Model created in COMSOL Multiphysics 6.4

Double-Headed Streamer in Parallel-Plate Electrodes

Introduction

Streamers are ephemeral, filamentary electric discharges that can form in a nonconductive environment when exposed to a strong electric field. These discharges can achieve a high density of electrons, leading to a concentration of chemically active species relevant in various applications.

Their propagation involves highly nonlinear dynamics characterized by steep density gradients and the presence of high space-charge density in thin layers. At the forefront of the streamer, charge separation generates intense electric fields, driving sharp ionization fronts into the surrounding neutral medium.

In negative streamers (directed toward the anode), ionizing electrons are propelled outward by the space charge, extending the streamer toward the anode. These electrons, often high in energy, can result from drift, diffusion, photoionization, or other mechanisms providing seed electrons ahead of the streamer. Conversely, in positive streamers (directed toward the cathode), the space-charge field accelerates electrons inward, necessitating an ionization mechanism for the production of ionizing electrons.

This case study investigates a double-headed streamer between parallel-plate electrodes. Initially, a cluster of electrons is positioned between two electrodes spaced 1 cm apart, subject to a 52 kV voltage, creating a background electric field of 52 kV/cm. Negative and positive streamers propagate toward the electrodes, exhibiting electric field and electron density consistent with simulation results in Ref. 1.

Model Definition

The model is two-dimensional and describes the transient behavior of an initial electron seed in the presence of a strong electric field. The Electric Discharge interface is used to simulate the streamer propagation. The built-in charge transport model is used:

$$
\frac{\partial n_i}{\partial t} + \nabla \cdot (\mathbf{w}_i n_i - D_i \nabla n_i) = R_i
$$

where

$$
i = \mathrm{e, p, n}
$$

$$
z_{\mathrm{e, p, n}} = -1, +1, -1
$$

$$
\mathbf{w}_i = z_i \mu_i \mathbf{E}
$$

$$
R_\mathrm{e} = \alpha|\mathbf{w}_\mathrm{e}|n_\mathrm{e} - \eta|\mathbf{w}_\mathrm{e}|n_\mathrm{e} - \beta_\mathrm{ep} n_\mathrm{e} n_\mathrm{p}
$$

$$
R_\mathrm{p} = \alpha|\mathbf{w}_\mathrm{e}|n_\mathrm{e} - \beta_\mathrm{ep} n_\mathrm{e} n_\mathrm{p} - \beta_\mathrm{pn} n_\mathrm{p} n_\mathrm{n}
$$

$$
R_\mathrm{n} = \eta|\mathbf{w}_\mathrm{e}|n_\mathrm{e} - \beta_\mathrm{pn} n_\mathrm{p} n_\mathrm{n}
$$

where

- e, p, n denote electrons, positive ions, and negative ions
- n_i is the number density of the charge carrier (SI unit: $1/\mathrm{m}^3$)
- $\mathbf{E}$ is the electric field (SI unit: V/m)
- z_i denotes the carrier charge (SI unit: 1)
- μ_i denotes the carrier mobility (SI unit: $\mathrm{m}^2/(\mathrm{V{\cdot}s})$)
- $\mathbf{w}_i$ is the drift velocity in the electric field (SI unit: m/s)
- D_i is the diffusion coefficient (SI unit: m^2/s)
- R_i is the reaction rate (SI unit: $1/(\mathrm{m}^3{\cdot}\mathrm{s})$)
- α is the ionization coefficient (SI unit: 1/m)
- η is the attachment coefficient (SI unit: 1/m)
- β_ep is the electron–ion recombination coefficient (SI unit: m^3/s)
- β_pn is the ion–ion recombination coefficient (SI unit: m^3/s)

The above transport equations are fully coupled with Poisson's equation through the electric field and the space charge:

$$
\nabla \cdot (\varepsilon_\mathrm{r} \varepsilon_0 \mathbf{E}) = \rho
$$

$$
\rho = e \sum_i z_i n_i
$$

where e is the elementary charge.

In this model, electrons and positive ions are modeled, while negative ions are neglected.

Results and Discussion

Figure 1 plots the z component of the electric field for several instants during the streamer simulation. Figure 2 shows the electron density distribution at 2.5 ns.

Starting from the center, two streamers develop toward the electrodes. These streamers have different propagation mechanisms that result in different morphologies and propagation speeds. The top streamer is anode directed and develops a negative space-charge density since the electric field pulls electrons ahead of the streamer. The bottom streamer is cathode directed and the electrons are drifting in the opposite direction of the streamer propagation. The propagation of the cathode-directed streamer is only possible because it is given a high enough background electron density. For simulating a positive streamer in a weak field, the photoionization effect has to be taken into account. See the library model Positive Streamer Propagation in a Weak Electric Field for such an example.

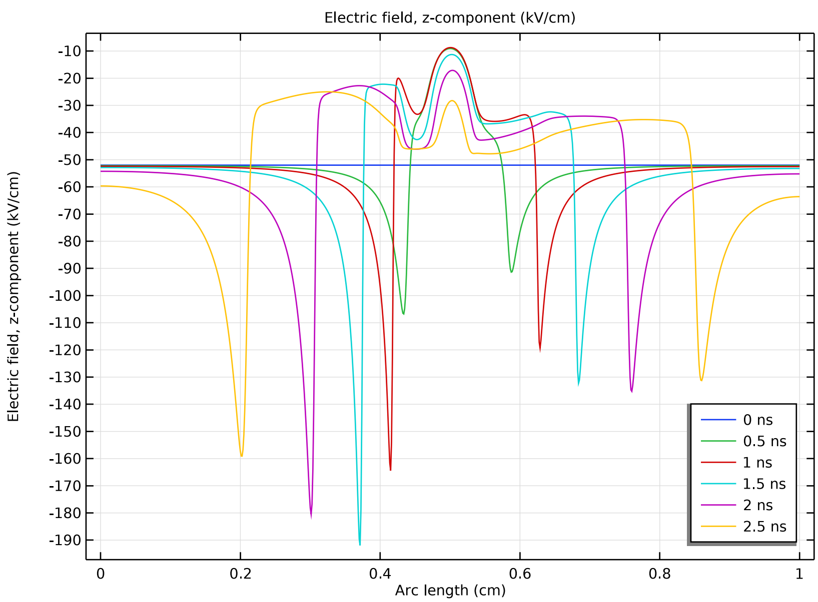


Figure 1: Spatial distribution along the axis of symmetry of the z-component of the electric field for several time instants during the streamer propagation. Compare with figure 7 of Ref. 1.

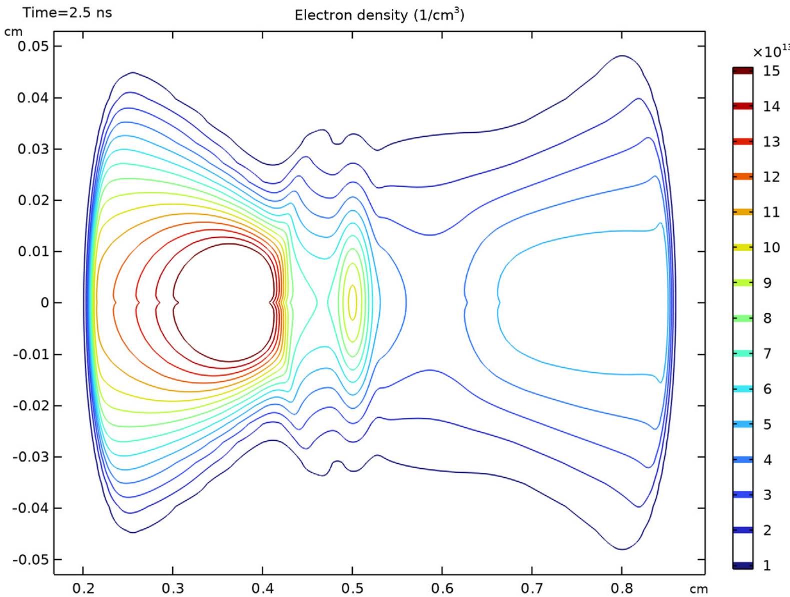


Figure 2: Contours of the electron number density at 2.5 ns. Compare with figure 6 of Ref. 1.

References

1. D. Bessières, J. Paillol, A. Bourdon, P. Segur, and E. Marode, "A new one-dimensional moving mesh method applied to the simulation of streamer discharges," *J. Phys. D: Appl. Phys.*, vol. 40, pp. 6559–6570, 2007.

Application Library path: `Electric_Discharge_Module/Streamer_Discharges/double_headed_streamer`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.
2 In the **Select Physics** tree, select **Electric Discharge** > **Electric Discharge (edis)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Time Dependent with Initialization**.
6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
V0	52[kV]	52000 V	Applied voltage

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
2 In the **Settings** window for **Variables**, locate the **Variables** section.
3 In the table, enter the following settings:

Name	Expression	Unit	Description
N0	(1e8+1e14*exp(-((z/(1[cm])-0.5)/0.027)^2-(r/(1[cm])/0.021)^2))[cm^-3]	1/m³	Initial number density

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
2 In the **Settings** window for **Geometry**, locate the **Units** section.
3 From the **Length unit** list, choose **cm**.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, click to expand the **Layers** section.

3 Clear the **Layers on bottom** checkbox.

4 Select the **Layers to the left** checkbox.

5 In the table, enter the following settings:

Layer name	Thickness (cm)
Layer 1	0.06

ELECTRIC DISCHARGE (EDIS)

Gas 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Electric Discharge (edis)** click **Gas 1**.

2 In the **Settings** window for **Gas**, locate the **Model Formulation** section.

3 From the **Charge carriers** list, choose **Electrons and positive ions**.

4 Locate the **Transport Properties** section. Find the **Drift** subsection. From the μ_p list, choose **User defined**. In the associated text field, type 0.

5 Find the **Diffusion** subsection. From the **Diffusion coefficient** list, choose **User defined**.

6 From the list, choose **Diagonal**.

7 Specify the D_e matrix as

1800[cm^2/s]	0	0
0	0	0
0	0	2190[cm^2/s]

8 In the D_p text field, type 0.

9 Locate the **Reactions** section. Find the **Recombination** subsection. From the β_{ep} list, choose **User defined**.

Electrode 1

1 In the **Physics** toolbar, click **Attributes** and choose **Electrode**.

2 Select Boundaries 2 and 5 only.

Gas 1

In the **Model Builder** window, click **Gas 1**.

Electrode 2

1 In the **Physics** toolbar, click **Attributes** and choose **Electrode**.

2 Select Boundaries 3 and 6 only.

3 In the **Settings** window for **Electrode**, locate the **Terminal** section.

4 In the V_0 text field, type `V0`.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the n_e text field, type `N0`.

4 In the n_p text field, type `N0`.

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Electric Discharge** > **Gases** > **N2 - Nitrogen**.

4 Right-click and choose **Add to Component 1 (comp1)**.

5 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 1 only.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 2 and 3 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 From the **Distribution type** list, choose **Predefined**.

5 In the **Number of elements** text field, type `20`.

6 In the **Element ratio** text field, type `10`.

7 Select the **Reverse direction** checkbox.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 1 and 4 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `800`.

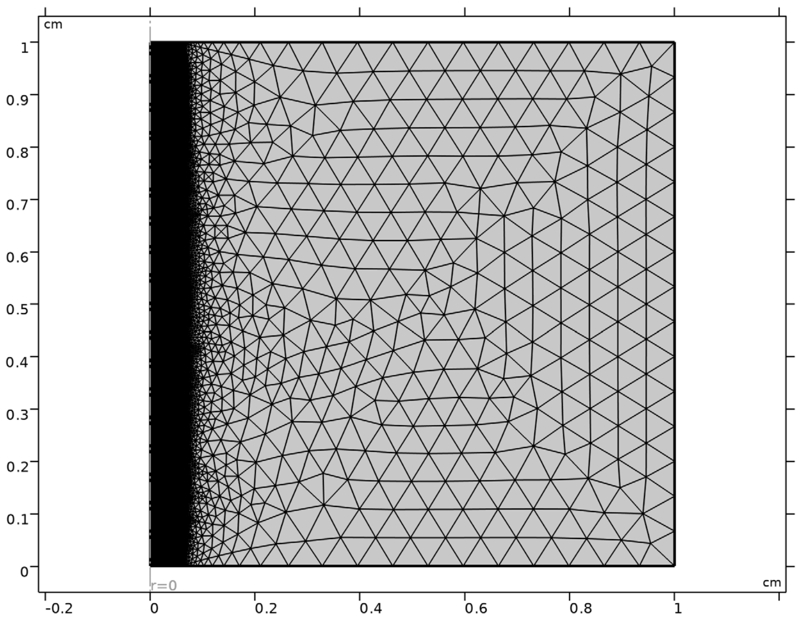
Free Triangular 1

1 In the **Mesh** toolbar, click **Free Triangular**.

2 In the **Settings** window for **Free Triangular**, click **Build All**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar.

4 In the **Model Builder** window, click **Mesh 1**.



STUDY 1

Step 2: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 2: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **ns**.

4 In the **Output times** text field, type `range(0,0.5,2.5)`.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.

4 From the **Maximum step constraint** list, choose **Constant**.

5 In the **Maximum step** text field, type `0.01`.

6 In the **Model Builder** window, click **Study 1**.

7 In the **Settings** window for **Study**, locate the **Study Settings** section.

8 Clear the **Generate default plots** checkbox.

9 In the **Study** toolbar, click **Compute**.

RESULTS

Electric Field

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electric Field` in the **Label** text field.

3 Locate the **Legend** section. From the **Position** list, choose **Lower right**.

Line Graph 1

1 Right-click **Electric Field** and choose **Line Graph**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

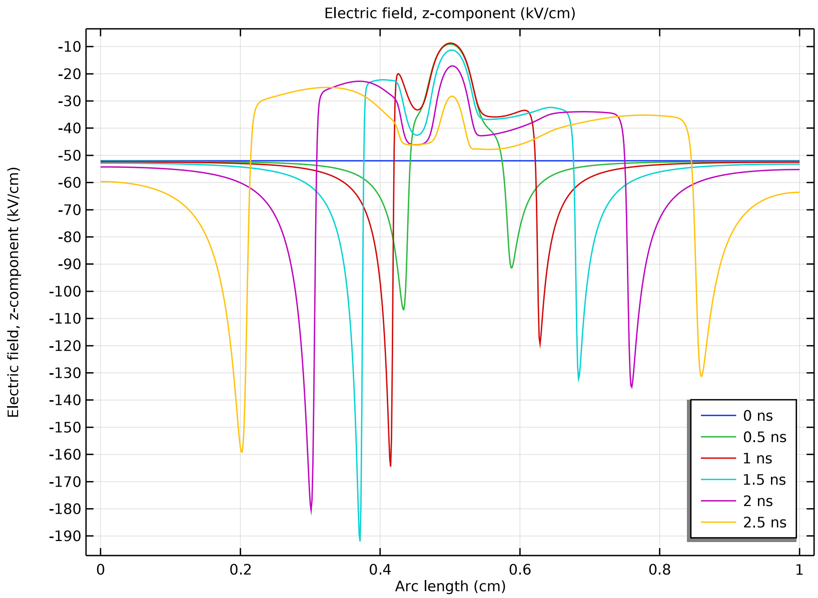
4 In the **Expression** text field, type `edis.Ez`.

5 In the **Unit** field, type `kV/cm`.

6 Click to expand the **Legends** section. Select the **Show legends** checkbox.

7 In the **Electric Field** toolbar, click **Plot**.

8 Click the **Zoom Extents** button in the **Graphics** toolbar.



2D Plot Group 2

In the **Results** toolbar, click **2D Plot Group**.

Mirror 2D 1

In the **Results** toolbar, click **More Datasets** and choose **Mirror 2D**.

Electron Density

1 In the **Model Builder** window, under **Results** click **2D Plot Group 2**.

2 In the **Settings** window for **2D Plot Group**, type `Electron Density` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

Contour 1

1 Right-click **Electron Density** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Expression** section.

3 In the **Expression** text field, type `edis.n_e`.

4 In the **Unit** field, type `1/cm^3`.

5 Locate the **Levels** section. From the **Entry method** list, choose **Levels**.

6 In the **Levels** text field, type `range(1.0e13,1.0e13,1.5e14)`.

Electron Density

1 In the **Model Builder** window, click **Electron Density**.

2 In the **Settings** window for **2D Plot Group**, locate the **Plot Settings** section.

3 Clear the **Plot dataset edges** checkbox.

4 From the **View** list, choose **View 1**.

5 Click **Go to Source**.

DEFINITIONS

Axis

1 Click the **Zoom Extents** button in the **Graphics** toolbar.

2 In the **Model Builder** window, expand the **View 1** node, then click **Axis**.

3 In the **Settings** window for **Axis**, locate the **Axis** section.

4 In the **r minimum** text field, type `0`.

5 In the **r maximum** text field, type `0.06`.

6 In the **z minimum** text field, type `0`.

7 In the **z maximum** text field, type `1`.

8 From the **View scale** list, choose **Automatic**.

RESULTS

Transformation 1

1 In the **Model Builder** window, right-click **Contour 1** and choose **Transformation**.

2 In the **Settings** window for **Transformation**, locate the **Transformation** section.

3 Select the **Rotate** checkbox.

4 In the **Angle** text field, type `-90`.

5 In the **Electron Density** toolbar, click **Plot**.

6 Click the  **Zoom Extents** button in the **Graphics** toolbar.