Introduction

This tutorial models the scattering of a plane wave off a sphere. It is a classic benchmark model for the boundary element method (BEM). When the sphere is modeled as sound hard, the problem has an analytical solution, as described in Ref. 1 and Ref. 2. The model compares the results using the Pressure Acoustics, Boundary Elements interface with the analytical solution for several frequencies. The results show very good agreement. The model results do not show any irregular modes; these can appear as numerical resonances for the inside of the sphere and are sometimes seen when using the BEM method.

Model Definition

The spherical scatterer of radius of 1 m, is depicted in Figure 1. The scatterer is surrounded by an infinite domain of air, with a speed of sound of 343 m/s and a density of 1.225 kg/m^3.

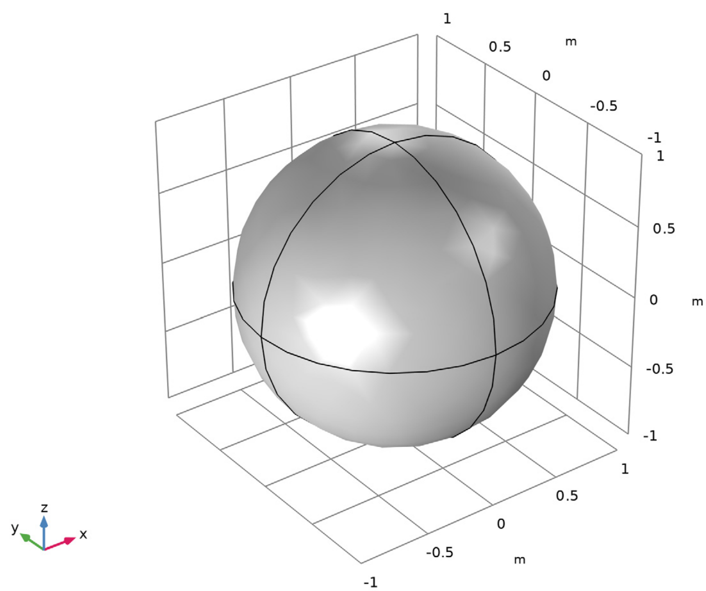

Figure 1: Figure 1 *Spherical scatterer geometry.*

The domain is subjected to an incident plane wave traveling in the global x direction. The *Background Pressure Field* feature is used to set up the incident plane wave and solve this problem. In this situation, the problem is automatically reduced to only solve for the scattered pressure field p_s. The total field p_t is the sum of the scattered field and the (known) background pressure field p_b:

$$p_t = p_b + p_s \tag{1}$$

The plane-wave option defines a background pressure field of the type

$$p_b = p_0 e^{-i(\mathbf{k}\cdot\mathbf{x})} = p_0 e^{-ik_s\left(\frac{\mathbf{x}\cdot\mathbf{e}_k}{\|\mathbf{e}_k\|}\right)} \tag{2}$$

where p_0 is the wave amplitude, $\mathbf{k}$ is the wave vector with amplitude $k_s = \omega/c$ and wave direction vector $\mathbf{e_k}$, and $\mathbf{x}$ is the location on the sphere boundary. The analytical solution for the scattered field p_s created by a spherical scatterer under a plane wave in an infinite acoustic domain is given by

$$p_s = \sum_{n=0}^{N} i^n (2n+1) \frac{j'_n(kR)}{h'_n(kR)} P_n[\cos(\theta)] h_n(kr) \tag{3}$$

Here, k is the wave number, R is the sphere radius, $r = |\mathbf{x} - \mathbf{x}_0|$ is the distance from the sphere center $\mathbf{x}_0$, and N should tend to infinity (we will include 100 terms in the sum). The angle θ is defined such that the shadow zone is located at $\theta = 0$. These expressions are defined in the **Variables** node in the model.

The expression uses the spherical Bessel functions j_n and h_n and their derivatives j'_n and h'_n, which are defined as analytical functions in the model. The Legendre polynomial P_n is a built-in function in COMSOL defined through the expression `legendre(l,x)`.

Results and Discussion

The sound pressure level at 1000 Hz is shown in Figure 2. The BEM solves only on the boundary, but the field obtained through the BEM can be evaluated in any point in the domain that it represents. The figure shows the scattered acoustic sound pressure level on the BEM boundary and its evaluation in the space surrounding the sphere.

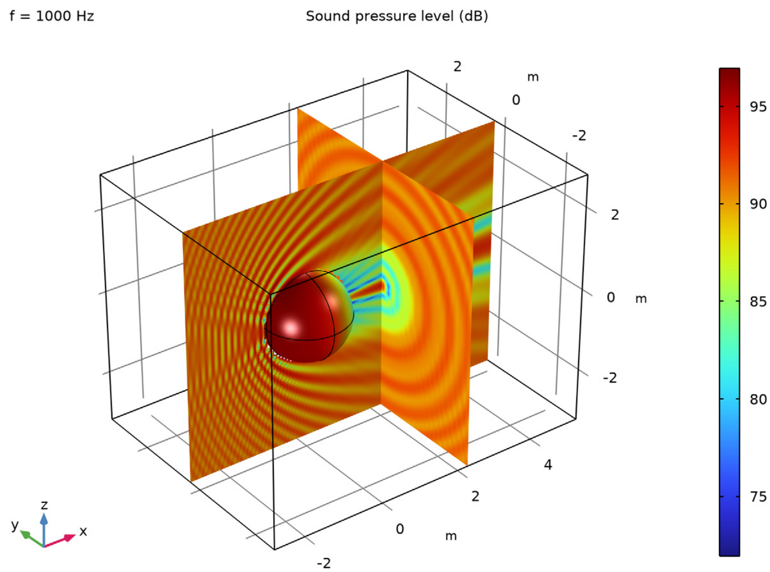

Figure 2: Sound pressure level on the BEM boundary and in the spatial domain.

Figure 3 plots the computed exterior-field pressure at a radial distance of 10 m and a frequency of 200 Hz versus azimuthal angle in the positive *xy*-plane and compares it to the analytical solution. As the plot shows, the computed solution is very close to the analytical solution.

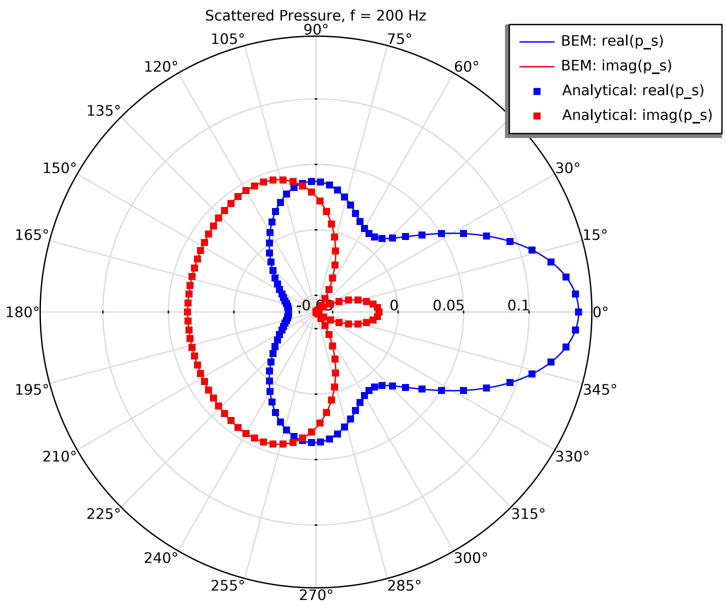

Figure 3: Scattered pressure at $f = 200$ Hz.

As described in Ref. 3, BEM can present nonuniqueness problems around certain frequencies. The absolute total pressure $|p_t|$ is depicted at the point $\mathbf{x} = (2R_0, 0, 0)$ in Figure 4. From this plot, it is evident that the BEM formulation used does not show any irregular modes, and the solution is in agreement with the analytical solution. Irregular modes would show up as peaks in the solution corresponding to the eigenmodes (resonances) of the interior domain of the sphere.

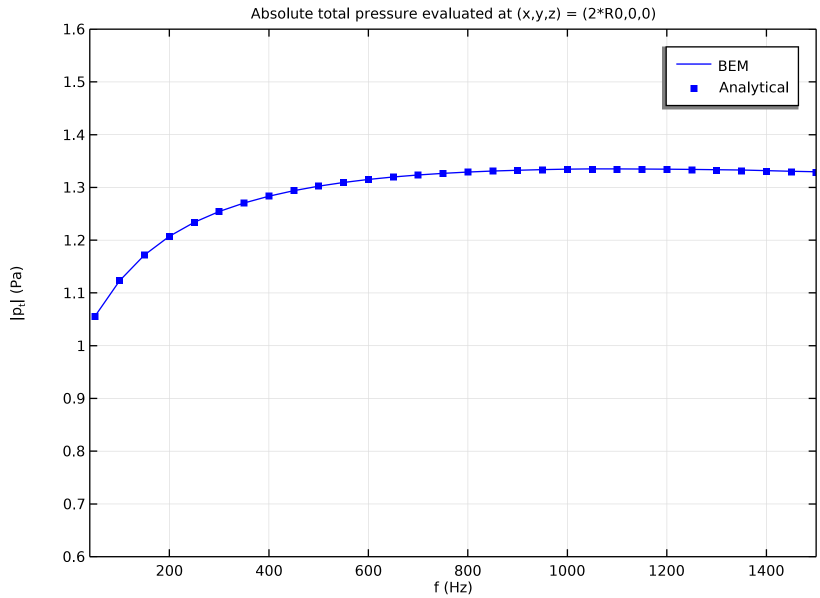

Figure 4: Absolute total pressure evaluated at $x = 2R_0$.

Notes about the COMSOL implementation

The model and the analytical results may disagree at high frequency if you plot the solution along one of the perimeters of the sphere. This is due to the implementation of the BEM quadrature settings in COMSOL, which are a compromise between performance and accuracy.

If you do want to increase the accuracy of the BEM solution beyond the default settings, you can manually tighten the quadrature settings for BEM feature. To do that, click on **Show More Options** to activate the **Advanced Physics Options** to access the quadrature options under the **Pressure Acoustics, Boundary Elements** physics. Set the Integration orders to manual and change the Integration order of the distant elements, close elements, elements with common vertex, elements with common edge and same element pairs to `4`, `6`, `6`, `6`, `6`, respectively. Additionally, tighten the tolerance for the solver from the default of `0.01` to `1e-4`. You can get a much better agreement with the analytical solution using these changes of the quadrature settings. However, the computational time and memory requirements will increase for the computation.