



Model created in COMSOL Multiphysics 6.4

# Electric Field Between Concentric Cylinders

## *Introduction*

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This introductory model treats the electrostatics problem of two concentric cylinders of infinite length, which is commonly found in textbooks. Since the problem can be solved analytically, the model can be used to compare theory with numerical simulation results. Two cases are considered:

- A fixed potential on each cylinder
- A surface charge density on one cylinder and a fixed potential on the other

## *Model Definition*

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First, consider the analytical solution to this problem to use for comparison. Let  $r_i$  be the radius of the inner cylinder and  $r_o$  the radius of the outer cylinder. In the first case, where we have a fixed potential  $V_0$  on the inner boundary and ground on the outer boundary, the electric potential satisfies

$$V = \frac{V_0}{\ln(r_i/r_o)} \ln(r/r_o)$$

which results in the electric field

$$\mathbf{E} = -\nabla V = \frac{-V_0}{r \ln(r_i/r_o)} \hat{r}$$

where  $\hat{r}$  is a unit vector in the radial direction.

In the case where there is a surface charge  $q_0$  on the inner cylinder, the electric potential instead reads

$$V = \frac{-q_0}{2\pi\epsilon_0} \ln(r/r_o)$$

which gives the electric field

$$\mathbf{E} = \frac{q_0}{2\pi\epsilon_0 r} \hat{r}$$

where  $\epsilon_0$  is the permittivity of vacuum.

To simplify the geometry of the model, take advantage of its twofold symmetry. First, since the cylinders are infinitely long, a perpendicular cross section is sufficient to represent the

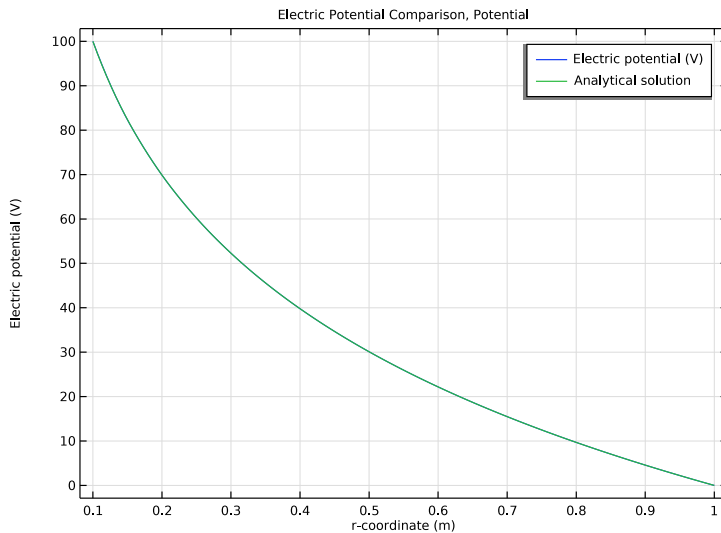
whole length. Furthermore, such a cross section is rotationally symmetric and it — and by extension, the full geometry — can thus be represented by a 1D component.

In the case with a fixed surface charge density on the inner boundary, an interesting note can be made about why it might be preferable to keep ground on the outer boundary instead of another surface charge  $q_0$  with the opposite sign. Since a surface charge is added on the inner cylinder, the latter could be considered to be more intuitive. However, there is one important difference: Having ground on a boundary sets the zero level of the electric potential, and therefore acts as a gauge fix. With only the surface charges specified, there is an infinite number of viable solutions for the electric potential. In order to solve such a problem numerically, one would have to change the default direct solver to an iterative one. Iterative solvers can solve ungauged problems, finding one solution out of the many possible ones. It is of course important to remember that the electric field will be the same no matter which potential is chosen.

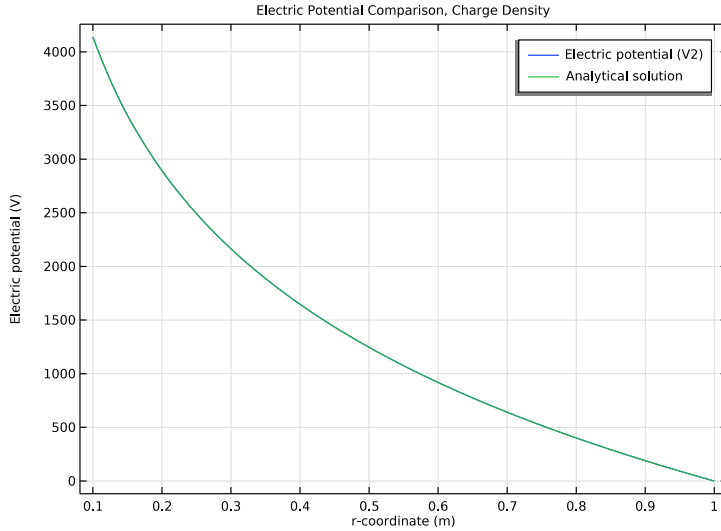
## Results

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The electric potential in the two cases is plotted in [Figure 1](#) and [Figure 2](#), where the numerical solution is compared with its analytical counterpart.

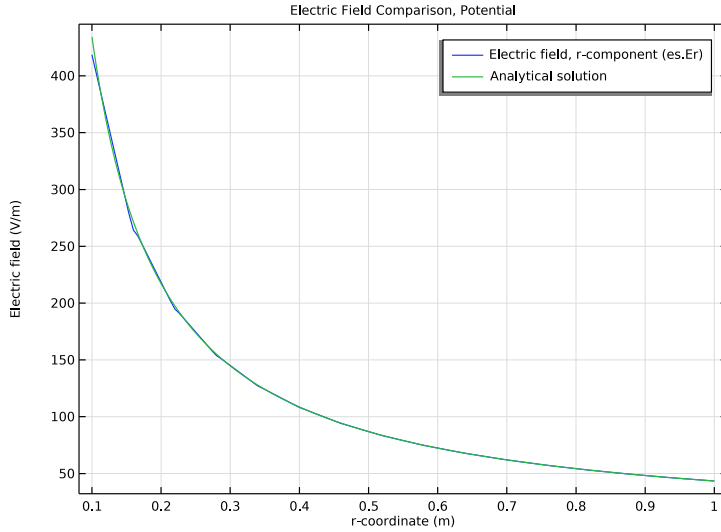


*Figure 1: The electric potential from the numerical and analytical solutions for the case with a specified potential.*

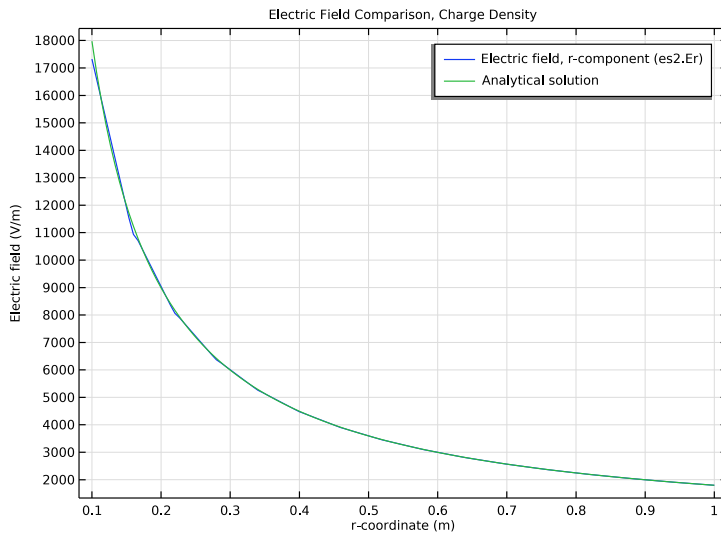


*Figure 2: The electric potential from the numerical and analytical solutions for the case with a specified surface charge density.*

Similarly, the solutions for the radial component of the electric field are plotted and compared in [Figure 3](#) and [Figure 4](#). In these plots it can be seen how well the numerical solutions for the two different approaches agree with the analytical solutions.



*Figure 3: The radial component of the electric field from the numerical and analytical solutions for the case with a specified potential.*



*Figure 4: The radial component of the electric field from the numerical and analytical solutions for the case with a specified surface charge density.*

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**Application Library path:** ACDC\_Module/Introductory\_Electrostatics/  
electric\_field\_concentric\_cylinders


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### *Modeling Instructions*




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From the **File** menu, choose **New**.

#### **NEW**

In the **New** window, click  **Model Wizard**.

#### **MODEL WIZARD**

- 1 In the **Model Wizard** window, click  **1D Axisymmetric**.
- 2 In the **Select Physics** tree, select **AC/DC > Electric Fields and Currents > Electrostatics (es)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies > Stationary**.
- 6 Click  **Done**.

First, define some parameters that will be used when building the model.

#### **GLOBAL DEFINITIONS**

##### *Parameters 1*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

<b>Name</b>	<b>Expression</b>	<b>Value</b>	<b>Description</b>
ri	0.1[m]	0.1 m	Radius of inner cylinder
ro	1[m]	1 m	Radius of outer cylinder
V0	100[V]	100 V	Potential at inner cylinder
q0	1e-7[C/m]	1E-7 C/m	Charge at inner cylinder

Building the geometry is simplified by using 1D axisymmetric, since only an interval is needed to create two concentric cylinders of infinite length.

## GEOMETRY I

### Interval I (iI)

- 1 In the **Model Builder** window, under **Component I (comp I)** right-click **Geometry I** and choose **Interval**.
- 2 In the **Settings** window for **Interval**, locate the **Interval** section.
- 3 In the table, enter the following settings:

Coordinates (m)
$r_i$
$r_o$

- 4 Click  **Build All Objects**.


In the first physics interface, add ground at the outer cylinder and a fixed potential at the inner cylinder.

## ELECTROSTATICS (ES)

### Ground I



- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Ground**.
- 2 Select Boundary 2 only.

### Electric Potential I

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Electric Potential**.
- 2 Select Boundary 1 only.
- 3 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.
- 4 In the  $V_0$  text field, type  $V_0$ .

Now, add a second Electrostatics physics interface. Here, a surface charge density will be added on the inner cylinder, while keeping ground on the outer cylinder.

## ADD PHYSICS


- 1 In the **Physics** toolbar, click  **Add Physics** to open the **Add Physics** window.
- 2 Go to the **Add Physics** window.
- 3 In the tree, select **Recently Used** > **Electrostatics (es)**.
- 4 Click the **Add to Component I** button in the window toolbar.
- 5 In the **Physics** toolbar, click  **Add Physics** to close the **Add Physics** window.

## ELECTROSTATICS 2 (ES2)


### *Ground 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Ground**.
- 2 Select Boundary 2 only.

### *Surface Charge Density 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Surface Charge Density**.
- 2 Select Boundary 1 only.
- 3 In the **Settings** window for **Surface Charge Density**, locate the **Surface Charge Density** section.
- 4 In the  $\rho_s$  text field, type  $q_0 / (2 \cdot \pi \cdot r_i)$ .

## STUDY 1

In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Electric Potential Comparison, Potential*

Now, it is time to compare the results with the known analytical solutions. Plots of the electric potential are added automatically, so for those only the comparison needs to be added. Then, add plots of the computed electric field and compare those with analytical solutions as well.

- 1 In the **Settings** window for **ID Plot Group**, type **Electric Potential Comparison, Potential** in the **Label** text field.
- 2 Locate the **Plot Settings** section. Select the **y-axis label** checkbox.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

### *Line Graph 2*

- 1 Right-click **Electric Potential Comparison, Potential** and choose **Line Graph**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 4 In the **Expression** text field, type  $V_0 \cdot \log(r/r_0) / \log(r_i/r_0)$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $r$ .
- 7 Click to expand the **Legends** section. Select the **Show legends** checkbox.
- 8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

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**Legends**

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Analytical solution

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*Line Graph 1*

- 1 In the **Model Builder** window, click **Line Graph 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **Legends** section.
- 3 Select the **Show legends** checkbox.
- 4 Find the **Include** subsection. Select the **Expression** checkbox.
- 5 Clear the **Solution** checkbox.
- 6 Select the **Description** checkbox.
- 7 In the **Electric Potential Comparison, Potential** toolbar, click  **Plot**.

*Electric Potential Comparison, Charge Density*

- 1 In the **Model Builder** window, under **Results** click **Electric Potential (es2)**.
- 2 In the **Settings** window for **ID Plot Group**, type Electric Potential Comparison, Charge Density in the **Label** text field.
- 3 Locate the **Plot Settings** section. Select the **y-axis label** checkbox.
- 4 Locate the **Title** section. From the **Title type** list, choose **Label**.

*Line Graph 2*

- 1 Right-click **Electric Potential Comparison, Charge Density** and choose **Line Graph**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 4 In the **Expression** text field, type  $-q_0 \cdot \log(r/r_0) / (2 \cdot \pi \cdot \epsilon_0 \cdot \text{const})$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $r$ .
- 7 Locate the **Legends** section. Select the **Show legends** checkbox.
- 8 From the **Legends** list, choose **Manual**.
- 9 In the table, enter the following settings:

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**Legends**

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Analytical solution

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### *Line Graph 1*

- 1 In the **Model Builder** window, click **Line Graph 1**.
- 2 In the **Settings** window for **Line Graph**, locate the **Legends** section.
- 3 Select the **Show legends** checkbox.
- 4 Find the **Include** subsection. Select the **Expression** checkbox.
- 5 Clear the **Solution** checkbox.
- 6 Select the **Description** checkbox.
- 7 In the **Electric Potential Comparison, Charge Density** toolbar, click  **Plot**.

### *Electric Field Comparison, Potential*

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Electric Field Comparison, Potential in the **Label** text field.
- 3 Locate the **Plot Settings** section.
- 4 Select the **y-axis label** checkbox. In the associated text field, type Electric field (V/m).
- 5 Locate the **Title** section. From the **Title type** list, choose **Label**.

### *Line Graph 1*


- 1 Right-click **Electric Field Comparison, Potential** and choose **Line Graph**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 4 In the **Expression** text field, type  $es \cdot Er$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $r$ .
- 7 Locate the **Legends** section. Select the **Show legends** checkbox.
- 8 Find the **Include** subsection. Select the **Expression** checkbox.
- 9 Clear the **Solution** checkbox.
- 10 Select the **Description** checkbox.

### *Line Graph 2*


- 1 In the **Model Builder** window, right-click **Electric Field Comparison, Potential** and choose **Line Graph**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

- 4 In the **Expression** text field, type  $-V_0/(r*\log(r_i/r_o))$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $r$ .
- 7 Locate the **Legends** section. Select the **Show legends** checkbox.
- 8 From the **Legends** list, choose **Manual**.
- 9 In the table, enter the following settings:

<b>Legends</b>
Analytical solution

- 10 In the **Electric Field Comparison, Potential** toolbar, click  **Plot**.

#### *Electric Field Comparison, Charge Density*

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Electric Field Comparison, Charge Density in the **Label** text field.
- 3 Locate the **Plot Settings** section.
- 4 Select the **y-axis label** checkbox. In the associated text field, type Electric field (V/m).
- 5 Locate the **Title** section. From the **Title type** list, choose **Label**.

#### *Line Graph 1*

- 1 Right-click **Electric Field Comparison, Charge Density** and choose **Line Graph**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 4 In the **Expression** text field, type  $\epsilon s_2 \cdot E_r$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $r$ .
- 7 Locate the **Legends** section. Select the **Show legends** checkbox.
- 8 Find the **Include** subsection. Select the **Expression** checkbox.
- 9 Clear the **Solution** checkbox.
- 10 Select the **Description** checkbox.

#### *Line Graph 2*

- 1 In the **Model Builder** window, right-click **Electric Field Comparison, Charge Density** and choose **Line Graph**.

- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
- 4 In the **Expression** text field, type  $q_0 / (2 \cdot \pi \cdot \epsilon_{0\_const} \cdot r)$ .
- 5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
- 6 In the **Expression** text field, type  $r$ .
- 7 Locate the **Legends** section. Select the **Show legends** checkbox.
- 8 From the **Legends** list, choose **Manual**.
- 9 In the table, enter the following settings:

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**Legends**

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Analytical solution

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- 10 In the **Electric Field Comparison, Charge Density** toolbar, click  **Plot**.