



Model created in COMSOL Multiphysics 6.4

DC Breakdown Voltage of Parallel Electrodes in Air

Introduction

This simulation model focuses on electrical breakdown in air at atmospheric pressure, where streamer breakdown is the dominant mechanism. Unlike glow discharges, streamer discharges are unstable and marked by a rapid rise in discharge current due to electron impact ionization (see Fig. 7.1 in [Ref. 1](#)).

The model computes the DC breakdown voltage between parallel electrodes in air using a charge transport formulation. While implemented in one dimension for simplicity, the approach can be generalized to other gases and extended to higher dimensions. Validation against published experimental data shows that the model reproduces breakdown behavior with high accuracy.

Model Definition

The Electric Discharge interface is used to compute the discharge current. The built-in charge transport model is used:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (\mathbf{w}_i n_i - D_i \nabla n_i) = R_i$$

where

$$i = e, p, n$$

$$z_{e, p, n} = -1, +1, -1$$

$$\mathbf{w}_i = z_i \mu_i \mathbf{E}$$

$$R_e = \alpha |\mathbf{w}_e| n_e - \eta |\mathbf{w}_e| n_e - \beta_{ep} n_e n_p$$

$$R_p = \alpha |\mathbf{w}_e| n_e - \beta_{ep} n_e n_p - \beta_{pn} n_p n_n$$

$$R_n = \eta |\mathbf{w}_e| n_e - \beta_{pn} n_p n_n$$

where

- e, p, n denote electrons, positive ions, and negative ions
- n_i is the number density of the charge carrier (SI unit: $1/\text{m}^3$)
- \mathbf{E} is the electric field (SI unit: V/m)
- z_i denotes the carrier charge (SI unit: 1)
- μ_i denotes the carrier mobility (SI unit: $\text{m}^2/(\text{V}\cdot\text{s})$)
- \mathbf{w}_i is the drift velocity in the electric field (SI unit: m/s)
- D_i is the diffusion coefficient (SI unit: m^2/s)
- R_i is the reaction rate (SI unit: $1/(\text{m}^3\cdot\text{s})$)
- α is the ionization coefficient (SI unit: $1/\text{m}$)
- η is the attachment coefficient (SI unit: $1/\text{m}$)
- β_{ep} is the electron-ion recombination coefficient (SI unit: m^3/s)
- β_{pn} is the ion-ion recombination coefficient (SI unit: m^3/s)

The above transport equations are fully coupled with Poisson's equation through the electric field and the space charge:

$$\nabla \cdot (\epsilon_r \epsilon_0 \mathbf{E}) = \rho$$

$$\rho = e \sum_i z_i n_i$$

where e is the elementary charge.

The breakdown is identified by a sharp increase in current caused by an electron avalanche. To capture this behavior, the model performs a parametric sweep of the applied voltage. The simulation automatically terminates once the discharge current exceeds a defined threshold.

Results and Discussion

Figure 1 shows the current–voltage (I–V) characteristics. As can be seen, there is a region where the discharge current increases sharply for each air gap distance. Figure 2 compares the breakdown voltages and corresponding electric fields as a function of gap distance for three different material datasets. The simulated results exhibit good agreement with experimental measurements (see Fig. 7.4 in Ref. 1).

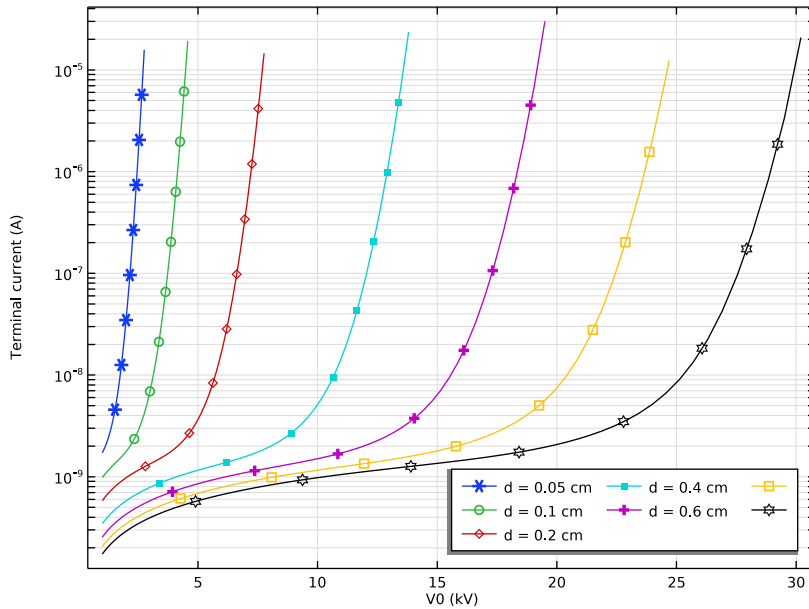


Figure 1: The current-voltage characteristics of the discharge for different air gaps.

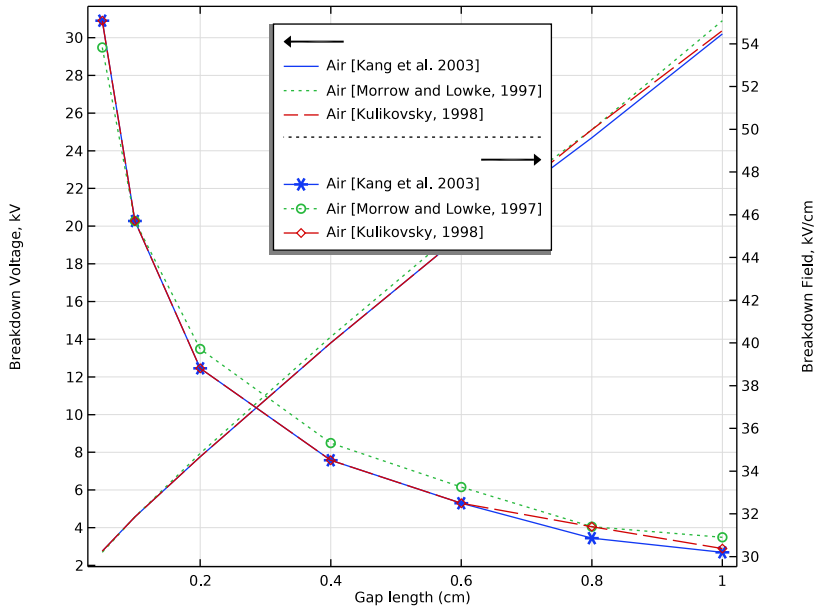


Figure 2: The simulated breakdown voltages as a function gap distance for three different material datasets.

References


1. Y.P. Raizer, *Gas Discharges Physics*, Springer, 1997.

Application Library path: Electric_Discharge_Module/Electrical_Breakdown/
breakdown_voltage_dc




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **ID**.
- 2 In the **Select Physics** tree, select **Electric Discharge** > **Electric Discharge (edis)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Stationary with Initialization**.
- 6 Click  **Done**.

GEOMETRY I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **cm**.

GLOBAL DEFINITIONS

Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
V0	10[kV]	10000 V	Applied voltage
d	1[cm]	0.01 m	Gap length

GEOMETRY I

Interval 1 (i1)

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Geometry 1** node.
- 2 Right-click **Geometry 1** and choose **Interval**.
- 3 In the **Settings** window for **Interval**, locate the **Interval** section.
- 4 In the table, enter the following settings:

Coordinates (cm)
0
d

5 Click  **Build All Objects**.

MATERIALS

Material Switch 1 (sw1)

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **More Materials > Material Switch**.

ADD MATERIAL FROM LIBRARY

In the **Home** toolbar, click  **Windows** and choose **Add Material from Library**.

ADD MATERIAL


- 1 Go to the **Add Material** window.
- 2 In the tree, select **Electric Discharge > Gases > Air > Air [Kang et al. 2003]**.
- 3 Right-click and choose **Add to Material Switch 1 (sw1)**.
- 4 In the tree, select **Electric Discharge > Gases > Air > Air [Morrow and Lowke, 1997]**.
- 5 Right-click and choose **Add to Material Switch 1 (sw1)**.
- 6 In the tree, select **Electric Discharge > Gases > Air > Air [Kulikovskiy, 1998]**.
- 7 Right-click and choose **Add to Material Switch 1 (sw1)**.

ELECTRIC DISCHARGE (EDIS)

Gas 1

In the **Model Builder** window, under **Component 1 (comp1) > Electric Discharge (edis)** click **Gas 1**.

Electrode 1

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Electrode**.
- 2 Select **Boundary 1** only.
- 3 In the **Settings** window for **Electrode**, locate the **Charge Transport** section.
- 4 From the **Boundary condition for electrons** list, choose **Number density**.
- 5 From the **Boundary condition for negative ions** list, choose **Number density**.

Gas 1

In the **Model Builder** window, click **Gas 1**.

Electrode 2


- 1 In the **Physics** toolbar, click  **Attributes** and choose **Electrode**.
- 2 Select **Boundary 2** only.

- 3 In the **Settings** window for **Electrode**, locate the **Terminal** section.
- 4 In the V_0 text field, type V_0 .
- 5 Locate the **Charge Transport** section. From the **Boundary condition for positive ions** list, choose **Number density**.

MESH 1


- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.
- 3 From the list, choose **User-controlled mesh**.

Distribution 1

- 1 In the **Model Builder** window, right-click **Edge 1** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 From the **Distribution type** list, choose **Predefined**.
- 4 In the **Number of elements** text field, type 200.
- 5 In the **Element ratio** text field, type 5.
- 6 Select the **Symmetric distribution** checkbox.
- 7 Click  **Build All**.


STUDY 1

Parametric Sweep

- 1 In the **Study** toolbar, click  **Parametric Sweep**.
- 2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
- 3 Click **+ Add**.
- 4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
d (Gap length)	0.05 0.1 0.2 0.4 0.6 0.8 1	cm

Material Sweep

- 1 In the **Study** toolbar, click  **More Study Extensions** and choose **Material Sweep**.
- 2 In the **Settings** window for **Material Sweep**, locate the **Study Settings** section.
- 3 Click **+ Add**.

Step 1: Electrostatics Initialization

- 1 In the **Model Builder** window, click **Step 1: Electrostatics Initialization**.

- 2 In the **Settings** window for **Electrostatics Initialization**, click to expand the **Study Extensions** section.
- 3 Select the **Auxiliary sweep** checkbox.
- 4 Click **+ Add**.
- 5 In the table, enter the following settings:


Parameter name	Parameter value list	Parameter unit
V0 (Applied voltage)	1000	V

Step 2: Stationary

- 1 In the **Model Builder** window, click **Step 2: Stationary**.
- 2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.
- 3 Select the **Auxiliary sweep** checkbox.
- 4 Click **+ Add**.
- 5 In the table, enter the following settings:


Parameter name	Parameter value list	Parameter unit
V0 (Applied voltage)	$10^{\{\text{range}(\log_{10}(1000), 1/100, \log_{10}(50000))\}}$	V

Solution 1 (sol1)

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node.
- 3 In the **Model Builder** window, expand the **Study 1 > Solver Configurations > Solution 1 (sol1) > Stationary Solver 2** node.
- 4 Right-click **Study 1 > Solver Configurations > Solution 1 (sol1) > Stationary Solver 2 > Parametric 1** and choose **Stop Condition**.
- 5 In the **Settings** window for **Stop Condition**, locate the **Stop Expressions** section.
- 6 Click **+ Add**.
- 7 In the table, enter the following settings:


Stop expression	Stop if	Active	Description
comp1.edis.IO_1>1e-5[A]	True (>=1)	√	Stop expression 1

- 8 Locate the **Output at Stop** section. From the **Add solution** list, choose **Step after stop**.

- 9 In the **Model Builder** window, under **Study 1 > Solver Configurations > Solution 1 (sol1) > Stationary Solver 2** click **Fully Coupled 1**.
- 10 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.
- 11 In the **Maximum number of iterations** text field, type 250.
- 12 In the **Model Builder** window, click **Study 1**.
- 13 In the **Settings** window for **Study**, locate the **Study Settings** section.
- 14 Clear the **Generate default plots** checkbox.
- 15 In the **Study** toolbar, click  **Compute**.

RESULTS

Breakdown Voltages & Fields

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type **Breakdown Voltages & Fields** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **None**.
- 4 Locate the **Plot Settings** section. Select the **Two y-axes** checkbox.
- 5 Select the **y-axis label** checkbox. In the associated text field, type **Breakdown Voltage**, kV.
- 6 Select the **Secondary y-axis label** checkbox. In the associated text field, type **Breakdown Field**, kV/cm.

Global 1

- 1 Right-click **Breakdown Voltages & Fields** and choose **Global**.
- 2 In the **Settings** window for **Global**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol3)**.
- 4 From the **Material Switch 1** list, choose **From list**.
- 5 In the **Values (Material Switch 1)** list box, select **Air [Kang et al. 2003]**.
- 6 From the **Parameter selection (V0)** list, choose **Last**.
- 7 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
V0	kV	Air [Kang et al. 2003]

- 8 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

- 9 In the **Expression** text field, type d.
- 10 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle**.

Global 2

- 1 Right-click **Global 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Global**, locate the **Data** section.
- 3 In the **Values (Material Switch 1)** list box, select **Air [Morrow and Lowke, 1997]**.
- 4 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
V0	kV	Air [Morrow and Lowke, 1997]

Global 3

- 1 Right-click **Global 2** and choose **Duplicate**.
- 2 In the **Settings** window for **Global**, locate the **Data** section.
- 3 In the **Values (Material Switch 1)** list box, select **Air [Kulikovsky, 1998]**.
- 4 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
V0	kV	Air [Kulikovsky, 1998]

Global 1, Global 2, Global 3

- 1 In the **Model Builder** window, under **Results > Breakdown Voltages & Fields**, Ctrl-click to select **Global 1**, **Global 2**, and **Global 3**.
- 2 Right-click and choose **Duplicate**.

Global 4

- 1 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 2 In the table, enter the following settings:

Expression	Unit	Description
V0/d	kV/cm	Air [Kang et al. 2003]

- 3 Locate the **y-Axis** section. Select the **Plot on secondary y-axis** checkbox.
- 4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle (reset)**.
- 5 From the **Color** list, choose **Cycle (reset)**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

Global 5

- 1 In the **Model Builder** window, click **Global 5**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
V0/d	kV/cm	Air [Morrow and Lowke, 1997]

- 4 Locate the **y-Axis** section. Select the **Plot on secondary y-axis** checkbox.
- 5 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.


Global 6


- 1 In the **Model Builder** window, click **Global 6**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:

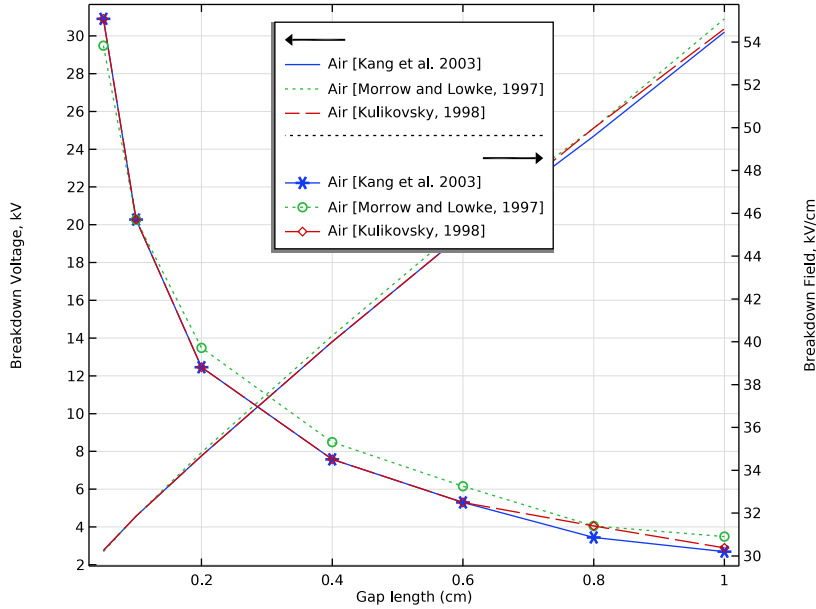
Expression	Unit	Description
V0/d	kV/cm	Air [Kulikovsky, 1998]

- 4 Locate the **y-Axis** section. Select the **Plot on secondary y-axis** checkbox.
- 5 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.


Breakdown Voltages & Fields

- 1 In the **Model Builder** window, click **Breakdown Voltages & Fields**.
- 2 In the **Settings** window for **ID Plot Group**, click to expand the **Title** section.
- 3 From the **Title type** list, choose **None**.
- 4 Locate the **Legend** section. From the **Position** list, choose **Upper middle**.
- 5 Click the  **Go to Default View** button in the **Graphics** toolbar.

6 Click the  **Zoom Extents** button in the **Graphics** toolbar.



IV Curve

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type IV Curve in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (sol3)**.
- 4 From the **Material Switch 1** list, choose **First**.
- 5 Locate the **Title** section. From the **Title type** list, choose **None**.
- 6 Locate the **Axis** section. Select the **y-axis log scale** checkbox.
- 7 Locate the **Legend** section. From the **Position** list, choose **Lower right**.
- 8 In the **Number of columns** text field, type 3.

Global 1

- 1 Right-click **IV Curve** and choose **Global**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
edis.IO_1	A	Terminal current

4 Locate the **x-Axis Data** section. From the **Unit** list, choose **kV**.


5 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.


6 From the **Positioning** list, choose **Interpolated**.

7 Click to expand the **Legends** section. Find the **Include** subsection. Clear the **Solution** checkbox.

8 Clear the **Description** checkbox.

9 Find the **Prefix and suffix** subsection. In the **Prefix** text field, type $d = \text{eval}(d, \text{cm}) \text{ cm}$.

10 In the **IV Curve** toolbar, click  **Plot**.

11 Click the  **Zoom Extents** button in the **Graphics** toolbar.

