



Model created in COMSOL Multiphysics 6.4

Helmholtz Resonator Analyzed with Different Frequency Domain Solvers

Introduction

This example simulates a simple three-dimensional axisymmetric Helmholtz resonator, a classic acoustics model of a resonating circuit with a known theoretical solution. The idealized version considered here, consists of a tube and a closed volume in series, which are exposed to a harmonically oscillating pressure. Real-world phenomena explained by the resonator include, among others, the resonance from blowing across the top of an empty bottle and the sound produced by closed-cavity drums such as the djembe and by subwoofers. This model illustrates the use of different numerical solvers. The model shows how to solve this pressure acoustics problem for a range of frequencies using the following solvers: (a) Frequency Domain, with and without Asymptotic Waveform Evaluation (AWE) for faster solution, and (b) Frequency Domain, Modal, which reconstructs the frequency response based on a specified set of eigenmodes.

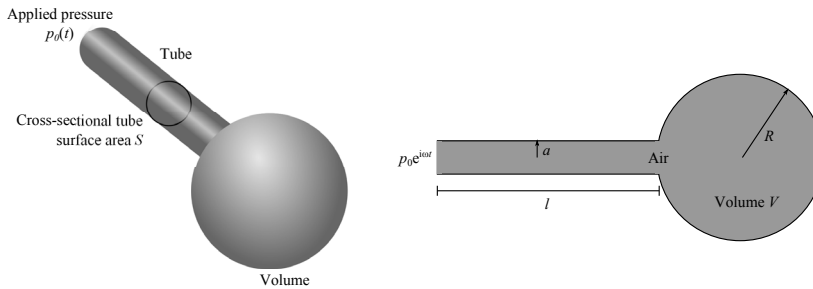


Figure 1: Illustrations of the Helmholtz resonator.

Model Definition

The model consists of a tube and a volume coupled in series driven by a harmonically oscillating pressure $p_0(t) = p_0 e^{i\omega t}$ at the tube inlet. The Helmholtz resonator (schematically depicted in Figure 1) is one of the simplest resonating circuits. This circuit is typically described using lumped-parameter (equivalent circuit) modeling (see, for example, Ref. 1) as a serial coupling of an acoustic inertance L (equivalent to inductance in electrical circuits and to mass in point-mass mechanics) caused by acceleration of the fluid in the tube and an acoustic compliance C (equivalent to capacitance in electrical

circuits and a spring in point-mass mechanics) arising from compression of the volume, see [Figure 1](#) and [Figure 3](#).

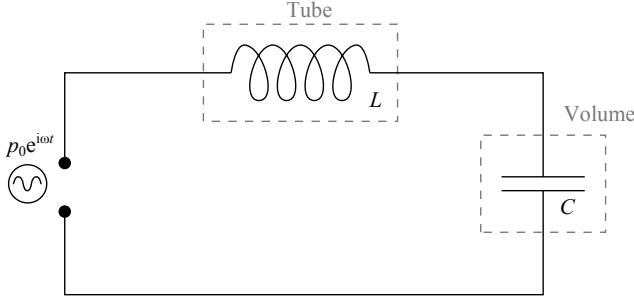


Figure 2: Diagram of the equivalent electrical circuit to the Helmholtz resonator.

Because compression of the fluid volume and acceleration of the fluid in the tube are not instantaneous events but instead occur on specific time scales (each given by the geometry together with the properties of the fluid), the response of the resonator depends on the frequency, and there exists a frequency that maximizes the response — in other words, a resonance frequency ω_R . This is of course to be expected given the direct analogy to electric LC circuits, and by the same analogy we find that the resonance frequency is given by

$$\omega_R = \frac{1}{\sqrt{LC}} \quad (1)$$

As detailed in [Ref. 1](#), the acoustic lumped-parameter elements are given by

$$\begin{aligned} L &= \frac{\rho_0(l + \gamma a)}{S} \\ C &= \frac{V}{\rho_0 c_0^2} \end{aligned} \quad (2)$$

where ρ_0 is the background quiescent density of the fluid, c_0 is the background quiescent speed of sound, l is the tube length, a is the tube radius, S is the cross-sectional area of the tube transverse to the direction of the flow, γ is the end correction factor (detailed below), and V is the closed volume. Thus, using [Equation 2](#) in [Equation 1](#) we find that the resonance frequency ω_R is given by

$$\omega_R = c_0 \sqrt{\frac{S}{V(l + \gamma a)}} = c_0 \sqrt{\frac{a^2}{\frac{4}{3}R^3(l + \gamma a)}} \quad (3)$$

where in the last equation, we have assumed that the volume is a sphere with $V = \frac{4}{3}\pi R^3$ and the tube is cylindrical so $S = \pi a^2$.

END CORRECTION FACTOR γ : PHYSICAL ORIGIN AND APPROXIMATE NUMERICAL VALUE

When the fluid exits the tube and enters the volume, the acoustic waves disperse and the acoustic pressure drops. However, the waves initially continue along the axis of the tube when they just leave it, and moreover, they cannot move into the region occupied by the tube. Consequently, they do not completely disperse immediately as they leave the tube and the immediate region downstream of the tube is therefore still felt by the fluid in the tube where it imposes an acoustic load. In ideal models, this load results in an additional acoustic inertance corresponding to an effective increase in the length of the tube by γa . In other words, the total length of the lumped-parameter inertance L in [Equation 2](#) is longer than the actual tube length l .

The factor γ depends on the specific geometry of the tube-volume connection and is of the order unity, [Ref. 1](#) and [2](#). We shall take for reference an infinite flange for which the correction factor is $\gamma = 0.82$ ([Ref. 2](#)); this is not fully correct because the acoustic pressure disperses less in the circular geometry resulting in a larger acoustic load.

SOLVER DESCRIPTIONS

Using the default settings for the Frequency Domain solver, it solves the problem subjected to harmonic excitation at a set of specified excitation frequencies. While this can be time consuming for larger frequency sweeps, the (numerically) exact solution is calculated explicitly at every frequency, and so the solutions from this solver can always be expected to be correct (assuming convergence of the model and appropriate meshing to resolve all length scales of the physics).

To accommodate larger sweeps, the Adaptive Frequency Sweep can be used. This solver uses asymptotic waveform evaluation (AWE). The approach does not explicitly compute the exact solution at all frequencies but instead performs a Taylor expansion of the solution about a few exact solutions and otherwise uses a lower-order approximation (Padé or Taylor) to estimate the solution across the required frequency range.

Finally, the Frequency Domain, Modal solver can also be used to perform a frequency sweep. For this, it first computes a set of system eigenfrequencies and associated eigenmodes (searching either within a user-defined range or for a user-defined number of frequencies). The full solution across the frequency sweep is then approximated by a linear combination of the basis set formed by the eigensolutions (see [Notes About the COMSOL Implementation](#)).

GOALS OF ANALYSIS

This pressure acoustics problem is solved in a specified frequency regime using different solvers, with the dual purpose of illustrating the capabilities of the solvers and also highlight the solver-specific settings to be aware of. The following preset solvers are used: (a) Frequency Domain, (b) Adaptive Frequency Sweep (with Asymptotic Waveform Evaluation (AWE) for faster solution), and (c) Frequency Domain, Modal, which reconstructs the frequency response based on the eigenfrequencies in the specified range.

RESISTANCE DUE TO ACOUSTIC RADIATION

As a final note, it should be mentioned that the equivalent circuit diagram in Figure 2 is incorrect. A full equivalent circuit description of the resonator should also include the acoustic radiation resistance R caused by the dissipated energy out of the tube when the fluid in the tube moves into the volume, see Figure 3. Mathematical formulas for the resistance in a number of situations may be found in Ref. 1. However, the value of the resistance does not affect the resonance frequency (the only read-out used here), only the absolute level of the impedance of the system, so we are well-justified in ignoring R .

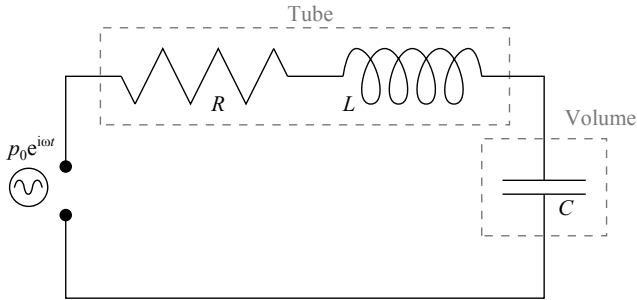


Figure 3: Diagram of the equivalent electrical circuit to the Helmholtz resonator including the radiation resistance R .

Results and Discussion

This model uses the absolute acoustic pressure averaged over the end volume

$$\langle p \rangle = \frac{1}{V} \int_V |p| dv \quad (4)$$

to investigate the response. We find that the lowest eigenfrequency indeed corresponds to the resonance predicted by the lumped-parameter model, and that this theoretical prediction indeed is in good agreement with the numerical results. However, the full

numerical COMSOL Multiphysics solutions illustrate the many higher modes that are ignored in the simple lumped-parameter model.

Comparing the different solvers, it is noted that they all produce the same response for the frequency sweep; see Figure 4 and Figure 5 on logarithmic and linear scales, respectively. The sound pressure level at 1123.1 Hz is depicted in Figure 6. The AWE and modal based frequency sweep methods are quite useful to speed up the running times of large models with a large number of frequencies requested. Due to the small number of degrees of freedom in this model, it is not possible to measure any speed-up in CPU time. See the next section for details.

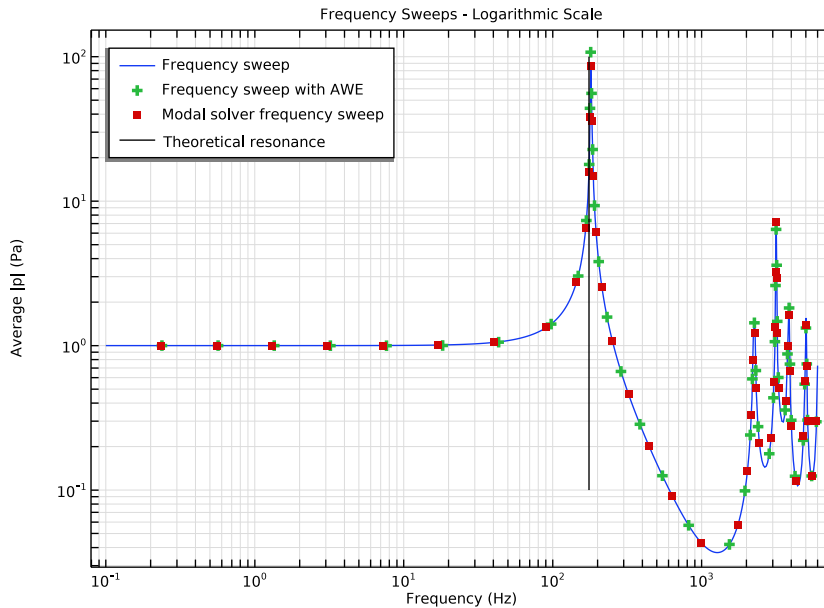


Figure 4: Frequency sweep for the readout $\langle p \rangle$ illustrating agreement between all solvers and the theory for the first eigenfrequency and also illustrating agreement between all solvers for the higher frequencies.

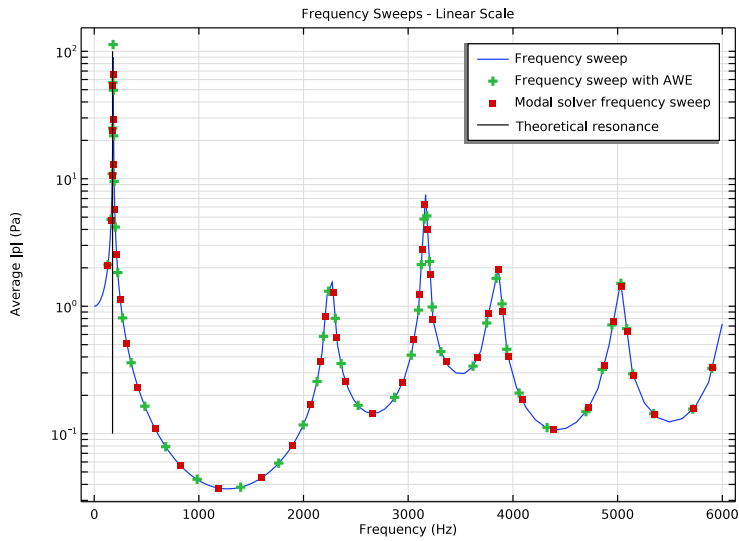


Figure 5: Same frequency sweep as in Figure 4 here shown on a linear frequency scale to emphasize the higher frequencies.

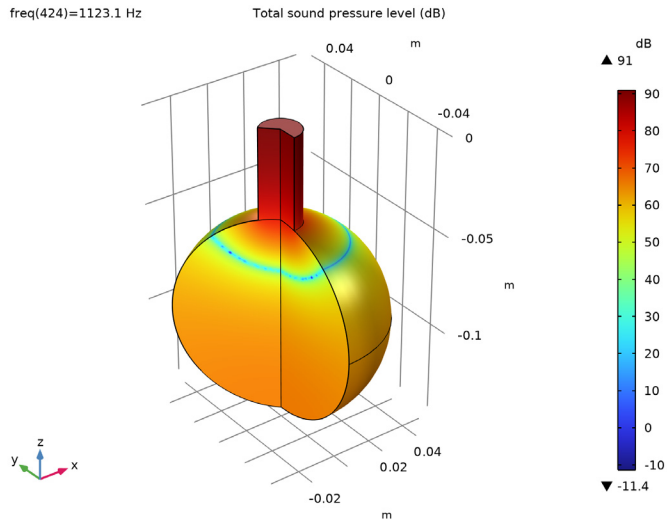


Figure 6: Sound pressure level computed as a default output from the simulations, here shown for $f = 1123.1$ Hz.

- As a general rule of thumb, at least five quadratic elements should be used to resolve each wavelength. Therefore, the maximum mesh element size is set to $1/5$ of the shortest wavelengths present, c_{air}/f_{max} . This mesh size is artificially refined to increase the running time.
- When several resonance frequencies are present, the default parameters for the AWE option may use some tuning, for example, using an adequate **Absolute tolerance** value (see under the **>AWE Solver 1** node). In this model, the default value of 0.001 is used and gives reasonable results (when compared to the other methods). Lowering the value to 0.0001 will improve the high frequency results slightly (see the linear scale plot). Other settings are in general less important, like changing the number of points to linearize about (**Evaluation points**), changing the number of terms in the Taylor expansion about each point (**Expansion size**), or changing from Padé to Taylor expansion of the approximating solution (**Expansion type**). In the current setup, we chose a relatively high upper frequency bound so about 10 resonance frequencies are present in the sweep. Had the upper frequency limit been lowered so only the first resonance was included (for instance, by setting $f_{max} = 100$ Hz), then the default relative AWE tolerance would have sufficed. Note that if either end of the frequency range is close to a resonance, the AWE solver can become unreliable.
- The `linper` operator informs the solver that the term in the expression is a perturbation (a source term) that must be included in the linearized problem. The modal solver will only use the pressure under the `linper` operator as a source, while the other solvers will ignore this perturbation term.
- To obtain good results with the Modal solver up to f_{max} , we must set the upper limit in its eigenfrequency search to $1.5 * f_{max}$ to capture modes that may have an influence on the highest part of the frequency of interest. Using only f_{max} as the upper limit results in poorer estimates of the solution at higher frequencies.
- The Adaptive Frequency Sweep with AWE and the Frequency Domain, Modal solver both rely on approximating the solution using a few exact solutions in the sweep range. Thus, these methods provide greater speed-up in CPU time if only few resonances fall in the sweep range, or if it is comparatively easier to find the eigensolutions relative to all the full solutions. Thus, these methods will be especially useful for large models and fine frequency sweeps over broad ranges. Solving the frequency sweep will take approximately 16 s, while the AWE solver will require 8 s and the modal based frequency sweep will require only 5 s. The speed up will depend on the number of frequencies requested and the number of resonances present in the frequency of interest.

- Not all resonance peaks may be captured with the current frequency resolution used in this tutorial, because it gets coarser as the frequency increases. For example, there is a narrow resonance peak at about 4.9 kHz that lies within the interval $[f_{\min}, f_{\max}]$, but not visible in Figure 4 or Figure 5. However, it is detected by the Eigenvalue solver used in the last study. Another indication is the warning message produced by the Adaptive Frequency Sweep with AWE solver which suggests that the frequency sampling should be refined.
- Finally, note that we define the model in terms of geometric parameters (α , L , and R). This makes it easy to quickly include parametric sweeps in the geometry, which, for instance, could be used to tune the lowest eigenfrequency.

References


1. D.T. Blackstock, *Fundamentals of Physical Acoustics*, John Wiley & Sons, 2000.
2. A.D. Pierce, *Acoustics: An introduction to its physical principles and applications*, Acoustical Society of America, 1989.

Application Library path: Acoustics_Module/Tutorials,_Pressure_Acoustics/helmholtz_resonator_solvers




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.


MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D Axisymmetric**.
- 2 In the **Select Physics** tree, select **Acoustics** > **Pressure Acoustics** > **Pressure Acoustics, Frequency Domain (acpr)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies** > **Frequency Domain**.
- 6 Click  **Done**.

GLOBAL DEFINITIONS

Parameters 1


Load all model parameters from a file; these include geometrical parameters and physical properties of the air.

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file `helmholtz_resonator_solvers_parameters.txt`.


GEOMETRY 1

Construct the simple 2D axisymmetric geometry of the resonator from [Figure 1](#) using rectangles and circles.


Rectangle 1 (r1)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
You start by making the tube. It has length L and radius a .
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type a .
- 4 In the **Height** text field, type L .
- 5 Locate the **Position** section. In the **z** text field, type $-L$.

Circle 1 (c1)

- 1 In the **Geometry** toolbar, click  **Circle**.
Proceed to make the volume which has radius R_v and is placed after the tube. You ensure an overlap of $a/2$ knowing that you will merge the tube and volume into one object (the resonator). Note that objects or parts of objects extending into the left-hand side of the symmetry line ($r = 0$) will be removed, so you will not have to manually remove the left half of the circle.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type R_v .
- 4 Locate the **Position** section. In the **z** text field, type $-(L+R_v-a/2)$.

Union 1 (un1)

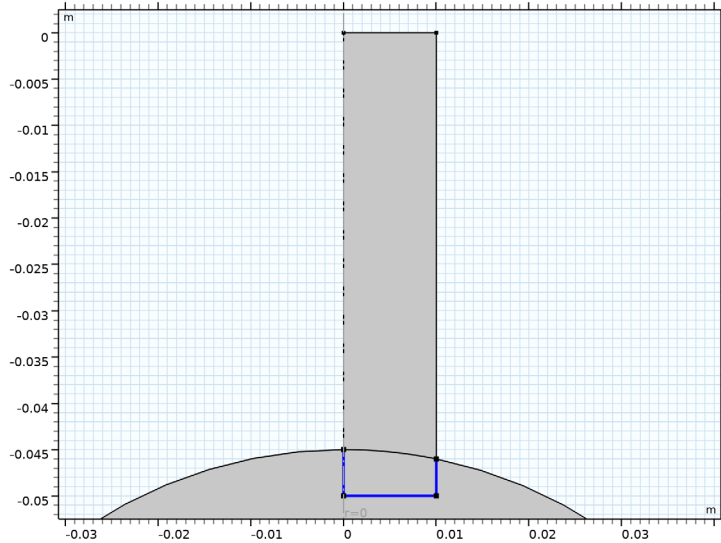
- 1 In the **Geometry** toolbar, click  **Booleans and Partitions** and choose **Union**.
Clean up the remaining parts of the geometry.

2 Click in the **Graphics** window and then press Ctrl+A to select both objects.


Delete Entities 1 (dell)


1 In the **Model Builder** window, right-click **Geometry 1** and choose **Delete Entities**.

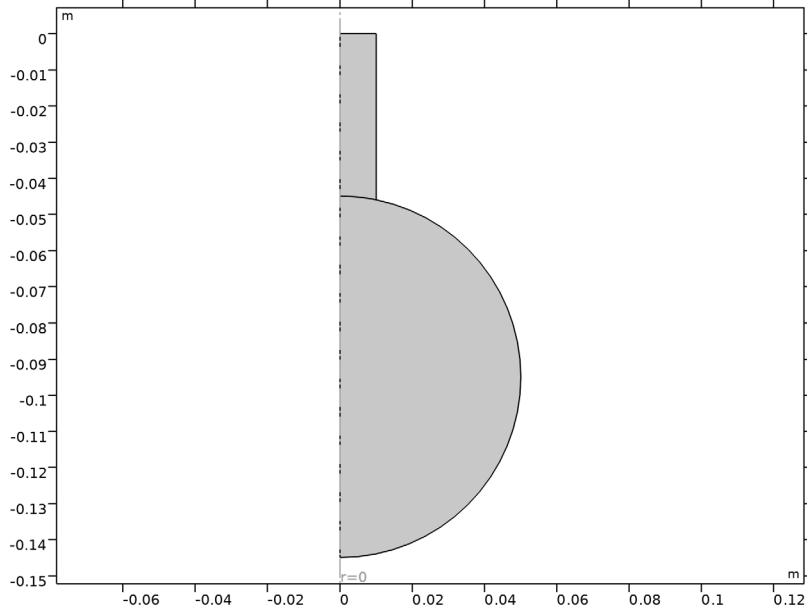
2 On the object **unil**, select Boundaries 1, 2, and 5 only.



3 In the **Settings** window for **Delete Entities**, click  **Build All Objects**.

4 Click the  **Zoom Extents** button in the **Graphics** toolbar.

5 In the **Geometry** toolbar, click  **Build All**.



MATERIALS

Now add the material. For this simple model it suffices to use tabulated values for the physical parameters of air, which can be defined manually by adding a blank material and assign to it the values `rho_air` and `c_air`. Alternatively load air from the Material Library.

My air

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, type `My air` in the **Label** text field.
- 3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	<code>rho</code>	<code>rho_air</code>	kg/m ³	Basic
Speed of sound	<code>c</code>	<code>c_air</code>	m/s	Basic

Next define an integration coupling that integrates over the volume V which will be used for evaluating the system response.

DEFINITIONS


Integration 1 (intop1)

- 1 In the **Definitions** toolbar, click  **Nonlocal Couplings** and choose **Integration**.
- 2 Select Domain 1 only.

PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

Pressure Acoustics 1

Next, you define that you will be solving Pressure Acoustics in the entire domain, and you apply the boundary conditions.

- 1 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Pressure 1


- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Pressure**.
- 2 Select Boundary 3 only.

The applied pressure is a combination of an constant part and a `linper` operator. The constant part will be used in the Frequency Domain steps and inactive in the rest, while the `linper` part will be active when running the Frequency Domain, Modal Step and inactive in the rest, as described in Notes About the COMSOL Implementation.

- 3 In the **Settings** window for **Pressure**, locate the **Pressure** section.
- 4 In the p_0 text field, type `1+linper(1)`.

Set up the AWE expression used for the adaptive frequency sweep solver.


AWE Expression 1

- 1 In the **Physics** toolbar, click  **Global** and choose **AWE Expression**.
- 2 In the **Settings** window for **AWE Expression**, locate the **Asymptotic Waveform Evaluation Expression** section.
- 3 In the L_{AWE} text field, type `intop1(abs(p)^2)/1[Pa^2]/intop1(1)`.

MESH

Proceed and generate the mesh using the **Physics-controlled mesh** functionality. The frequency controlling the maximum element size is entered manually to accommodate the different solvers to be used. In general, 5 to 6 second-order elements per wavelength are needed to resolve the waves. For more details, see *Meshing (Resolving the Waves)* in the *Acoustics Module User's Guide*. The default **Automatic** option gives 5 elements per wavelength. In this case, use a mesh size 8 times finer to artificially increase the run time and demonstrate differences in the running time with the different solvers.

MESH 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Pressure Acoustics, Frequency Domain (acpr)** section.
- 3 From the **Maximum mesh element size control parameter** list, choose **Frequency**.
- 4 In the f_{\max} text field, type f_{\max} .
- 5 From the **Number of mesh elements per wavelength** list, choose **User defined**.
- 6 In the text field, type 40.
- 7 Click  **Build All**.

STUDY 1 - FREQUENCY SWEEP


Next, set up the first of the three solvers (Frequency Domain solver). You already included this solver (this study) when you started this analysis, so you can immediately set it up.

- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Study 1 - Frequency sweep in the **Label** text field.
- 3 Locate the **Study Settings** section. Clear the **Generate default plots** checkbox.

Step 1: Frequency Domain

- 1 In the **Model Builder** window, under **Study 1 - Frequency sweep** click **Step 1: Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type $10^{\{\text{range}(\log_{10}(f_{\min}), (\log_{10}(f_{\max}) - (\log_{10}(f_{\min}))) / 499, \log_{10}(f_{\max}))\}}$.

This command selects 500 frequencies in the range $f_{\min} - f_{\max}$ which will be evenly spaced when shown on a logarithmic axis.


- 4 In the **Study** toolbar, click  **Compute**.

ROOT

Now use the **Adaptive Frequency Sweep** study which uses the asymptotic waveform evaluation (AWE) method. You perform this as a separate study to be able to compare the two solutions.


ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.



- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Adaptive Frequency Sweep**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Adaptive Frequency Sweep

- 1 In the **Settings** window for **Adaptive Frequency Sweep**, locate the **Study Settings** section.
- 2 In the **Frequencies** text field, type $10^{\{\text{range}(\log_{10}(f_{\min}), (\log_{10}(f_{\max}) - (\log_{10}(f_{\min}))) / 499, \log_{10}(f_{\max}))\}}$.
- 3 From the **AWE expression type** list, choose **Physics controlled**.
- 4 In the **Relative tolerance** text field, type 0.005.
The tolerance of the AWE solver is reduced to ensure the accuracy of the results.
- 5 In the **Model Builder** window, click **Study 2**.
- 6 In the **Settings** window for **Study**, locate the **Study Settings** section.
- 7 Clear the **Generate default plots** checkbox.
- 8 In the **Label** text field, type Study 2 - Frequency sweep with AWE.
- 9 In the **Study** toolbar, click  **Compute**.

ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces > Frequency Domain, Modal**.
- 4 Click the **Add Study** button in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

You then set up the third solver; the Frequency Domain Modal study.


STUDY 3 - MODAL SOLVER FREQUENCY SWEEP

- 1 In the **Settings** window for **Study**, locate the **Study Settings** section.
- 2 Clear the **Generate default plots** checkbox.
- 3 In the **Label** text field, type Study 3 - Modal solver frequency sweep.

Step 1: Eigenfrequency


- 1 In the **Model Builder** window, under **Study 3 - Modal solver frequency sweep** click **Step 1: Eigenfrequency**.
- 2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.
- 3 From the **Eigenfrequency search method** list, choose **Rectangle**.
Next, change the upper limit of the real part of the frequencies you want to investigate to make sure the entire frequency range f_{\min} - f_{\max} is appropriately resolved.
- 4 Find the **Rectangle search region** subsection. In the **Largest real part (Eigenfrequency)** text field, type $1.5 * f_{\max}$.

Step 2: Frequency Domain, Modal


- 1 In the **Model Builder** window, click **Step 2: Frequency Domain, Modal**.
- 2 In the **Settings** window for **Frequency Domain, Modal**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type $10^{\{\text{range}(\log_{10}(f_{\min}), (\log_{10}(f_{\max}) - (\log_{10}(f_{\min}))) / 499, \log_{10}(f_{\max}))\}}$.
- 4 In the **Study** toolbar, click  **Compute**.

RESULTS

Frequency Sweeps - Logarithmic Scale

- 1 In the **Results** toolbar, click  **ID Plot Group**.
Next, add plots of the system response across all investigated frequencies. These will look like [Figure 3](#) and [Figure 4](#).
- 2 In the **Settings** window for **ID Plot Group**, type **Frequency Sweeps - Logarithmic Scale** in the **Label** text field.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.
- 4 Locate the **Plot Settings** section.
- 5 Select the **x-axis label** checkbox. In the associated text field, type **Frequency (Hz)**.
- 6 Select the **y-axis label** checkbox. In the associated text field, type **Average |p| (Pa)**.
- 7 Locate the **Axis** section. Select the **x-axis log scale** checkbox.
- 8 Select the **y-axis log scale** checkbox.
- 9 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Octave Band 1

- 1 In the **Frequency Sweeps - Logarithmic Scale** toolbar, click  **More Plots** and choose **Octave Band**.

- 2 In the **Settings** window for **Octave Band**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 1 - Frequency sweep/Solution 1 (sol1)**.
- 4 Locate the **Selection** section. From the **Geometric entity level** list, choose **Domain**.
- 5 Select Domain 1 only.
- 6 Locate the **y-Axis Data** section. In the **Expression** text field, type `abs(acpr.p_t)`.
- 7 From the **Expression type** list, choose **General (non-dB)**.
- 8 Locate the **Plot** section. From the **Quantity** list, choose **Continuous power spectral density**.
- 9 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 10 From the **Legends** list, choose **Manual**.
- 11 In the table, enter the following settings:

Legends

Frequency sweep

Octave Band 2

- 1 Right-click **Octave Band 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Octave Band**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2 - Frequency sweep with AWE/Solution 2 (sol2)**.
- 4 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
- 5 Find the **Line markers** subsection. From the **Marker** list, choose **Plus sign**.
- 6 From the **Positioning** list, choose **Interpolated**.
- 7 In the **Number** text field, type 50.
- 8 Locate the **Legends** section. In the table, enter the following settings:

Legends

Frequency sweep with AWE

Octave Band 3

- 1 Right-click **Octave Band 2** and choose **Duplicate**.
- 2 In the **Settings** window for **Octave Band**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 3 - Modal solver frequency sweep/Solution 3 (sol3)**.
- 4 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Modal solver frequency sweep

Frequency Sweeps - Logarithmic Scale

In the **Frequency Sweeps - Logarithmic Scale** toolbar, click  **Line Segments**.

Line Segments 1

- 1 In the **Settings** window for **Line Segments**, locate the **Data** section.
- 2 From the **Dataset** list, choose **Study 1 - Frequency sweep/Solution 1 (sol1)**.
- 3 From the **Parameter selection (freq)** list, choose **First**.
- 4 Locate the **x-Coordinates** section. In the table, enter the following settings:

Expression	Unit	Description
f_theo	1/s	Lumped-parameter resonance frequency
f_theo	1/s	Lumped-parameter resonance frequency

5 Locate the **y-Coordinates** section. In the table, enter the following settings:

Expression	Unit	Description
0.1	1	
100	1	

- 6 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.
- 7 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 8 From the **Legends** list, choose **Manual**.
- 9 In the table, enter the following settings:

Legends
Theoretical resonance

10 In the **Frequency Sweeps - Logarithmic Scale** toolbar, click  **Plot**.

Frequency Sweeps - Linear Scale

- 1 In the **Model Builder** window, right-click **Frequency Sweeps - Logarithmic Scale** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Axis** section.
- 3 Clear the **x-axis log scale** checkbox.

- 4 In the **Label** text field, type Frequency Sweeps - Linear Scale.
- 5 Locate the **Legend** section. From the **Position** list, choose **Upper right**.