



Model created in COMSOL Multiphysics 6.4

Large Eddy Simulation of a 3D Hill Geometry

Introduction

Flow prediction in the case of separating flows from blunt surfaces are computationally challenging. Such scenarios are frequently encountered in a variety of engineering simulations of external and internal flows, for instance, flow around highly-loaded swept wings and duct flows with surface deformations and curved geometries. They demonstrate a high sensitivity to upstream flow and turbulence conditions for detachment and reattachment phenomena, both spatially and temporally. Moreover, three-dimensional flows admit convoluted separation patterns and leaner recirculation regions.

The present model demonstrates the application of the Large Eddy Simulation (LES) method with automatic wall modeling to predict flow quantities for a well-studied laboratory experiment of flow around an axisymmetric 3D hill feature in a duct system; see [Ref. 1](#) and [Ref. 2](#). The Reynolds number of the flow, Re_H is equal to 130,000 based on the hill height, H . Synthetic turbulence inlet condition option is enabled to generate isotropic turbulence at the inlet boundary. The nondimensional quantity, coefficient of pressure, denoted as C_p , is validated with those reported in the experiments. Velocity fields at the leeward side and in the wake region of the hill are compared with those obtained in the experiments.

Model Definition

The present section describes the details of the geometry and the flow problem setup. Details required to build the geometry, mesh the domain of interest, and apply suitable solution postprocessing are presented.

GEOMETRY

The domain of interest is a block of dimensions $19.8H$, $3.2H$, and $11.7H$ in the x , y , and z directions. It represents a long rectangular duct with large aspect ratio. The base of the axisymmetric hill is located in the plane defined by $y = 0$. It is centered at a distance of $4.1H$ and $5.85H$ along the x and z directions. The hill has a radius of $2H$ at the base and the crest is located at $y = H$. The hill profile, h , as a function of the hill radius, r , is given by

$$h(r) = \frac{-H}{6.04844} \left[J_0(3.1962) I_0\left(\frac{3.1962 r}{2H}\right) - I_0(3.1962) J_0\left(\frac{3.1962 r}{2H}\right) \right]. \quad (1)$$

Here, J_0 is the Bessel function of the first kind and I_0 is the modified Bessel function of the first kind. The final geometry is shown in [Figure 1](#).

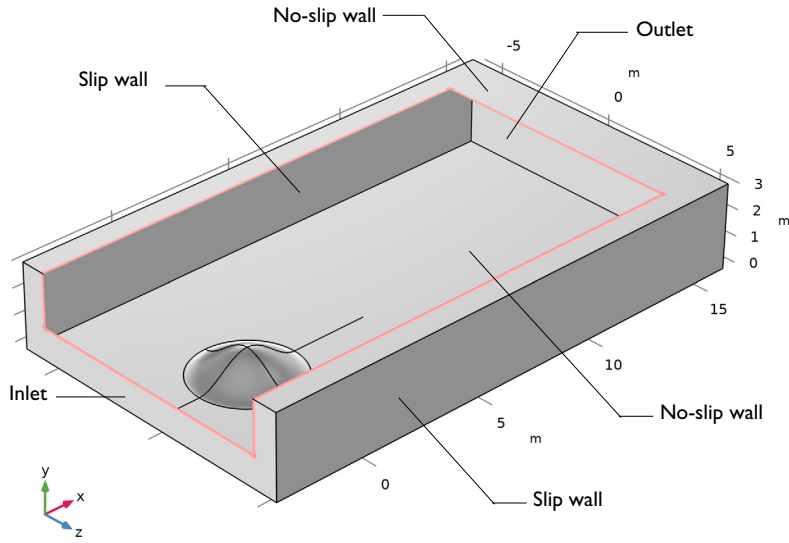


Figure 1: Geometry setup of the model shown with a cut-box to reveal the hill profile. Boundary conditions for the LES problem are also annotated.

PHYSICS INTERFACE SETTINGS

In the model, $H = 1$ m, the density of the fluid, $\rho = 1$ kg/m³, the velocity of the fluid, $U = 2$ m/s, and the dynamic viscosity of the fluid is calculated from the Reynolds number as $\mu = \rho U H / (\text{Re}_H)$ kg/(m·s).

As a first step, the potential flow problem which describes an incompressible, irrotational flow is solved. The potential flow problem takes the form of the Laplace equation for the velocity potential, ϕ , as

$$\nabla \cdot \nabla \phi = 0. \quad (2)$$

The boundary conditions are:

$$\mathbf{n} \cdot \nabla \phi = -U \quad \text{at the inlet boundary,} \quad (3)$$

$$\phi = 0 \quad \text{at the outlet boundary,} \quad (4)$$

$$\mathbf{n} \cdot \nabla \phi = 0 \quad \text{at remaining boundaries.} \quad (5)$$

The LES problem uses the Residual Based Variational Multiscale (RBVM) LES model with automatic wall treatment. The initial values of the velocity and pressure fields for the LES problem are obtained from the potential flow solution as

$$\mathbf{u}_{\text{init}} = \nabla \phi, \quad (6)$$

$$p_{\text{init}} = \frac{\rho}{2}(U^2 - (\nabla \phi \cdot \nabla \phi)). \quad (7)$$

Figure 1 shows the boundary conditions as applied to the problem. A normal inflow velocity condition is prescribed at the inlet. Since the inlet boundary is located close to the hill feature, it has the prescribed velocity profile of the (1/7)th power law for turbulent flows, given by

$$U_0 = \frac{U}{0.75644} \left| \frac{y}{3.2H} \right|^{\frac{1}{7}} \left[1 - \left(\frac{y}{3.2H} \right) \right]^{\frac{1}{7}} \quad (8)$$

The velocity profile is adjusted to ensure that the mass flow rate is equivalent to that produced by a uniform inlet velocity of value U . The synthetic turbulence option at the inlet is enabled and the turbulence length scale is specified as $0.1H$. The number of Fourier modes is chosen to be 600. The outlet condition has the “suppress backflow” option disabled. This permits backflow, which is a common feature in turbulent wake regions.

MESHING

Element sizes close to the hill are chosen so as to obtain a good spatial resolution to capture the geometric feature. The LES simulation requires high resolution mesh elements to transport turbulence generated at the inlet, obtain a good flow representation in the boundary layers, and capture flow features in the wake of the hill. In this context, triangular elements for the hill boundary and mapped quadrilateral elements for the boundary at $y = 0$ are constructed. Further on, a swept mesh for the volume is generated with appropriate element spacing for the boundary layers, see Figure 2. Although such high resolution mesh is not necessary for the potential flow solution, the same mesh is employed as the computational penalty is not restrictive.

The first boundary layer element is of thickness, measured in wall units, equal to $y^+ \approx 4$. The distance from the wall in wall units, y^+ , is computed using the friction velocity, u_τ , the kinematic viscosity, ν , the wall shear stress, τ_w , and the skin-friction coefficient, c_f , as

$$y^+ = \frac{y u_\tau}{\nu}, \quad u_\tau = \sqrt{\frac{\tau_w}{\rho}}, \quad \tau_w = c_f \left(\frac{1}{2} \rho U^2 \right). \quad (9)$$

The skin-friction coefficient as a function of the Reynolds number for channel flows is taken from [Ref. 5](#).

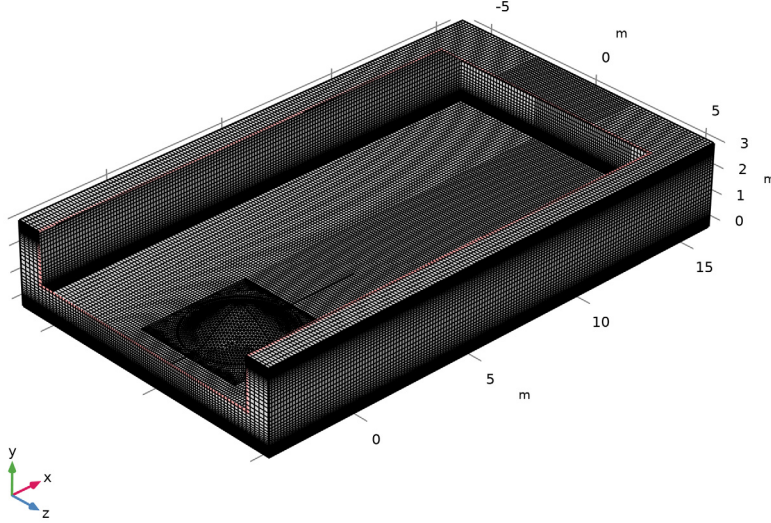


Figure 2: The mesh shown with a cut-box to reveal the surface mesh on the hill profile.

STUDY

The potential flow problem is solved with a stationary study step. The LES problem is solved with a time-dependent study step with the end time equal to $40H/U$, which is the average time taken for a fluid parcel to traverse nearly twice the length of the domain. Four hundred solution time steps are recorded in the latter half of the time interval. These are time-averaged and used for comparison with experiments.

Results and Discussion

The first quantity of postprocess is the coefficient of pressure, given by

$$C_p = \frac{p}{\frac{1}{2}\rho U^2} \quad (10)$$

A contour plot of C_p on the hill surface is shown in [Figure 3](#). The variation of C_p along the line defined by $z = 0$, as a function of x/H , is plotted in [Figure 4](#). The experimental data

reported in Ref. 1 is obtained from Ref. 6, courtesy of Gwibo Byun, Research Scientist, Department of Aerospace and Ocean Engineering, Virginia Tech. The simulation correctly predicts the zone of high suction pressure at the crest of the hill and the zone of the low suction pressure at the base. These zones are slightly larger in the simulation when compared to experiments in Ref. 1. However, they show good correspondence to simulations reported in Ref. 4.

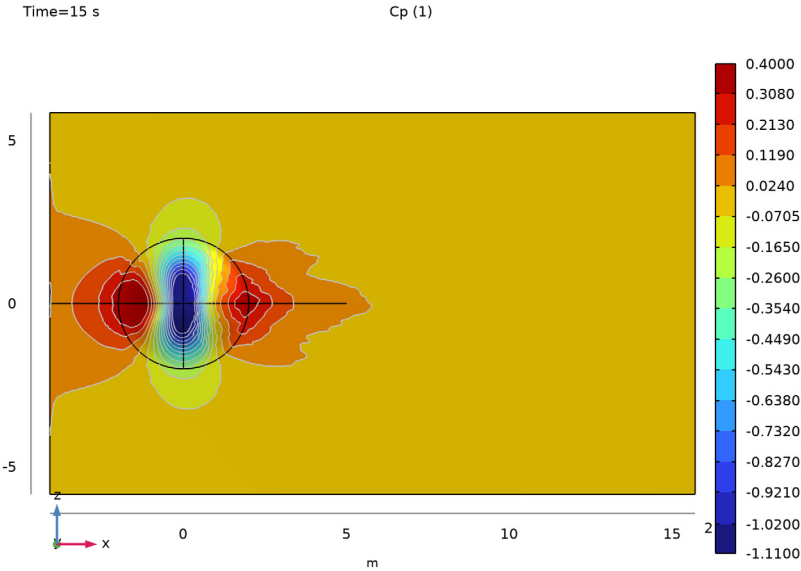


Figure 3: Contour plot of C_p on the hill surface.

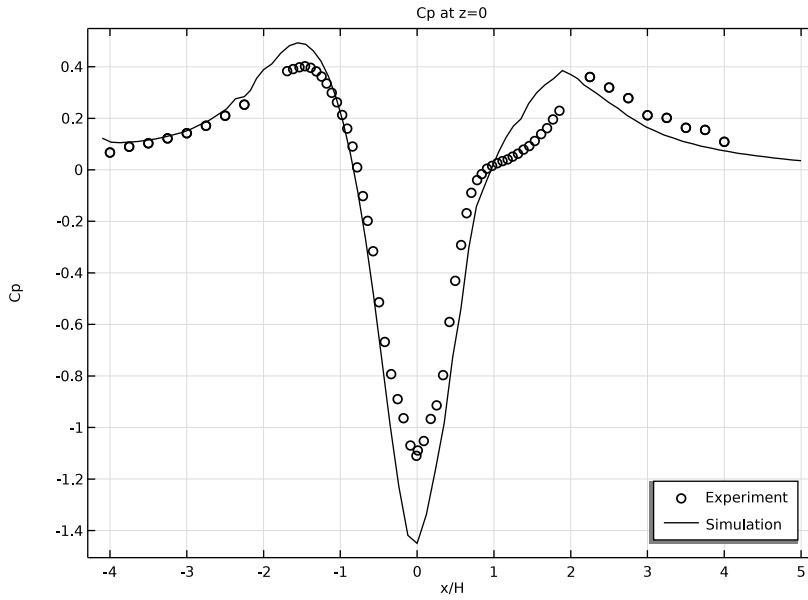


Figure 4: One-dimensional plot of C_p as a function of x/H along the line $z = 0$.

The nondimensional vorticity flux vector, given by the quantity $-(\mathbf{n} \times \nabla C_p)$, is plotted in Figure 5 and shows good qualitative correlation to experimental measurements in Ref. 1. It demonstrates nonuniformity of vorticity on the hill which is observed in the experiments in the form of streamwise and spanwise variations in the separation pattern on the hill surface.

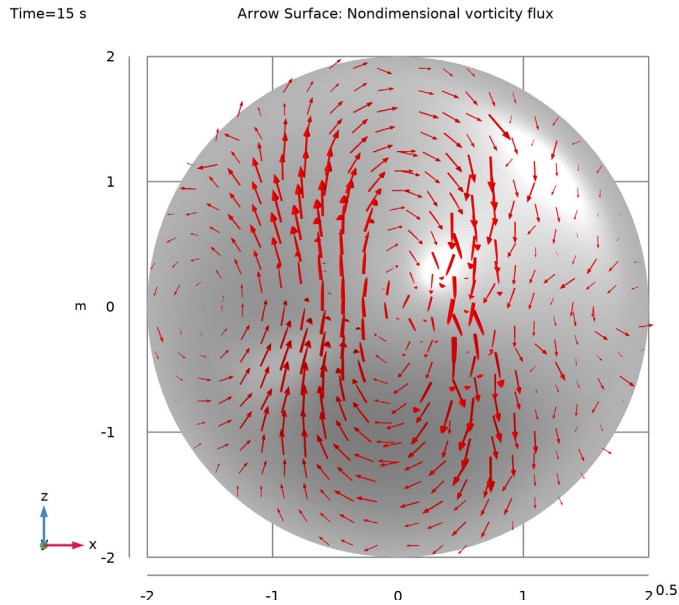


Figure 5: Vector plot of the nondimensional vorticity flux on the hill surface.

The velocity field on the plane defined by $z = 0$ is visualized in [Figure 6](#). The flow is shown to accelerate at the crest of the hill where the suction pressure is the highest. The adverse pressure gradient in the leeward side of the hill results in a backflow region, shown here in magenta color. The simulation overpredicts the length and underpredicts the height of the separation zone at the hill base as reported in experiments (see [Ref. 2](#)).

Arrow Surface: Velocity field at $z=0$. Backflow region in magenta.

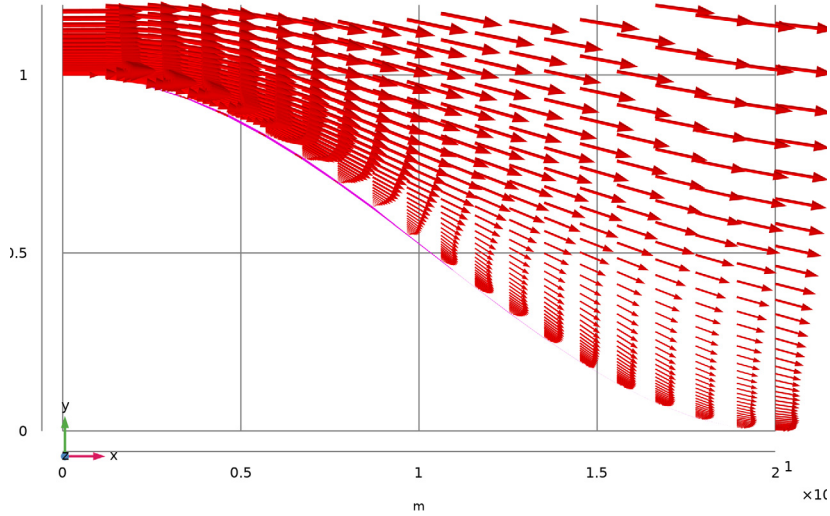


Figure 6: Vector plot of the velocity on the plane defined by $z = 0$. The region of backflow is shown in magenta.

Similarly, the velocity field in the wake region on the plane defined by $x = 3.69H$ is shown in Figure 7. The recirculation bubble is seen attached to the bottom surface as a result of fluid roll-up from the sides of the hill and the down-wash from the top of the hill. The position of the recirculation region is predicted correctly, however, the width is under-predicted when compared to Ref. 4.

Time=15 s

Arrow Surface: Velocity field at $x=3.69H$.

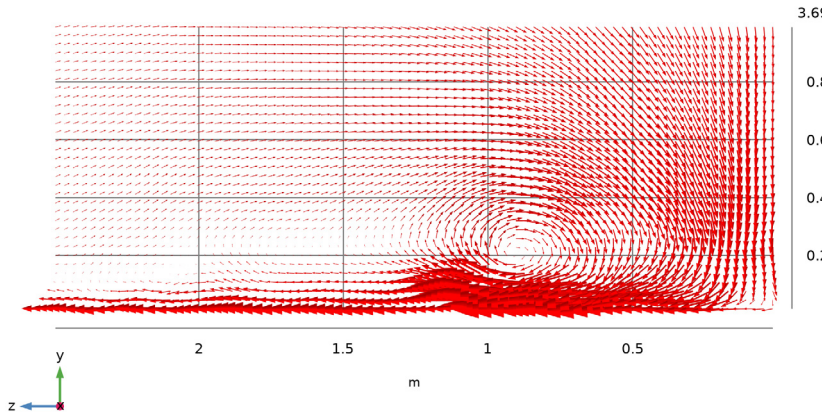


Figure 7: Vector plot of the velocity on the plane defined by $x = 3.69H$.

Finally, the unsteady nature of the flow-field is demonstrated in the plot of instantaneous streamlines, corresponding to the end of the time interval, given in [Figure 8](#). Also shown in the plot is the velocity magnitude on the plane defined by $z = 0$.

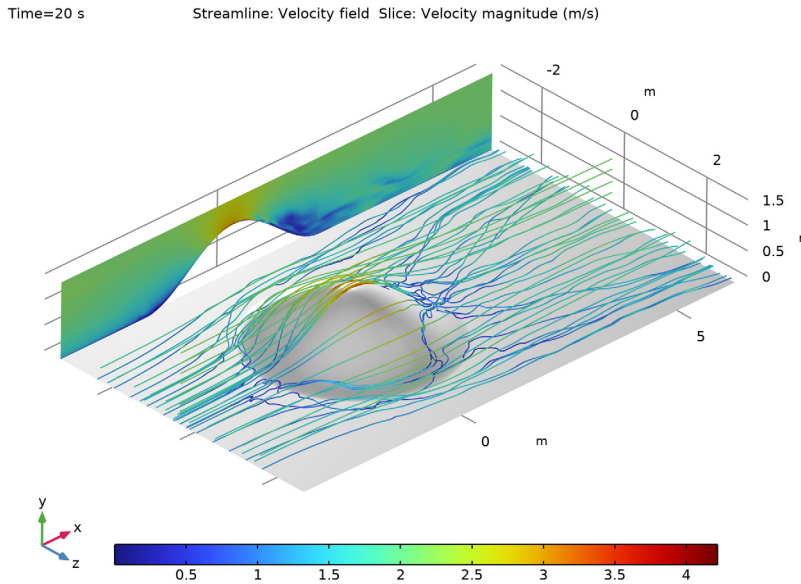


Figure 8: Instantaneous streamlines corresponding to the solution at the end of the simulation. Also shown is the velocity magnitude at the domain midplane defined by $z = 0$.

References


1. R.L. Simpson, C.H. Long, and G. Byun, “Study of Vortical Separation from an Axisymmetric Hill,” *Int. J. Heat Fluid Flow*, vol. 23, pp. 582–591, 2002.
2. G. Byun and R.L. Simpson, “Structure of Three-Dimensional Separated Flow on an Axisymmetric Bump,” *AIAA J.*, vol. 44, no. 5, 2006.
3. L. Davidson, “Using Isotropic Synthetic Fluctuations as Inlet Boundary Conditions for Unsteady Simulations,” *Adv. Appl. Fluid Mech.*, vol. 1, no. 1, pp. 1–35, 2007.
4. T. Persson and others, “Numerical Investigation of the Flow over an Axisymmetric Hill Using LES, DES, and RANS,” *J. Turbul.*, vol. 7, no. 4, 2006.
5. S. Pope, “Turbulent Flows,” *Cambridge University Press*, 2000.
6. G. Byun, *Structure of Vortical Separations*, archive.aoc.vt.edu/gbyun/.

Application Library path: CFD_Module/Single-Phase_Flow/les_3d_hill




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **3D**.
- 2 In the **Select Physics** tree, select **Fluid Flow** > **Single-Phase Flow** > **Large Eddy Simulation** > **LES RBVM (spf)**.
- 3 Click **Add**.
- 4 In the **Select Physics** tree, select **Fluid Flow** > **Single-Phase Flow** > **Potential Flow** > **Incompressible Potential Flow (ipf)**.
- 5 Click **Add**.
- 6 Click  **Study**.
- 7 In the **Select Study** tree, select **Preset Studies for Some Physics Interfaces** > **Stationary**.
- 8 Click  **Done**.

GLOBAL DEFINITIONS

Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:


Name	Expression	Value	Description
H	1[m]	1 m	Hill height
ReH	1.3e5	1.3E5	Reynolds number based on H
rho	1[kg/m^3]	1 kg/m ³	Density of the fluid
U	2[m/s]	2 m/s	Velocity of the fluid

Name	Expression	Value	Description
mu	$\rho \cdot U \cdot H / \text{Re}H$	1.5385E-5 kg/(m·s)	Viscosity of the fluid
LTin	$0.1 \cdot H$	0.1 m	Turbulence length scale at the inlet

DEFINITIONS

Create an analytic function that defines the hill profile.


Axisymmetric Hill Profile

- 1 In the **Definitions** toolbar, click  **Analytic**.
- 2 In the **Settings** window for **Analytic**, type Axisymmetric Hill Profile in the **Label** text field.
- 3 In the **Function name** text field, type hill.
- 4 Locate the **Definition** section. In the **Expression** text field, type $-(\text{besselj}(0, 3.1962) * \text{besseli}(0, 3.1962 * x / (2 * H)) - \text{besseli}(0, 3.1962) * \text{besselj}(0, 3.1962 * x / (2 * H))) * H / 6.04844$.
- 5 Locate the **Plot Parameters** section. In the table, enter the following settings:



Plot	Argument	Lower limit	Upper limit	Fixed value	Unit
√	x	0	2*H	0	

GEOMETRY I


Block 1 (blk1)

- 1 In the **Geometry** toolbar, click  **Block**.
- 2 In the **Settings** window for **Block**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type $19.8 * H$.
- 4 In the **Depth** text field, type $3.2 * H$.
- 5 In the **Height** text field, type $11.7 * H$.
- 6 Locate the **Position** section. In the **x** text field, type $-4.1 * H$.
- 7 In the **z** text field, type $-11.7 * H / 2$.


Work Plane 1 (wpl)

- 1 In the **Geometry** toolbar, click  **Work Plane**.
- 2 In the **Settings** window for **Work Plane**, locate the **Plane Definition** section.
- 3 From the **Plane** list, choose **xz-plane**.
- 4 Click  **Go to Plane Geometry**.


Work Plane 1 (wp1) > Square 1 (sq1)

- 1 In the **Work Plane** toolbar, click  **Square**.
- 2 In the **Settings** window for **Square**, locate the **Size** section.
- 3 In the **Side length** text field, type 4.5*H.
- 4 Locate the **Position** section. From the **Base** list, choose **Center**.



Work Plane 1 (wp1) > Rectangle 1 (r1)

- 1 In the **Work Plane** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type 19.8*H.
- 4 In the **Height** text field, type 4.5*H.
- 5 Locate the **Position** section. In the **yw** text field, type -2.25*H.
- 6 In the **xw** text field, type -4.1*H.


Line Segment 1 (ls1)

- 1 In the **Model Builder** window, right-click **Geometry 1** and choose **More Primitives > Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 Click to clear the  **Activate Selection** toggle button for **Start vertex**.
- 4 From the **Specify** list, choose **Coordinates**.
- 5 In the **x** text field, type -4.1*H.
- 6 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 7 In the **x** text field, type 5*H.
- 8 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** checkbox.

Work Plane 2 (wp2)


- 1 In the **Geometry** toolbar, click  **Work Plane**.
- 2 In the **Settings** window for **Work Plane**, locate the **Plane Definition** section.
- 3 From the **Plane** list, choose **zy-plane**.
- 4 Click  **Go to Plane Geometry**.

Work Plane 2 (wp2) > Parametric Curve 1 (pc1)


- 1 In the **Work Plane** toolbar, click  **More Primitives** and choose **Parametric Curve**.
- 2 In the **Settings** window for **Parametric Curve**, locate the **Parameter** section.
- 3 In the **Maximum** text field, type 2*H.

- 4 Locate the **Expressions** section. In the **xw** text field, type **s**.
- 5 In the **yw** text field, type **hill(s)**.



Work Plane 2 (wp2) > Line Segment 1 (ls1)

- 1 In the **Work Plane** toolbar, click  **More Primitives** and choose **Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 In the **xw** text field, type **2*H**.

Work Plane 2 (wp2) > Line Segment 2 (ls2)

- 1 In the **Work Plane** toolbar, click  **More Primitives** and choose **Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 In the **yw** text field, type **H**.




Work Plane 2 (wp2) > Convert to Solid 1 (csol1)

- 1 In the **Work Plane** toolbar, click  **Conversions** and choose **Convert to Solid**.
- 2 In the **Settings** window for **Convert to Solid**, locate the **Input** section.
- 3 Click the  **Paste Selection** button for **Input objects**.
- 4 In the **Paste Selection** dialog, type **ls1 ls2 pc1** in the **Selection** text field.
- 5 Click **OK**.

Revolve 1 (rev1)

In the **Model Builder** window, under **Component 1 (comp1) > Geometry 1** right-click **Work Plane 2 (wp2)** and choose **Revolve**.

Difference 1 (dif1)


- 1 In the **Geometry** toolbar, click  **Booleans and Partitions** and choose **Difference**.
- 2 In the **Settings** window for **Difference**, locate the **Difference** section.
- 3 Click the  **Paste Selection** button for **Objects to add**.
- 4 In the **Paste Selection** dialog, type **blk1 wp1 ls1** in the **Selection** text field.
- 5 Click **OK**.
- 6 In the **Settings** window for **Difference**, locate the **Difference** section.
- 7 Click the  **Paste Selection** button for **Objects to subtract**.

8 In the **Paste Selection** dialog, type rev1 in the **Selection** text field.

9 Click **OK**.

Mesh Control Edges I (mce I)

Designate as mesh control entities those edges that are only used in mesh creation.


1 In the **Geometry** toolbar, click  **Virtual Operations** and choose **Mesh Control Edges**.

2 In the **Settings** window for **Mesh Control Edges**, locate the **Input** section.

3 Click the  **Paste Selection** button for **Edges to include**.

4 In the **Paste Selection** dialog, type 5, 9, 15-17, 19, 29-31, 33 in the **Selection** text field.

5 Click **OK**.

6 In the **Geometry** toolbar, click  **Build All**.

DEFINITIONS

Create an edge selection for plotting C_p values.

Edge for C_p plot

1 In the **Definitions** toolbar, click  **Explicit**.

2 In the **Settings** window for **Explicit**, type Edge for C_p plot in the **Label** text field.

3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Edge**.

4 Select Edges 5, 13, 18, and 19 only.

INCOMPRESSIBLE POTENTIAL FLOW (IPF)

Set up the Incompressible Potential Flow interface to solve for the velocity potential and to get an initial approximation for the velocity and pressure. Use a linear approximation to reduce the memory requirements.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Incompressible Potential Flow (ipf)**.

2 In the **Settings** window for **Incompressible Potential Flow**, locate the **Pressure** section.

3 In the U_{scale} text field, type U.

4 Click to expand the **Discretization** section. From the **Velocity potential** list, choose **Linear**.

Fluid Properties I

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Incompressible Potential Flow (ipf)** click **Fluid Properties I**.

2 In the **Settings** window for **Fluid Properties**, locate the **Fluid Properties** section.

3 From the ρ list, choose **User defined**. In the associated text field, type rho.

Velocity 1

1 In the **Physics** toolbar, click  **Boundaries** and choose **Velocity**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Velocity**, locate the **Velocity** section.

4 In the U_{in} text field, type U.

Open Boundary 1

1 In the **Physics** toolbar, click  **Boundaries** and choose **Open Boundary**.

2 Select Boundary 10 only.

LES RBVM (SPF)

Set up the Large Eddy Simulation problem.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **LES RBVM (spf)**.

2 In the **Settings** window for **LES RBVM**, locate the **Turbulence** section.

3 From the **Wall treatment** list, choose **Automatic**.

Fluid Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **LES RBVM (spf)** click **Fluid Properties 1**.

2 In the **Settings** window for **Fluid Properties**, locate the **Fluid Properties** section.

3 From the ρ list, choose **User defined**. In the associated text field, type rho.

4 From the μ list, choose **User defined**. In the associated text field, type mu.

Initial Values 1

Use the stationary solution of the potential-flow problem to compute the initial values.

Initial Values 2

1 In the **Model Builder** window, under **Component 1 (comp1)** > **LES RBVM (spf)** right-click **Initial Values 1** and choose **Duplicate**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 Specify the **u** vector as

ipf.u	x
ipf.v	y
ipf.w	z

4 In the p text field, type `ipf.p`.

Wall 2

1 In the **Physics** toolbar, click  **Boundaries** and choose **Wall**.

Use a slip wall condition on the lateral sides of the channel.

2 In the **Settings** window for **Wall**, locate the **Boundary Condition** section.

3 From the **Wall condition** list, choose **Slip**.

4 Select Boundaries 3 and 4 only.

Inlet 1

Specify the inlet velocity profile based on the (1/7)th power law for turbulent flows. Include synthetic turbulence effects.

1 In the **Physics** toolbar, click  **Boundaries** and choose **Inlet**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Inlet**, locate the **Velocity** section.

4 In the U_0 text field, type $U \cdot \text{abs}(y / (3.2 \cdot H))^{(1/7)} \cdot \text{abs}(1 - (y / (3.2 \cdot H)))^{(1/7)} / 0.75644$.

5 Select the **Include synthetic turbulence** checkbox.

6 Locate the **Turbulence Conditions** section. From the I_T list, choose **Low (0.01)**.

7 From the L_T list, choose **User defined**.

8 In the text field, type `LT.in`.

9 In the N text field, type `600`.

Outlet 1

1 In the **Physics** toolbar, click  **Boundaries** and choose **Outlet**.

2 Select Boundary 10 only.

3 In the **Settings** window for **Outlet**, locate the **Pressure Conditions** section.

4 Clear the **Suppress backflow** checkbox.

MESH 1


Create a mesh suitable for the LES simulation.

Identical Mesh 1

1 In the **Mesh** toolbar, click  **Sizing** and choose **Identical Mesh**.

2 Select Boundaries 7, 9, and 14 only.

3 In the **Settings** window for **Identical Mesh**, locate the **Second Entity Group** section.


- 4 Click to select the  **Activate Selection** toggle button.
- 5 Select Boundaries 6, 8, and 15 only.

Size

- 1 In the **Model Builder** window, click **Size**.
- 2 In the **Settings** window for **Size**, locate the **Element Size** section.
- 3 From the **Calibrate for** list, choose **Fluid dynamics**.
- 4 Click the **Custom** button.
- 5 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 0.1222*H.
- 6 In the **Minimum element size** text field, type 0.0390*H.

Mapped 1

Create a boundary mesh with Mapped and Triangular meshes.

- 1 In the **Mesh** toolbar, click  **More Generators** and choose **Mapped**.
- 2 Select Boundaries 2, 11–13, and 16 only.

Distribution 1


- 1 Right-click **Mapped 1** and choose **Distribution**.
- 2 Select Edges 4, 24, 36, and 38 only.
- 3 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 4 In the **Number of elements** text field, type 25.

Distribution 2


- 1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.
- 2 Select Edge 1 only.
- 3 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 4 From the **Distribution type** list, choose **Predefined**.
- 5 In the **Number of elements** text field, type 25.
- 6 In the **Element ratio** text field, type 2.5.
- 7 From the **Growth rate** list, choose **Exponential**.
- 8 Select the **Reverse direction** checkbox.

Distribution 3

- 1 Right-click **Distribution 2** and choose **Duplicate**.
- 2 In the **Settings** window for **Distribution**, locate the **Edge Selection** section.


- 3 Click  **Clear Selection**.
- 4 Select Edge 25 only.
- 5 Locate the **Distribution** section. Clear the **Reverse direction** checkbox.

Free Triangular I

- 1 In the **Mesh** toolbar, click  **More Generators** and choose **Free Triangular**.
- 2 Select Boundaries 6–9, 14, and 15 only.

Swept I

Sweep the boundary mesh to obtain the volume mesh.

In the **Mesh** toolbar, click  **Swept**.

Distribution I

- 1 Right-click **Swept I** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 From the **Distribution type** list, choose **Predefined**.
- 4 In the **Number of elements** text field, type 82.
- 5 In the **Element ratio** text field, type 82.
- 6 From the **Growth rate** list, choose **Exponential**.
- 7 Select the **Symmetric distribution** checkbox.
- 8 In the **Model Builder** window, right-click **Mesh I** and choose **Build All**.



STUDY 1: STATIONARY POTENTIAL-FLOW SOLUTION

Solve for a stationary solution to the potential-flow problem. This solution is used in the initial value for the LES problem.

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, locate the **Study Settings** section.
- 3 Clear the **Generate default plots** checkbox.
- 4 In the **Label** text field, type Study 1: Stationary Potential-Flow Solution.



Step 1: Stationary

- 1 In the **Model Builder** window, under **Study 1: Stationary Potential-Flow Solution** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 3 In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **LES RBVM (spf)**.

- 4 In the **Study** toolbar, click  **Compute**.
Disable the **Initial Values 2** node of the LES RBVM interface in the potential-flow study.
- 5 Select the **Modify model configuration for study step** checkbox.
- 6 In the tree, select **Component 1 (comp1) > LES RBVM (spf) > Initial Values 2**.
- 7 Click  **Disable**.

ADD STUDY

Solve the time-dependent study for the LES problem.

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** checkbox for **Incompressible Potential Flow (ipf)**.
- 4 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies > Time Dependent**.
- 5 Click the **Add Study** button in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

STUDY 2: TIME-DEPENDENT LES SOLUTION

- 1 In the **Settings** window for **Study**, locate the **Study Settings** section.
- 2 Clear the **Generate default plots** checkbox.
- 3 In the **Label** text field, type Study 2: Time-Dependent LES Solution.
- 4 Locate the **Study Settings** section. Select the **Store solution for all intermediate study steps** checkbox.


Step 1: Time Dependent

Set the end time of the time-dependent study to $40H/U = 20$ seconds. Store solutions in the range 10–20 s

- 1 In the **Model Builder** window, under **Study 2: Time-Dependent LES Solution** click **Step 1: Time Dependent**.
- 2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 3 In the **Output times** text field, type range (0, 20*H/U, 20*H/U) range (20*H/U, (20*H/U) / 400, 40*H/U).
- 4 Click to expand the **Values of Dependent Variables** section. Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.



- 5 From the **Study** list, choose **Study 1: Stationary Potential-Flow Solution, Stationary**.
- 6 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 7 From the **Method** list, choose **Solution**.
- 8 From the **Study** list, choose **Study 1: Stationary Potential-Flow Solution, Stationary**.

Step 2: Combine Solutions

- 1 In the **Study** toolbar, click  **More Study Extensions** and choose **Combine Solutions**.
- 2 In the **Settings** window for **Combine Solutions**, locate the **Combine Solutions Settings** section.
- 3 From the **Solution operation** list, choose **Remove solutions**.
- 4 From the **Selection** list, choose **Manual**.
- 5 In the **Index** text field, type 1 2.
- 6 Select the **Clear source solution** checkbox.



Solution 2 (sol2)

Use automatic time-stepping to speed up the simulation.

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node, then click **Time-Dependent Solver 1**.
- 3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.
- 4 From the **Maximum step constraint** list, choose **Automatic**.
- 5 In the **Study** toolbar, click  **Compute**.

ADD STUDY



Compute the average of the stored solutions from the previous step.

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

STUDY 3: TIME-AVERAGED LES SOLUTION

- 1 In the **Settings** window for **Study**, type Study 3: Time-Averaged LES Solution in the **Label** text field.
- 2 Locate the **Study Settings** section. Clear the **Generate convergence plots** checkbox.
- 3 Clear the **Generate default plots** checkbox.


Step 1: Combine Solutions

- 1 In the **Study** toolbar, click  **More Study Extensions** and choose **Combine Solutions**.
- 2 In the **Settings** window for **Combine Solutions**, locate the **Combine Solutions Settings** section.
- 3 From the **Solution operation** list, choose **Weighted summation**.
- 4 From the **Solution** list, choose **Study 2: Time-Dependent LES Solution/Solution 2 (sol2)**.
- 5 In the **Expression** text field, type 1.0/401.
- 6 In the **Study** toolbar, click  **Compute**.

STUDY 2: TIME-DEPENDENT LES SOLUTION

Remove the solution except for the last time step to reduce file size.

Step 3: Combine Solutions 2

- 1 In the **Study** toolbar, click  **More Study Extensions** and choose **Combine Solutions**.
- 2 In the **Settings** window for **Combine Solutions**, locate the **Combine Solutions Settings** section.
- 3 From the **Solution operation** list, choose **Remove solutions**.
- 4 From the **Exclude or include** list, choose **Include**.
- 5 From the **Time (s)** list, choose **Manual**.
- 6 In the **Index** text field, type 401.
- 7 Select the **Clear source solution** checkbox.
- 8 Right-click **Step 3: Combine Solutions 2** and choose **Reset Solver to Default for Selected Step**.
- 9 Right-click **Step 3: Combine Solutions 2** and choose **Compute Selected Step**.

RESULTS



Create 1D and 2D datasets from the solution dataset for use in plotting.

Exterior Walls


- 1 In the **Model Builder** window, expand the **Results** node.

- 2 Right-click **Results** > **Datasets** and choose **Surface**.
- 3 In the **Settings** window for **Surface**, type Exterior Walls in the **Label** text field.
- 4 Locate the **Data** section. From the **Dataset** list, choose **Study 3: Time-Averaged LES Solution/Solution 4 (sol4)**.
- 5 Locate the **Selection** section. From the **Selection** list, choose **All boundaries**.


Edge at z=0

- 1 In the **Results** toolbar, click  **More Datasets** and choose **Edge 3D**.
- 2 In the **Settings** window for **Edge 3D**, type Edge at z=0 in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 3: Time-Averaged LES Solution/Solution 4 (sol4)**.
- 4 Locate the **Selection** section. Click  **Paste Selection**.
- 5 In the **Paste Selection** dialog, type 5 13 18 19 in the **Selection** text field.
- 6 Click **OK**.

Cut Plane at z=0

- 1 In the **Results** toolbar, click  **Cut Plane**.
- 2 In the **Settings** window for **Cut Plane**, type Cut Plane at z=0 in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 3: Time-Averaged LES Solution/Solution 4 (sol4)**.
- 4 Locate the **Plane Data** section. From the **Plane** list, choose **xy-planes**.

*Cut Plane at x=3.69*H*

- 1 In the **Results** toolbar, click  **Cut Plane**.
- 2 In the **Settings** window for **Cut Plane**, type Cut Plane at x=3.69*H in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 3: Time-Averaged LES Solution/Solution 4 (sol4)**.
- 4 Locate the **Plane Data** section. In the **x-coordinate** text field, type 3.69*H.

Experimental data

Import experimental data.

- 1 In the **Results** toolbar, click  **Table**.
- 2 In the **Settings** window for **Table**, type Experimental data in the **Label** text field.
- 3 Locate the **Data** section. Click  **Import**.

4 Browse to the model's Application Libraries folder and double-click the file `les_3d_hill_cp.txt`.


Create a 1D plot of the coefficient of pressure along the edge at $z=0$.

5 In the **Model Builder** window, click **Results**.

6 In the **Settings** window for **Results**, locate the **Update of Results** section.

7 Select the **Only plot when requested** checkbox.

C_p at z=0

1 In the **Results** toolbar, click  **ID Plot Group**.

2 In the **Settings** window for **ID Plot Group**, type *C_p at z=0* in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 2: Time-Dependent LES Solution/Solution 2 (sol2)**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

5 In the **Title** text area, type *C_p at z=0*.

6 Locate the **Plot Settings** section.

7 Select the **x-axis label** checkbox. In the associated text field, type x/H .

8 Select the **y-axis label** checkbox. In the associated text field, type *C_p*.

9 Locate the **Legend** section. From the **Position** list, choose **Lower right**.

Experiment

1 Right-click **C_p at z=0** and choose **Table Graph**.

2 In the **Settings** window for **Table Graph**, type *Experiment* in the **Label** text field.

3 Locate the **Data** section. From the **x-axis data** list, choose x/H .

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 From the **Color** list, choose **Black**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

7 Click to expand the **Legends** section. Select the **Show legends** checkbox.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
<i>Experiment</i>


Simulation

- 1 In the **Model Builder** window, right-click **Cp at z=0** and choose **Line Graph**.
- 2 In the **Settings** window for **Line Graph**, type **Simulation** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Edge at z=0**.
- 4 Locate the **y-Axis Data** section. In the **Expression** text field, type $p / (0.5 * \rho * U^2)$.
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type x/H .
- 7 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.
- 8 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 9 From the **Legends** list, choose **Manual**.
- 10 In the table, enter the following settings:

Legends

Simulation



Cp at z=0

- 1 In the **Model Builder** window, click **Cp at z=0**.
- 2 In the **Cp at z=0** toolbar, click  **Plot**.

Cp Contours on the hill

Create a 2D contour plot of the coefficient of pressure on the 3D hill.

Create a 2D contour plot of the coefficient of pressure on the 3D hill.

- 1 In the **Results** toolbar, click  **3D Plot Group**.
- 2 In the **Settings** window for **3D Plot Group**, type **Cp Contours on the hill** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 3: Time-Averaged LES Solution/Solution 4 (sol4)**.
- 4 Locate the **Plot Settings** section. From the **View** list, choose **New view**.
- 5 In the **Cp Contours on the hill** toolbar, click  **Plot**.
- 6 Click to expand the **Number Format** section. Select the **Manual color legend settings** checkbox.
- 7 Select the **Show trailing zeros** checkbox.
- 8 In the **Precision** text field, type 5.


Contour 1

- 1 Right-click **Cp Contours on the hill** and choose **Contour**.
- 2 In the **Settings** window for **Contour**, locate the **Expression** section.
- 3 In the **Expression** text field, type $p / (0.5 * \rho * U^2)$.
- 4 Select the **Description** checkbox. In the associated text field, type Cp.
- 5 Locate the **Levels** section. From the **Entry method** list, choose **Levels**.
- 6 In the **Levels** text field, type 3.08E-01 2.13E-01 1.19E-01 2.40E-02 -7.05E-02 -1.65E-01 -2.60E-01 -3.54E-01 -4.49E-01 -5.43E-01 -6.38E-01 -7.32E-01 -8.27E-01 -9.21E-01 -1.02E+00.
- 7 Locate the **Coloring and Style** section. From the **Contour type** list, choose **Filled**.
- 8 From the **Legend type** list, choose **Filled**.

Contour 2

- 1 In the **Model Builder** window, right-click **Cp Contours on the hill** and choose **Contour**.
- 2 In the **Settings** window for **Contour**, locate the **Expression** section.
- 3 In the **Expression** text field, type $p / (0.5 * \rho * U^2)$.
- 4 Select the **Description** checkbox. In the associated text field, type Cp.
- 5 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 6 Locate the **Levels** section. From the **Entry method** list, choose **Levels**.
- 7 In the **Levels** text field, type 3.08E-01 2.13E-01 1.19E-01 2.40E-02 -7.05E-02 -1.65E-01 -2.60E-01 -3.54E-01 -4.49E-01 -5.43E-01 -6.38E-01 -7.32E-01 -8.27E-01 -9.21E-01 -1.02E+00.
- 8 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 9 From the **Color** list, choose **Gray**.
- 10 Clear the **Color legend** checkbox.

Cp Contours on the hill



- 1 In the **Model Builder** window, click **Cp Contours on the hill**.
- 2 In the **Cp Contours on the hill** toolbar, click  **Plot**.

Modify the view to obtain [Figure 3](#).

Nondimensional Vorticity Flux

Create a 2D contour plot of the nondimensional vorticity flux.

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **3D Plot Group**.

- 2 In the **Settings** window for **3D Plot Group**, type **Nondimensional Vorticity Flux** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 3: Time-Averaged LES Solution/Solution 4 (sol4)**.
- 4 Locate the **Plot Settings** section. From the **View** list, choose **New view**.
- 5 In the **Nondimensional Vorticity Flux** toolbar, click  **Plot**.
- 6 Clear the **Plot dataset edges** checkbox.
- 7 Click to expand the **Selection** section. From the **Geometric entity level** list, choose **Boundary**.
- 8 Click  **Paste Selection**.
- 9 In the **Paste Selection** dialog, type 6 7 8 9 in the **Selection** text field.
- 10 Click **OK**.

Surface 1


- 1 Right-click **Nondimensional Vorticity Flux** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type 1.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 6 From the **Color** list, choose **Gray**.

Arrow Surface 1

- 1 In the **Model Builder** window, right-click **Nondimensional Vorticity Flux** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.
- 3 In the **x-component** text field, type $(nz*dtang(\rho, y) - ny*dtang(\rho, z)) / (0.5*\rho*U^2)$.
- 4 In the **y-component** text field, type $(nx*dtang(\rho, z) - nz*dtang(\rho, x)) / (0.5*\rho*U^2)$.
- 5 In the **z-component** text field, type $(ny*dtang(\rho, x) - nx*dtang(\rho, y)) / (0.5*\rho*U^2)$.
- 6 From the **Components to plot** list, choose **Tangential**.
- 7 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
- 8 In the **Title** text area, type **Arrow Surface: Nondimensional vorticity flux**.

- 9 Locate the **Arrow Positioning** section. In the **Number of arrows** text field, type 400.
- 10 Locate the **Coloring and Style** section.
- 11 Select the **Scale factor** checkbox. In the associated text field, type 0.25.



Nondimensional Vorticity Flux

- 1 In the **Model Builder** window, click **Nondimensional Vorticity Flux**.
- 2 In the **Nondimensional Vorticity Flux** toolbar, click  **Plot**.

Modify the view to obtain [Figure 5](#).

Velocity Vectors at $z=0$

Create a 2D plot showing the velocity vectors on the leeward side of the hill and the region of backward flow on the plane defined by $z = 0$.

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **3D Plot Group**.
- 2 In the **Settings** window for **3D Plot Group**, type Velocity Vectors at $z=0$ in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **None**.
- 4 Locate the **Plot Settings** section. From the **View** list, choose **New view**.
- 5 In the **Velocity Vectors at $z=0$** toolbar, click  **Plot**.

Arrow Surface 1

- 1 Right-click **Velocity Vectors at $z=0$** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Cut Plane at $z=0$** .
- 4 Locate the **Expression** section. In the **x-component** text field, type u .
- 5 In the **y-component** text field, type v .
- 6 In the **z-component** text field, type w .
- 7 From the **Components to plot** list, choose **Tangential**.
- 8 Locate the **Title** section. From the **Title type** list, choose **Manual**.
- 9 In the **Title** text area, type Arrow Surface: Velocity field at $z=0$.
- 10 Locate the **Arrow Positioning** section. From the **Placement** list, choose **Mesh vertices**.
- 11 Locate the **Coloring and Style** section.
- 12 Select the **Scale factor** checkbox. In the associated text field, type 0.08.

Filter 1

- 1 Right-click **Arrow Surface 1** and choose **Filter**.

- 2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
- 3 In the **Logical expression for inclusion** text field, type $(x >= 0) \ \&\& \ (x <= 2 * H) \ \&\& \ (y <= 1.2 * H)$.


Surface 1

- 1 In the **Model Builder** window, right-click **Velocity Vectors at z=0** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Cut Plane at z=0**.
- 4 Locate the **Expression** section. In the **Expression** text field, type 1.
- 5 Locate the **Title** section. From the **Title type** list, choose **Manual**.
- 6 In the **Title** text area, type Backflow region in magenta..
- 7 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 8 From the **Color** list, choose **Magenta**.

Filter 1

- 1 Right-click **Surface 1** and choose **Filter**.
- 2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
- 3 In the **Logical expression for inclusion** text field, type $(u <= 0) \ \&\& \ (x >= 0) \ \&\& \ (x <= 2 * H) \ \&\& \ (y <= 1.2 * H)$.



Velocity Vectors at z=0

- 1 In the **Model Builder** window, under **Results** click **Velocity Vectors at z=0**.
- 2 In the **Velocity Vectors at z=0** toolbar, click  **Plot**.

Modify the view to obtain [Figure 6](#).

Velocity Vectors at x=3.69H

Create a 2D plot showing the velocity vectors in the wake region of the hill on the plane defined by $x = 3.69H$.

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **3D Plot Group**.
- 2 In the **Settings** window for **3D Plot Group**, type Velocity Vectors at x=3.69H in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **None**.
- 4 Locate the **Plot Settings** section. From the **View** list, choose **New view**.
- 5 In the **Velocity Vectors at x=3.69H** toolbar, click  **Plot**.


Arrow Surface 1

- 1 Right-click **Velocity Vectors at $x=3.69H$** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Cut Plane at $x=3.69*H$** .
- 4 Locate the **Expression** section. In the **x-component** text field, type u .
- 5 In the **y-component** text field, type v .
- 6 In the **z-component** text field, type w .
- 7 From the **Components to plot** list, choose **Tangential**.
- 8 Locate the **Title** section. From the **Title type** list, choose **Manual**.
- 9 In the **Title** text area, type Arrow Surface: Velocity field at $x=3.69H$.
- 10 Locate the **Arrow Positioning** section. In the **Number of arrows** text field, type $4e4$.
- 11 Locate the **Coloring and Style** section.
- 12 Select the **Scale factor** checkbox. In the associated text field, type 0.6 .

Filter 1

- 1 Right-click **Arrow Surface 1** and choose **Filter**.
- 2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
- 3 In the **Logical expression for inclusion** text field, type $(y >= 0) \ \&\& \ (y <= H) \ \&\& \ (z >= 0) \ \&\& \ (z <= 2.5 * H)$.



Velocity Vectors at $x=3.69H$

- 1 In the **Model Builder** window, under **Results** click **Velocity Vectors at $x=3.69H$** .
- 2 In the **Velocity Vectors at $x=3.69H$** toolbar, click  **Plot**.

Modify the view to obtain [Figure 7](#).

Instantaneous Streamlines

Create a 3D plot showing instantaneous streamlines.

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **3D Plot Group**.
- 2 In the **Settings** window for **3D Plot Group**, type Instantaneous Streamlines in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 2: Time-Dependent LES Solution/Solution 2 (sol2)**.
- 4 Locate the **Plot Settings** section. From the **View** list, choose **New view**.
- 5 In the **Instantaneous Streamlines** toolbar, click  **Plot**.

6 Locate the **Color Legend** section. From the **Position** list, choose **Bottom**.

Surface 1

- 1 Right-click **Instantaneous Streamlines** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type 1.
- 4 Locate the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 6 From the **Color** list, choose **White**.

Filter 1

- 1 Right-click **Surface 1** and choose **Filter**.
- 2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
- 3 In the **Logical expression for inclusion** text field, type $(x \geq -4 \cdot H) \ \&\& \ (x \leq 6 \cdot H) \ \&\& \ (z > -3 \cdot H) \ \&\& \ (z \leq 3 \cdot H) \ \&\& \ (y > 0) \ \&\& \ (y \leq 1.5 \cdot H)$.

Instantaneous Streamlines

- 1 In the **Model Builder** window, under **Results** click **Instantaneous Streamlines**.
- 2 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.
- 3 Clear the **Plot dataset edges** checkbox.

Streamline 1

- 1 Right-click **Instantaneous Streamlines** and choose **Streamline**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 From the **Positioning** list, choose **Starting-point controlled**.
- 4 From the **Entry method** list, choose **Coordinates**.
- 5 In the **x** text field, type $-4 \cdot H$.
- 6 In the **y** text field, type $0.01 \cdot H$.
- 7 In the **z** text field, type $\text{range}(-2 \cdot H, 4 \cdot H / 10, 2 \cdot H)$.
- 8 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **Tube**.
- 9 In the **Tube radius expression** text field, type $0.01 \cdot H$.
- 10 Select the **Radius scale factor** checkbox.

Color Expression 1

Right-click **Streamline 1** and choose **Color Expression**.

Filter 1

- 1 In the **Model Builder** window, right-click **Streamline 1** and choose **Filter**.
- 2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
- 3 In the **Logical expression for inclusion** text field, type $(x >= -4 * H) \ \&\& \ (x <= 6 * H) \ \&\& \ (z > = -3 * H) \ \&\& \ (z <= 3 * H) \ \&\& \ (y >= 0) \ \&\& \ (y <= 1.5 * H)$.

Streamline 2

- 1 Right-click **Streamline 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Streamline**, click to expand the **Title** section.
- 3 From the **Title type** list, choose **None**.
- 4 Locate the **Streamline Positioning** section. In the **y** text field, type range $(0, 0.2 * H, 1.6 * H)$.
- 5 In the **z** text field, type 0.
- 6 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Streamline 1**.

Streamline 3

- 1 Right-click **Streamline 2** and choose **Duplicate**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **x** text field, type $2 * H$.
- 4 In the **y** text field, type $0.01 * H$.
- 5 In the **z** text field, type range $(-2 * H, 4 * H / 10, 2 * H)$.

Streamline 4

- 1 Right-click **Streamline 3** and choose **Duplicate**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **y** text field, type $0.1 * H$.

Streamline 5

- 1 Right-click **Streamline 4** and choose **Duplicate**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **y** text field, type $0.3 * H$.

Slice 1

- 1 In the **Model Builder** window, right-click **Instantaneous Streamlines** and choose **Slice**.
- 2 In the **Settings** window for **Slice**, locate the **Plane Data** section.
- 3 From the **Plane** list, choose **xy-planes**.

- 4 In the **Planes** text field, type 1.
- 5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Streamline I**.


Transformation I

- 1 Right-click **Slice I** and choose **Transformation**.
- 2 In the **Settings** window for **Transformation**, locate the **Transformation** section.
- 3 In the **z** text field, type $-2.9 * H$.

Filter I

- 1 In the **Model Builder** window, right-click **Slice I** and choose **Filter**.
- 2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
- 3 In the **Logical expression for inclusion** text field, type $(x >= -4 * H) \ \&\& \ (x <= 6 * H) \ \&\& \ (z > = -3 * H) \ \&\& \ (z <= 3 * H) \ \&\& \ (y >= 0) \ \&\& \ (y <= 1.5 * H)$.

Instantaneous Streamlines

- 1 In the **Model Builder** window, under **Results** click **Instantaneous Streamlines**.
- 2 In the **Instantaneous Streamlines** toolbar, click  **Plot**.

Modify the view to obtain [Figure 8](#).

