Created in COMSOL Multiphysics 6.2

View Factor Computation

The computation of view factors is central when performing heat transfer simulations using the radiosity method to account for surface-to-surface radiation. This benchmark demonstrates how to compute geometrical view factors for two concentric spheres that emit and receive radiation from each other. It compares simulation results to exact analytical values, and illustrates how to use the symmetry of the geometry to reduce the computational cost.

Introduction

The surface-to-surface radiation method in the Heat Transfer module relies on the radiosity method. When surface-to-surface radiation is activated, operators are available to compute the view factors between diffuse surfaces that irradiate each other. For some standard configurations one can determine the view factors analytically, but in engineering applications this is rarely possible. A benchmark example that compares exact analytical values with simulation results shows the accuracy of the numerical method. Here, two concentric spheres are used, for which the analytical view factors are known.

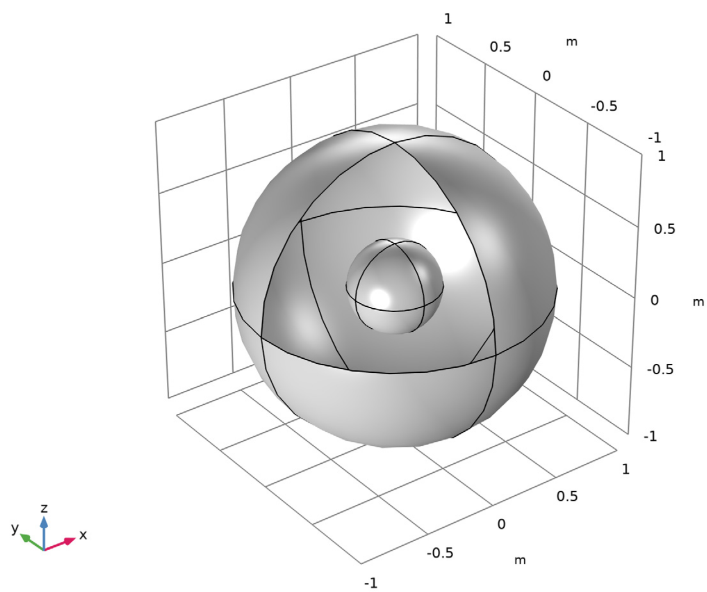


Figure 1: Model geometry.

Model Definition

The model consists of two concentric radiating spheres (Figure 1) acting as perfect emitters (surface emissivities both equal to 1). The radiation direction of each sphere is such that they irradiate each other.

ANALYTICAL VIEW FACTORS

A detailed introduction on view factors is given in (Ref. 1). An arbitrary view factor $F_{1 \to 2}$ is defined as the proportion of radiation leaving surface S_1 and intercepting surface S_2, that is

$$F_{1 \to 2} = \frac{\text{Radiation leaving } S_1 \text{ and intercepting } S_2}{\text{Total radiation leaving } S_1}$$

If the radiosity is the same on each surface, it only depends on the geometrical configuration and thus can be calculated from geometrical properties. For two concentric spheres, labeled ext for the outer sphere and int for the inner sphere, the view factors are:

$$F_{\mathrm{ext} \to \mathrm{ext}} = 1 - \left(\frac{R_{\mathrm{int}}}{R_{\mathrm{ext}}}\right)^2$$

$$F_{\mathrm{ext} \to \mathrm{int}} = \left(\frac{R_{\mathrm{int}}}{R_{\mathrm{ext}}}\right)^2$$

$$F_{\mathrm{int} \to \mathrm{int}} = 0$$

$$F_{\mathrm{int} \to \mathrm{ext}} = 1$$

Refer to Table 1 for numerical values of these quantities.

VIEW FACTOR COMPUTATION IN COMSOL MULTIPHYSICS

The view factor computation is performed at each evaluation point on the boundaries. A boundary has an *upside* and a *downside* and both can be exposed to radiation. In this model the outer sphere radiates inward, which is the downward direction and the inner sphere radiates outward, which is the upward direction. Two operators are available for evaluation of the mutual surface irradiation. These are:

- `radopu(`*`expression_upside`*`,`*`expression_downside`*`)`
- `radopd(`*`expression_upside`*`,`*`expression_downside`*`)`

Both are available on all boundaries where surface-to-surface features are active. The `radopu(,)` and `radopd(,)` operators perform the computation on the upside and downside of the boundary where they are evaluated. The *`expression_upside`* and *`expression_downside`* arguments are the radiosity expression on the upside and downside of the boundaries that irradiate the boundary where the operators are evaluated.

To compute the geometrical view factor, for example, $F_{\mathrm{ext} \to \mathrm{int}}$, in COMSOL Multiphysics, the following integration needs to be defined:

$$F_{\mathrm{ext} \to \mathrm{int}} = \frac{\int_{S_{\mathrm{int}}} \mathrm{radopu}(0, \mathrm{ext}) ds}{A_{\mathrm{ext}}}$$

where S_{int} and S_{ext} denote interior and exterior surfaces, A_{int} and A_{ext} are corresponding surface areas.

The integration at the numerator is defined over the inner sphere, which radiates only on the upside of its surface. Hence, `radopd(,)` evaluates to zero and the integrand reduces to `radopu(0,`*`expression_upside`*`)`. Moreover since the outer sphere radiates only on the downside of its surface the *`expression_downside`* argument should be `ext`.

Results and Discussion

The view factors computed with COMSOL Multiphysics, by using or not the symmetry in the geometry, are listed in Table 1 together with analytical values and absolute errors.

TABLE 1: VIEW FACTOR COMPUTATION.

VIEW FACTOR	ANALYTICAL VALUE	COMPUTED VALUE	ERROR (NO SYMMETRY)	ERROR (SYMMETRY)
$F_{\mathrm{int} \to \mathrm{int}}$	0	0	0	0
$F_{\mathrm{int} \to \mathrm{ext}}$	1	1	$1.1 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$
$F_{\mathrm{ext} \to \mathrm{ext}}$	0.91	0.91	$8.8 \cdot 10^{-4}$	$8.5 \cdot 10^{-4}$
$F_{\mathrm{ext} \to \mathrm{int}}$	0.09	0.09	$1.9 \cdot 10^{-8}$	$2.5 \cdot 10^{-16}$

In this table, analytical and computed values are slightly different but rounded to hundredths. The very small errors are listed in the last columns and reflect accurate results of the view factors provided by the simulation.

Reference

1. Michael F. Modest, *Radiative Heat Transfer*, 3rd ed., Academic Press, 2013

Application Library path: `Heat_Transfer_Module/Verification_Examples/view_factor`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

You are only interested in the view factors, hence Surface-to-Surface Radiation is the only needed interface.

2 In the **Select Physics** tree, select **Heat Transfer>Radiation>Surface-to-Surface Radiation (rad)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GEOMETRY 1

Define the radii of the spheres and the analytical values for the view factors as parameters.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r_int	0.3[m]	0.3 m	Radius, interior sphere
r_ext	1[m]	1 m	Radius, exterior sphere

Name	Expression	Value	Description
F_ext_ext	1-(r_int/r_ext)^2	0.91	Analytical view factor from exterior sphere to exterior sphere
F_ext_int	(r_int/r_ext)^2	0.09	Analytical view factor from exterior sphere to interior sphere
F_int_int	0	0	Analytical view factor from interior sphere to interior sphere
F_int_ext	1	1	Analytical view factor from interior sphere to exterior sphere

GEOMETRY 1

No domain information are required, define the geometry as surface objects.

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.
2 In the **Settings** window for **Sphere**, locate the **Object Type** section.
3 From the **Type** list, choose **Surface**.
4 Locate the **Size** section. In the **Radius** text field, type `r_int`.

   Create a selection of this sphere to easily access its entities throughout the modeling process.

5 Locate the **Selections of Resulting Entities** section. Find the **Cumulative selection** subsection. Click **New**.
6 In the **New Cumulative Selection** dialog box, type `Inner sphere` in the **Name** text field.
7 Click **OK**.

Sphere 2 (sph2)

1 In the **Geometry** toolbar, click **Sphere**.
2 In the **Settings** window for **Sphere**, locate the **Object Type** section.
3 From the **Type** list, choose **Surface**.
4 Locate the **Size** section. In the **Radius** text field, type `r_ext`.
5 Locate the **Selections of Resulting Entities** section. Find the **Cumulative selection** subsection. Click **New**.
6 In the **New Cumulative Selection** dialog box, type `Outer sphere` in the **Name** text field.
7 Click **OK**.

Thanks to the symmetries of the model, only one eighth of the geometry is needed. Remove the unnecessary parts.

Delete Entities 1 (del1)

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Delete Entities**.

2 On the object **sph1**, select Boundaries 1–7 only.

3 On the object **sph2**, select Boundaries 1–7 only.

4 In the **Settings** window for **Delete Entities**, click **Build All Objects**.

Add a material to the boundaries to define their emissivities.

MATERIALS

Blackbody

1 In the **Materials** toolbar, click **Blank Material**.

2 In the **Settings** window for **Material**, type `Blackbody` in the **Label** text field.

3 Click to expand the **Material Properties** section. In the **Material properties** tree, select **Basic Properties>Surface Emissivity**.

4 Click **Add to Material**.

5 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Surface emissivity	epsilon_rad	1	1	Basic

SURFACE-TO-SURFACE RADIATION (RAD)

Diffuse Surface 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Surface-to-Surface Radiation (rad)** click **Diffuse Surface 1**.

2 In the **Settings** window for **Diffuse Surface**, locate the **Radiation Direction** section.

3 From the list, choose **Positive normal direction**.

Diffuse Surface 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Diffuse Surface**.

2 In the **Settings** window for **Diffuse Surface**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Outer sphere**.

4 Locate the **Radiation Direction** section. From the list, choose **Negative normal direction**.

Implement a symmetry condition for the view factor computation.

Symmetry for Surface-to-Surface Radiation 1

1 In the **Physics** toolbar, click **Global** and choose **Symmetry for Surface-to-Surface Radiation**.

2 In the **Settings** window for **Symmetry for Surface-to-Surface Radiation**, locate the **Symmetry for Surface-to-Surface Radiation** section.

3 From the **Type of symmetry** list, choose **Three perpendicular symmetry planes**.

Define new variables and nonlocal integration couplings to evaluate the view factors directly after running the study. First, define variables that are used to identify the surfaces.

DEFINITIONS

Identifiers, Inner Sphere

1 In the **Home** toolbar, click **Variables** and choose **Local Variables**.

2 In the **Settings** window for **Variables**, type `Identifiers, Inner Sphere` in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Inner sphere**.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
ext	0		Exterior surface indicator
int	1		Interior surface indicator

Identifiers, Outer Sphere

1 In the **Home** toolbar, click **Variables** and choose **Local Variables**.

2 In the **Settings** window for **Variables**, type `Identifiers, Outer Sphere` in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **Outer sphere**.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
ext	1		Exterior surface indicator
int	0		Interior surface indicator

Now, define nonlocal integration couplings for both spheres.

Integration, Inner Sphere

1. In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.
2. In the **Settings** window for **Integration**, type `Integration, Inner Sphere` in the **Label** text field.
3. In the **Operator name** text field, type `intop_int`.
4. Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Boundary**.
5. From the **Selection** list, choose **Inner sphere**.

Integration, Outer Sphere

1. In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.
2. In the **Settings** window for **Integration**, type `Integration, Outer Sphere` in the **Label** text field.
3. In the **Operator name** text field, type `intop_ext`.
4. Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Boundary**.
5. From the **Selection** list, choose **Outer sphere**.

Adjust the mesh size for both spheres manually to get approximately the same number of elements on each sphere. This is an efficient mesh for view factor computation.

MESH 1

Free Triangular 1

1. In the **Mesh** toolbar, click **More Generators** and choose **Free Triangular**.
2. In the **Settings** window for **Free Triangular**, locate the **Boundary Selection** section.
3. From the **Selection** list, choose **All boundaries**.

Size 1

1. Right-click **Free Triangular 1** and choose **Size**.
2. In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.
3. From the **Selection** list, choose **Inner sphere**.
4. Locate the **Element Size** section. Click the **Custom** button.
5. Locate the **Element Size Parameters** section.
6. Select the **Maximum element size** check box. In the associated text field, type `r_int/5`.

Size 2

1 In the **Model Builder** window, right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

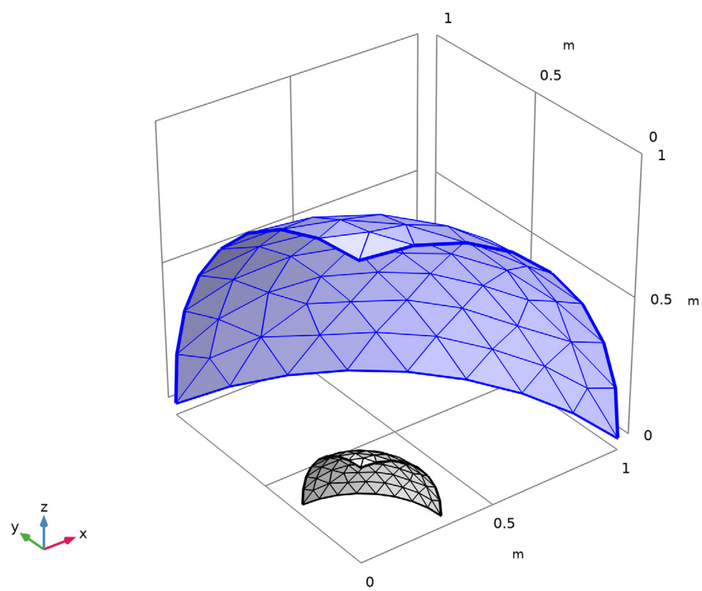
3 From the **Selection** list, choose **Outer sphere**.

4 Locate the **Element Size** section. Click the **Custom** button.

5 Locate the **Element Size Parameters** section.

6 Select the **Maximum element size** check box. In the associated text field, type `r_ext/5`.

7 Click **Build All**.



The view factors are computed automatically, before the actual study runs. To evaluate the geometrical view factors, it is sufficient to obtain the initial values.

STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

View Factors

1 In the **Model Builder** window, expand the **Results** node.

2 Right-click **Results>Derived Values** and choose **Global Evaluation**.

3 In the **Settings** window for **Global Evaluation**, type `View Factors` in the **Label** text field.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`intop_int(rad.radopu(int,0))/intop_int(1)`	1	`Interior to interior view factor`
`intop_ext(rad.radopd(int,0))/intop_int(1)`	1	`Interior to exterior view factor`
`intop_ext(rad.radopd(0,ext))/intop_ext(1)`	1	`Exterior to exterior view factor`
`intop_int(rad.radopu(0,ext))/intop_ext(1)`	1	`Exterior to interior view factor`

Absolute Error, View Factors

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, type `Absolute Error, View Factors` in the **Label** text field.

3 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`abs(intop_int(rad.radopu(int,0))/intop_int(1)-F_int_int)`	1	`Absolute error, interior to interior view factor`
`abs(intop_ext(rad.radopd(int,0))/intop_int(1)-F_int_ext)`	1	`Absolute error, interior to exterior view factor`
`abs(intop_ext(rad.radopd(0,ext))/intop_ext(1)-F_ext_ext)`	1	`Absolute error, exterior to exterior view factor`
`abs(intop_int(rad.radopu(0,ext))/intop_ext(1)-F_ext_int)`	1	`Absolute error, exterior to interior view factor`

4 In the **Results** toolbar, click **Evaluate** and choose **Evaluate All**.

The results are displayed in the **Table** window.

Then, verify that a computation on the full geometry yields the same values.

GEOMETRY 1

Delete Entities 1 (del1)

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** right-click **Delete Entities 1 (del1)** and choose **Disable**.

2 In the **Settings** window for **Delete Entities**, click **Build All Objects**.

3 Click the **Go to Default View** button in the **Graphics** toolbar.

Create a new view which hides one of the front boundaries so that you can look inside the outer sphere.

DEFINITIONS

View 2

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **View**.

Hide for Geometry 1

1 In the **Model Builder** window, right-click **View 2** and choose **Hide for Geometry**.

2 In the **Settings** window for **Hide for Geometry**, locate the **Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 On the object **sph2**, select Boundary 2 only.

SURFACE-TO-SURFACE RADIATION (RAD)

Symmetry for Surface-to-Surface Radiation 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Surface-to-Surface Radiation (rad)** right-click **Symmetry for Surface-to-Surface Radiation 1** and choose **Disable**.

Finally, compute the view factors with the new configuration and compare them with the previous results to validate the use of symmetry.

2 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

View Factors

In the **Settings** window for **Global Evaluation**, click **Evaluate**.

Absolute Error, View Factors

1 In the **Model Builder** window, click **Absolute Error, View Factors**.

2 In the **Settings** window for **Global Evaluation**, click **Evaluate**.