Model created in COMSOL Multiphysics 6.4

Iron Sphere in a Magnetic Field — Static Field

Introduction

This tutorial is part of a series on modeling an iron sphere in a background magnetic field within the Introduction to Electromagnetics tutorial group. The focus here is on the case of a magnetically permeable iron sphere in a static, spatially uniform magnetic field. Two approaches to numerically modeling the sphere are investigated and compared to the analytical solution for a range of permeability values of the sphere material.

This tutorial starts with the base model constructed in the introduction section of this tutorial series. The user can begin either by opening the completed introduction model or by following the model construction steps outlined in the introduction.

Model Definition

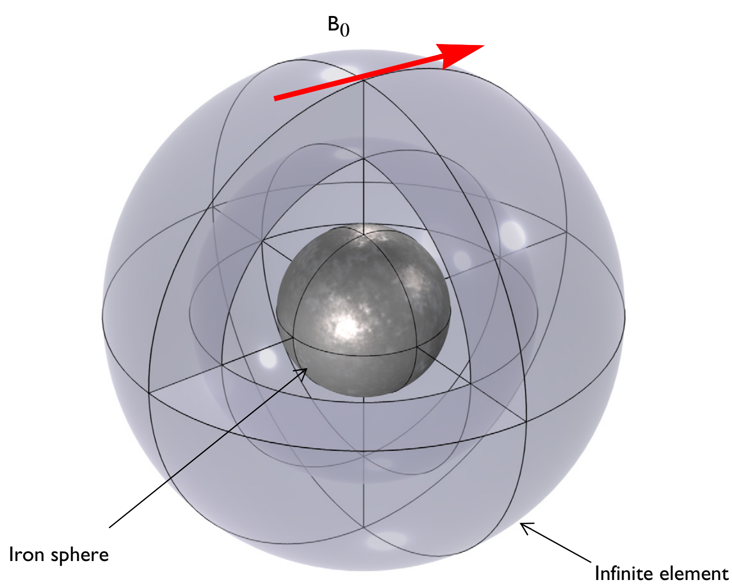


Figure 1: A magnetically permeable iron sphere in a spatially uniform background magnetic field. The sphere at the center is surrounded by air and enclosed in a region of Infinite Elements. The middle layer is the region of interest and defined as the analysis domain.

This model, as well as the others in this series, uses the same basic structure illustrated in Figure 1. It consists of a 0.25 mm diameter iron sphere placed in a spatially uniform background magnetic field of strength $B_0 = 1$ mT.

In this tutorial, the relative permeability of the sphere is varied from $\mu_r = 2$ to $\mu_r = 4000$. The analytic solution for the field inside a permeable sphere exposed to a uniform magnetic field is:

$$\mathbf{B} = \mathbf{B}_0\left(\frac{3\mu_r}{\mu_r + 2}\right), \qquad (1)$$

where B_0 is the background magnetic field.

There are two ways in which this problem can be formulated using either a scalar or vector potential approach.

Scalar Potential Formulation

The scalar potential formulation, used in the Magnetic Fields, No Currents (mfnc) interface, solves the magnetic flux conservation equation:

$$\nabla \cdot \mathbf{B} = 0. \qquad (2)$$

It uses a partial differential equation for the magnetic scalar potential field, V_m:

$$\nabla \cdot \mu_r\mu_0(-\nabla V_m + \mathbf{H}_b) = 0, \qquad (3)$$

where the background field is specified in terms of the **H**-field, $\mathbf{H}_b$. The **B**-field is then computed from the **H**-field: $\mathbf{B} = \mu_r\mu_0\mathbf{H}$. The magnetic field is in turn computed from the gradient of the magnetic scalar potential. Because the governing equation evaluates the gradients of a scalar field, the Lagrange element formulation is used. In this formulation, the background field and boundary conditions for this problem are specified purely in terms of derivatives of the V_m field, and the solution is unique up to a constant. To remove this indeterminacy, the value of the magnetic scalar potential must be constrained at one point in the model, to fix the value of the constant.

Vector Potential Formulation

The vector potential formulation, used in the Magnetic Fields (mf) interface, solves an equation for the magnetic vector potential, **A**:

$$\nabla\times\mu_r^{-1}\mu_0^{-1}\nabla\times(\mathbf{A} + \mathbf{A}_b) = 0, \qquad (4)$$

where the **B**-field is the curl of the $(\mathbf{A} + \mathbf{A}_b)$ field. In this approach, the background field and boundary conditions are specified directly in terms of the **A**-field. Here, the governing equation takes the curl of a vector valued field, and this problem is solved using a Curl element formulation. This formulation does not require as fine of a mesh as the Lagrange element formulation to achieve the same accuracy.

Results and Discussion

Figure 2 plots the magnetic flux density calculated using the vector potential formulation from the Magnetic Fields (mf) interface, and Figure 3 shows the results computed using the scalar potential formulation from Magnetic Fields, No Currents (mfnc) interface, both for the μ_r = 4000 case. The fields in the Infinite Element region are not plotted, as these do not have any physical significance.

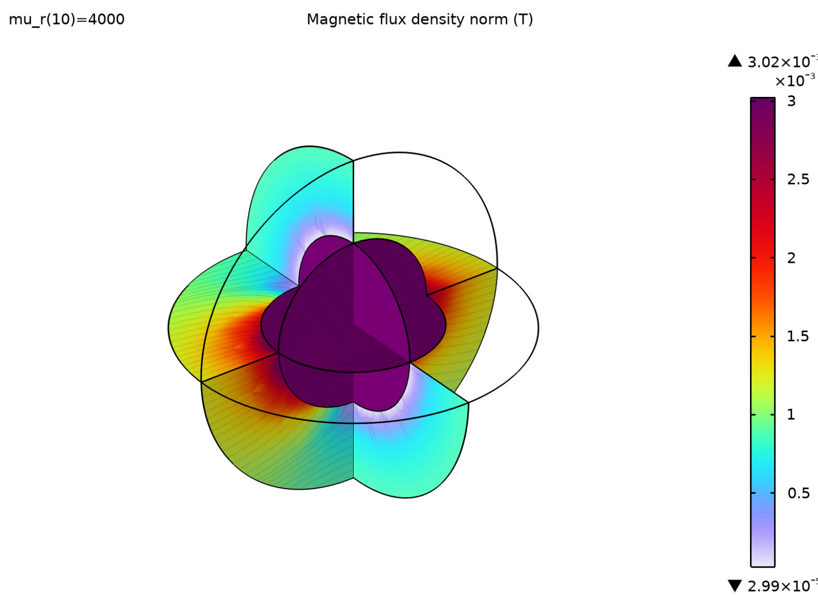


Figure 2: The magnetic field calculated using the vector potential formulation from the Magnetic Fields (mf) interface. Note: three segments of the multislice are hidden in this plot to improve visibility of all the regions. This was accomplished by hiding a region of the sphere from view during the model construction in the introduction.

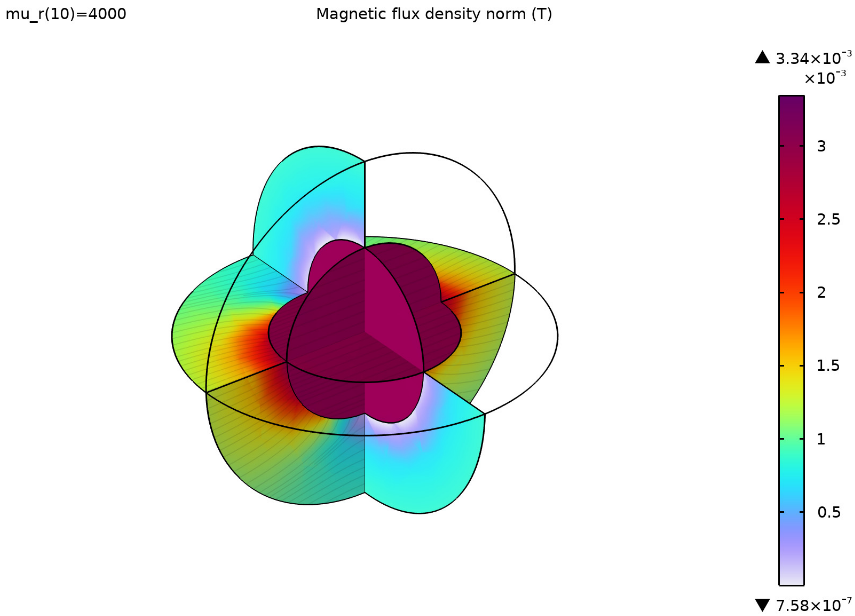


Figure 3: The magnetic field calculated using the scalar potential formulation from the Magnetic Fields, No Currents (mfnc) interface.

FLUX DENSITY AS A FUNCTION OF μ_r

Figure 4 shows the field enhancement versus the permeability for both cases, along with the analytic solution. The relative difference is plotted in Figure 5. In the limit as the mesh is refined the solutions agree within numerical precision.

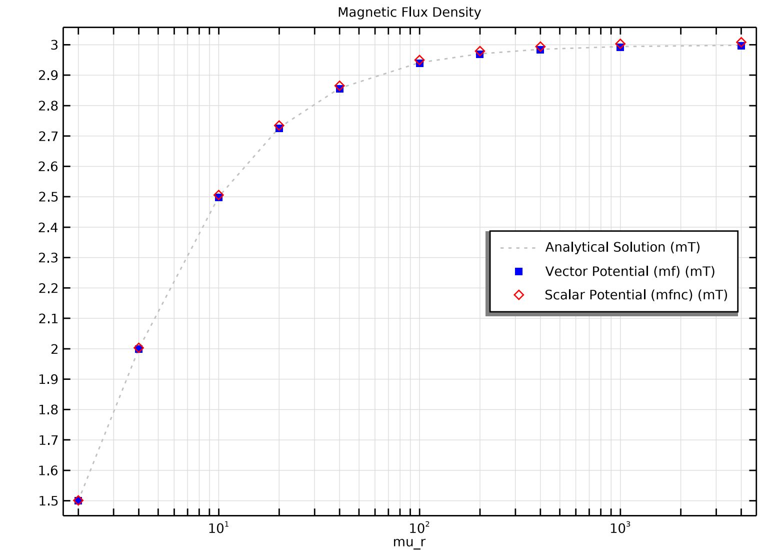


Figure 4: The magnetic flux density calculated using all three methods.

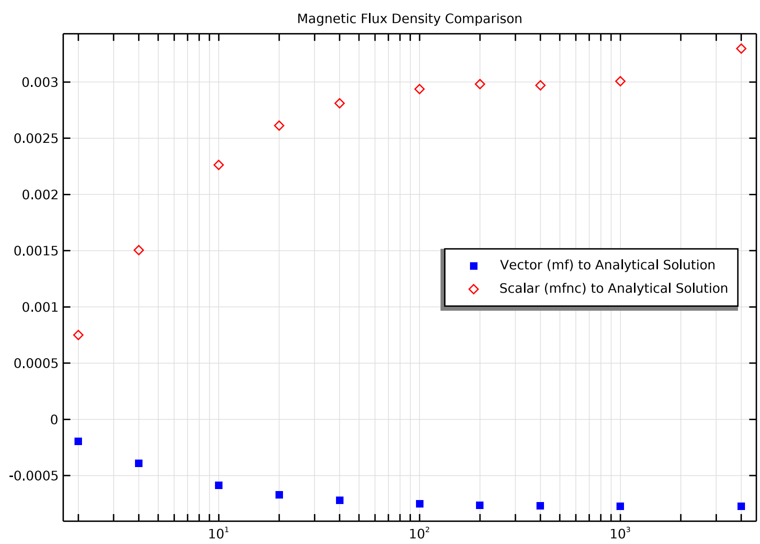


Figure 5: Relative difference of the flux density compared to the analytical solution.

There are some differences between the two formulations. In this case, the Magnetic Fields interface slightly underestimates the field strength, while the Magnetic Fields, No Current interface overestimates it. The agreement with the analytic solution for both formulations improves with increasing mesh refinement. Although the Magnetic Fields, No Currents interface requires a finer mesh for approximately the same level of accuracy, it does use less total memory. Its drawback is that it cannot be used to model situations where there is any current flowing in the model, or any variation with respect to time.

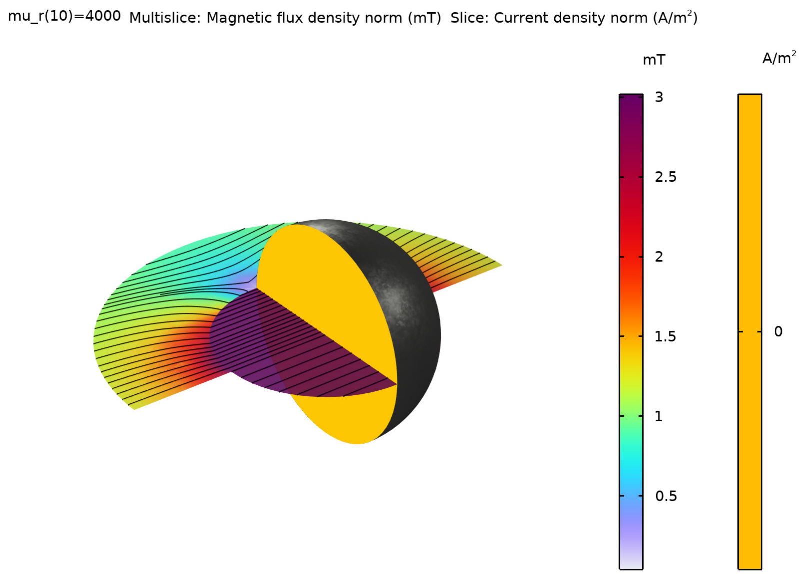


Figure 6: A cross section of the iron sphere showing no current flowing in the sphere and the magnetic flux of the background magnetic field.

Application Library path: `ACDC_Module/Introductory_Electromagnetics/iron_sphere_bfield_01_static`

Modeling Instructions

This tutorial will demonstrate the physics of an iron sphere in a static, spatially uniform magnetic field. The instructions on the following pages will help you to build, configure, solve, and analyze the model. If anything seems out of order, please retrace your steps. The finalized model — available in the model's Application Libraries folder — can help you out. You can compare it directly to your current model by means of the **Compare** option in the **Developer** toolbar.

The geometry, materials, and selections have been prepared in the *Introduction* tutorial (chapter 1). They have been saved in the file `iron_sphere_bfield_00_introduction.mph`. You can start by opening this file and saving it under a new name.

Hint: if you are new to COMSOL Multiphysics, it is worthwhile to check out the Introduction tutorial first.

1 From the **File** menu, choose **Open**.
2 Browse to the model's Application Libraries folder and double-click the file `iron_sphere_bfield_00_introduction.mph`.
3 From the **File** menu, choose **Save As**.
4 Browse to a suitable folder and type the filename `iron_sphere_bfield_01_static.mph`.

GLOBAL DEFINITIONS

Parameters 1

To begin, enter the parameters specific for this model. Namely, the value for the relative permeability of the iron sphere and the analytic calculation of the magnetic field in the sphere. The study will sweep these parameters to get a range of results.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
mu_r	4000	4000	Relative permeability, iron sphere
B_analytic	((3*mu_r)/(mu_r+2))*B0	0.0029985 T	

MATERIALS

Iron (mat2)

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Materials** click **Iron (mat2)**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	mu_r	1	Basic

This vector potential formulation does not require much detail in the outer layers so we can adjust the mesh to optimize calculation time.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Fine**.

4 Locate the **Sequence Type** section. From the list, choose **User-controlled mesh**.

Distribution 1

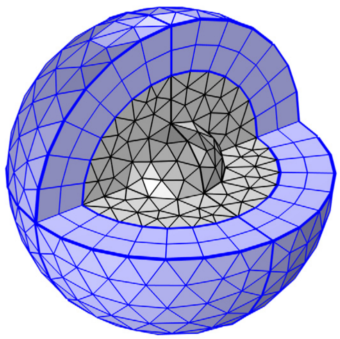
1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Mesh 1** > **Swept 1** node, then click **Distribution 1**.

2 In the **Settings** window for **Distribution**, locate the **Distribution** section.

3 In the **Number of elements** text field, type 2.

4 Click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



Note: In the introduction modeling steps, the nearest upper quarter sphere was hidden to improve visibility in the result plots. This allows the visibility of the mesh layers of the Infinite Element Domain and the Analysis Domain.

STUDY 1

Step 1: Frequency Domain

As this particular model has a static background magnetic field, the Frequency Domain study is removed and replaced it with a Stationary study.

1 In the **Model Builder** window, expand the **Study 1** node.

2 Right-click **Study 1** > **Step 1: Frequency Domain** and choose **Disable**.

Step 2: Stationary

1 In the **Study** toolbar, click **Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** checkbox.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
mu_r (Relative permeability, iron sphere)	2 4 10 20 40 100 200 400 1000 4000	

6 In the Study toolbar, click **Compute**.

RESULTS

Next, add a selection to the output plots to only look at the analysis domain as this is the area of interest.

Study 1/Solution 1 (sol1)

In the **Model Builder** window, expand the **Results** > **Datasets** node, then click **Study 1/ Solution 1 (sol1)**.

Selection

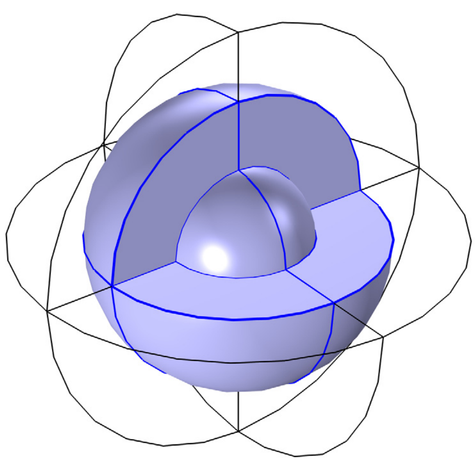
1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Analysis domain**.

5 Click the **Zoom to Selection** button in the **Graphics** toolbar.



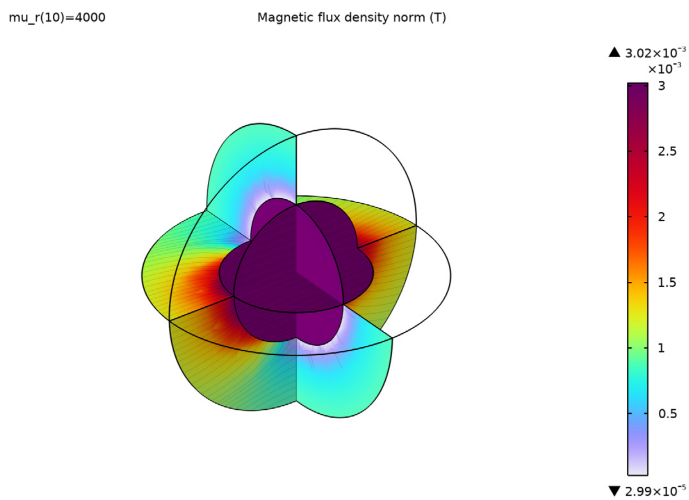
This sets the default plot to only show the analysis domain.

Magnetic Flux Density (mf)

1 In the **Model Builder** window, under **Results** click **Magnetic Flux Density (mf)**.

2 In the **Magnetic Flux Density (mf)** toolbar, click **Plot**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar.



Make a Cut Point at the origin position. This will be used to evaluate the magnetic flux density at the center of the iron sphere.

Cut Point 3D 1

1 In the **Results** toolbar, click **Cut Point 3D**.

2 In the **Settings** window for **Cut Point 3D**, locate the **Point Data** section.

3 In the **x** text field, type `0`.

4 In the **y** text field, type `0`.

5 In the **z** text field, type `0`.

Magnetic Flux Density

1 In the **Results** toolbar, click **1D Plot Group**.

The Cut Point can be used to plot the magnetic flux density evaluated at that point from both the analytic value, B_analytic, and the numerically computed result, mf.normB, for each value of mu_r.

2 In the **Settings** window for **1D Plot Group**, type `Magnetic Flux Density` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

4 Locate the **Axis** section. Select the **x-axis log scale** checkbox.

5 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

Analytical Solution

1 Right-click **Magnetic Flux Density** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, type `Analytical Solution` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 3D 1**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `B_analytic`.

5 From the **Unit** list, choose **mT**.

6 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dotted**.

7 From the **Color** list, choose **Gray**.

8 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

9 Click to expand the **Legends** section. Select the **Show legends** checkbox.

10 Find the **Include** subsection. Select the **Label** checkbox.

11 Clear the **Solution** checkbox.

12 Clear the **Point** checkbox.

13 Select the **Unit** checkbox.

14 In the **Magnetic Flux Density** toolbar, click **Plot**.

Vector Potential (mf)

1 In the **Model Builder** window, right-click **Magnetic Flux Density** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, type `Vector Potential (mf)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 3D 1**.

4 Locate the **y-Axis Data** section. From the **Unit** list, choose **mT**.

5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

7 From the **Color** list, choose **Blue**.

8 Locate the **Title** section. From the **Title type** list, choose **Label**.

9 Locate the **Legends** section. Select the **Show legends** checkbox.

10 Find the **Include** subsection. Select the **Label** checkbox.

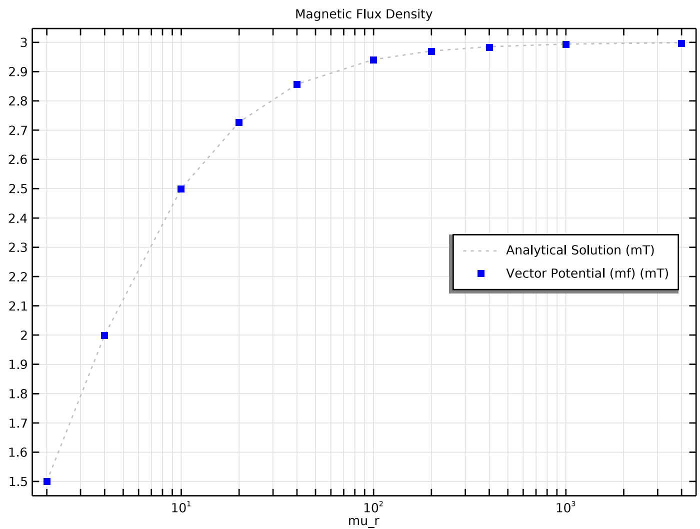
11 Clear the **Point** checkbox.

12 Clear the **Solution** checkbox.

13 Select the **Unit** checkbox.

14 In the **Magnetic Flux Density** toolbar, click **Plot**.

15 Click the **Zoom Extents** button in the **Graphics** toolbar.



The Magnetic Flux Density increases with the permeability of the iron sphere, reaching a maximum value of ~3 mT at mu_r = 4000. This is the permeability value used in the other tutorials in this series. The numerical results are very close to the analytical solution. To look at the difference more closely, we can now plot the difference between these data points.

Magnetic Flux Density Comparison

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Magnetic Flux Density Comparison` in the **Label** text field.

3 Locate the **Title** section. From the **Title type** list, choose **Label**.

4 Locate the **Axis** section. Select the **x-axis log scale** checkbox.

5 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

Vector (mf) to Analytical Solution

1 Right-click **Magnetic Flux Density Comparison** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, type `Vector (mf) to Analytical Solution` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 3D 1**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `(mf.normB-B_analytic)/B_analytic`.

5 Locate the **Title** section. From the **Title type** list, choose **Label**.

6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

7 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

8 From the **Color** list, choose **Blue**.

9 Locate the **Legends** section. Select the **Show legends** checkbox.

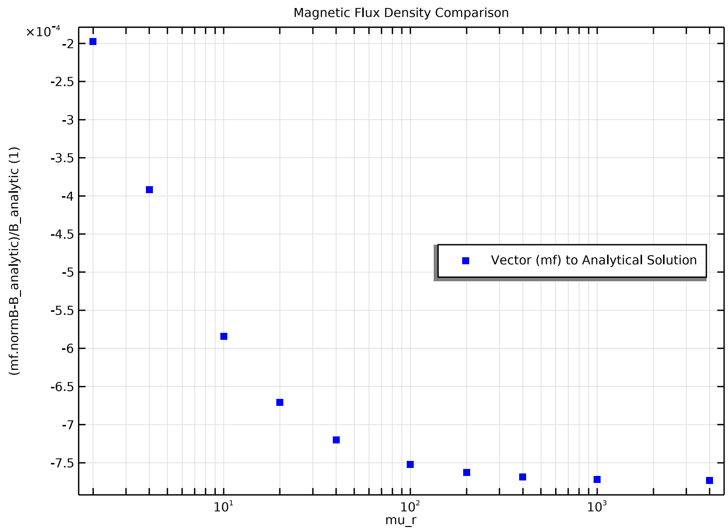
10 Find the **Include** subsection. Select the **Label** checkbox.

11 Clear the **Point** checkbox.

12 Clear the **Solution** checkbox.

13 In the **Magnetic Flux Density Comparison** toolbar, click **Plot**.

14 Click the **Zoom Extents** button in the **Graphics** toolbar.



The vector potential formulation slightly underestimates the magnetic flux density when compared to the analytically solved case.

ADD PHYSICS

The vector potential formulation is very close to the analytical solution. However, using a scalar formulation for the magnetic flux density can be less computationally expensive. The next section will demonstrate how to compute the solution with this.

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 In the tree, select **AC/DC** > **Magnetic Fields, No Currents** > **Magnetic Fields, No Currents (mfnc)**.

4 Click the **Add to Component 1** button in the window toolbar.

5 In the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.

MAGNETIC FIELDS, NO CURRENTS (MFNC)

1 In the **Settings** window for **Magnetic Fields, No Currents**, locate the **Background Magnetic Field** section.

2 From the **Solve for** list, choose **Reduced field**.

3 Specify the $\mathbf{H}_b$ vector as

B0/mu0_const	x

Magnetic Flux Conservation in Solids 1

1 In the **Physics** toolbar, click **Domains** and choose **Magnetic Flux Conservation in Solids**.

2 In the **Settings** window for **Magnetic Flux Conservation in Solids**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Iron Sphere**.

So far, the magnetic potential is not constrained anywhere and the solution can only be computed up to a constant. This model uses a zero magnetic scalar potential applied to a point on the surface of the air domain. This provides a reference point enabling the numerical solver to produce a unique solution.

Zero Magnetic Scalar Potential 1

1 In the **Physics** toolbar, click **Points** and choose **Zero Magnetic Scalar Potential**.

2 Select Point 8 only.

COMPONENT 1 (COMP1)

For the second study a mesh with greater detail is required. The default mesh parameters using a "fine" element size provides suitable detail for the scalar potential formulation.

MESH 2

1 In the **Mesh** toolbar, click **Add Mesh** and choose **Add Mesh**.

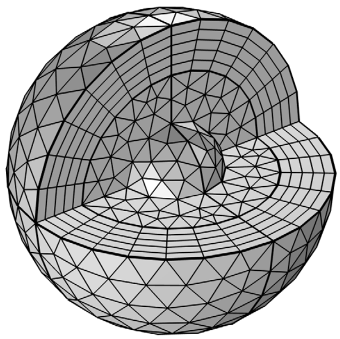
2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Fine**.

4 In the table, clear the **Use** checkbox for **Magnetic Fields (mf)**.

5 Click **Build All**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.



STUDY 1

The mfnc interface is only used in the second study of this tutorial. This interface can be removed from the first solver in case the user wishes to rerun the first study at a later point.

Step 2: Stationary

1. In the **Model Builder** window, under **Study 1** click **Step 2: Stationary**.
2. In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
3. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Magnetic Fields, No Currents (mfnc)**.

ADD STUDY

Next, add a study that only uses the mfnc interface to obtain the solution using the vector potential formulation.

1. In the **Study** toolbar, click **Add Study** to open the **Add Study** window.
2. Go to the **Add Study** window.
3. Find the **Studies** subsection. In the **Select Study** tree, select **General Studies** > **Stationary**.
4. Click the **Add Study** button in the window toolbar.
5. In the **Study** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Scalar potential formulation

1 In the **Settings** window for **Stationary**, type `Scalar potential formulation` in the **Label** text field.

2 Locate the **Study Extensions** section. Select the **Auxiliary sweep** checkbox.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
mu_r (Relative permeability, iron sphere)	2 4 10 20 40 100 200 400 1000 4000	

5 Locate the **Physics and Variables Selection** section. In the **Solve for** column of the table, under **Component 1 (comp1)**, clear the checkbox for **Magnetic Fields (mf)**.

Magnetic Flux Density (mfnc)

In the **Study** toolbar, click **Compute**.

Study 2/Solution 2 (sol2)

In the **Model Builder** window, under **Results** > **Datasets** click **Study 2/Solution 2 (sol2)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

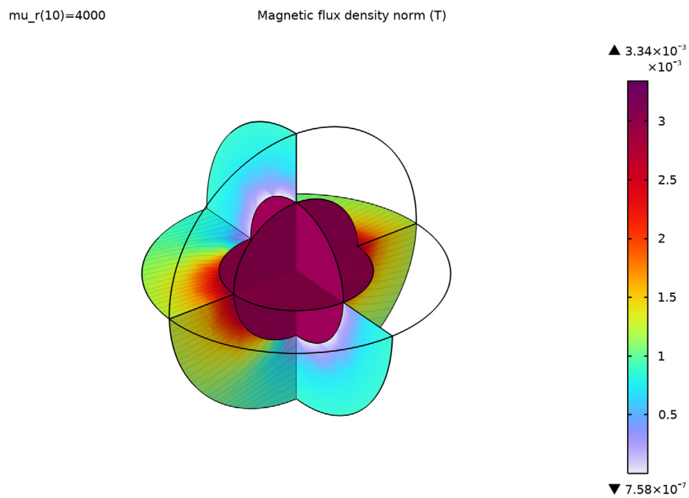
4 From the **Selection** list, choose **Analysis domain**.

Magnetic Flux Density (mfnc)

1 In the **Model Builder** window, under **Results** click **Magnetic Flux Density (mfnc)**.

2 In the **Magnetic Flux Density (mfnc)** toolbar, click **Plot**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar.



Create a second Cut Point to evaluate the second data set from the new study.

Cut Point 3D 2

1. In the **Results** toolbar, click **Cut Point 3D**.
2. In the **Settings** window for **Cut Point 3D**, locate the **Data** section.
3. From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.
4. Locate the **Point Data** section. In the **x** text field, type `0`.
5. In the **y** text field, type `0`.
6. In the **z** text field, type `0`.
7. Click **Plot**.

Scalar Potential (mfnc)

1. In the **Model Builder** window, right-click **Magnetic Flux Density** and choose **Point Graph**.
2. In the **Settings** window for **Point Graph**, type `Scalar Potential (mfnc)` in the **Label** text field.
3. Locate the **Data** section. From the **Dataset** list, choose **Cut Point 3D 2**.
4. Locate the **y-Axis Data** section. In the **Expression** text field, type `mfnc.normB`.
5. From the **Unit** list, choose **mT**.
6. Locate the **Title** section. From the **Title type** list, choose **Label**.

7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

8 Find the **Line markers** subsection. From the **Marker** list, choose **Diamond**.

9 From the **Color** list, choose **Red**.

10 Locate the **Legends** section. Select the **Show legends** checkbox.

11 Find the **Include** subsection. Select the **Label** checkbox.

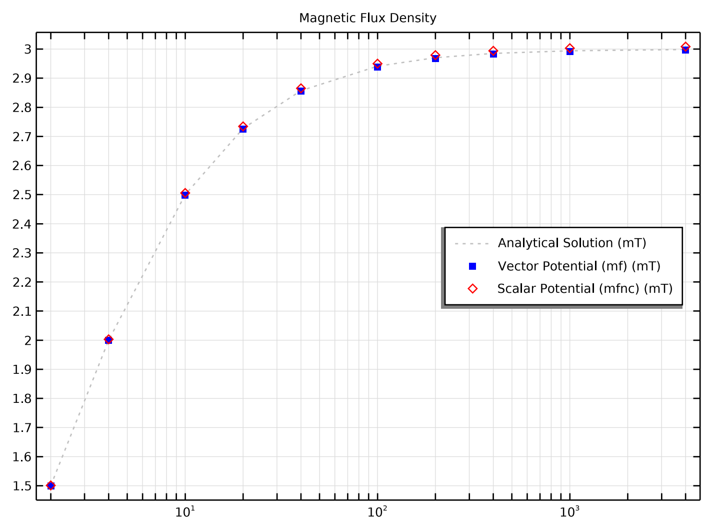
12 Clear the **Point** checkbox.

13 Clear the **Solution** checkbox.

14 Select the **Unit** checkbox.

15 In the **Magnetic Flux Density** toolbar, click **Plot**.

16 Click the **Zoom Extents** button in the **Graphics** toolbar.



Add the difference between the scalar potential formulation and the analytical solution

Scalar (mfnc) to Analytical Solution

1 In the **Model Builder** window, right-click **Magnetic Flux Density Comparison** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, type `Scalar (mfnc) to Analytical Solution` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 3D 2**.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `(mfnc.normB-B_analytic)/B_analytic`.

5 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Diamond**.

7 From the **Color** list, choose **Red**.

8 Locate the **Legends** section. Select the **Show legends** checkbox.

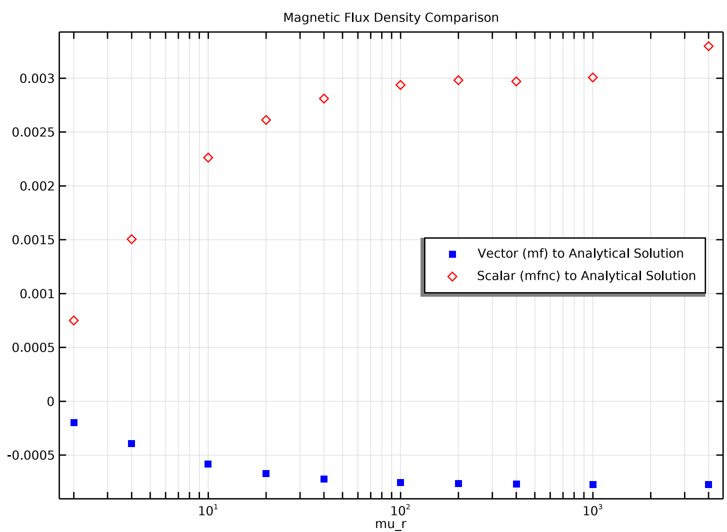
9 Find the **Include** subsection. Select the **Label** checkbox.

10 Clear the **Point** checkbox.

11 Clear the **Solution** checkbox.

12 In the **Magnetic Flux Density Comparison** toolbar, click **Plot**.

13 Click the **Zoom Extents** button in the **Graphics** toolbar.



This plot shows that the Scalar Potential formulation overestimates the magnetic flux density compared to the analytical solution.

The Scalar Potential formulation used by the Magnetic Fields, No Currents (mfnc) physics interface has the advantages of having a faster computing time. These can be compared by looking in the information in the respective Study Settings windows. However, this comes at the cost requiring a finer mesh while still achieving a less accurate result when compared to the Vector Potential formulation used by the Magnetic Fields (mf) physics interface. The mfnc interface also is limited to stationary studies and does not include current changes overtime. These are factors that require considering when designing simulations in using these physics modules.