



Model created in COMSOL Multiphysics 6.4

# Inductive Heating of a Copper Cylinder

## Introduction

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The induced currents in a copper cylinder produce heat, and when the temperature rises, the electric conductivity of the copper changes. Solving the heat transfer problem simultaneously with the field propagation problem is therefore crucial for an accurate description of this process.

The heating caused by the induced currents is called inductive heating. Generally, heating due to currents is also called resistive heating or ohmic heating.

A challenge in induction heating is that the high current in the induction coils requires active cooling. This can be obtained by making the coil conductors hollow and circulating water inside. Even for rather modest flow rates, the coolant flow becomes highly turbulent which makes the heat transfer between conductor and fluid very efficient. This example illustrates a simplified way of modeling water cooling based on the assumption of turbulent flow and instantaneous mixing.

For mechanical support and electrical insulation, the cylinder and coil are embedded in FR4 composite material.

## Model Definition

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The system to be solved is given by

$$\begin{aligned}j\omega\sigma(T)\mathbf{A} + \nabla \times (\mu^{-1}\nabla \times \mathbf{A}) &= \mathbf{0} \\ \rho C_p \frac{\partial T}{\partial t} - \nabla \cdot k \nabla T &= Q(T, \mathbf{A})\end{aligned}$$

where  $\rho$  is the density,  $C_p$  is the specific heat capacity,  $k$  is the thermal conductivity, and  $Q$  is the inductive heating. Note that in 2D axisymmetric models when solving the out-of-plane vector potential (as in this model), the **Magnetic Fields** interface uses *The Covariant Formulation* described in the *AC/DC Module User's Guide*, where the dependent variable is  $\Psi = rA_\phi$ , instead of  $A_\phi$ .

The electric conductivity of copper,  $\sigma$ , is given by the expression

$$\sigma = \frac{1}{[\rho_0(1 + \alpha(T - T_0))]}$$

where  $\rho_0$  is the resistivity at the reference temperature  $T_0 = 293$  K,  $\alpha$  is the temperature coefficient of the resistivity, and  $T$  is the actual temperature in the domain.

The time average of the inductive heating over one period, is given by

$$Q = \frac{1}{2}\sigma|E|^2$$

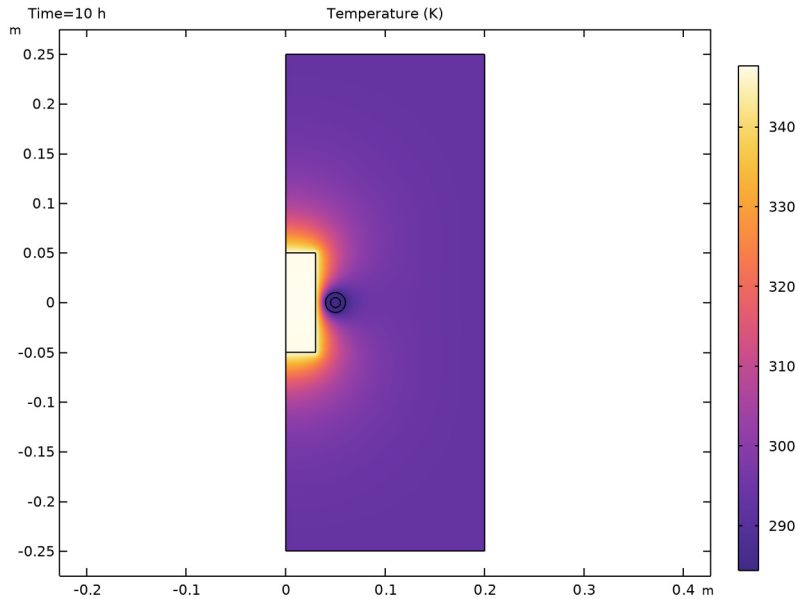
The coil conductor is cooled by a turbulent water flow in an internal cooling channel. This is emulated by a combination of a high effective thermal conductivity and a homogenized out-of-plane convective loss term:

$$Q_c = \frac{\frac{dM}{dt} C_p (T_{in} - T)}{2\pi r A_c}$$

where  $\frac{dM}{dt}$  is the water mass flow,  $T_{in}$  is the water inlet temperature,  $r$  is the radial coordinate, and  $A_c$  is the cross-section area of the cooling channel.

### *Results and Discussion*

The temperature after 10 h is shown in Figure 1. The average temperature of the copper cylinder has increased from 293 K to 346 K during this time, and the current in the coil has an amplitude of 2 kA.



*Figure 1: Temperature distribution after 10 h.*

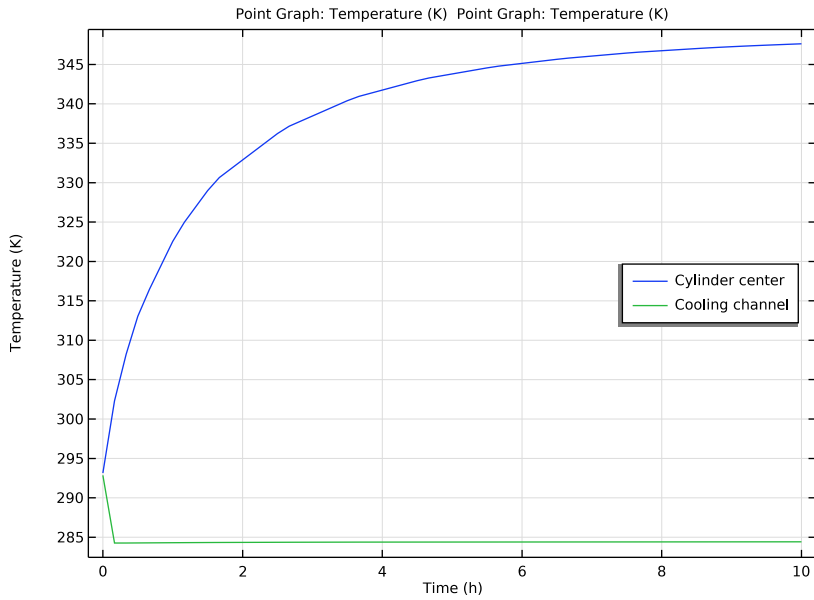


Figure 2: The plot shows the temperature evolution in the center of the copper cylinder and in the cooling channel.

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**Application Library path:** ACDC\_Module/Electromagnetic\_Heating/  
inductive\_heating


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### *Modeling Instructions*

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From the **File** menu, choose **New**.

#### **NEW**

In the **New** window, click  **Model Wizard**.

#### **MODEL WIZARD**

- 1 In the **Model Wizard** window, click  **2D Axisymmetric**.
- 2 In the **Select Physics** tree, select **Heat Transfer > Electromagnetic Heating > Induction Heating**.

- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **Preset Studies for Selected Multiphysics > Frequency-Transient**.
- 6 Click  **Done**.

## GLOBAL DEFINITIONS



### Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:



Name	Expression	Value	Description
I0	2e3[A]	2000 A	Current
T0	293[K]	293 K	Reference temperature
r0	1.754e-8[ohm*m]	1.754E-8 Ω·m	Resistivity at T=T0
a1	0.0039[1/K]	0.0039 1/K	Temperature coefficient
Rc	5[mm]	0.005 m	Cooling channel radius
Ac	pi*Rc^2	7.854E-5 m <sup>2</sup>	Cooling channel x-section
Mt	1[kg/min]	0.016667 kg/s	Cooling water mass flow rate
Tin	10[degC]	283.15 K	Cooling water inlet temperature

## GEOMETRY 1



### Rectangle 1 (r1)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type 0.2.
- 4 In the **Height** text field, type 0.5.
- 5 Locate the **Position** section. In the **z** text field, type -0.25.
- 6 Click  **Build Selected**.


### Rectangle 2 (r2)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type 0.03.
- 4 In the **Height** text field, type 0.1.
- 5 Locate the **Position** section. In the **z** text field, type -0.05.
- 6 Click  **Build Selected**.


### Circle 1 (c1)

- 1 In the **Geometry** toolbar, click  **Circle**.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type 0.01.
- 4 Locate the **Position** section. In the **r** text field, type 0.05.
- 5 Click  **Build Selected**.



### Circle 2 (c2)

- 1 Right-click **Circle 1 (c1)** and choose **Duplicate**.
- 2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
- 3 In the **Radius** text field, type R<sub>c</sub>.
- 4 Click  **Build Selected**.

### Form Union (fin)

- 1 In the **Model Builder** window, click **Form Union (fin)**.
- 2 In the **Settings** window for **Form Union/Assembly**, click  **Build Selected**.

### ADD MATERIAL

- 1 In the **Materials** toolbar, click  **Add Material** to open the **Add Material** window.
- 2 Go to the **Add Material** window.
- 3 In the tree, select **Built-in > FR4 (Circuit Board)**.
- 4 Click the **Add to Component** button in the window toolbar.
- 5 In the tree, select **AC/DC > Copper**.
- 6 Click the **Add to Component** button in the window toolbar.
- 7 In the tree, select **Built-in > Water, liquid**.
- 8 Click the **Add to Component** button in the window toolbar.
- 9 In the **Materials** toolbar, click  **Add Material** to close the **Add Material** window.

## MATERIALS

### *Copper (mat2)*

- 1 Select Domains 2 and 3 only.
- 2 In the **Model Builder** window, expand the **Copper (mat2)** node, then click **Linearized resistivity (ltr)**.
- 3 In the **Settings** window for **Linearized Resistivity**, locate the **Output Properties** section.
- 4 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Reference resistivity	rho0	r0	$\Omega\cdot\text{m}$	x
Resistivity temperature coefficient	alpha	a1	1/K	x
Reference temperature	Tref	T0	K	x

### *Water, liquid (mat3)*

- 1 In the **Model Builder** window, click **Water, liquid (mat3)**.
- 2 Select Domain 4 only.  
The built-in water material does not provide the electric permittivity and the magnetic permeability. Add those values.
- 3 In the **Settings** window for **Material**, locate the **Material Contents** section.
- 4 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1		Basic
Relative permittivity	epsilononr_iso ; epsilononrii = epsilononr_iso, epsilononrij = 0	80		Basic

Increase the thermal conductivity of the water to model the efficient heat transport in turbulent flow.


5 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Thermal conductivity	$k_{iso}$ ; $k_{ii} = k_{iso}$ , $k_{ij} = 0$	1e3	W/(m·K)	Basic


### MAGNETIC FIELDS (MF)

Add a separate **Ampère's Law** feature in the copper regions to specify a temperature-dependent resistivity.

#### *Ampère's Law in Solids 2*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Ampère's Law in Solids**.
- 2 Select Domains 2 and 3 only.
- 3 In the **Settings** window for **Ampère's Law in Solids**, locate the **Constitutive Relation Jc-E** section.
- 4 From the **Conduction model** list, choose **Linearized resistivity**.


#### *Domain Coil 1*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Domain Coil**.
- 2 Select Domain 3 only.
- 3 In the **Settings** window for **Domain Coil**, locate the **Coil** section.
- 4 In the  $I_{coil}$  text field, type I0.


### HEAT TRANSFER IN SOLIDS (HT)

Set up the Heat Transfer boundary conditions.

#### *Temperature 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Temperature**.
- 2 Select Boundaries 2, 7, and 9 only.
- 3 In the **Settings** window for **Temperature**, locate the **Temperature** section.
- 4 In the  $T_0$  text field, type T0.

#### *Heat Source 1*



- 1 In the **Physics** toolbar, click  **Domains** and choose **Heat Source**.
- 2 Select Domain 4 only.
- 3 In the **Settings** window for **Heat Source**, locate the **Heat Source** section.
- 4 In the  $Q_0$  text field, type  $Mt \cdot ht \cdot Cp \cdot (T_{in} - T) / (2 \cdot \pi \cdot r \cdot A_c)$ .

## MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.

## STUDY 1

*Step 1: Frequency–Transient*

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency–Transient**.
- 2 In the **Settings** window for **Frequency–Transient**, locate the **Study Settings** section.
- 3 From the **Time unit** list, choose **h**.
- 4 Click  **Range**.
- 5 In the **Range** dialog, type 10[**min**] in the **Step** text field.
- 6 In the **Stop** text field, type 10[**h**].
- 7 Click **Replace**.
- 8 In the **Settings** window for **Frequency–Transient**, locate the **Study Settings** section.
- 9 In the **Frequency** text field, type 500[**Hz**].
- 10 In the **Study** toolbar, click  **Compute**.

## RESULTS

*Temperature (ht)*

The 2D plot shows the temperature distribution after 10 hours; compare with [Figure 1](#).

Create point datasets for plotting the temperature evolution in the center of the copper cylinder and cooling channel.


*Cut Point 2D 1*

- 1 In the **Model Builder** window, expand the **Results > Datasets** node.
- 2 Right-click **Results > Datasets** and choose **Cut Point 2D**.
- 3 In the **Settings** window for **Cut Point 2D**, locate the **Point Data** section.
- 4 In the **r** text field, type 0.
- 5 In the **z** text field, type 0.

*Cut Point 2D 2*

- 1 Right-click **Cut Point 2D 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Cut Point 2D**, locate the **Point Data** section.
- 3 In the **r** text field, type 0.05.

### *Point Temperatures*

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type **Point Temperatures** in the **Label** text field.
- 3 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

### *Point Graph 1*

- 1 Right-click **Point Temperatures** and choose **Point Graph**.
- 2 In the **Settings** window for **Point Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Cut Point 2D 1**.
- 4 From the **Solution parameters** list, choose **From parent**.
- 5 Click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1) > Heat Transfer in Solids > Temperature > T - Temperature - K**.
- 6 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 7 From the **Legends** list, choose **Manual**.
- 8 In the table, enter the following settings:

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**Legends**

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Cylinder center

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### *Point Graph 2*

- 1 Right-click **Point Graph 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Point Graph**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Cut Point 2D 2**.
- 4 Locate the **Legends** section. In the table, enter the following settings:



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**Legends**

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Cooling channel

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- 5 In the **Point Temperatures** toolbar, click  **Plot**.
- 6 Click the  **Zoom Extents** button in the **Graphics** toolbar.  
The plot shows the temperature evolution in the center of the copper cylinder and in the cooling channel; compare with [Figure 2](#).