Model created in COMSOL Multiphysics 6.4

# Eigenmodes in Air Bubble with Surface Tension

## *Introduction*

This model analyses the eigenmodes and eigenfrequencies of a spherical air bubble in water. The model evaluates the pulsation mode and a set of surface modes. The eigenfrequencies of the surface modes are determined by the surface tension of the air-water interface. The model compares the numerical results with the analytical results in Ref. 1. Lastly, the impact of the surroundings fluids viscosity on the eigenfrequencies are investigated.

## *Model Definition*

The vibrations of a spherically symmetric bubble are modeled in a 2D axisymmetric model for various azimuthal mode numbers. The model consists of an air bubble with radius $a$ = 10 µm in surrounded by a water domain; see Figure 1. The air bubble and surrounding fluid domain is modeled with Thermoviscous Acoustics and then coupled to a Pressure Acoustics domain to which a Spherical Wave Radiation boundary is applied.

The interior boundary condition Surface Tension is used to include the effects of the surface tension on the bubble surface. An eigenfrequency study is used to find the resonances of a 10 µm air bubble in water.

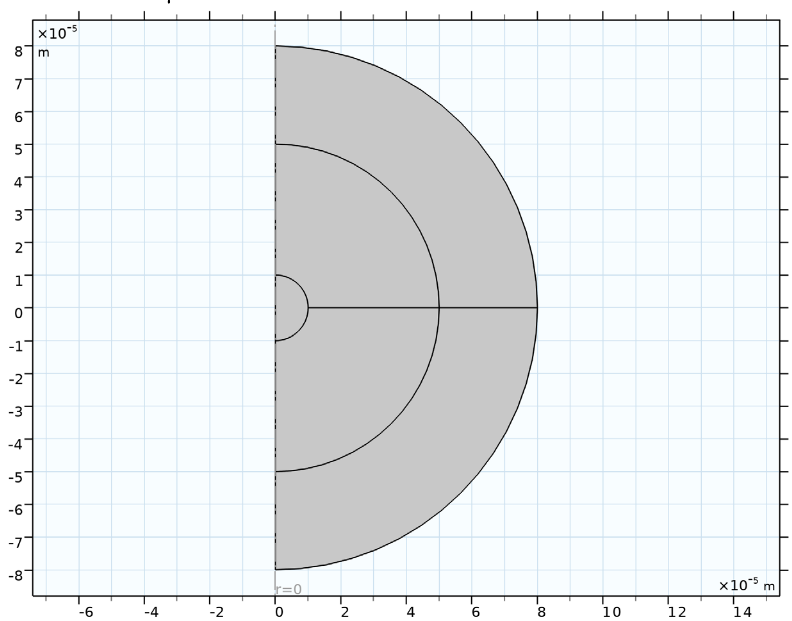


*Figure 1: Model geometry of the air bubble (inner circle) and surrounding fluid.*

The pulsation or compression resonance of an air bubble in water is given by the sound speed. It therefore depends on whether the pressure field in the bubble is isothermal or adiabatic. For small bubbles the wave will be isothermal and for large bubbles mainly adiabatic. The resonance frequency is given in Ref. 1 as

$$f_0 = \frac{1}{2\pi a}\left[\frac{3\kappa P_0}{\rho_0} - \frac{2\gamma}{\rho_0 a}\right]^{1/2} \tag{1}$$

where $\kappa$ is the polytropic coefficient ranging between 1 for isothermal and 1.4 for an isotropic acoustic field, $\gamma$ is the surface tension coefficient, and $P_0$ is the steady pressure inside the bubble.

The surface modes are governed by the surface tension. If the fluid viscosities are negligible, the resonance frequencies for mode numbers $l > 1$ are given in Ref. 1 as

$$f_l = \frac{1}{2\pi a}\left[\frac{(l-1)(l+1)(l+2)\gamma}{\rho_0 a}\right]^{1/2} \tag{2}$$

Including the viscosity of the surrounding fluid tends to lower the resonance frequencies. The model investigates mode numbers up to $l = 4$ and the azimuthal mode numbers $m = 0, 1, 2$. A resonance mode will be identified by these two mode numbers as $(l, m)$.

When investigating the viscosity the fluid parameters of water will be manipulated such that the viscosity will be

$$\mu = N_\mu \mu_0 \tag{3}$$

Where $N_\mu$ is a dimensionless scaling parameter and $\mu_0$ is the viscosity of water.

# Results and Discussion

The resulting pressure field for the eigenmodes will be discontinuous across the air-water interface due to the surface tension. In Figure 2 the pressure field for the mode ($l$ = 4, $m$ = 0) is shown. Here, $l$ is the total mode number and $m$ is the azimuthal mode number.

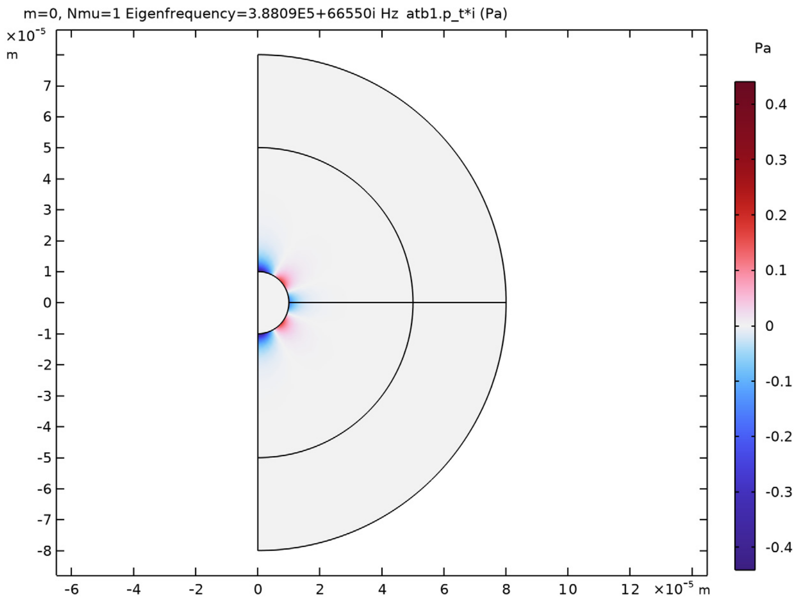


*Figure 2: Acoustic pressure for the mode (4,0). The pressure is discontinuous across the air-water interface.*

The vibrations of the 10 micrometer bubble can be visualized with the Revolving 2D dataset. In Figure 3 the displacement along the normal vector is plotted for the mode (2,2).

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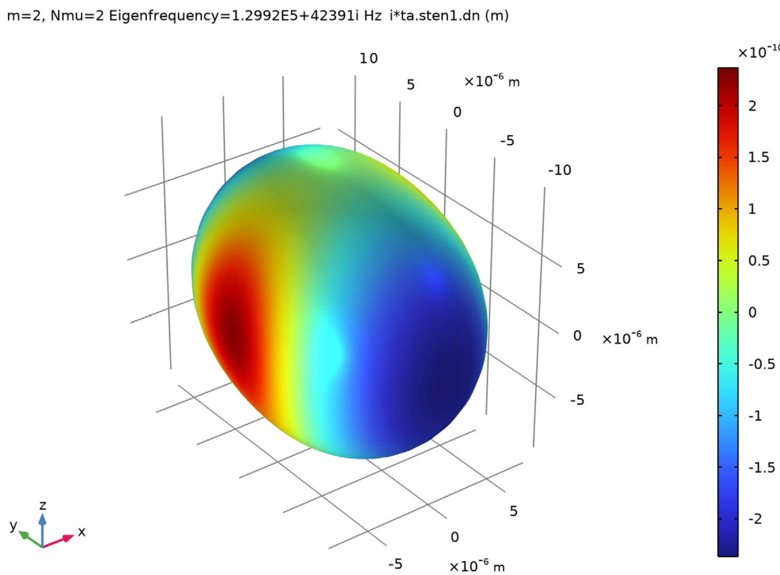


*Figure 3: Color plot of the displacement of the air-water interface for the eigenmode (2,2).*

The different eigenmodes correspond to the spherical harmonics. A selection of the modes are shown in Figure 4 for the mode numbers $l$ = 0, 2, 3, 4 and azimuthal mode numbers $m$ = 0, 1, 2.

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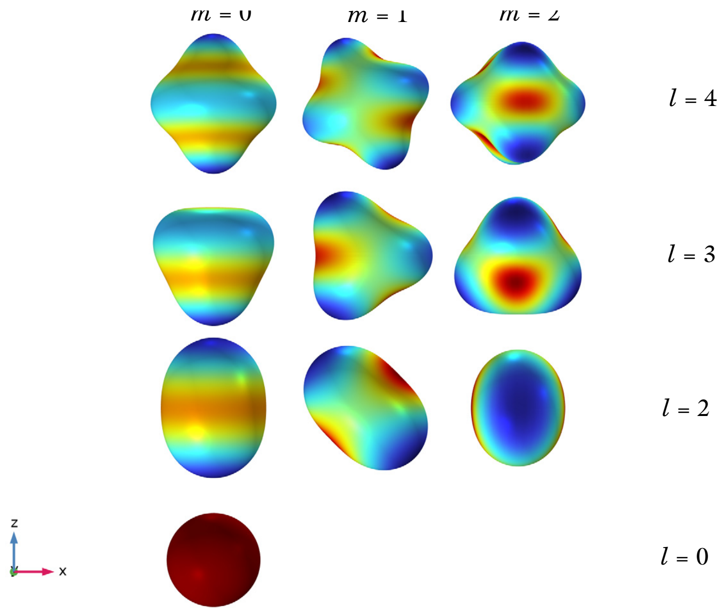



*Figure 4: The eigenmodes of the air bubble for a set of mode numbers (l, m).*

Lastly, the eigenfrequencies are compared to the analytical expressions in Equation 1 and Equation 2. The two red lines show the isothermal and isotropic resonance frequency for the pulsation mode (0, 0) given by Equation 1. The black line show the resonance frequency for the surface modes $l$ = 2, 3, 4 given by Equation 2. Modes with the same wave number but different azimuthal wave number has the same eigenfrequencies. For a negligible viscosity $N_{\mu} = 0.02$ the eigenfrequencies are in good agreement with the analytical expressions, see Figure 5. For higher viscosity the resonance frequency of the surface modes are decreasing significantly while the resonance of the pulsation mode is constant.

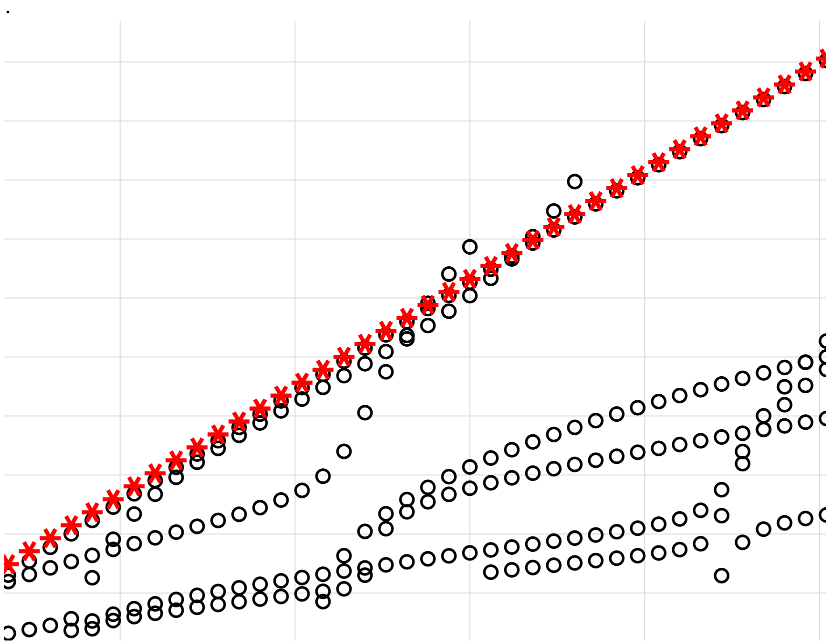


*Figure 5: Eigenfrequencies for an air bubble in water. The eigenfrequency of the surface modes depends on the viscosity of the surrounding fluid while the pulsation mode is constant.*

## *Notes About the COMSOL Implementation*

For the eigenfrequency solver it is important that the equations do not depend on the reciprocal of the frequency. Since the surface tension depends on the displacement of the air-water interface it depends on $1/\omega$. Therefore, the scaling parameter `ta.delta`, which is applied to all equations in the Thermoviscous Acoustics interface, is set equal to `ta.iomega`. Thereby, the equations no longer depend on $1/\omega$. This is done by setting **Equation form** to **Frequency domain** and then setting the scaling factor manually to `1/ta.iomega`.

When modeling the azimuthal mode number it is important to set the value in both the Thermoviscous Acoustics and Pressure Acoustics interfaces as well as in the **Revolution 2D** node under the node **Datasets**.

## *Reference*


1. S. Temkin, *Suspension Acoustics*, Cambridge University Press, 2005

**Application Library path:** `Acoustics_Module/Tutorials,_Thermoviscous_Acoustics/eigenmodes_air_bubble`

## *Modeling Instructions*

From the **File** menu, choose **New**.

### NEW

In the **New** window, click **Model Wizard**.

### MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Acoustics** > **Thermoviscous Acoustics** > **Thermoviscous Acoustics, Frequency Domain (ta)**.

3 Click **Add**.

4 In the **Select Physics** tree, select **Acoustics** > **Pressure Acoustics** > **Pressure Acoustics, Frequency Domain (acpr)**.

5 Click **Add**.

6 Click **Study**.

7 In the **Select Study** tree, select **General Studies** > **Eigenfrequency**.

8 Click **Done**.

### GLOBAL DEFINITIONS

*Parameters 1*

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

| Name | Expression | Value | Description |
|---|---|---|---|
| R0 | 10[µm] | 1E-5 m | Radius |
| gamma | 72.9[mN/m] | 0.0729 N/m | Surface tension coefficient |
| PAWater | 1[atm] | 1.0133E5 Pa | Pressure in water |

| Name | Expression | Value | Description |
|---|---|---|---|
| PABubble | PAWater+gamma*2/R0 | 1.1591E5 $N/m^2$ | Pressure in air bubble |
| m | 0 | 0 | Azimuthal mode number |
| rhowater | 998.2[kg/m^3] | 998.2 $kg/m^3$ | Density, water |
| Nmu | 1 | 1 | Viscosity scaling parameter |

## GEOMETRY 1

*Circle 1 (c1)*

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type 8*R0.

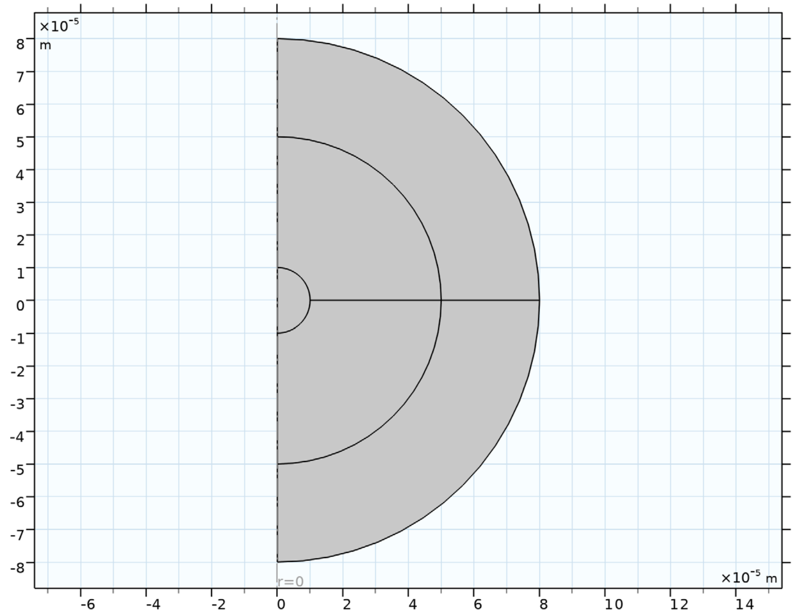
4 In the **Sector angle** text field, type 180.

5 Locate the **Rotation Angle** section. In the **Rotation** text field, type -90.

6 Click to expand the **Layers** section. In the table, enter the following settings:

| Layer name | Thickness (m) |
|---|---|
| Layer 1 | 3*R0 |
| Layer 2 | 4*R0 |

**7** Click **Build All Objects**.




## ADD MATERIAL

**1** In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

**2** Go to the **Add Material** window.

**3** In the tree, select **Built-in** > **Water, liquid**.

**4** Click the **Add to Component** button in the window toolbar.

**5** In the tree, select **Built-in** > **Air**.

**6** Click the **Add to Component** button in the window toolbar.

**7** In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

## MATERIALS

*Air (mat2)*

Select Domain 3 only.

*Water, liquid (mat1)*

**1** In the **Model Builder** window, click **Water, liquid (mat1)**.

**2** In the **Settings** window for **Material**, locate the **Material Contents** section.

**3** In the table, enter the following settings:

| Property | Variable | Value | Unit | Property group |
|---|---|---|---|---|
| Dynamic viscosity | mu | `eta(T)*Nmu` | Pa·s | Basic |

The parameter Nmu is introduced to study the resonance frequencies dependency on the viscosity of the surrounding fluid.

## THERMOVISCOUS ACOUSTICS, FREQUENCY DOMAIN (TA)

**1** In the **Model Builder** window, under **Component 1 (comp1)** click **Thermoviscous Acoustics, Frequency Domain (ta)**.

**2** In the **Settings** window for **Thermoviscous Acoustics, Frequency Domain**, locate the **Thermoviscous Acoustics Equation Settings** section.

**3** Select the **Out-of-plane mode extension** checkbox.

**4** In the $m$ text field, type `m`.

**5** Click to expand the **Equation** section. From the **Equation form** list, choose **Frequency domain**.

**6** Locate the **Thermoviscous Acoustics Equation Settings** section. In the Δ text field, type `ta.iomega`.

The parameter `ta.delta` is manually changed so that the final equations do not depend the reciprocal of ta.omega.

### *Thermoviscous Acoustics Model 1*

**1** In the **Model Builder** window, under **Component 1 (comp1)** > **Thermoviscous Acoustics, Frequency Domain (ta)** click **Thermoviscous Acoustics Model 1**.

**2** In the **Settings** window for **Thermoviscous Acoustics Model**, locate the **Model Input** section.

**3** In the $p_0$ text field, type `PAWater`.

### *Thermoviscous Acoustics Model 2*

**1** In the **Physics** toolbar, click **Domains** and choose **Thermoviscous Acoustics Model**.

**2** Select Domain 3 only.

**3** In the **Settings** window for **Thermoviscous Acoustics Model**, locate the **Model Input** section.

**4** In the $p_0$ text field, type `PABubble`.

**5** In the **Model Builder** window, click **Thermoviscous Acoustics, Frequency Domain (ta)**.

**6** Select Domains 2–4 only.

*Surface Tension 1*

1 In the **Physics** toolbar, click **Boundaries** and choose **Surface Tension**.

2 Select Boundaries 11 and 12 only.

3 In the **Settings** window for **Surface Tension**, locate the **Surface Tension** section.

4 From the **Surface tension coefficient** list, choose **User defined**. In the σ text field, type `gamma`.

The **Surface Tension** feature is added on the interior boundary between the air bubble and the surrounding water domain.

#### PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Pressure Acoustics, Frequency Domain (acpr)**.

2 Select Domains 1 and 5 only.

3 In the **Settings** window for **Pressure Acoustics, Frequency Domain**, locate the **Pressure Acoustics Equation Settings** section.

4 In the $m$ text field, type `m`.

*Pressure Acoustics 1*

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Pressure Acoustics, Frequency Domain (acpr)** click **Pressure Acoustics 1**.

2 In the **Settings** window for **Pressure Acoustics**, locate the **Model Input** section.

3 In the $p_A$ text field, type `PAWater`.

4 Locate the **Pressure Acoustics Model** section. From the **Fluid model** list, choose **Thermally conducting and viscous**.

*Spherical Wave Radiation 1*

1 In the **Physics** toolbar, click **Boundaries** and choose **Spherical Wave Radiation**.

2 Select Boundaries 9 and 14 only.

#### MULTIPHYSICS

*Acoustic–Thermoviscous Acoustic Boundary 1 (atb1)*

1 In the **Physics** toolbar, click **Multiphysics Couplings** and choose **Boundary** > **Acoustic–Thermoviscous Acoustic Boundary**.

2 Select Boundaries 10 and 13 only.

## MESH 1

*Free Triangular 1*

In the **Mesh** toolbar, click **Free Triangular**.

*Size 1*

1 Right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundaries 11 and 12 only.

5 Locate the **Element Size** section. Click the **Custom** button.

6 Locate the **Element Size Parameters** section.

7 Select the **Maximum element size** checkbox. In the associated text field, type `R0/20`.

*Size*

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 From the **Predefined** list, choose **Fine**.

4 Click **Build All**.

## STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** checkbox.

*Parametric Sweep*

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 From the **Sweep type** list, choose **All combinations**.

4 Click **Add**.

5 In the table, enter the following settings:

| Parameter name | Parameter value list | Parameter unit |
|---|---|---|
| m (Azimuthal mode number) | {0,1,2} | |

6 Click **Add**.

**7** In the table, enter the following settings:

| Parameter name | Parameter value list | Parameter unit |
|---|---|---|
| Nmu (Viscosity scaling parameter) | {0.02,0.5,1,2} | |

Parametric sweep set up to study different angular mode numbers m and different viscosities.

*Step 1: Eigenfrequency*

**1** In the **Model Builder** window, click **Step 1: Eigenfrequency**.

**2** In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.

**3** Select the **Desired number of eigenfrequencies** checkbox. In the associated text field, type 4.

**4** In the **Search for eigenfrequencies around shift** text field, type 80[kHz].

**5** From the **Search method around shift** list, choose **Larger real part**.

*Solution 1 (sol1)*

**1** In the **Study** toolbar, click **Show Default Solver**.

**2** In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Eigenvalue Solver 1**.

**3** In the **Settings** window for **Eigenvalue Solver**, locate the **General** section.

**4** Find the **Eigenvalue linearization point** subsection. In the **Value of eigenvalue linearization point** text field, type 250[kHz].

**5** In the **Study** toolbar, click **Compute**.

## RESULTS

In the **Model Builder** window, expand the **Results** node.

*Revolution 2D 1*

**1** In the **Model Builder** window, expand the **Results** > **Datasets** node.

**2** Right-click **Results** > **Datasets** and choose **Revolution 2D**.

**3** In the **Model Builder** window, click **Revolution 2D 1**.

**4** In the **Settings** window for **Revolution 2D**, locate the **Data** section.

**5** From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

**6** Click to expand the **Revolution Layers** section. In the **Revolution angle** text field, type 380.

7 Click to expand the **Advanced** section. Find the **phi** subsection. In the **Azimuthal mode number** text field, type `m`.

## RESULT TEMPLATES

1 In the **Results** toolbar, click **Result Templates** to open the **Result Templates** window.

2 Go to the **Result Templates** window.

3 In the tree, select **Study 1/Parametric Solutions 1 (sol2)** > **Acoustic–Thermoviscous Acoustic Boundary 1** > **Acoustic Pressure (atb1)**.

4 Click the **Add Result Template** button in the window toolbar.

5 In the **Results** toolbar, click **Result Templates** to close the **Result Templates** window.

## RESULTS

### *Acoustic Pressure (atb1)*

1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

2 From the **Parameter value (m)** list, choose **0**.

3 From the **Parameter value (Nmu)** list, choose **1**.

4 From the **Eigenfrequency (Hz)** list, choose **3.8809E5+66550i**.

### *Surface 1*

1 In the **Model Builder** window, expand the **Acoustic Pressure (atb1)** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `atb1.p_t*i`.

4 In the **Acoustic Pressure (atb1)** toolbar, click **Plot**.

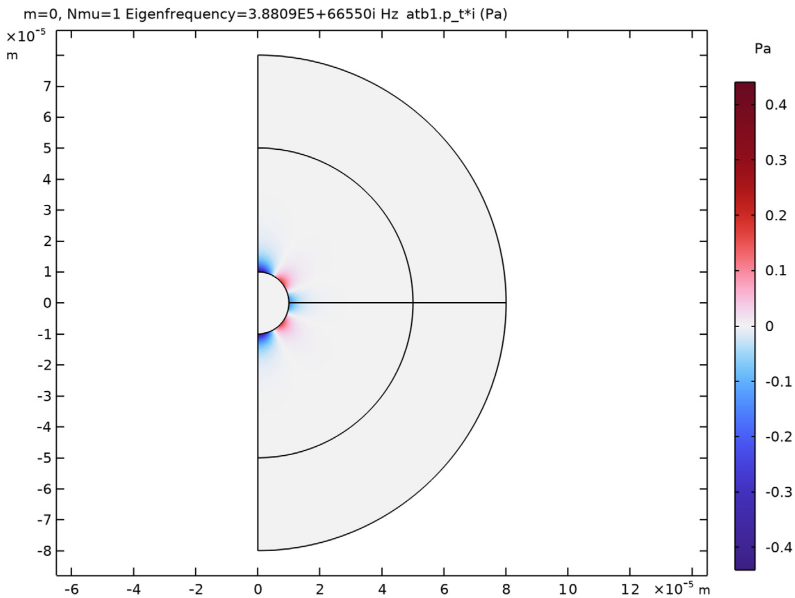



Figure of the acoustic pressure for one eigenfrequency with m=0.

*Bubble Displacement*

1 In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, type `Bubble Displacement` in the **Label** text field.

3 Locate the **Plot Settings** section. Clear the **Plot dataset edges** checkbox.

*Surface 1*

1 Right-click **Bubble Displacement** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `i*ta.sten1.dn`.

4 Locate the **Coloring and Style** section. From the **Scale** list, choose **Linear symmetric**.

*Deformation 1*

1 Right-click **Surface 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Expression** section.

3 In the **R-component** text field, type `i*ta.sten1.dn*nr`.

4 In the **PHI-component** text field, type `0`.

**5** In the **Z-component** text field, type `i*ta.sten1.dn*nz`.

**6** In the **Bubble Displacement** toolbar, click **Plot**.

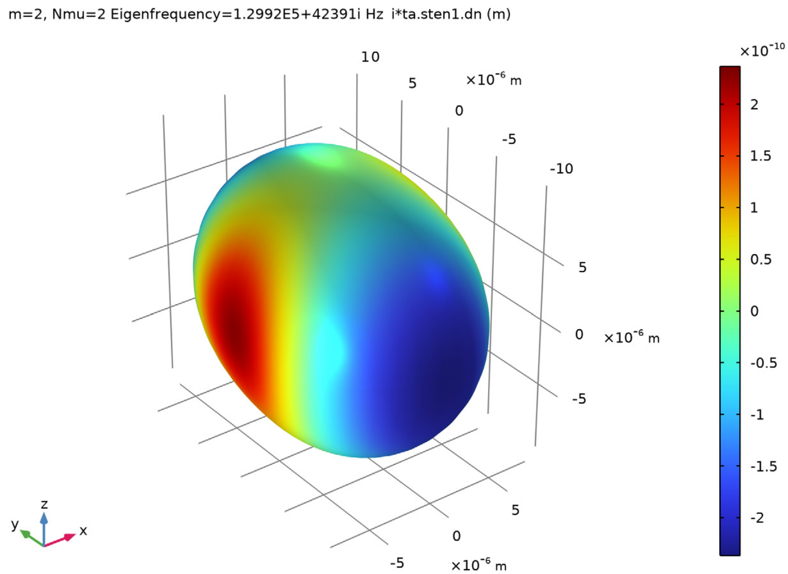



Figure of the surface displacement and bubble deformation for one eigenfrequency with m=2.

*Bubble Displacement Array*

**1** In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

**2** In the **Settings** window for **3D Plot Group**, type `Bubble Displacement Array` in the **Label** text field.

**3** Locate the **Plot Settings** section. Clear the **Plot dataset edges** checkbox.

**4** Click to expand the **Plot Array** section. From the **Array type** list, choose **Square**.

**5** From the **Array plane** list, choose **xz**.

*Surface 1*

**1** Right-click **Bubble Displacement Array** and choose **Surface**.

**2** In the **Settings** window for **Surface**, locate the **Data** section.

**3** From the **Dataset** list, choose **Revolution 2D 1**.

**4** From the **Parameter value (m)** list, choose **0**.

**5** From the **Parameter value (Nmu)** list, choose **1**.

6 From the **Eigenfrequency (Hz)** list, choose **3.0399E5+24043i**.

7 Locate the **Expression** section. In the **Expression** text field, type `i*ta.sten1.dn`.

8 Locate the **Coloring and Style** section. From the **Scale** list, choose **Linear symmetric**.

9 Click to expand the **Plot Array** section. Select the **Manual indexing** checkbox.

10 In the **Row index** text field, type `0`.

11 In the **Column index** text field, type `0`.

12 Locate the **Coloring and Style** section. Clear the **Color legend** checkbox.

13 Click to expand the **Title** section. From the **Title type** list, choose **None**.

*Deformation 1*

1 Right-click **Surface 1** and choose **Deformation**.

2 In the **Settings** window for **Deformation**, locate the **Expression** section.

3 In the **R-component** text field, type `i*ta.sten1.dn*nr`.

4 In the **PHI-component** text field, type `0`.

5 In the **Z-component** text field, type `i*ta.sten1.dn*nz`.

*Surface 2*

1 In the **Model Builder** window, under **Results** > **Bubble Displacement Array** right-click **Surface 1** and choose **Duplicate**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Eigenfrequency (Hz)** list, choose **1.4169E5+24879i**.

4 Locate the **Plot Array** section. In the **Row index** text field, type `1`.

*Surface 3*

1 Right-click **Surface 2** and choose **Duplicate**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Eigenfrequency (Hz)** list, choose **2.5929E5+43471i**.

4 Locate the **Plot Array** section. In the **Row index** text field, type `2`.

*Surface 4*

1 Right-click **Surface 3** and choose **Duplicate**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Eigenfrequency (Hz)** list, choose **3.8809E5+66550i**.

4 Locate the **Plot Array** section. In the **Row index** text field, type `3`.

*Surface 5*

1 Right-click **Surface 4** and choose **Duplicate**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Parameter value (m)** list, choose **1**.
4 From the **Eigenfrequency (Hz)** list, choose **1.4167E5+24866i**.
5 Locate the **Plot Array** section. In the **Row index** text field, type 1.
6 In the **Column index** text field, type 1.

*Surface 6*

1 Right-click **Surface 5** and choose **Duplicate**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Eigenfrequency (Hz)** list, choose **2.593E5+43472i**.
4 Locate the **Plot Array** section. In the **Row index** text field, type 2.

*Surface 7*

1 Right-click **Surface 6** and choose **Duplicate**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Eigenfrequency (Hz)** list, choose **3.8806E5+66538i**.
4 Locate the **Plot Array** section. In the **Row index** text field, type 3.

*Surface 8*

1 Right-click **Surface 7** and choose **Duplicate**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Parameter value (m)** list, choose **2**.
4 From the **Eigenfrequency (Hz)** list, choose **1.4169E5+24866i**.
5 Locate the **Plot Array** section. In the **Row index** text field, type 1.
6 In the **Column index** text field, type 2.

*Surface 9*

1 Right-click **Surface 8** and choose **Duplicate**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Eigenfrequency (Hz)** list, choose **2.5928E5+43459i**.
4 Locate the **Plot Array** section. In the **Row index** text field, type 2.

*Surface 10*

1 Right-click **Surface 9** and choose **Duplicate**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

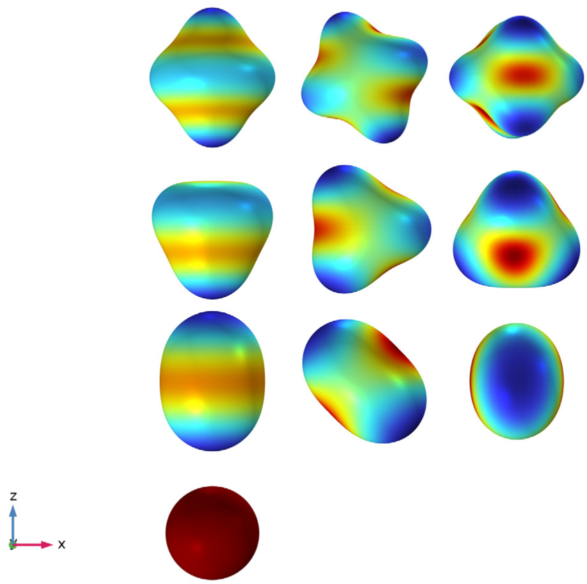
3 From the **Eigenfrequency (Hz)** list, choose **3.8808E5+66532i**.

4 Locate the **Plot Array** section. In the **Row index** text field, type 3.

5 In the **Bubble Displacement Array** toolbar, click **Plot**.

6 Click the **Show Grid** button in the **Graphics** toolbar.

7 Click the **Go to XZ View** button in the **Graphics** toolbar.




Array of the pulsation and surface modes for m = 0, 1, and 2.

*Evaluation Group 1*

In the **Results** toolbar, click **Evaluation Group**.

*Global Evaluation 1*

1 Right-click **Evaluation Group 1** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.

4 Locate the **Expressions** section. In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| freq | Hz | Frequency |

**5** In the **Evaluation Group 1** toolbar, click **Evaluate**.

*Bubble Resonances*

**1** In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

**2** In the **Settings** window for **1D Plot Group**, type `Bubble Resonances` in the **Label** text field.

**3** Locate the **Axis** section. Select the **Manual axis limits** checkbox.

**4** In the **x minimum** text field, type `120e3`.

**5** In the **x maximum** text field, type `430e3`.

**6** In the **y minimum** text field, type `-0.05`.

**7** In the **y maximum** text field, type `2.05`.

*Table Graph 1*

**1** Right-click **Bubble Resonances** and choose **Table Graph**.

**2** In the **Settings** window for **Table Graph**, locate the **Data** section.

**3** From the **Source** list, choose **Evaluation group**.

**4** From the **Plot columns** list, choose **Manual**.

**5** From the **x-axis data** list, choose **Eigenfrequency (Hz)**.

**6** In the **Columns** list box, select **Nmu**.

**7** Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

**8** Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

*Line Segments 1*

**1** In the **Model Builder** window, right-click **Bubble Resonances** and choose **Line Segments**.

**2** In the **Settings** window for **Line Segments**, locate the **x-Coordinates** section.

**3** In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| `1/(2*pi*R0)*sqrt((2-1)*(2+1)*(2+2)*gamma/((rhowater)*R0))` | 1/s | |
| `1/(2*pi*R0)*sqrt((2-1)*(2+1)*(2+2)*gamma/((rhowater)*R0))` | 1/s | |

**4** Locate the **y-Coordinates** section. In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| 0 | 1 | |
| 2 | 1 | |

**5** Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.

**6** In the **Bubble Resonances** toolbar, click **Plot**.

*Line Segments 2*

**1** Right-click **Bubble Resonances** and choose **Line Segments**.

**2** In the **Settings** window for **Line Segments**, locate the **x-Coordinates** section.

**3** In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| `1/(2*pi*R0)*sqrt((3-1)*(3+1)*(3+2)*gamma/((rhowater)*R0))` | 1/s | |
| `1/(2*pi*R0)*sqrt((3-1)*(3+1)*(3+2)*gamma/((rhowater)*R0))` | 1/s | |

**4** Locate the **y-Coordinates** section. In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| 0 | 1 | |
| 2 | 1 | |

**5** Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.

**6** In the **Bubble Resonances** toolbar, click **Plot**.

*Line Segments 3*

**1** Right-click **Bubble Resonances** and choose **Line Segments**.

**2** In the **Settings** window for **Line Segments**, locate the **x-Coordinates** section.

**3** In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| `1/(2*pi*R0)*sqrt((4-1)*(4+1)*(4+2)*gamma/((rhowater)*R0))` | 1/s | |
| `1/(2*pi*R0)*sqrt((4-1)*(4+1)*(4+2)*gamma/((rhowater)*R0))` | 1/s | |

**4** Locate the **y-Coordinates** section. In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| 0 | 1 | |
| 2 | 1 | |

**5** Click to expand the **Coloring and Style** section. From the **Color** list, choose **Black**.

Plotting the theoretical eigenfrequencies for an inviscid surrounding fluid.

*Line Segments 4*

**1** Right-click **Bubble Resonances** and choose **Line Segments**.

**2** In the **Settings** window for **Line Segments**, locate the **x-Coordinates** section.

**3** In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| `1/(2*pi*R0)*sqrt(3*1.4*PAWater/rhowater)* sqrt(1+2*gamma/(PAWater*R0)*(1-1/(3*1.4)))` | 1/s | |
| `1/(2*pi*R0)*sqrt(3*1.4*PAWater/rhowater)* sqrt(1+2*gamma/(PAWater*R0)*(1-1/(3*1.4)))` | 1/s | |

**4** Locate the **y-Coordinates** section. In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| 0 | 1 | |
| 2 | 1 | |

**5** Click to expand the **Coloring and Style** section. From the **Color** list, choose **Red**.

*Line Segments 5*

**1** Right-click **Bubble Resonances** and choose **Line Segments**.

**2** In the **Settings** window for **Line Segments**, locate the **x-Coordinates** section.

**3** In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| `1/(2*pi*R0)*sqrt(3*1*PAWater/rhowater)* sqrt(1+2*gamma/(PAWater*R0)*(1-1/(3*1)))` | 1/s | |
| `1/(2*pi*R0)*sqrt(3*1*PAWater/rhowater)* sqrt(1+2*gamma/(PAWater*R0)*(1-1/(3*1)))` | 1/s | |

**4** Locate the **y-Coordinates** section. In the table, enter the following settings:

| Expression | Unit | Description |
|---|---|---|
| 0 | 1 | |
| 2 | 1 | |

**5** Click to expand the **Coloring and Style** section. From the **Color** list, choose **Red**.

Plotting the theoretical eigenfrequencies for the surface modes (black) and the analytical prediction for the pulsating mode for an adiabatic and isothermal acoustic field (red).

**6** In the **Bubble Resonances** toolbar, click **Plot**.