Created in COMSOL Multiphysics 6.3

Transmission Line Parameters of a Coaxial Cable

Introduction

Transmission lines are used to guide waves of alternating current and voltage at radio frequencies and have been used for more than 150 years, starting with the first telegraph cables in the 1850s. Today, transmission lines exist in a variety of forms, many of which are adapted for easy fabrication and employment in printed circuit board designs. Transmission lines are key elements in most modern electronic devices. They are used to carry information, at minimal loss and distortion, from one place to another within a device and between devices. The theoretical foundation for designing and analyzing transmission lines was laid by James Clerk Maxwell, Lord Kelvin, and Oliver Heaviside. Transmission line theory is a cornerstone in the teaching of RF and microwave engineering.

Electromagnetic fields propagate along transmission lines, to a fair approximation, as transverse electromagnetic (TEM) waves. The 1D, frequency domain wave equation for the electric potential on a transmission line is written in the form:

$$\frac{\partial}{\partial x}\left(\frac{1}{R+i\omega L}\frac{\partial V}{\partial x}\right)-(G+i\omega C)V = 0$$

where R, L, G, and C are the distributed resistance, inductance, conductance, and capacitance, respectively. A similar wave equation can be deduced for the current flowing along the transmission line. Below is an equivalent circuit model of a transmission line terminated by a load impedance.

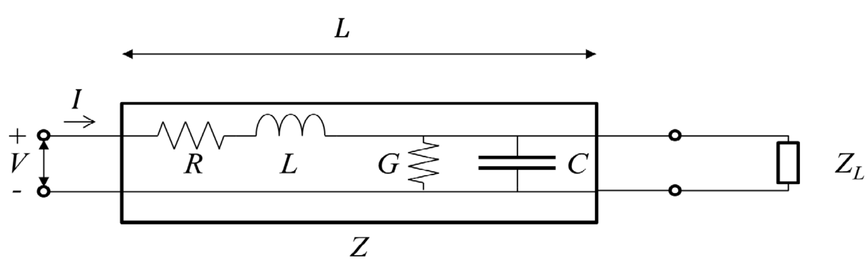


Figure 1: Schematic of a transmission line with a load impedance.

The solution to the wave equation represents a forward- and a backward-propagating wave:

$$V(x) = V_+e^{-\gamma x}+V_-e^{\gamma x}$$

where γ, the complex propagation constant is given by:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

If only a forward-propagating wave is present in the transmission line (no reflections), dividing the voltage V by the current I gives the characteristic impedance Z, or wave impedance of the transmission line:

$$Z = \frac{V}{I} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

The design of a transmission line is to a large extent about achieving a value of the characteristic impedance that matches the impedance of the load or whatever device the transmission line is connected to. By matching the impedance, reflections are kept to a minimum and the transferred RF power is maximized.

The Transmission Line, RLGC Parameters, a predefined multiphysics interface, offers a couple of physics interfaces and multiphysics coupling for computing the transmission line parameters R, L, G, and C, as well as γ and Z for a cross section of various types of transmission lines.

Model Definition

The analysis performed by the Transmission Line, RLGC parameters interface is based on the assumption of quasi-TEM modes, meaning that longitudinal components of electric and magnetic fields are nonzero but small. Then, the main properties of the propagating mode can be deduced from separate magnetic and electric analyses, the former yielding the R and L parameters and the latter yielding the G and C parameters. The equations solved for are firstly a magnetic formulation for out-of-plane currents where the out-of-plane magnetic vector potential A_z is calculated from the Magnetic Fields interface in 2D. The equation solved is:

$$(j\omega\sigma - \omega^2\varepsilon_r\varepsilon_0)A_z + \nabla\times\left(\frac{1}{\mu}\nabla\times A_z\right) = \sigma\frac{V_0}{L}$$

where σ is the conductivity, $\varepsilon_0\varepsilon_r$ the permittivity, μ the permeability, and ω is the angular frequency. Furthermore, V_0/L represents the applied out-of-plane electric field, which is applied only to the conductors. An important parameter is the skin depth:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

The skin depth is a measure of the exponential drop in current density with the distance to the surface inside conductors. It can be very small and needs to be resolved by the finite element mesh. This is obtained by using a special meshing method known as boundary layer meshing automatically provided by the physics-controlled mesh. After solving, the net current I flowing in the out-of-plane direction is measured by integration of the current density J:

$$J = V_0\frac{\sigma}{L} - (j\omega\sigma - \omega^2\varepsilon_r\varepsilon_0)A_z$$

over the forward conductor. The resulting out-of-plane impedance Z is computed as:

$$Z = \frac{V_0}{I} = R + j\omega L$$

yielding the R and L parameters.

Secondly, a current balance for the in-plane conduction and displacement currents with the electric potential V as the unknown is solved:

$$-\nabla \cdot ((\sigma + j\omega\varepsilon_r\varepsilon_0)\nabla V) = 0$$

A potential difference V_1 is applied between the forward and return conductors and the net in-plane current I_1 flowing between the conductors is measured by integration of the normal component of the in-plane current density J_1:

$$J_1 = -(\sigma + j\omega\varepsilon_r\varepsilon_0)\nabla V$$

over the surface (boundary) of the electrode with the applied potential. The resulting in-plane admittance Y_1 is then computed as:

$$Y_1 = \frac{I_1}{V_1} = G + j\omega C$$

yielding the G and C parameters.

Results and Discussion

After completing the computation, the transmission line parameters in Table 1 are automatically evaluated by default, including series resistance R, series inductance L, shunt conductance G, shunt capacitance C, all calculated per unit length, as well as characteristic impedance Z and propagation constant γ.

TABLE 1: COMPUTED APPROXIMATE TRANSMISSION LINE PARAMETERS

	VALUES
Series resistance R	4.2 Ω/m
series inductance L	2.56E-7 H/m
shunt conductance G	4.93E-14 S/m
shunt capacitance C	9.8E-11 F/m
characteristic impedance Z	51-0.067j Ω
propagation constant γ	0.041+31.5j Ω

Several default plots help to understand the electromagnetic field behavior within the transmission line structure.

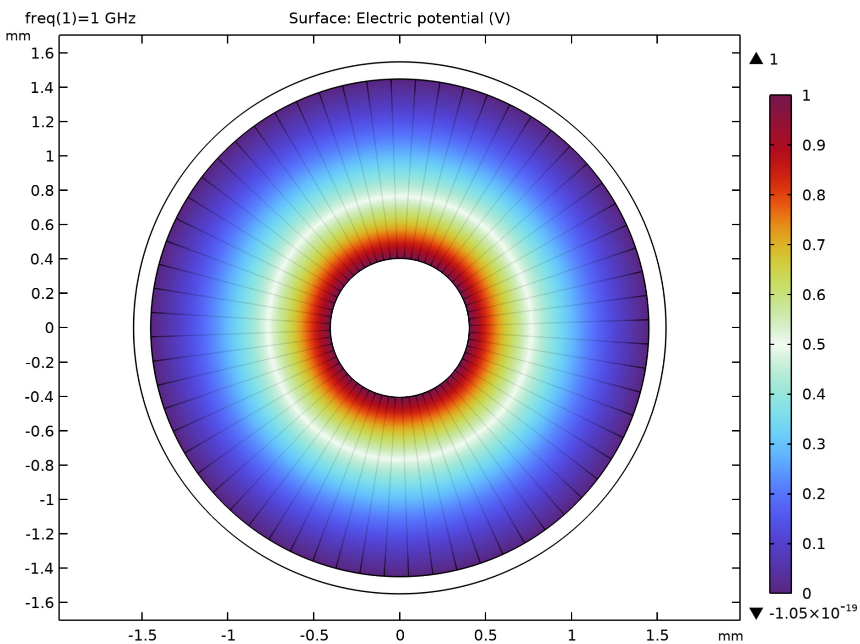


Figure 2: Surface plot of the electric potential along with a streamline plot of the electric field.

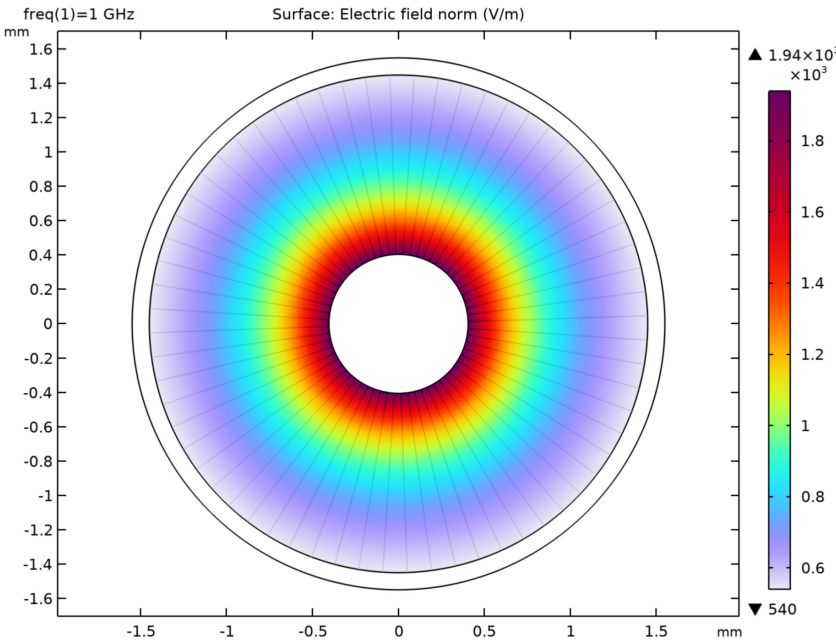


Figure 3: Surface plot of the electric field norm along with a streamline plot of the electric field.

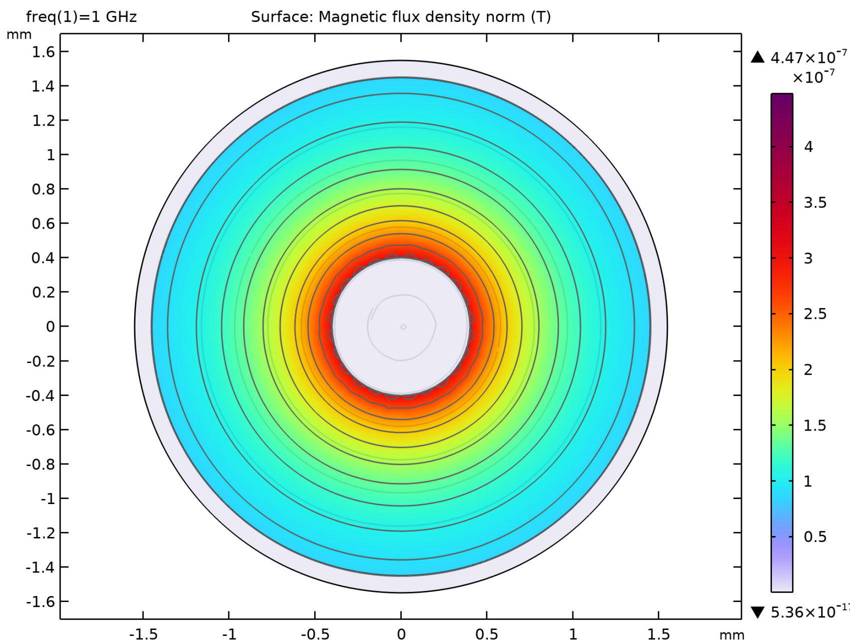


Figure 4: Surface plot of the magnetic flux density norm along with a streamline plot of the magnetic flux density and a contour plot of the z-component of the magnetic vector potential.

Notes About the COMSOL Implementation

The TEM waves assumption underlying the Transmission Line, RLGC Parameters multiphysics interface is only valid if the distance between forward (signal path) and return (ground) conductors is substantially smaller than the wavelength in the medium (<10%).

Application Library path: `RF_Module/Transmission_Lines_and_Waveguides/transmission_line_coaxial`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.
2 In the **Select Physics** tree, select **Radio Frequency** > **Transmission Line, RLGC Parameters**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **Preset Studies for Selected Multiphysics** > **Frequency Domain**.
6 Click **Done**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.
2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
3 In the **Frequencies** text field, type `f0`.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	1[GHz]	1E9 Hz	Frequency
Ri_coax	0.405[mm]	4.05E-4 m	Inner radius
d_s_coax	0.1[mm]	1E-4 m	Screen thickness
epsr_coax	2.25	2.25	Relative permittivity of dielectric
mur_coax	1	1	Relative permeability of dielectric
sigma_d_coax	1e-14[S/m]	1E-14 S/m	Conductivity of dielectric

Name	Expression	Value	Description
sigma_c_coax	5.98e7[S/m]	5.98E7 S/m	Conductivity of conductors
Ro_coax	1.45[mm]	0.00145 m	Outer radius
delta_coax	sqrt(2/(2*pi*f0* mur_coax* mu0_const* sigma_c_coax))	2.0581E-6 m	

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Ro_coax+d_s_coax`.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Ro_coax`.

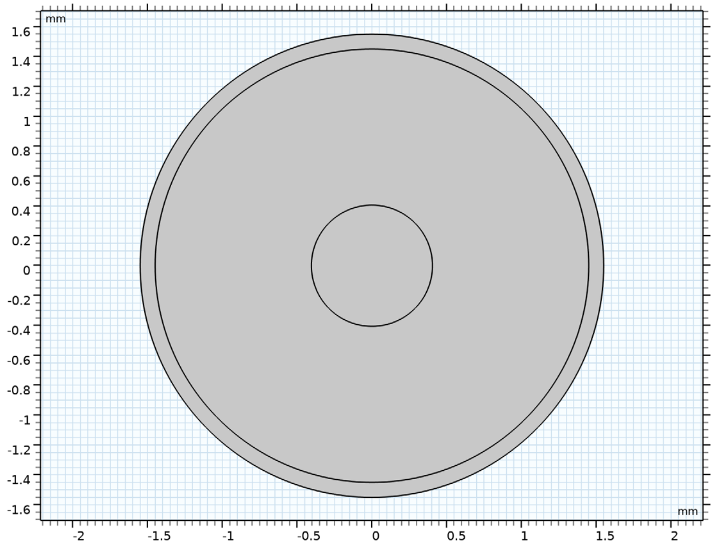
Circle 3 (c3)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Ri_coax`.

4 Click **Build All Objects**.



MATERIALS

Conductor Material

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Conductor Material` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

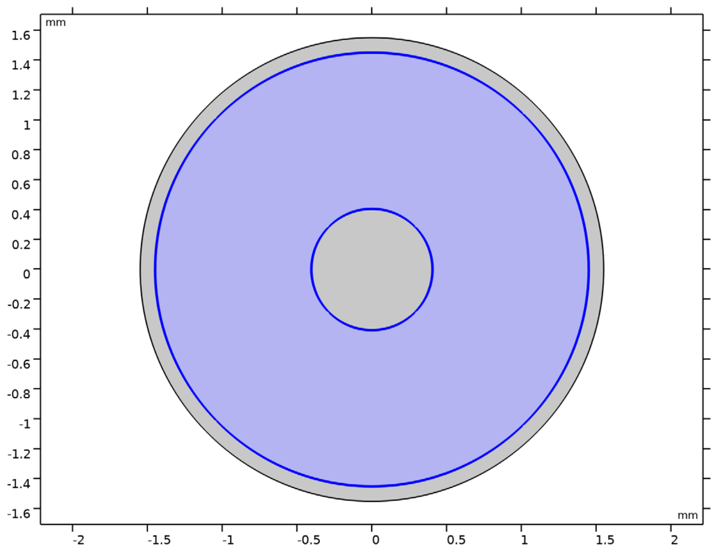
Property	**Variable**	**Value**	**Unit**	**Property group**
Electric conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	`sigma_coax`	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic

Dielectric Material

1 Right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Dielectric Material` in the **Label** text field.

3 Select Domain 2 only.



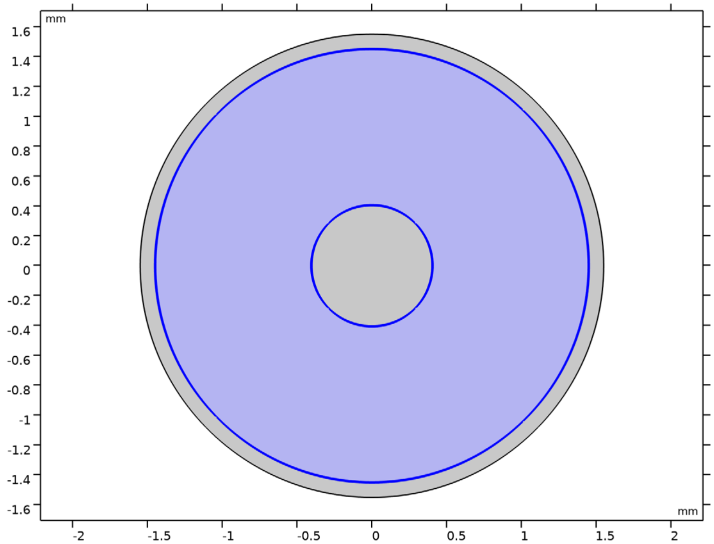
4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Electric conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	`sigma_d_coax`	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	`epsr_coax`	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	`mur_coax`	1	Basic

ELECTRIC CURRENTS (EC)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electric Currents (ec)**.

2 Select Domain 2 only.



Electric Potential 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Electric Currents (ec)** click **Electric Potential 1**.

2 In the **Settings** window for **Electric Potential**, locate the **Boundary Selection** section.

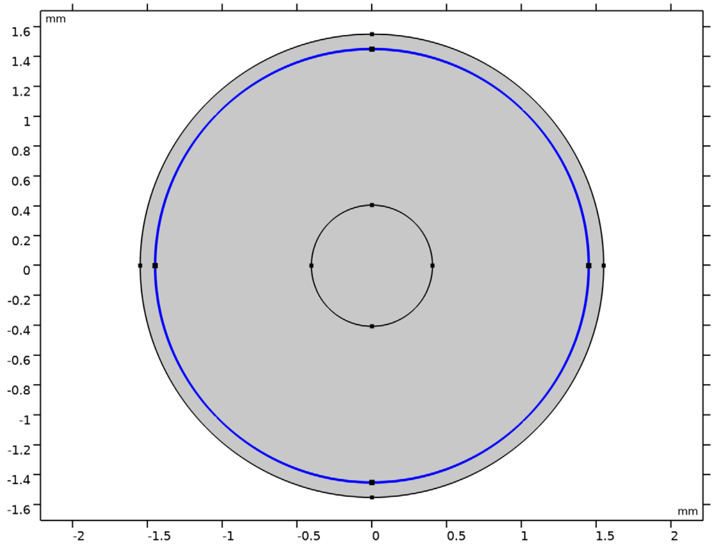
3 Click **Clear Selection**.

4 Select Boundaries 5, 6, 9, and 10 only.

Ground 1

1 In the **Model Builder** window, click **Ground 1**.

2 Select Boundaries 3, 4, 8, and 11 only.



MAGNETIC FIELDS (MF)

External Current Density 1

1 In the **Model Builder** window, under **Component 1 (comp1)** > **Magnetic Fields (mf)** click **External Current Density 1**.

2 Select Domain 3 only.

3 In the **Settings** window for **External Current Density**, locate the **External Current Density** section.

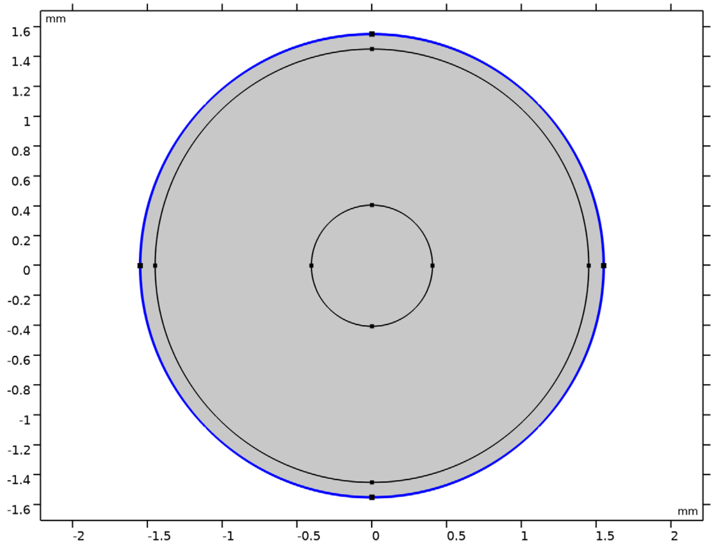
4 Specify the $\mathbf{J}_e$ vector as

sigma_c_coax*1[V/m]	z

Perfect Magnetic Conductor 1

1 In the **Model Builder** window, click **Perfect Magnetic Conductor 1**.

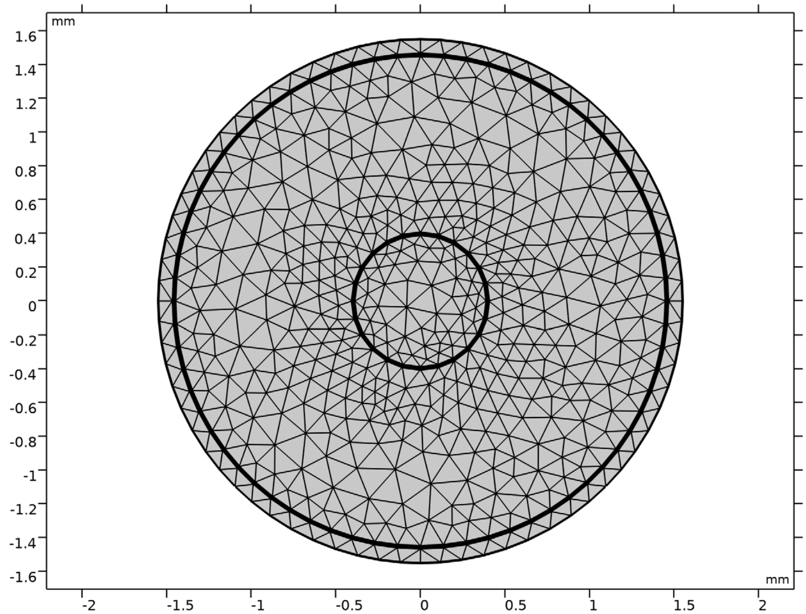
2 Select Boundaries 1, 2, 7, and 12 only.



3 Right-click **Perfect Magnetic Conductor 1** and choose **Disable**.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.



2 In the **Home** toolbar, click **Compute**.

RESULTS

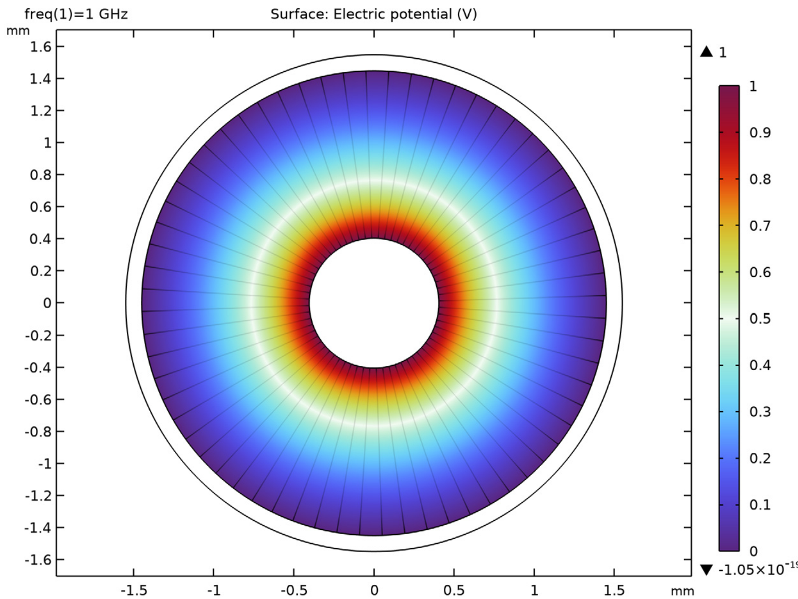
Transmission Line Parameters (tlp1)

1 In the **Model Builder** window, under **Results** click **Transmission Line Parameters (tlp1)**.

2 In the **Transmission Line Parameters (tlp1)** toolbar, click **Evaluate**.

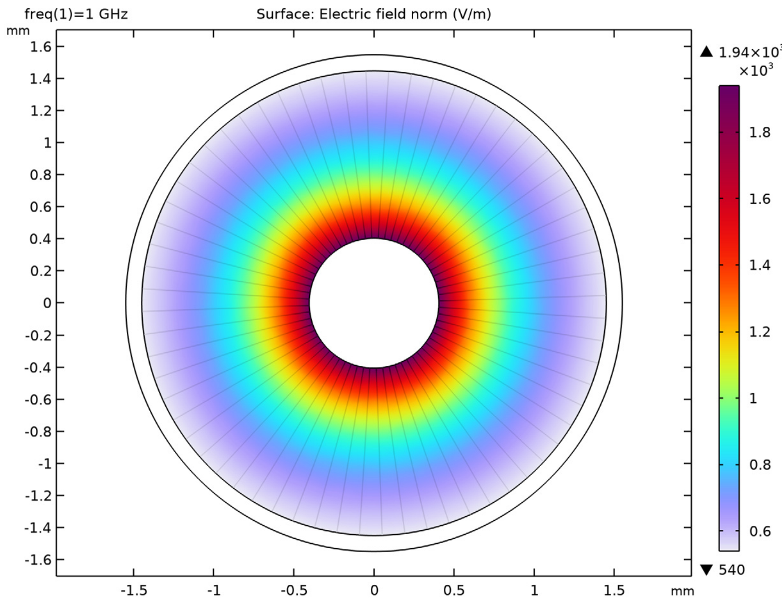
Electric Potential (ec)

In the **Model Builder** window, click **Electric Potential (ec)**.



Electric Field (ec)

In the **Model Builder** window, click **Electric Field (ec)**.



Magnetic Flux Density (mf)

In the **Model Builder** window, click **Magnetic Flux Density (mf)**.