Model created in COMSOL Multiphysics 6.4

Optimizing the Shape of a Horn

Introduction

This example shows how to apply boundary shape optimization to a simple axisymmetric horn. For the sake of simplicity, the on-axis far-field sound pressure level is maximized for a single frequency and in a single direction. The focus is on the optimization procedure and optimization solver settings. The objective function is computed using the built-in exterior field optimization function. The deformation of the geometry is introduced using the *Shape Optimization* functionality of COMSOL Multiphysics.

The model was inspired by the work of Erik Bängtsson, Daniel Noreland, and Martin Berggren (Ref. 1).

Note: This application requires the Acoustics Module and the Optimization Module.

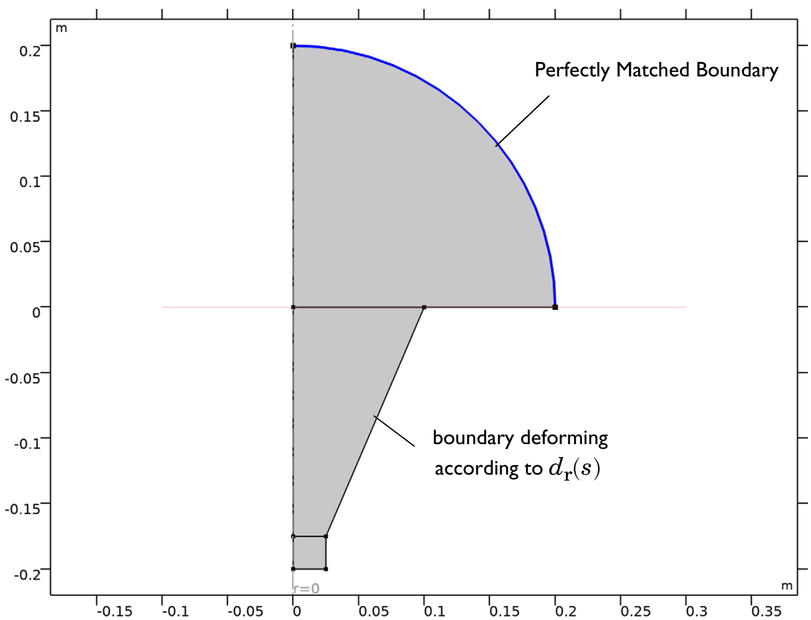


Figure 1: The initial configuration is a simple cone (the z < 0 part of the gray area) in an infinite baffle.

Model Definition

A plane-wave mode feeds an axisymmetric horn radiating from an infinite baffle toward an open half space; see Figure 1. The radius of the waveguide attached to the horn throat is assumed to be fixed, as well as the depth of the horn and the size of the horn mouth attached to the baffle. By varying the curvature of the initially conical neck of the horn, its directivity and impedance can be changed.

The surface of the horn neck is parameterized assuming that the radius and the z position of the horn deviate from the simple cone by a set of 8th-order Bernstein polynomials. The maximum displacement (in the two directions) is given by the $d_{\max}$ parameter. The number of optimization variables is determined by the order of the polynomial. Using a higher order gives more freedom and potentially a better final value of the objective function, but it also makes the optimization process more sensitive and can generate a shape that is less suitable for production.

The dedicated built-in optimization function for the exterior-field sound pressure level is evaluated on axis and used as objective function (the expression `Lp_pext_opt(0, Lfar)`, where L_{far} is the far-field evaluation distance). The optimization problem is solved with the MMA optimization solver with an iteration limit of 50 and no move limit.

Results and Discussion

By changing the shape of the horn within the limits of the selected parameterization, the on-axis sound pressure level can be raised by about 1 dB compared to the simple cone in Figure 1.

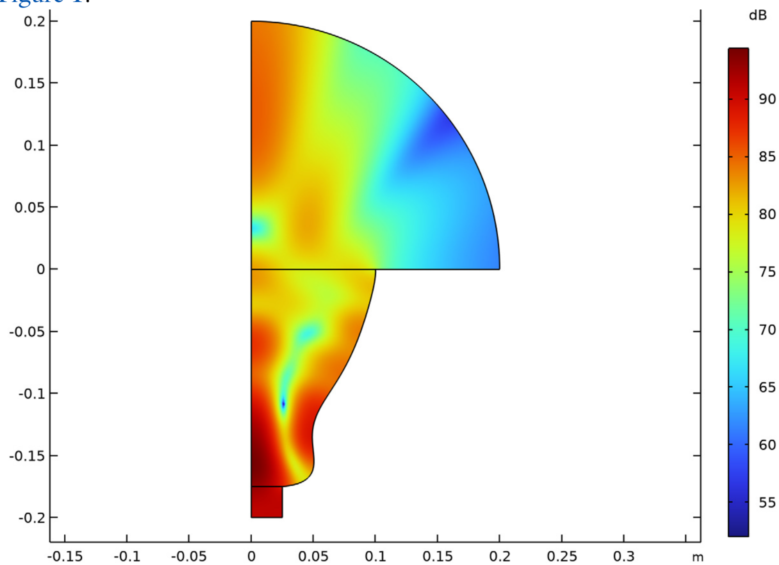


Figure 2: The final shape of the horn, optimized for on-axis SPL at 5000 Hz.

The improvement is rather small, because the initial design also shows a marked directivity, as can be seen from Figure 3. Obviously, the optimal shape with respect to on-axis SPL leads to deep undesirable minima in other directions.

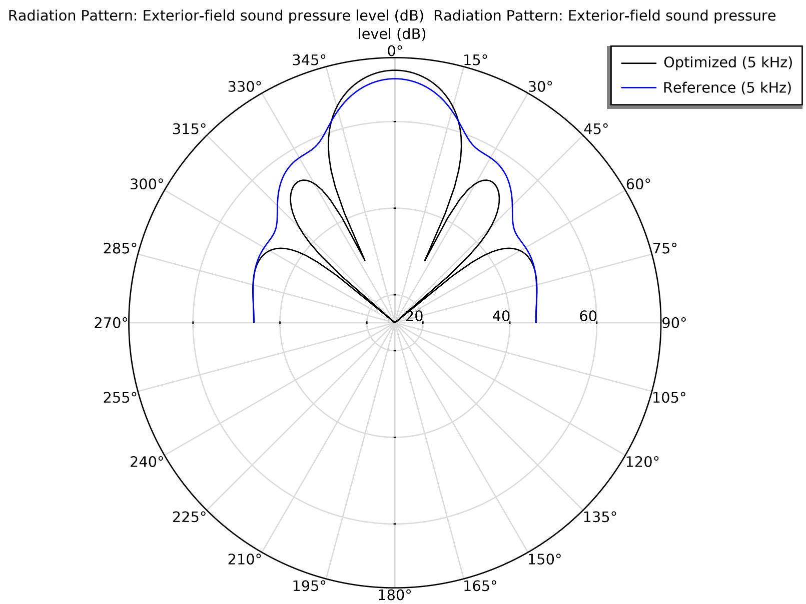


Figure 3: Radiation plot of the original (dashed blue) and final (solid black) designs.

Optimizing with respect to a slight off-axis direction can give you a more uniform far-field pattern, but may also result in a deep minimum on the axis. Try for example to set the off-axis angle to 22°.

Varying the frequency reveals that the on-axis improvement is robust over a wide frequency band; see Figure 4.

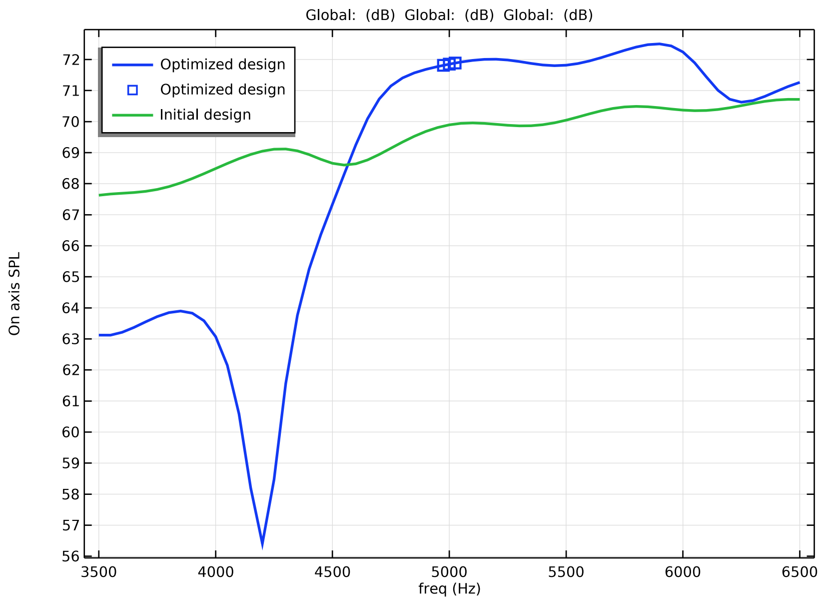


Figure 4: The objective function is plotted as a function of the frequency for the initial as well as the optimized design.

To search for a stable and practically useful horn design, you might instead create a composite objective function as a weighted sum of transmission values evaluated for a number of discrete directions, or choose to minimize the deviation from the mean SPL over a range of angles. In addition, you would also want to optimize with respect to more than one frequency, and experiment with different parameterizations.

Notes About the COMSOL Implementation

COMSOL Multiphysics implements the parameterization using a **Polynomial Boundary** feature.The mesh is allowed to move freely in the conical part of the horn, but otherwise kept fix.

The amplitude of the boundary displacement is restricted by the maximum displacement and the polynomial order. These constraints are intended to keep the mesh element volumes positive at all times.

A time-harmonic Pressure Acoustics, Frequency Domain interface solves for the pressure field inside the horn and in a small spherical domain surrounding its opening.

Set up the optimization problem in the **Shape Optimization** study node.

Reference

1. E. Bängtsson, D. Noreland, and M. Berggren, "Shape Optimization of an Acoustic Horn," *Technical Report 2002-019*, Department of Information Technology, Uppsala University, May 2002.

Application Library path: `Acoustics_Module/Optimization/horn_shape_optimization`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Acoustics** > **Pressure Acoustics** > **Pressure Acoustics, Frequency Domain (acpr)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies** > **Frequency Domain**.

6 Click **Done**.

GEOMETRY 1

Square 1 (sq1)

1 In the **Geometry** toolbar, click **Square**.

2 In the **Settings** window for **Square**, locate the **Size** section.

3 In the **Side length** text field, type `0.025`.

4 Locate the **Position** section. In the **z** text field, type `-0.2`.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.2`.

4 In the **Sector angle** text field, type `90`.

5 Click **Build Selected**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.

Polygon 1 (pol1)

1 In the **Geometry** toolbar, click **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 From the **Data source** list, choose **Vectors**.

4 In the **r** text field, type `0 0.1 0.1 0.025 0.025 0`.

5 In the **z** text field, type `0 0 0 -0.175 -0.175 -0.175`.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	5[kHz]	5000 Hz	Optimization frequency
df	50[Hz]	50 Hz	Optimization frequency bandwidth
Lfar	1[m]	1 m	Far-field evaluation distance

COMPONENT 1 (COMP1)

Free Shape Domain 1

1 In the **Physics** toolbar, click **Optimization** and choose **Shape Optimization**.

2 Select Domain 2 only.

Polynomial Boundary 1

1 In the **Shape Optimization** toolbar, click **Polynomial Boundary**.

2 Select Boundary 8 only.

3 In the **Settings** window for **Polynomial Boundary**, locate the **Control Variable Settings** section.

4 From the d_{max} list, choose **Box**.

5 In the text field, type 0.03.

6 Locate the **Polynomial** section. In the **Order** text field, type 8.

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Click the **Add to Component** button in the window toolbar.

5 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

Port 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Port**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Port**, locate the **Port Properties** section.

4 From the **Type of port** list, choose **Circular**.

5 Locate the **Incident Mode Settings** section. In the A_p^{in} text field, type 1.

This gives you a plane wave with the amplitude 1 Pa propagating in the positive z direction.

Exterior Field Calculation 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Exterior Field Calculation**.

2 Select Boundary 6 only.

3 In the **Settings** window for **Exterior Field Calculation**, locate the **Exterior Field Calculation** section.

4 From the **Condition in the z = z_0 plane** list, choose **Symmetric/ Infinite sound hard boundary**.

Perfectly Matched Boundary 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Perfectly Matched Boundary**.

2 Select Boundary 10 only.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Pressure Acoustics, Frequency Domain (acpr)** section.

3 From the **Maximum mesh element size control parameter** list, choose **Frequency**.

4 In the f_{max} text field, type `f0`.

5 From the **Number of mesh elements per wavelength** list, choose **User defined**.

6 In the text field, type `10`.

7 Click **Build All**.

STUDY 1 - REFERENCE SOLUTION

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Study 1 - Reference Solution` in the **Label** text field.

Before starting the actual optimization, it is a good practice to check the model setup by solving the one with the default parameters. This way, you can also study the reference state which you intend to improve.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1 - Reference Solution** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `range(3500,50,6500)`.

4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Deformed Geometry**.

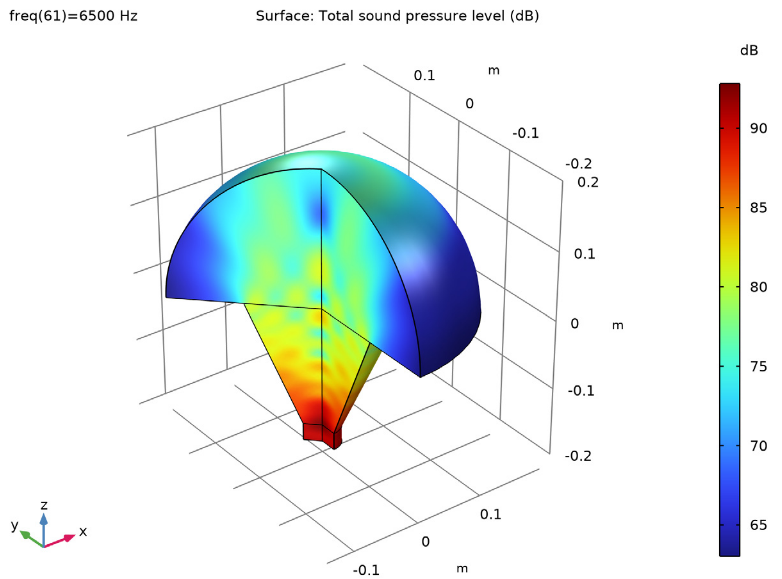
5 In the **Study** toolbar, click **Compute**.

RESULTS

Sound Pressure Level, 3D (acpr)

1 Click the **Zoom Extents** button in the **Graphics** toolbar.

2 In the **Model Builder** window, under **Results** click **Sound Pressure Level, 3D (acpr)**.



Radiation Pattern 1

1 In the **Model Builder** window, expand the **Exterior-Field Sound Pressure Level (acpr)** node, then click **Radiation Pattern 1**.

2 In the **Settings** window for **Radiation Pattern**, locate the **Evaluation** section.

3 Find the **Evaluation distance** subsection. In the **Radius** text field, type `Lfar`.

Radiation Pattern 1

1 In the **Model Builder** window, expand the **Exterior-Field Pressure (acpr)** node, then click **Radiation Pattern 1**.

2 In the **Settings** window for **Radiation Pattern**, locate the **Evaluation** section.

3 Find the **Evaluation distance** subsection. In the **Radius** text field, type `Lfar`.

Shape Optimization

In the **Model Builder** window, under **Results** right-click **Shape Optimization** and choose **Delete**.

Acoustic Pressure (acpr), Acoustic Pressure, 3D (acpr), Exterior-Field Pressure (acpr), Exterior-Field Sound Pressure Level (acpr), Sound Pressure Level (acpr), Sound Pressure Level, 3D (acpr)

1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Acoustic Pressure (acpr)**, **Sound Pressure Level (acpr)**, **Acoustic Pressure, 3D (acpr)**, **Sound Pressure Level, 3D (acpr)**, **Exterior-Field Sound Pressure Level (acpr)**, and **Exterior-Field Pressure (acpr)**.

2 Right-click and choose **Group**.

Reference solution

In the **Settings** window for **Group**, type `Reference solution` in the **Label** text field.

ROOT

Next, add a new study for the optimization.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Frequency Domain**.

4 Click the **Add Study** button in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `f0-df/2 f0 f0+df/2`.

3 In the **Model Builder** window, click **Study 2**.

4 In the **Settings** window for **Study**, type `Study 2 - Optimized Solution` in the **Label** text field.

Shape Optimization

1 In the **Study** toolbar, click **Optimization** and choose **Shape Optimization**.

2 In the **Settings** window for **Shape Optimization**, locate the **Optimization Solver** section.

3 In the **Maximum number of iterations** text field, type `50`.

4 Clear the **Move limits** checkbox.

5 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description
comp1.Lp_pext_opt(0, Lfar)	

Set up a MaxMin problems such that frequency associated with the worst objective function is prioritized.

6 From the **Type** list, choose **Maximization**.

7 From the **Solution** list, choose **Minimum of objectives**.

8 Find the **Objective settings** subsection. From the **Objective scaling** list, choose **Initial solution based**.

9 In the **Study** toolbar, click **Get Initial Value**.

By making the nonlinear tolerance stricter than that of the optimization solver, you ensure that the optimization does not fail because each solution is not sufficiently converged. An optimality tolerance of 1e-4 is still stricter than the accuracy of this low-resolution finite element model.

Step 1: Frequency Domain

1 In the **Model Builder** window, expand the **Study 2 - Optimized Solution** > **Solver Configurations** node, then click **Study 2 - Optimized Solution** > **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 From the **Tolerance** list, choose **User controlled**.

4 In the **Relative tolerance** text field, type `1e-6`.

Solution 2 (sol2)

1 In the **Model Builder** window, expand the **Solution 2 (sol2)** node.

2 In the **Model Builder** window, expand the **Study 2 - Optimized Solution** > **Solver Configurations** > **Solution 2 (sol2)** > **Optimization Solver 1** node, then click **Stationary Solver 1**.

3 In the **Settings** window for **Stationary Solver**, locate the **General** section.

4 From the **Linearity** list, choose **Nonlinear**.

5 In the **Model Builder** window, expand the **Study 2 - Optimized Solution** > **Solver Configurations** > **Solution 2 (sol2)** > **Optimization Solver 1** > **Stationary Solver 1** node, then click **Fully Coupled 1**.

6 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.

7 In the **Minimum damping factor** text field, type `1e-4`.

Shape Optimization

1 In the **Model Builder** window, under **Study 2 - Optimized Solution** click **Shape Optimization**.

2 In the **Settings** window for **Shape Optimization**, click to expand the **Output** section.

3 Select the **Plot** checkbox.

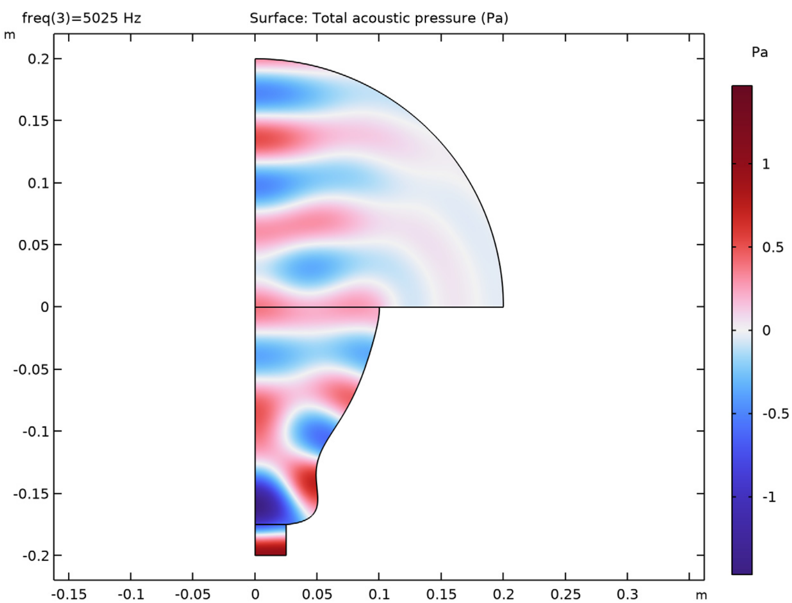
4 From the **Plot group** list, choose **Shape Optimization**.

5 In the **Study** toolbar, click **= Compute**.

RESULTS

Acoustic Pressure (acpr) 1

The first default plot is the acoustic pressure. It should look like the image below.



Sound Pressure Level (acpr) 1

1 In the **Model Builder** window, click **Sound Pressure Level (acpr) 1**.

2 In the **Sound Pressure Level (acpr) 1** toolbar, click **Plot**.

Your plot of the sound pressure level should now look like Figure 2.

To see a direct comparison of the exterior-field polar pattern before and after optimization, modify the exterior-field plot of the sound pressure level in the rz-plane. Modify it to only plot the results in the positive half plane ($z > 0$), increase the resolution, and change some Coloring and Style options. The resulting plot of the exterior field should look like Figure 3. Note that 0 deg on the polar graph corresponds to the vertical z-axis.

Exterior-Field Sound Pressure Level (acpr) 1

1 In the **Model Builder** window, click **Exterior-Field Sound Pressure Level (acpr) 1**.

2 In the **Settings** window for **Polar Plot Group**, locate the **Data** section.

3 From the **Parameter selection (freq)** list, choose **Manual**.

4 In the **Parameter indices (1-3)** text field, type `2`.

Radiation Pattern 1

1 In the **Model Builder** window, expand the **Exterior-Field Sound Pressure Level (acpr) 1** node, then click **Radiation Pattern 1**.

2 In the **Settings** window for **Radiation Pattern**, locate the **Evaluation** section.

3 Find the **Angles** subsection. From the **Restriction** list, choose **Manual**.

4 In the ϕ **start** text field, type `-90`.

5 In the ϕ **range** text field, type `180`.

6 Find the **Evaluation distance** subsection. In the **Radius** text field, type `Lfar`.

7 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

Legends
`Optimized (5 kHz)`

9 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.

Radiation Pattern 2

1 Right-click **Results** > **Exterior-Field Sound Pressure Level (acpr) 1** > **Radiation Pattern 1** and choose **Duplicate**.

2 In the **Settings** window for **Radiation Pattern**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1 - Reference Solution/Solution 1 (sol1)**.

4 From the **Parameter selection (freq)** list, choose **From list**.

5 In the **Parameter values (freq (Hz))** list box, select **5000**.

6 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Reference (5 kHz)`

7 Locate the **Coloring and Style** section. From the **Color** list, choose **Blue**.

8 In the **Exterior-Field Sound Pressure Level (acpr) 1** toolbar, click **Plot**.

Radiation Pattern 1

1 In the **Model Builder** window, expand the **Exterior-Field Pressure (acpr) 1** node, then click **Radiation Pattern 1**.

2 In the **Settings** window for **Radiation Pattern**, locate the **Evaluation** section.

3 Find the **Evaluation distance** subsection. In the **Radius** text field, type `Lfar`.

Arrow Line 1

1 In the **Model Builder** window, expand the **Shape Optimization** node, then click **Arrow Line 1**.

2 In the **Settings** window for **Arrow Line**, locate the **Arrow Positioning** section.

3 From the **Placement** list, choose **Mesh vertices**.

Color Expression 1

1 In the **Model Builder** window, expand the **Arrow Line 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Coloring and Style** section.

3 From the **Color table** list, choose **Rainbow**.

Acoustic Pressure (acpr) 1, Acoustic Pressure, 3D (acpr) 1, Exterior-Field Pressure (acpr) 1, Exterior-Field Sound Pressure Level (acpr) 1, Shape Optimization, Sound Pressure Level (acpr) 1, Sound Pressure Level, 3D (acpr) 1

1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Acoustic Pressure (acpr) 1**, **Sound Pressure Level (acpr) 1**, **Acoustic Pressure, 3D (acpr) 1**, **Sound Pressure Level, 3D (acpr) 1**, **Exterior-Field Sound Pressure Level (acpr) 1**, **Exterior-Field Pressure (acpr) 1**, and **Shape Optimization**.

2 Right-click and choose **Group**.

Optimized solution

In the **Settings** window for **Group**, type `Optimized solution` in the **Label** text field.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies > Frequency Domain**.

4 Click the **Add Study** button in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `range(3500,50,6500)`.

3 Click to expand the **Results While Solving** section. From the **Probes** list, choose **None**.

4 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Deformed Geometry**.

5 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

6 From the **Method** list, choose **Solution**.

7 From the **Study** list, choose **Study 2 - Optimized Solution, Frequency Domain**.

8 In the **Model Builder** window, click **Study 3**.

9 In the **Settings** window for **Study**, type `Study 3 - Frequency Sweep (Optimized)` in the **Label** text field.

10 Locate the **Study Settings** section. Clear the **Generate default plots** checkbox.

11 In the **Study** toolbar, click **Compute**.

RESULTS

Optimized solution

Add **1D Plot Group** showing the objective function as a function of the frequency for the initial as well as the optimized design.

Response

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Response` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 3 - Frequency Sweep (Optimized)/Solution 3 (sol3)**.

4 Locate the **Plot Settings** section.

5 Select the **y-axis label** checkbox. In the associated text field, type `On axis SPL`.

6 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Global 1

1 Right-click **Response** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
Lp_pext_opt(0, Lfar)	dB	

4 Click to expand the **Coloring and Style** section. From the **Width** list, choose **2**.

5 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

6 In the table, enter the following settings:

Legends
Optimized design

Global 2

1 Right-click **Global 1** and choose **Duplicate**.

2 In the **Settings** window for **Global**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2 - Optimized Solution/Solution 2 (sol2)**.

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 From the **Color** list, choose **Cycle (reset)**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Square**.

Global 3

1 In the **Model Builder** window, under **Results** > **Optimized solution** > **Response** right-click **Global 1** and choose **Duplicate**.

2 In the **Settings** window for **Global**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1 - Reference Solution/Solution 1 (sol1)**.

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Initial design`

5 In the **Response** toolbar, click **Plot**.