Created in COMSOL Multiphysics 6.3

Evaluation of Dynamic Coefficients of a Cylindrical Journal Bearing

Introduction

When analyzing rotors, it is common that bearings are modeled through their effective dynamic coefficients about a static equilibrium position. This example demonstrates a method to compute such coefficients for a cylindrical journal bearing. Computed coefficients are compared to analytical values obtained from solving Reynolds equation, using a short bearing approximation. To make the comparison meaningful, the length of the bearing is taken to be much smaller than its diameter.

Model Definition

The cylindrical hydrodynamic journal bearing has a radius of 10 cm, and a length of 2 cm. The angular velocity of the journal is 1000 rad/s, and the clearance between the journal and the bearing is 100 μm. The viscosity and density of the lubricant are taken as 20 mPa·s and 866 kg/m^3, respectively. To find the equilibrium position corresponding to different static loads, the journal loading is varied from 10 N to 100,000 N.

Bearing stiffness and damping coefficients are computed for the equilibrium positions by solving a perturbed form of Reynolds equation.

The dimensionless stiffness and damping coefficients obtained from an analytical solution of Reynolds equation (Ref. 1) are given by

$$k_{22} = \frac{4[16\varepsilon^2 + \pi^2(2-\varepsilon^2)]}{Q}$$

$$k_{23} = \frac{\pi[\pi^2(1-\varepsilon^2) - 16\varepsilon^4]}{Q\varepsilon\sqrt{1-\varepsilon^2}}$$

$$k_{32} = -\frac{\pi[\pi^2(1+2\varepsilon^2)(1-\varepsilon^2) + 32\varepsilon^2(1+\varepsilon^2)]}{Q(1-\varepsilon^2)}$$

$$k_{33} = \frac{4[\pi^2(1+2\varepsilon^2)(1-\varepsilon^2) + 32\varepsilon^2(1+\varepsilon^2)]}{Q(1-\varepsilon^2)}$$

and

$$c_{22} = \frac{2\pi[\pi^2(1+2\varepsilon^2) - 16\varepsilon^2]\sqrt{1-\varepsilon^2}}{Q\varepsilon}$$

$$c_{23} = \frac{8[16\varepsilon^2 - \pi^2(1+2\varepsilon^2)]}{Q}$$

$$c_{32} = c_{23}$$

$$c_{33} = \frac{2\pi[48\varepsilon^2 + \pi^2(1-\varepsilon^2)^2]}{Q\varepsilon\sqrt{1-\varepsilon^2}}$$

where ε is the relative eccentricity of the journal. The parameter Q is given by

$$Q = [\pi^2 + (16 - \pi^2)\varepsilon^2]^{3/2}$$

To get the physical values of the dynamic coefficients, the dimensionless parameters must be scaled. This can be done by using the scaling factors

$$k_0 = \frac{W}{C} \quad \text{and} \quad c_0 = \frac{W}{C\Omega}$$

for the stiffness and damping, respectively. The parameter W is the bearing load, C is the clearance, and Ω is the angular speed of the journal.

Results and Discussion

Figure 1 shows how the journal eccentricity changes with the static load on the bearing. The figure shows that with increasing load, its effect on eccentricity decreases. This clearly depicts the nonlinear behavior of the bearing.

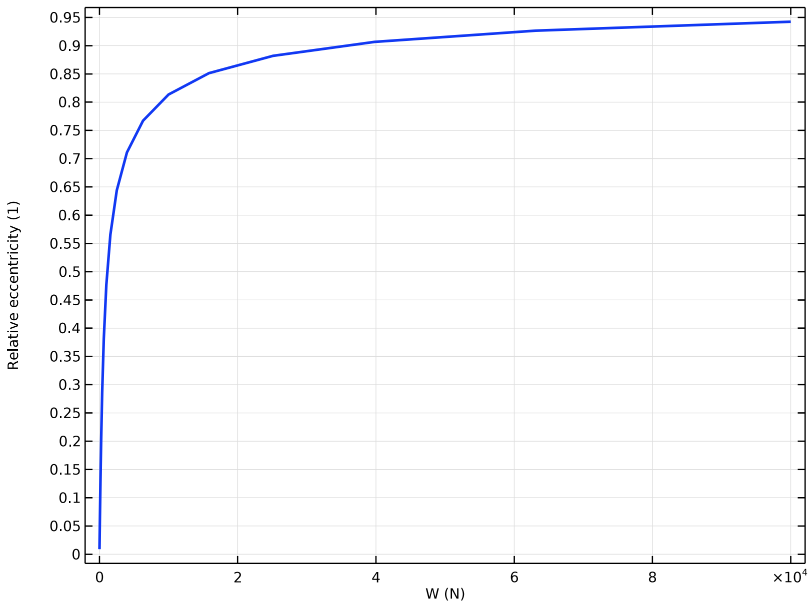

Figure 1: Eccentricity versus load.

Figure 2 shows the computed attitude angle with respect to loading direction, compared to the analytical curve. For small loads the curves coincide. With increasing loads, the journal becomes increasingly eccentric in the bearing. This produces a difference in shear forces at the minimum and maximum film thickness locations. The difference results in a net force on the journal. In high eccentricity cases, the journal equilibrium location is determined by the balance of external loads on the bearing, and the pressure and shear forces.

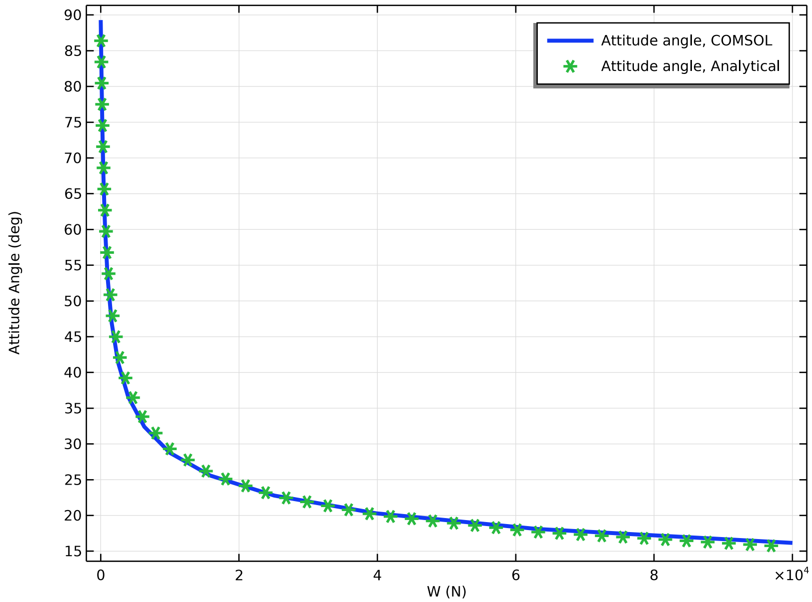

Figure 2: Attitude angle versus load.

The maximum film pressure and minimum film thickness are two important performance parameters for a bearing. These are plotted in Figure 3.

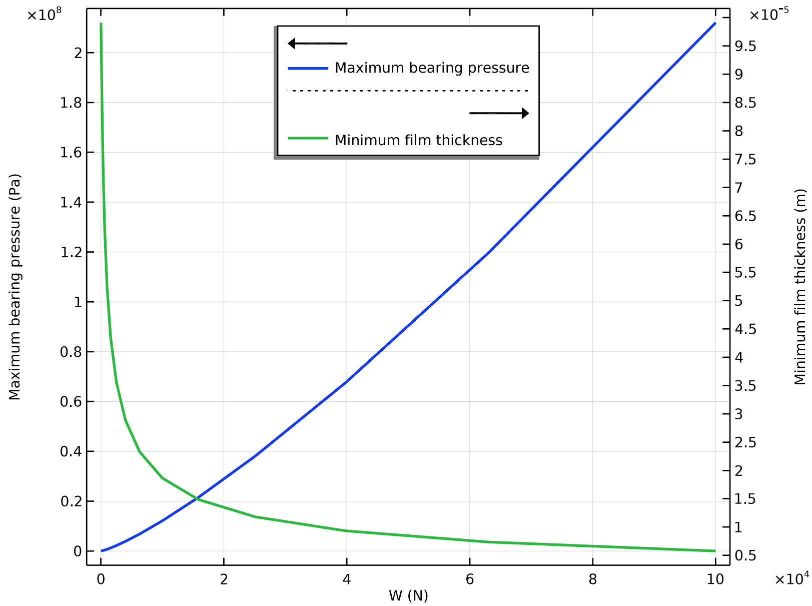

Figure 3: Maximum pressure and minimum film thickness versus load.

Figure 4 and Figure 5 compare the computed values of the dimensionless stiffness and dimensionless damping coefficient with the corresponding analytical values. The computed values match the analytical values.

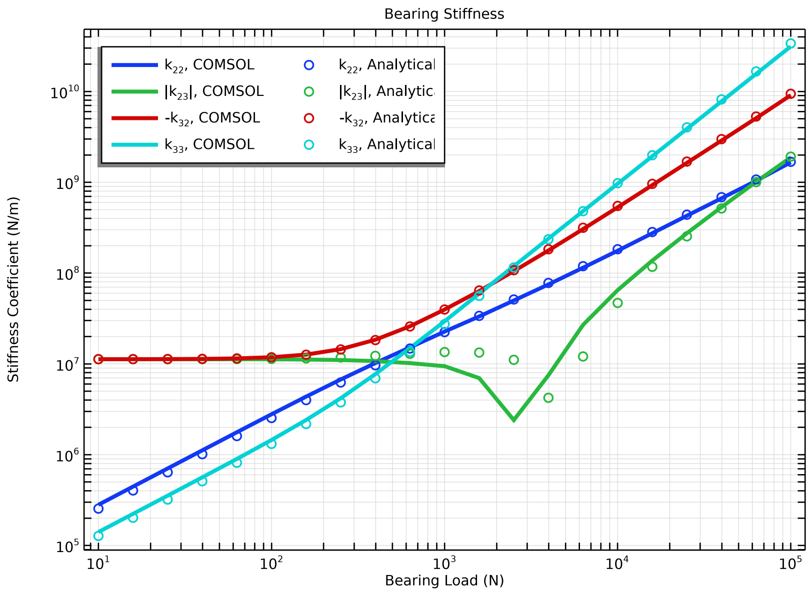

Figure 4: Bearing stiffness.

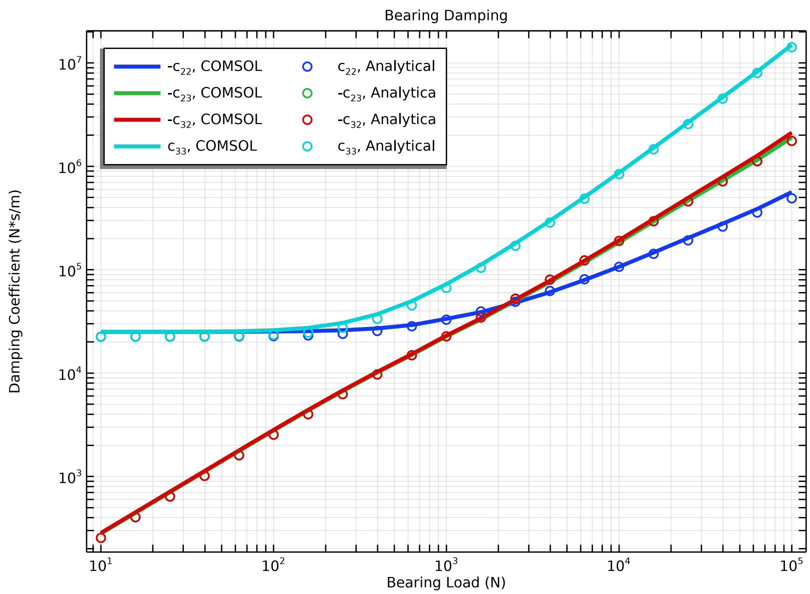

Figure 5: Bearing damping.

Reference

1. J.S. Rao, *Rotor Dynamics*, section 7.6, pp. 179–191, New Age International (P) Limited, 2014.

Application Library path: `Rotordynamics_Module/Verification_Examples/journal_bearing_dynamic_coefficients`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.
2 In the **Select Physics** tree, select **Structural Mechanics** > **Rotordynamics** > **Hydrodynamic Bearing (hdb)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **General Studies** > **Stationary**.
6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
R	10[cm]	0.1 m	Journal radius
L	2[cm]	0.02 m	Journal length
C	100[um]	1E-4 m	Clearance
Omega	1000[rad/s]	1000 rad/s	Angular velocity
mu0	20[mPa*s]	0.02 Pa·s	Lubricant viscosity
rho0	866[kg/m^3]	866 kg/m³	Lubricant density
W	500[N]	500 N	Static load on bearing

GEOMETRY 1

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.
2 In the **Settings** window for **Cylinder**, locate the **Object Type** section.
3 From the **Type** list, choose **Surface**.
4 Locate the **Size and Shape** section. In the **Radius** text field, type R.
5 In the **Height** text field, type L.
6 Locate the **Axis** section. From the **Axis type** list, choose **x-axis**.
7 Click **Build All Objects**.

Define the variables for the analytical stiffness and damping.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)** > **Definitions** node.

2 Right-click **Definitions** and choose **Variables**.

3 In the **Settings** window for **Variables**, locate the **Variables** section.

4 In the table, enter the following settings:

Name	Expression	Unit	Description
k0	W/C	N/m	Stiffness scaling
c0	W/(C*Omega)	N·s/m	Damping scaling
e	hdb.hjb1.ec_rel		Eccentricity
phi0	atan2(pi*sqrt(1-e^2), 4*e)	rad	Attitude angle
Q	(pi^2+(16-pi^2)*e^2)^1.5		Auxiliary variable
k22	4*(16*e^2+pi^2*(2-e^2))/Q		Dimensionless stiffness, 22 component
k23	pi*(pi^2*(1-e^2)-16*e^4)/(e*sqrt(1-e^2)*Q)		Dimensionless stiffness, 23 component
k32	-pi*(pi^2*(1+2*e^2)*(1-e^2)+32*e^2*(1+e^2))/(e*sqrt(1-e^2)*Q)		Dimensionless stiffness, 32 component
k33	4*(pi^2*(1+2*e^2)*(1-e^2)+32*e^2*(1+e^2))/((1-e^2)*Q)		Dimensionless stiffness, 33 component
c22	2*pi*sqrt(1-e^2)*(pi^2*(1+2*e^2)-16*e^2)/(e*Q)		Dimensionless damping, 22 component
c23	8*(16*e^2-pi^2*(1+2*e^2))/Q		Dimensionless damping, 23 component
c32	c23		Dimensionless damping, 32 component
c33	2*pi*(48*e^2+pi^2*(1-e^2)^2)/(e*sqrt(1-e^2)*Q)		Dimensionless damping, 33 component

HYDRODYNAMIC BEARING (HDB)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Hydrodynamic Bearing (hdb)**.

2 In the **Settings** window for **Hydrodynamic Bearing**, locate the **Dynamic Coefficients** section.

3 Select the **Calculate dynamic coefficients** checkbox.

Hydrodynamic Journal Bearing 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Hydrodynamic Bearing (hdb)** click **Hydrodynamic Journal Bearing 1**.

2 In the **Settings** window for **Hydrodynamic Journal Bearing**, locate the **Bearing Properties** section.

3 In the C text field, type `C`.

4 From the $\mathbf{X}_c$ list, choose **From geometry**.

5 Locate the **Journal Properties** section. From the **Specify** list, choose **Load**.

6 Specify the $\mathbf{W}_j$ vector as

0	x
0	y
-W	z

7 Specify the $\mathbf{u}_{j0}$ vector as

0	x
0	y
0	z

8 In the Ω text field, type `Omega`.

9 Locate the **Fluid Properties** section. From the μ list, choose **User defined**. In the associated text field, type `mu0`.

10 From the ρ list, choose **User defined**. In the associated text field, type `rho0`.

Use a mapped mesh to resolve the pressure.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click **More Generators** and choose **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **All boundaries**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Edges 1, 2, 4, and 6 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type 50.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Select Edge 7 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type 12.

5 Click **Build All**.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** checkbox.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
W (Static load on bearing)	10^range(1,0.2,5)	N

6 In the **Study** toolbar, click **Compute**.

RESULTS

Fluid Pressure (hdb)

In the **Fluid Pressure (hdb)** toolbar, click **Plot**.

Use the following instructions to plot the eccentricity versus load curve shown in Figure 1.

Eccentricity

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Eccentricity` in the **Label** text field.

Global 1

1 Right-click **Eccentricity** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Hydrodynamic Bearing** > **Hydrodynamic Journal Bearing 1** > **Eccentricity and attitude angle** > **hdb.hjb1.ec_rel - Relative eccentricity - 1**.

3 Click to expand the **Coloring and Style** section. From the **Width** list, choose **2**.

4 In the **Eccentricity** toolbar, click **Plot**.

5 Click the **Show Legends** button in the **Graphics** toolbar.

Eccentricity

1 In the **Model Builder** window, click **Eccentricity**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 In the **Eccentricity** toolbar, click **Plot**.

To compare the computed and analytical attitude angles shown in Figure 2, follow the below instructions.

Attitude Angle

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Attitude Angle` in the **Label** text field.

Global 1

1 Right-click **Attitude Angle** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Hydrodynamic Bearing** > **Hydrodynamic Journal Bearing 1** > **Eccentricity and attitude angle** > **hdb.hjb1.phia - Attitude angle - rad**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.phia	deg	Attitude angle, COMSOL

4 Locate the **Coloring and Style** section. From the **Width** list, choose **3**.

Global 2

1 In the **Model Builder** window, right-click **Attitude Angle** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
phi0	deg	Attitude angle, Analytical

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 From the **Positioning** list, choose **Interpolated**.

7 In the **Number** text field, type 50.

8 In the **Attitude Angle** toolbar, click **Plot**.

Attitude Angle

1 In the **Model Builder** window, click **Attitude Angle**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **y-axis label** checkbox. In the associated text field, type `Attitude Angle (deg)`.

4 Locate the **Title** section. From the **Title type** list, choose **None**.

Duplicate the eccentricity plot and follow the instructions below to plot the maximum pressure, and minimum film thickness curves, as shown in Figure 3.

Pressure and Film Thickness

1 In the **Model Builder** window, right-click **Eccentricity** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Pressure and Film Thickness` in the **Label** text field.

Global 1

1 In the **Model Builder** window, expand the **Pressure and Film Thickness** node, then click **Global 1**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Hydrodynamic Bearing** > **Pressure** > **hdb.hjb1.p_max - Maximum bearing pressure - Pa**.

Global 2

1 Right-click **Results** > **Pressure and Film Thickness** > **Global 1** and choose **Duplicate**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Hydrodynamic Bearing** > **Journal and bearing properties** > **Film thickness and clearance** > **hdb.hjb1.h_min - Minimum film thickness - m**.

Pressure and Film Thickness

1 In the **Model Builder** window, click **Pressure and Film Thickness**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **Two y-axes** checkbox.

4 In the table, select the **Plot on secondary y-axis** checkbox for **Global 2**.

5 Click the **Show Legends** button in the **Graphics** toolbar.

6 Locate the **Legend** section. From the **Position** list, choose **Upper middle**.

7 In the **Pressure and Film Thickness** toolbar, click **Plot**.

Figure 4 compares the computed stiffness to its analytical counterpart. Follow the instructions below to generate this plot.

Bearing Stiffness

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Bearing Stiffness` in the **Label** text field.

Global 1

1 Right-click **Bearing Stiffness** and choose **Global**.

2 In the **Settings** window for **Global**, click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Hydrodynamic Bearing** > **Dynamic coefficients** > **Translational stiffness coefficient - N/m** > **hdb.hjb1.k22 - Translational stiffness coefficient, 22-component**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.k22	N/m	k₂₂, COMSOL
abs(hdb.hjb1.k23)	N/m	|k₂₃|, COMSOL
-hdb.hjb1.k32	N/m	-k₃₂, COMSOL
hdb.hjb1.k33	N/m	k₃₃, COMSOL

4 Locate the **Coloring and Style** section. From the **Width** list, choose **3**.

5 Click the **y-Axis Log Scale** button in the **Graphics** toolbar.

6 Click the **x-Axis Log Scale** button in the **Graphics** toolbar.

Global 2

1 In the **Model Builder** window, right-click **Bearing Stiffness** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	**Unit**	**Description**
k22*k0	N/m	k₂₂, Analytical
abs(k23*k0)	N/m	|k₂₃|, Analytical
-k32*k0	N/m	-k₃₂, Analytical
k33*k0	N/m	k₃₃, Analytical

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 From the **Color** list, choose **Cycle (reset)**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

Bearing Stiffness

1 In the **Model Builder** window, click **Bearing Stiffness**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **Manual**.

4 In the **Title** text area, type `Bearing Stiffness`.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** checkbox. In the associated text field, type `Bearing Load (N)`.

7 Select the **y-axis label** checkbox. In the associated text field, type `Stiffness Coefficient (N/m)`.

8 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

9 In the **Number of columns** text field, type `2`.

Figure 5 compares the computed damping to its analytical counterpart. Follow the instructions below to generate this plot.

Bearing Damping

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Bearing Damping` in the **Label** text field.

Global 1

1 Right-click **Bearing Damping** and choose **Global**.

2 In the **Settings** window for **Global**, click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)** > **Hydrodynamic Bearing** > **Dynamic coefficients** > **Translational damping coefficient - N·s/m** > **hdb.hjb1.c22 - Translational damping coefficient, 22-component**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.c22	1	-c₂₂, COMSOL
-hdb.hjb1.c23	1	-c₂₃, COMSOL
-hdb.hjb1.c32	1	-c₃₂, COMSOL
hdb.hjb1.c33	1	c₃₃, COMSOL

4 Locate the **Coloring and Style** section. From the **Width** list, choose **3**.

5 Click the **y-Axis Log Scale** button in the **Graphics** toolbar.

6 Click the **x-Axis Log Scale** button in the **Graphics** toolbar.

Global 2

1 In the **Model Builder** window, right-click **Bearing Damping** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
c22*c0	1	c₂₂, Analytical
-c23*c0	1	-c₂₃, Analytical
-c32*c0	1	-c₃₂, Analytical
c33*c0	1	c₃₃, Analytical

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 From the **Color** list, choose **Cycle (reset)**.

6 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

7 In the **Bearing Damping** toolbar, click **Plot**.

Bearing Damping

1 In the **Model Builder** window, click **Bearing Damping**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **Manual**.

4 In the **Title** text area, type `Bearing Damping`.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** checkbox. In the associated text field, type `Bearing Load (N)`.

7 Select the **y-axis label** checkbox. In the associated text field, type `Damping Coefficient (N*s/m)`.

8 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

9 In the **Number of columns** text field, type `2`.