Created in COMSOL Multiphysics 6.3

Eigenmodes in a Muffler

Introduction

In this example, you compute the propagating modes in the chamber of an automotive muffler. The geometry is a cross section of the chamber in the Absorptive Muffler example.

The purpose of the model is to study the shape of the propagating modes and to find their cutoff frequencies. As discussed in the documentation of the Absorptive Muffler example, some of the modes significantly affect the damping of the muffler at frequencies above their cutoff. In this model, you study modes with cutoff frequencies up to 1500 Hz.

Model Definition

The muffler chamber has a race track shaped cross section, as seen in Figure 1. In this model, the chamber is considered to be hollow and field with air at atmospheric pressure.

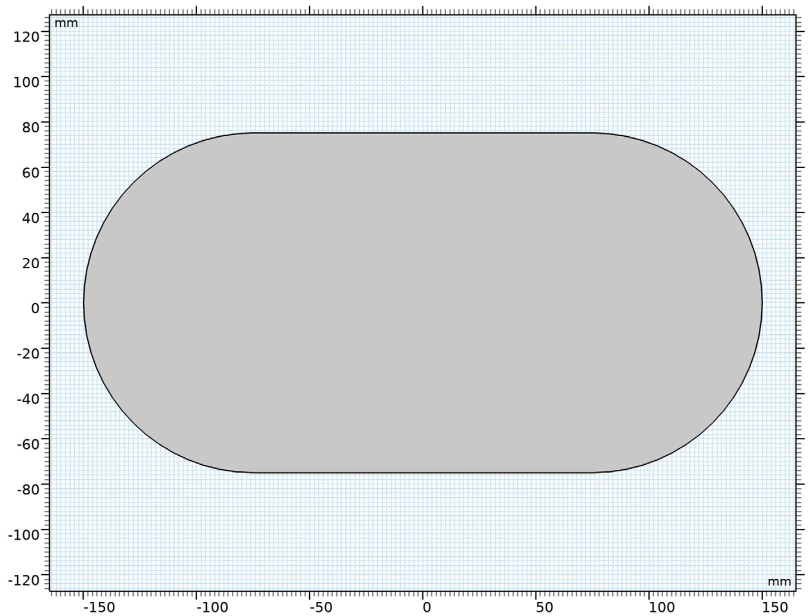

Figure 1: The model geometry.

The wave numbers and mode shapes through a cross section of the chamber are found as the solution of an eigenvalue problem for the acoustic pressure p:

$$\nabla \cdot \left(-\frac{\nabla p(x, y)}{\rho_0}\right) - \left(\frac{\omega^2}{\rho_0 c^2} - \frac{\kappa_z^2}{\rho_0}\right) p(x, y) = 0$$

where ρ_0 is the density, c the speed of sound, κ_z the out-of-plane wave number, and $\omega = 2\pi f$ the angular frequency. For a given angular frequency, only modes such that κ_z^2 is positive can propagate. The cutoff frequency of each mode is calculated as

$$f_j = \frac{\sqrt{\omega^2 - c^2\kappa_z^2}}{2\pi}$$

Results and Discussion

The model finds five propagating modes, whose characteristics are summed up in the table here below. The values are fund in the evaluation group in the Results section of the model. Note that both positive and negative wave numbers are presented, representing propagation in- and out-of-plane.

Cutoff frequency (Hz)	**Characteristics**
0	Plane wave
635	Antisymmetric with respect to x, symmetric with respect to y
1210	Symmetric with respect to x, antisymmetric with respect to y
1240	Symmetric with respect to x and y
1467	Antisymmetric with respect to x and y

For a muffler with a centered tube leading into the chamber, the first mode that is symmetric with respect to both the x-axis and the y-axis is propagating when the frequency is higher than 1240 Hz. Figure 2 shows this mode, which for an infinitely long chamber occurs at 1240 Hz. The mode is visualized in 3D in Figure 3.

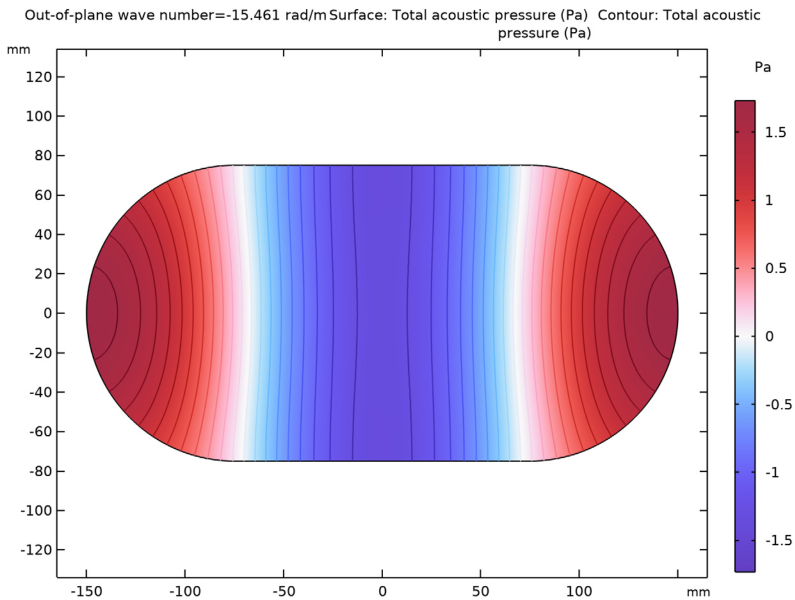

Figure 2: The chamber's first fully symmetric propagation mode. The plot shows the real part of the acoustic pressure.

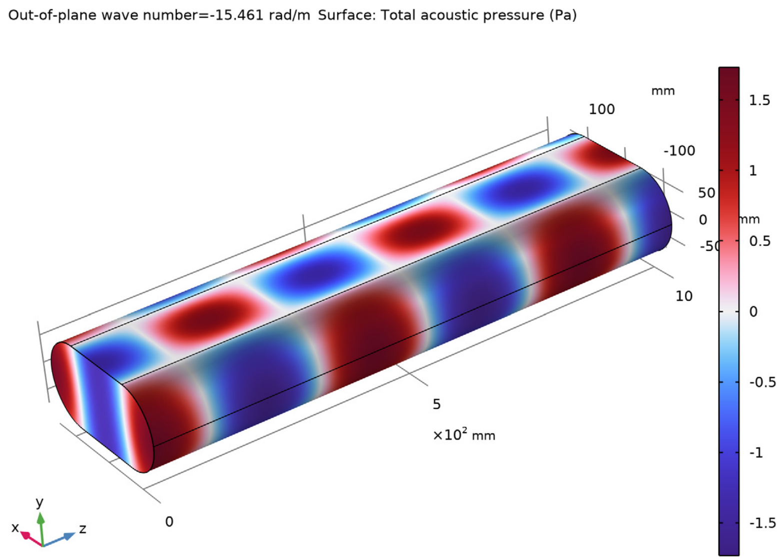

Figure 3: The first fully symmetric mode visualized in a section of waveguide.

Application Library path: `Acoustics_Module/Automotive/eigenmodes_in_muffler`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Acoustics** > **Pressure Acoustics** > **Pressure Acoustics, Frequency Domain (acpr)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces** > **Mode Analysis**.

6 Click **Done**.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Square 1 (sq1)

1 In the **Geometry** toolbar, click **Square**.

2 In the **Settings** window for **Square**, locate the **Size** section.

3 In the **Side length** text field, type `150`.

4 Locate the **Position** section. From the **Base** list, choose **Center**.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `75`.

4 Locate the **Position** section. In the **x** text field, type `-75`.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `75`.

4 Locate the **Position** section. In the **x** text field, type `75`.

Union 1 (uni1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Union**.

2 Click in the **Graphics** window and then press Ctrl+A to select all objects.

3 In the **Settings** window for **Union**, locate the **Union** section.

4 Clear the **Keep interior boundaries** checkbox.

5 Click **Build All Objects**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.

ADD MATERIAL

1 In the **Materials** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in** > **Air**.

4 Click the **Add to Component** button in the window toolbar.

5 In the **Materials** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Air (mat1)

By default, the boundaries of the geometry will be considered to be sound hard walls. No other physics settings are needed.

MESH 1

In this model, the mesh is set up manually. Add a free triangular mesh and then proceed to the solver settings.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

STUDY 1

Step 1: Mode Analysis

1 In the **Model Builder** window, under **Study 1** click **Step 1: Mode Analysis**.

2 In the **Settings** window for **Mode Analysis**, locate the **Study Settings** section.

3 Select the **Desired number of modes** checkbox. In the associated text field, type `8`.

4 Select the **Search for modes around shift** checkbox. In the associated text field, type `20`.

   The free-space propagation mode has an out-of-plane wave number equal to `omega/c = 27.5 rad/m`. With these settings, the solver returns the 8 modes with propagation constants closest to 20 rad/m first in the list.

5 In the **Mode analysis frequency** text field, type `1500[Hz]`.

   This setting makes the software look for propagating modes with cutoff frequencies up to 1500 Hz.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Acoustic Pressure (acpr)

The solver has found the free-space mode and all other propagating modes. There is a total of 5 different propagating modes. Because the waves can propagate both into and out of the modeling plane, each mode gets reported twice, with positive and negative out-of-plane wave numbers.

For the positive out-of-plane wave numbers, it holds that the higher the mode, the lower the wave number. However, the solver does not stop at zero. Because you asked for more than the 5 existing propagating modes, you get additional modes with imaginary out-of-plane wave numbers. This indicates that they are evanescent. The default plot shows the acoustic pressure distribution for a mode with a wave number of -17.95i rad/m.

1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
2 From the **Out-of-plane wave number (rad/m)** list, choose **-15.461**.
3 In the **Acoustic Pressure (acpr)** toolbar, click **Plot**.
4 Click the **Zoom Extents** button in the **Graphics** toolbar.

   This is the lowest fully symmetric mode, which is shown in Figure 2. You can compute the cutoff frequency of this mode using the expression in the model introduction. In order to refer to the speed of sound in air, use an arbitrary point in the geometry for this evaluation.

Evaluation Group 1 - Mode Cutoff Frequency

1 In the **Results** toolbar, click **Evaluation Group**.
2 In the **Settings** window for **Evaluation Group**, type `Evaluation Group 1 - Mode Cutoff Frequency` in the **Label** text field.
3 Locate the **Data** section. From the **Out-of-plane wave number selection** list, choose **From list**.
4 In the **Out-of-plane wave number (rad/m)** list, select **-15.461**.

Point Evaluation 1

1 Right-click **Evaluation Group 1 - Mode Cutoff Frequency** and choose **Point Evaluation**.
2 Select Point 1 only.
3 In the **Settings** window for **Point Evaluation**, locate the **Expressions** section.

4 In the table, enter the following settings:

Expression	**Unit**	**Description**
`sqrt(acpr.omega^2-acpr.kz^2*acpr.c^2)/(2*pi)`	`rad/s`	`Mode Cutoff Frequency`

5 In the **Evaluation Group 1 - Mode Cutoff Frequency** toolbar, click **Evaluate**.

Finally, create a plot to visualize the mode in 3D using the **2D Extrusion** dataset.

Extrusion 2D 1

1. In the **Results** toolbar, click **More Datasets** and choose **Extrusion 2D**.
2. In the **Settings** window for **Extrusion 2D**, locate the **Extrusion** section.
3. In the **z maximum** text field, type `1000`.
4. In the **Resolution** text field, type `100`.
5. Click to expand the **Advanced** section. In the **Out-of-plane wave number** text field, type `acpr.kz`.

Mode 3D

1. In the **Results** toolbar, click **3D Plot Group**.
2. In the **Settings** window for **3D Plot Group**, type `Mode 3D` in the **Label** text field.
3. Locate the **Data** section. From the **Out-of-plane wave number (rad/m)** list, choose **-15.461**.

Surface 1

Right-click **Mode 3D** and choose **Surface**.

Mode 3D

Rotate and zoom to get the desired 3D view.