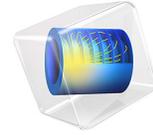# Michelson Interferometer

Introduction

A Michelson interferometer is composed of five elements: two mirrors, one beam splitter, an imaging device (screen), and a coherent light source. Figure 1 shows and describes a basic Michelson interferometer arrangement, the interference pattern is generated when the rays reflected by mirrors M1 and M2 arrive at the screen with different optical path lengths.

Changing, even slightly, the optical path length of either beam results in a change of the interference pattern at the screen. This change of optical path length can be accomplished by moving one of the mirrors. Unexpected changes in optical path length can also occur if any of the device's optical components undergo deformation, such as thermal expansion. This model illustrates the effects of thermal expansion on the interference pattern obtained at the screen of the interferometer.
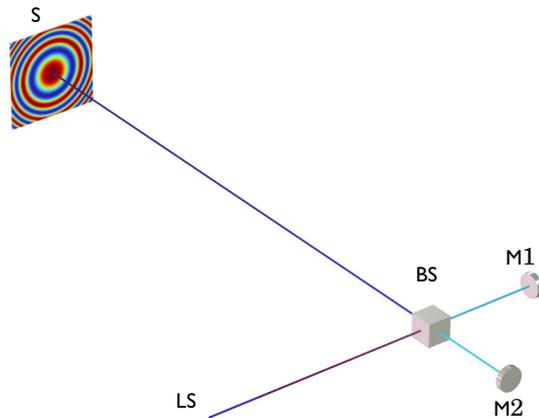

Figure 1: An illuminated Michelson interferometer arrangement with the resulting interference pattern displayed on the screen (S). The light comes from the light source (LS), hits the beam splitter (BS) before being equally diverted toward mirror 1 (M1) and mirror 2 (M2). Once reflected at the mirrors, the two composing beams return to the beam splitter (BS) where they recombine to travel toward the screen (S) or the light source (LS).

Model Definition

In this model the rays propagate through the beam splitter and the air enclosing the optical components. The mirrors are treated as Wall boundary conditions at which specular reflection occurs. The beam splitter (BS) is composed of two BK7 glass prisms separated by a thin dielectric film (BS interface). The beam splitter is surrounded by an anti-reflective (AR) coating. The AR coating and the BS interface are modeled by applying the Thin Dielectric Film subnode to the Material Discontinuity node.

The Settings window for the Material Discontinuity node includes an option to automatically set up a single-layer coating with the desired reflectance. In this case, a reflectance of 0.5 is desired for the interior boundary of the beam splitter so that an incoming ray is divided into two rays of equal intensity.

The light source used in the model is a He-Ne laser. For sake of simplicity only one ray is initially released from a point on a face of the enclosure. The initial ray is polarized such that it forms an s-polarized wave at the beam splitter surface. For the interference pattern to be accurate, it must correspond to a wavefront that subtends a very small solid angle. This can be accomplished by setting initial radius of curvature of the wavefront to −1 m and setting the initial optical path length difference between the beam splitter and the two mirrors to a large multiple of the free-space wavelength.

The interference pattern obtained at the screen depends on the radii of curvature, phase, and angle of incidence of the two rays as they arrive at the screen. For a spherical wavefront with principal radius of curvature $r_{1,0}$ with normal incidence at a surface, the change in phase corresponding to a shift x_p in position on the screen is

$$\Delta\Psi_1 = k\left(\sqrt{x_p^2 + r_{1,0}^2} - r_{1,0}\right)$$

where k is the wave number. Similarly, the change in phase for a wavefront with principal radius of curvature $r_{2,0}$ is

$$\Delta\Psi_2 = k\left(\sqrt{x_p^2 + r_{2,0}^2} - r_{2,0}\right)$$

For small values of x_p, an approximate solution for the difference in phase between the two wavefronts can be obtained by taking a Taylor series expansion of the expressions for $\Delta\Psi_1$ and $\Delta\Psi_2$ and retaining terms of up to second order in x_p,

$$\Delta\Psi_1 - \Delta\Psi_2 \cong kx_p^2\left(\frac{1}{2r_{1,0}} - \frac{1}{2r_{2,0}}\right)$$

If the two rays interfere constructively at $x_p = 0$, they also interfere constructively where the phase difference between the two wavefronts is an integer multiple of 2π. The first such point occurs where

$$x_p = \sqrt{\lambda_0 \left(\frac{1}{2r_{1,0}} - \frac{1}{2r_{2,0}} \right)^{-1}} \tag{1}$$

which is the distance from the center of the interference pattern to a point of maximum intensity on the first fringe.
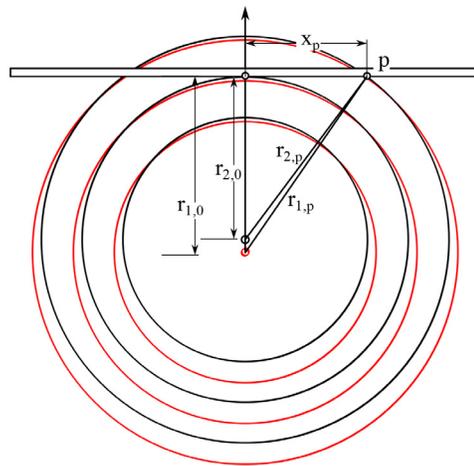

Figure 2: Interference of coherent light emitted from two point sources separated by a small distance.

Results and Discussion

Figure 3 shows the interference pattern resulting from the combination of two rays having an optical path length difference of $\delta_d = 8000\,\lambda_0$. As expected for spherical waves, the interference fringes are circular. On the figure it is possible to approximate the distance

from the center of the screen to the radial position of greatest intensity on the first circular fringe.
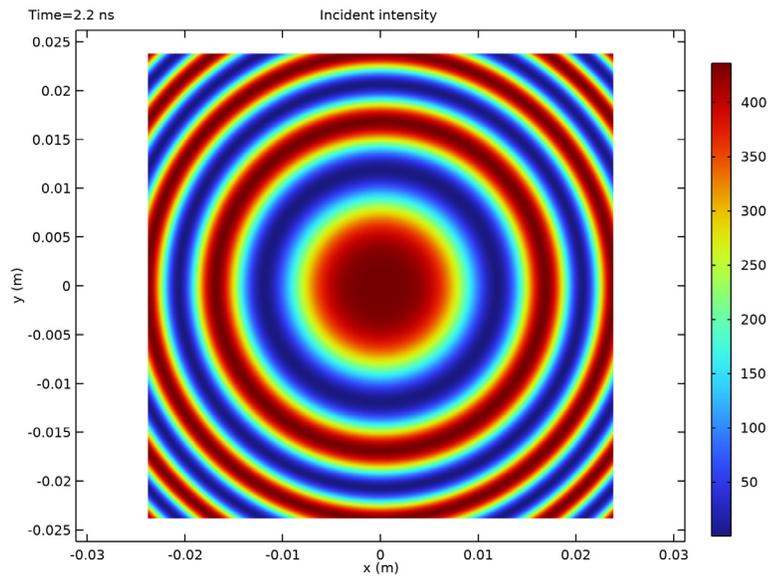
Figure 3: Interference pattern obtained for an optical path length difference of $\delta_d = 8000\,\lambda_0$.

Reference

1. M. Born and E. Wolf, *Principle of Optics*, 7th ed., Cambridge University Press, 2011.

Application Library path: `Ray_Optics_Module/Spectrometers_and_Monochromators/michelson_interferometer`

Modeling Instructions

From the **File** menu, choose **New**.

N E W

In the **New** window, click ◈ **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics** > **Ray Optics** > **Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
lam	632.8[nm]	6.328E-7 m	Wavelength of He-Ne laser
delta_d	8000*lam	0.0050624 m	Optical path length difference
th	0.12[in]	0.003048 m	Thickness of mirrors
dia	0.5[in]	0.0127 m	Diameter of mirrors
d1	2.335[in]	0.059309 m	Distance between mirror M1 and the center of the beam splitter
d2	d1-delta_d	0.054247 m	Distance between mirror M2 and the center of the beam splitter
dE	12[in]	0.3048 m	Distance between the beam splitter and the screen
n_int	3.9641	3.9641	Refractive index of the beam-splitter interface
n_coat	1.2354	1.2354	Refractive index of the anti-reflective coating

PART LIBRARIES

1 In the **Home** toolbar, click **Part Libraries**.

2 In the **Part Libraries** window, select **Ray Optics Module** > **3D** > **Beam Splitters** > **beam_splitter_cube** in the tree.

3 Click ⬅ **Add to Geometry**.

GEOMETRY 1

Beam Splitter Cube 1 (pi1)

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Geometry 1** click **Beam Splitter Cube 1 (pi1)**.

2 In the **Settings** window for **Part Instance**, locate the **Input Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
d	dia	0.0127 m	Side length

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click ⬜ **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type dia/2.

4 In the **Height** text field, type th.

5 Locate the **Position** section. In the **y** text field, type -d2-th.

6 Locate the **Axis** section. From the **Axis type** list, choose **y-axis**.

Cylinder 2 (cyl2)

1 In the **Geometry** toolbar, click ⬜ **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type dia/2.

4 In the **Height** text field, type th.

5 Locate the **Position** section. In the **x** text field, type d1.

6 Locate the **Axis** section. From the **Axis type** list, choose **x-axis**.

7 Click 🔲 **Build Selected**.

8 Click the ⤢ **Zoom Extents** button in the **Graphics** toolbar.

Block 1 (blk1)

1 In the **Geometry** toolbar, click 🔳 **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type 4*dia.

4 In the **Depth** text field, type th.

5 In the **Height** text field, type 4*dia.

6 Locate the **Position** section. From the **Base** list, choose **Center**.

7 In the **y** text field, type dE+th/2.

Block 2 (blk2)

1 In the **Geometry** toolbar, click ⬜ **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type 200[mm].

4 In the **Depth** text field, type 450[mm].

5 In the **Height** text field, type 100[mm].

6 Locate the **Position** section. From the **Base** list, choose **Center**.

7 In the **y** text field, type 100[mm].

Form Union (fin)

1 In the **Geometry** toolbar, click ▥ **Build All**.

2 Click the ↙ **Go to Default View** button in the **Graphics** toolbar.

DEFINITIONS

View 1

In the **Model Builder** window, expand the **Component 1 (comp1)** > **Definitions** node, then click **View 1**.

Hide for Geometry 1

1 In the **View 1** toolbar, click 👁 **Hide**.

2 In the **Settings** window for **Hide for Geometry**, locate the **Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Click the ▦ **Transparency** button in the **Graphics** toolbar.

5 On the object **fin**, select Boundaries 1–5 and 33 only.

ADD MATERIAL

1 In the **Home** toolbar, click ▦ **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Click the **Add to Component** button in the window toolbar.

5 In the tree, select **Optical** > **Schott Glass** > **Schott N-BK7 Glass**.

6 Click the **Add to Component** button in the window toolbar.

7 In the **Home** toolbar, click ![icon] **Add Material** to close the **Add Material** window.

MATERIALS

Schott N-BK7 Glass (mat2)
Select Domains 4 and 5 only.

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Intensity Computation** section.

3 From the **Intensity computation** list, choose **Compute intensity and power**.

4 Select the **Compute phase** checkbox.

Material Discontinuity 1
Apply an anti-reflective coating to the outside of the beam splitter.

1 In the **Model Builder** window, under **Component 1 (comp1) > Geometrical Optics (gop)**
click **Material Discontinuity 1**.

2 In the **Settings** window for **Material Discontinuity**, locate the **Coatings** section.

3 From the **Thin dielectric films on boundary** list, choose **Anti-reflective coating**.

4 Select the **Treat as single layer dielectric film** checkbox.

5 In the λ_0 text field, type lam.

Ray Properties 1

1 In the **Model Builder** window, click **Ray Properties 1**.

2 In the **Settings** window for **Ray Properties**, locate the **Ray Properties** section.

3 In the λ_0 text field, type lam.

Wall 1

1 In the **Physics** toolbar, click ![icon] **Boundaries** and choose **Wall**.

2 Select Boundary 7 only.

Mirror 1

1 In the **Physics** toolbar, click ![icon] **Boundaries** and choose **Mirror**.

2 Select Boundaries 14 and 27 only.

Beam Splitter
Use a second **Material Discontinuity** node for the interior boundary of the beam splitter.

1 In the **Physics** toolbar, click ▬ **Boundaries** and choose **Material Discontinuity**.

2 In the **Settings** window for **Material Discontinuity**, type Beam Splitter in the **Label** text field.

3 Locate the **Coatings** section. From the **Thin dielectric films on boundary** list, choose **Specify reflectance**.

4 In the R text field, type 0.5.

5 Select the **Treat as single layer dielectric film** checkbox.

6 In the n text field, type n_int.

7 In the λ_0 text field, type lam.

8 In the θ_i text field, type 45[deg].

9 Select Boundary 19 only.

Release from Grid 1

Specify how the ray is going to be released. Release a single ray from the face of the enclosure facing mirror M1. Generate a fully polarized, -1 m radius of curvature spherical wave.

1 In the **Physics** toolbar, click ⚛ **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 In the $q_{x,0}$ text field, type -100[mm].

4 Locate the **Ray Direction Vector** section. Specify the $\mathbf{L}_0$ vector as

1	x
0	y
0	z

5 Locate the **Initial Radii of Curvature** section. From the **Wavefront shape** list, choose **Spherical wave**.

6 In the r_0 text field, type -1[m].

7 Locate the **Initial Polarization** section. From the **Initial polarization type** list, choose **Fully polarized**.

8 From the **Initial polarization** list, choose **User defined**.

9 Specify the **u** vector as

0	x

0	y
1	z

Ray Termination 1

1 In the **Physics** toolbar, click ⚛ **Global** and choose **Ray Termination**.

2 In the **Settings** window for **Ray Termination**, locate the **Termination Criteria** section.

3 From the **Spatial extents of ray propagation** list, choose **Bounding box, from geometry**.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Coarser**.

4 Click ▦ **Build All**.

STUDY 1

Step 1: Ray Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 Click ᴧᴧ **Range**.

4 In the **Range** dialog, type 0.1 in the **Step** text field.

5 In the **Stop** text field, type 2.2.

6 Click **Replace**.

7 In the **Study** toolbar, click ═ **Compute**.

RESULTS

Create a dataset to display the interference pattern on the screen.

Cut Plane 1

1 In the **Results** toolbar, click 🔲 **Cut Plane**.

2 In the **Settings** window for **Cut Plane**, locate the **Data** section.

3 From the **Dataset** list, choose **Ray 1**.

4 Locate the **Plane Data** section. From the **Plane** list, choose **xz-planes**.

5 In the **y-coordinate** text field, type dE.

Interference Pattern

1 In the **Results** toolbar, click ■ **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Interference Pattern` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Plane 1**.

Interference Pattern 1

1 In the **Interference Pattern** toolbar, click ▮ **More Plots** and choose **Interference Pattern**.

2 In the **Settings** window for **Interference Pattern**, locate the **Coordinate Range** section.

3 From the **Origin location specification** list, choose **At ray of greatest intensity**.

4 In the **Interference Pattern** toolbar, click ▣ **Plot**. The plot should look like Figure 3.