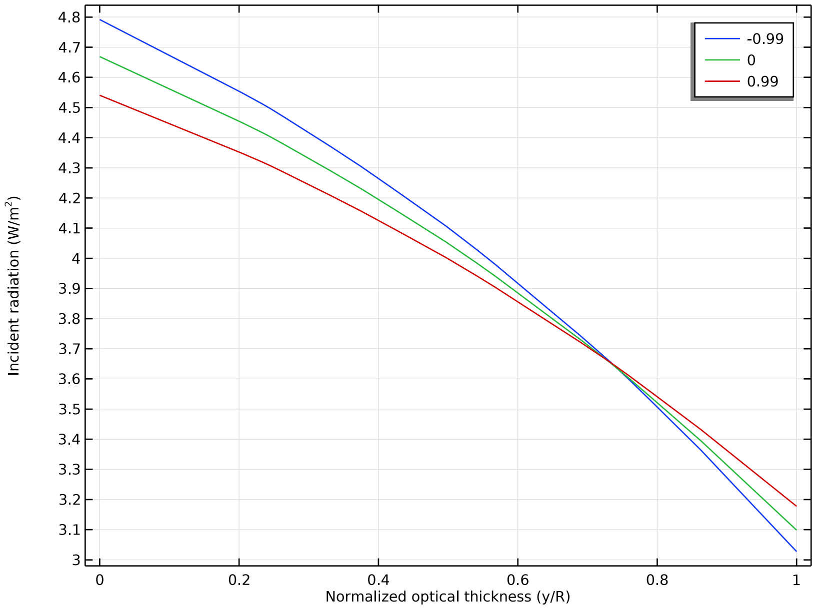


Figure 6: The effects of linear anisotropic scattering coefficient a_1 on the distribution of incident radiation $G(0, y, L/2)$ with respect to normalized optical thickness $-y/R$ for Case 2.

Unlike the discrete ordinates method, the P1 method is computationally inexpensive and gives fast results at the expense of accuracy in most cases. For large optical thickness the results show good agreement with the results obtained from the discrete ordinates method and hence with the results presented in the literature.

References

1. X.L. Chen and W.H. Sutton, "Radiative Transfer in Finite Cylindrical Media Using Transformed Integral Equations," *J. Quantitative Spectroscopy and Radiative Transfer*, vol. 77, pp. 233–271, 2003.

2. X. Chen, *Transformed Integral Equations of Radiative Transfer and Combined Convection-radiation Heat Transfer Enhancement with Porous Insert*, Doctoral Thesis, University of Oklahoma, 2003.

3. S.T. Thynell and M.N. Ozisik, "Radiation Transfer in Absorbing, Emitting, Isotropically Scattering, Homogeneous Cylindrical Media," *J. Quantitative Spectroscopy and Radiative Transfer*, vol. 38, no. 6, pp. 413–426, 1987.

4. H.Y. Li, M.N. Ozisik, and J.R. Tsai, "Two-dimensional radiation in a cylinder with spatially varying albedo," *J. Thermophysics and Heat Transfer*, vol. 6, no. 1, pp. 180–182, 1992.

Application Library path: `Heat_Transfer_Module/Verification_Examples/cylinder_participating_media_p1`

Modeling Instructions — Validation Case

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Heat Transfer** > **Radiation** > **Radiation in Participating Media (rpm)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies** > **Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
T0	(1[W/m^2]*pi/sigma_const)^(1/4)	86.275 K	Body temperature
Tw	0[K]	0 K	Wall temperature
ew	0.5	0.5	Wall emissivity
omega	0.5	0.5	Single-scattering albedo
sigma_s	omega	0.5	Scattering coefficient
kappa	sigma_s*(1/omega-1)	0.5	Absorption coefficient
R	0.5[m]	0.5 m	Cylinder radius
L	1[m]	1 m	Cylinder length

GEOMETRY 1

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type R.

4 In the **Height** text field, type L.

5 Click **Build All Objects**.

MATERIALS

Add a material to specify the absorption and scattering coefficients inside the cylinder.

Domain

1 In the **Materials** toolbar, click **Blank Material**.

2 In the **Settings** window for **Material**, type Domain in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Absorption coefficient	kappaR	kappa	1/m	Basic
Scattering coefficient	sigmaS	sigma_s	1/m	Basic

Analogously, specify the emissivity of the walls using a material.

Walls

1 In the **Materials** toolbar, click **Blank Material**.
2 In the **Settings** window for **Material**, type `Walls` in the **Label** text field.
3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Boundary**.
4 From the **Selection** list, choose **All boundaries**.
5 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Surface emissivity	epsilon_rad	ew	1	Basic

RADIATION IN PARTICIPATING MEDIA (RPM)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Radiation in Participating Media (rpm)**.
2 In the **Settings** window for **Radiation in Participating Media**, locate the **Participating Media Settings** section.
3 Find the **Radiation settings** subsection. From the **Radiation discretization method** list, choose **P1 approximation**.

Participating Medium 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Radiation in Participating Media (rpm)** click **Participating Medium 1**.
2 In the **Settings** window for **Participating Medium**, locate the **Model Input** section.
3 In the T text field, type `T0`.

Opaque Surface 1

1 In the **Model Builder** window, click **Opaque Surface 1**.
2 In the **Settings** window for **Opaque Surface**, locate the **Model Input** section.
3 In the T text field, type `Tw`.

STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.
2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
3 Click **Add**.

4 In the table, enter the following settings:

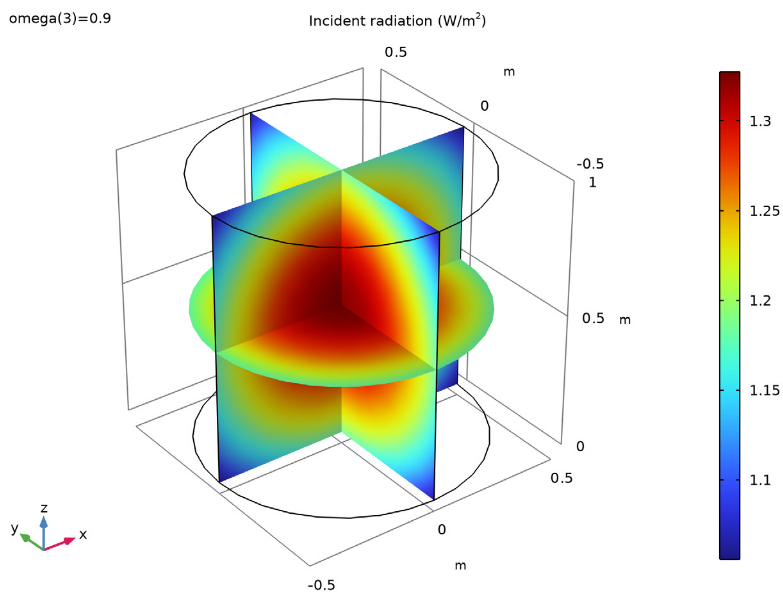
Parameter name	Parameter value list	Parameter unit
omega (Single-scattering albedo)	`0.1 0.5 0.9`	

5 In the **Study** toolbar, click **Compute**.

RESULTS

Incident Radiation (rpm)

The default plot shows a 3D distribution of incident radiation on surface slices.



Add a new 1D Plot to represent the net radiative heat flux along the z-coordinate and compare with Figure 2.

Net Radiative Heat Flux vs. z, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Net Radiative Heat Flux vs. z, 1D` in the **Label** text field.

Line Graph 1

1 In the **Net Radiative Heat Flux vs. z, 1D** toolbar, click **Line Graph**.

2 Click the **Transparency** button in the **Graphics** toolbar.

3 Select Edge 12 only.

4 Click the **Transparency** button in the **Graphics** toolbar again to return to the original state.

5 In the **Settings** window for **Line Graph**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Radiation in Participating Media** > **Boundary fluxes** > **rpm.qr_net - Net radiative heat flux - W/m²**.

6 Click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Geometry** > **Coordinate** > **z - z-coordinate**.

7 Click to expand the **Legends** section. Select the **Show legends** checkbox.

Finish the plot by adjusting the title and axis labels.

Net Radiative Heat Flux vs. z, 1D

1 In the **Model Builder** window, click **Net Radiative Heat Flux vs. z, 1D**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **x-axis label** checkbox.

5 Select the **y-axis label** checkbox.

6 In the **Net Radiative Heat Flux vs. z, 1D** toolbar, click **Plot**.

Cut Line 3D 1

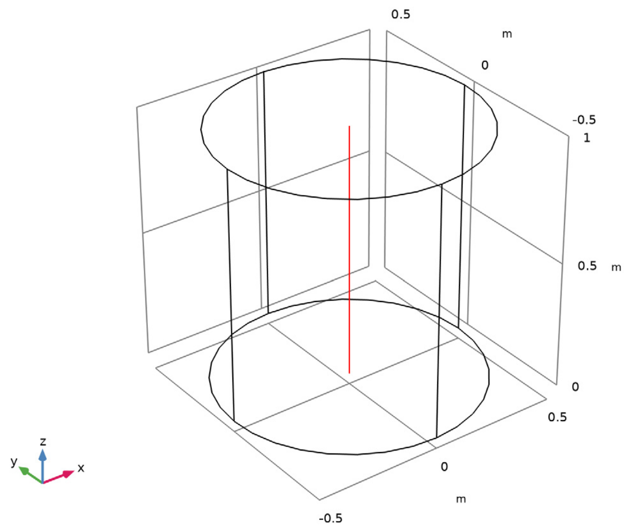
1 In the **Results** toolbar, click **Cut Line 3D**.

2 In the **Settings** window for **Cut Line 3D**, locate the **Line Data** section.

3 In row **Point 2**, set **X** to 0, and **z** to L.

4 Click **Plot**.

The Graphics window shows the location of the line in the model geometry.



Add a new 1D Plot to represent the incident radiation along the *z*-coordinate and compare with Figure 3.

Incident Radiation vs. z, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. z, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Line 3D 1**.

Line Graph 1

1 In the **Incident Radiation vs. z, 1D** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Geometry** > **Coordinate** > **z - z-coordinate**.

3 Locate the **Legends** section. Select the **Show legends** checkbox.

Incident Radiation vs. z, 1D

1 In the **Model Builder** window, click **Incident Radiation vs. z, 1D**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **x-axis label** checkbox.

5 Select the **y-axis label** checkbox.

6 In the **Incident Radiation vs. z, 1D** toolbar, click **Plot**.

Cut Line 3D 2

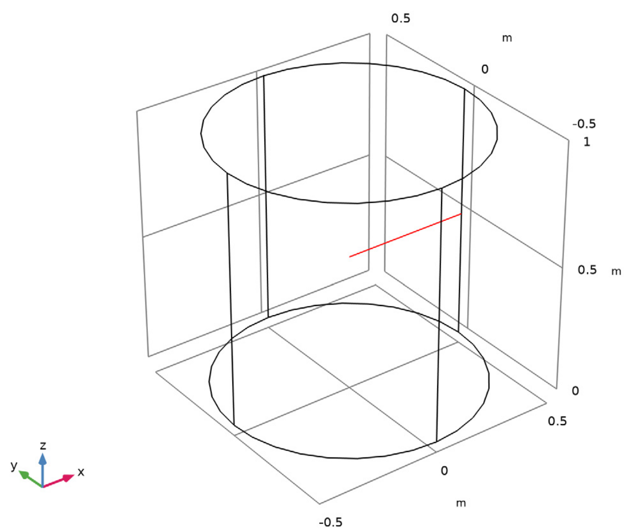
1 In the **Results** toolbar, click **Cut Line 3D**.

2 In the **Settings** window for **Cut Line 3D**, locate the **Line Data** section.

3 In row **Point 1**, set **Z** to `L/2`.

4 In row **Point 2**, set **X** to `R`, and **z** to `L/2`.

5 Click **Plot**.



Add a new 1D Plot to represent the incident radiation along the x-coordinate and compare with Figure 4.

Incident Radiation vs. x, 1D

1 Right-click **Incident Radiation vs. z, 1D** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. x, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Line 3D 2**.

4 Locate the **Plot Settings** section. In the **x-axis label** text field, type `x-coordinate (m)`.

Line Graph 1

1 In the **Model Builder** window, expand the **Incident Radiation vs. x, 1D** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Geometry** > **Coordinate** > **x - x-coordinate**.

3 In the **Incident Radiation vs. x, 1D** toolbar, click **Plot**.

Modeling Instructions — Case 1

RADIATION IN PARTICIPATING MEDIA (RPM)

Opaque Surface 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Opaque Surface**.

2 In the **Settings** window for **Opaque Surface**, locate the **Model Input** section.

3 In the T text field, type `Tw`.

4 Select Boundaries 1, 2, 5, and 6 only.

These are the vertical wall boundaries. To reach all of them, you can rotate the geometry or click either the **Transparency** button or the **Wireframe Rendering** button in the **Graphics** toolbar.

5 Locate the **Surface Radiative Properties** section. From the ε list, choose **User defined**. In the associated text field, type `ew*(1-y/R)`.

Now, disable **Opaque Surface 2** in **Study 1** to be able to rerun the same **Study 1** configuration.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** checkbox.

4 In the tree, select **Component 1 (comp1)** > **Radiation in Participating Media (rpm)** > **Opaque Surface 2**.

5 Click **Disable**.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Click the **Add Study** button in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

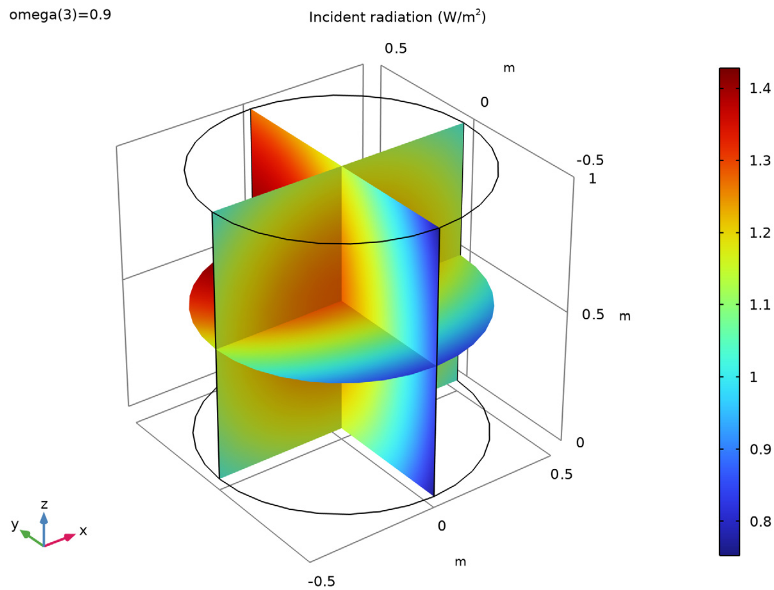
Parameter name	Parameter value list	Parameter unit
omega (Single-scattering albedo)	0.1 0.5 0.9	

5 In the **Study** toolbar, click **Compute**.

RESULTS

Incident Radiation (rpm) 1

The default plot shows a 3D distribution of incident radiation on surface slices.



Parametric Curve 3D 1

1 In the **Results** toolbar, click **More Datasets** and choose **Parametric Curve 3D**.

2 In the **Settings** window for **Parametric Curve 3D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

4 Locate the **Parameter** section. In the **Name** text field, type `phi`.

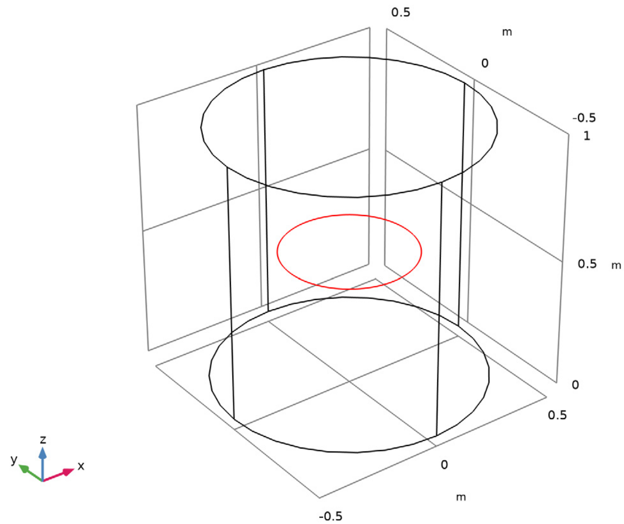
5 In the **Maximum** text field, type `2*pi`.

6 Locate the **Expressions** section. In the **x** text field, type `R*cos(phi)/2`.

7 In the **y** text field, type `R*sin(phi)/2`.

8 In the **z** text field, type `L/2`.

9 Click **Plot**.



Add a new 1D Plot to represent the incident radiation in function of the azimuthal angle and compare with Figure 5.

Incident Radiation vs. Azimuthal Angle, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. Azimuthal Angle, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Parametric Curve 3D 1**.

Line Graph 1

1 In the **Incident Radiation vs. Azimuthal Angle, 1D** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** checkbox.

Incident Radiation vs. Azimuthal Angle, 1D

1 In the **Model Builder** window, click **Incident Radiation vs. Azimuthal Angle, 1D**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section.

5 Select the **x-axis label** checkbox. In the associated text field, type `Azimuthal angle (rad)`.

6 Select the **y-axis label** checkbox.

Line Graph 1

1 In the **Model Builder** window, click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.

3 From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `phi`.

5 In the **Incident Radiation vs. Azimuthal Angle, 1D** toolbar, click **Plot**.

Modeling Instructions — Case 2

GLOBAL DEFINITIONS

Parameters 1

For case 2, you need to add a parameter for the Legendre coefficient `a1` in the scattering phase function.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
a1	0.99	0.99	Legendre coefficient

RADIATION IN PARTICIPATING MEDIA (RPM)

Add a new **Radiation in Participating Media** node for **Study 3** only.

Participating Medium 2

1 In the **Physics** toolbar, click **Domains** and choose **Participating Medium**.

2 Select Domain 1 only.

3 In the **Settings** window for **Participating Medium**, locate the **Model Input** section.

4 In the T text field, type `T0`.

5 Locate the **Scattering** section. From the **Scattering type** list, choose **Linear anisotropic**.

6 In the a_1 text field, type `a1`.

STUDY 1

Step 1: Stationary

Now, disable **Radiation in Participating Media 2** in **Study 1** and **Study 2** to be able to rerun the same configurations for these studies.

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 In the tree, select **Component 1 (comp1)** > **Radiation in Participating Media (rpm)** > **Participating Medium 2**.

4 Right-click and choose **Disable**.

STUDY 2

1 In the **Model Builder** window, under **Study 2** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** checkbox.

4 In the tree, select **Component 1 (comp1)** > **Radiation in Participating Media (rpm)** > **Participating Medium 2**.

5 Right-click and choose **Disable**.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.

4 Click the **Add Study** button in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
a1 (Legendre coefficient)	-0.99 0 0.99	

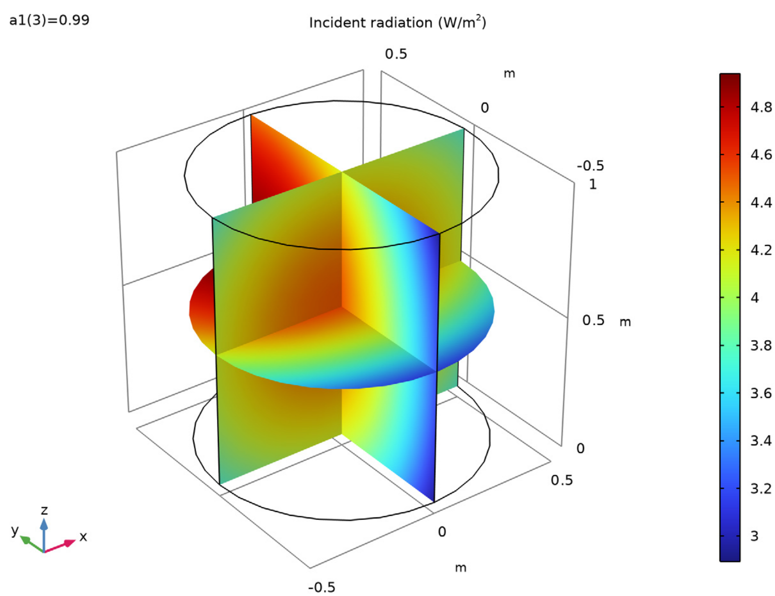
Note that the value $a_1 = 0$ gives the same solution as for $\omega = 0.5$ in case 1.

5 In the **Study** toolbar, click **Compute**.

RESULTS

Incident Radiation (rpm) 2

The default plot shows a 3D distribution of incident radiation on surface slices.



Parametric Curve 3D 2

1 In the **Results** toolbar, click **More Datasets** and choose **Parametric Curve 3D**.

2 In the **Settings** window for **Parametric Curve 3D**, locate the **Data** section.

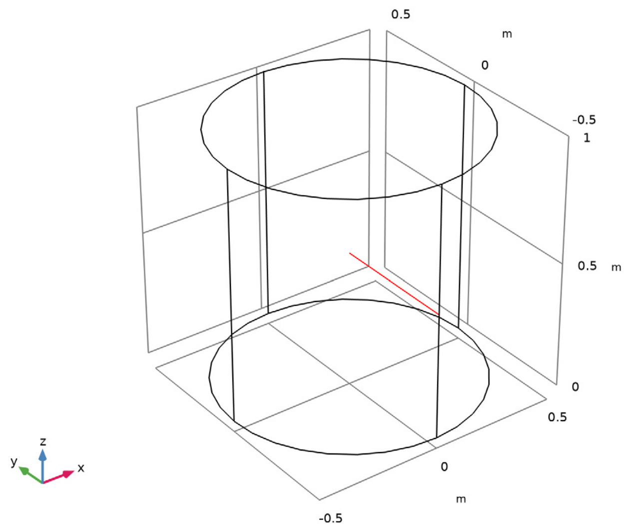
3 From the **Dataset** list, choose **Study 3/Solution 3 (sol3)**.

4 Locate the **Expressions** section. In the **y** text field, type `-s*R`.

5 In the **z** text field, type `L/2`.

With the above definition, $s = -y/R$ equals the optical thickness along the negative y-axis for the given constant values of x and z.

6 Click **Plot**.



Add a new 1D Plot to represent the incident radiation as function of the normalized optical thickness and compare with Figure 6.

Incident Radiation vs. Normalized Optical Thickness, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. Normalized Optical Thickness, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Parametric Curve 3D 2**.

Line Graph 1

1 In the **Incident Radiation vs. Normalized Optical Thickness, 1D** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.

3 From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `s`.

5 Locate the **Legends** section. Select the **Show legends** checkbox.

Incident Radiation vs. Normalized Optical Thickness, 1D

1 In the **Model Builder** window, click **Incident Radiation vs. Normalized Optical Thickness, 1D**.
2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.
3 From the **Title type** list, choose **None**.
4 Locate the **Plot Settings** section.
5 Select the **x-axis label** checkbox. In the associated text field, type `Normalized optical thickness (y/R)`.
6 Select the **y-axis label** checkbox.
7 In the **Incident Radiation vs. Normalized Optical Thickness, 1D** toolbar, click **Plot**.