

# Stress-Optical Effects with Generalized Plane Strain

#### Introduction

The assumptions made for plane strain in the previous analysis of the waveguide structure (see the Application Library model Stress-Optical Effects in a Photonic Waveguide, the model name is stress optical) do not hold in a situation where the silicon-silica laminate is free to expand in the z direction. Instead, it is necessary to use a generalized plane strain model that allows for free expansion in the z direction. The boundary conditions in the xy-plane already allow the structure to expand freely in all directions in the plane. When the different materials in a laminate expand with different expansion coefficients, the laminate bends. In this model, the silica-silicon laminate bends in both the x and z directions.

**Note:** This application requires the Wave Optics Module and the Structural Mechanics Module.

# Model Definition

#### GENERALIZED PLANE STRAIN

One possible extension of the plane strain formulation is to assume that the strains are independent of the out-of -plane coordinate z, that is

$$\varepsilon_{ij} = \varepsilon_{ij}(x, y)$$

Under the small strain assumption, the above equations have the following 3D solution:

$$u = u_0(x, y) - \frac{e_1}{2}z^2$$

$$v = v_0(x, y) - \frac{e_2}{2}z^2$$

$$w = (e_0 + e_1x + e_2y)z$$

where  $e_0$ ,  $e_1$ , and  $e_2$  are constants. Thus, at the cross-section z=0, one has

$$u = u_0(x, y)$$
$$v = v_0(x, y)$$
$$w = 0$$

and

$$\begin{split} \varepsilon_z &= e_0 + e_1 x + e_2 y \\ \varepsilon_{\gamma z} &= \varepsilon_{\gamma z} = 0 \end{split}$$

The above conditions differ from the plane strain state only by the fact that the normal out-of -plane strain component can vary linearly throughout the cross section. This approximation is expected to be good when the structure is free to expand in the out-ofplane direction, and possible bending curvature is small with respect to the extents of the structure in the xy-plane. It corresponds to a small rotation that is representative of each cross section of the structure along the z-axis.

In COMSOL Multiphysics, the Generalized Plane Strain option is available under Structural Mechanics interface in 2D. Coefficients  $e_0$ ,  $e_1$ , and  $e_2$  in the expression for the  $\varepsilon_z$  strain are modeled as extra degrees of freedom that are constant throughout the model (global variables). See the Structural Mechanics Module User's Guide for more details.

Figure 1 shows the von Mises stress distribution together with the deformed shape of the waveguide.

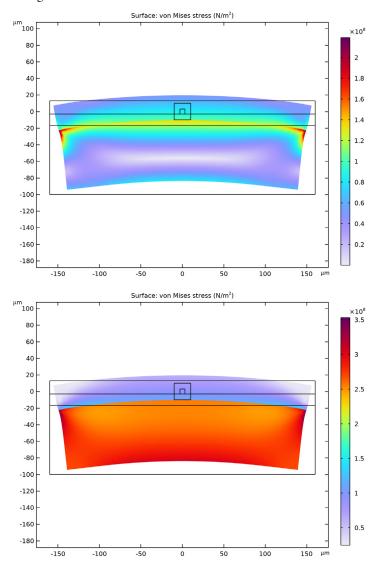
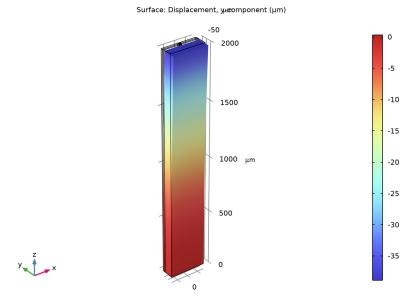


Figure 1: von Mises equivalent stress computed with (upper) and without (lower) the generalized plane strain assumption.

A 3D representation of the generalized plane strain solution is shown in this figure:



For symmetry reasons, the strain components  $\varepsilon_x$  and  $\varepsilon_z$  should be equal. The plot in Figure 2 visualizes the area where the relative difference between  $\varepsilon_x$  and  $\varepsilon_z$  is within 5%. The model is most accurate in the regions close to the core, far from the boundaries on the far left and right.

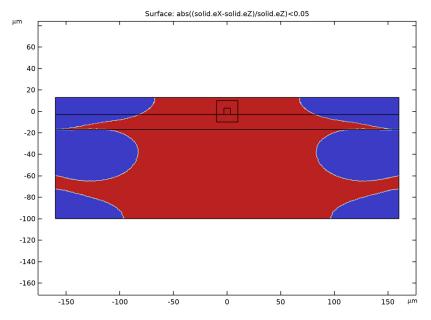


Figure 2: In the red-colored region, the relative difference between the x and z strain components is within 5%.

Figure 3 compares the effective mode indices for the first four propagating modes using the generalized plane strain equations with those obtained from the analysis in the previous model. As the plot shows, there is a systematic shift in the propagation constants when the strain in the z direction is taken into account.

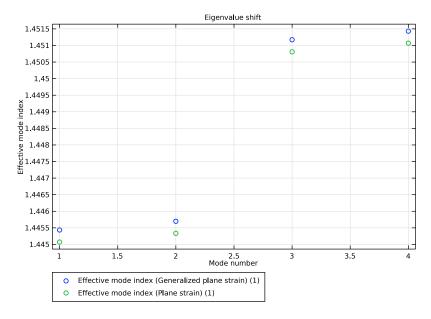


Figure 3: Effective mode indices assuming plane strain and generalized plane strain.

Figure 4 shows the stress-induced birefringence along the symmetry line within the waveguide. Nonzero out-of-plane strain leads to an increase in the birefringence effect.

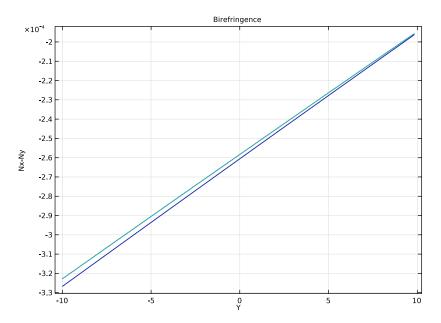


Figure 4: Birefringence along the vertical symmetry line within the waveguide for plane strain (lower curve) and generalized plane strain (upper curve).

Figure 5 gives the details of the eigenmode with the lowest effective mode index. It presents the visualization of the power flow, also called the optical intensity or the Poynting vector, in the out-of-plane direction.

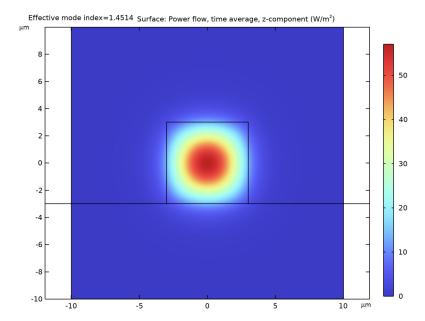


Figure 5: Eigenmode with lowest mode index, computed with the stress-optical effect under generalized plane strain assumption.

**Application Library path:** Wave\_Optics\_Module/Waveguides\_and\_Couplers/stress\_optical\_generalized

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

# MODEL WIZARD

- I In the Model Wizard window, click **Q** 2D.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.

- 4 In the Select Physics tree, select Optics>Wave Optics>Electromagnetic Waves, Frequency Domain (ewfd).
- 5 Click Add.
- 6 Click Study.
- 7 In the Select Study tree, select Preset Studies for Some Physics Interfaces>Stationary.
- 8 Click M Done.

#### **GLOBAL DEFINITIONS**

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file stress\_optical\_parameters.txt.

#### GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose µm.

#### Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 320.
- 4 In the Height text field, type 83.
- **5** Locate the **Position** section. In the **x** text field, type -160.
- 6 In the y text field, type -100.

## Rectangle 2 (r2)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type **320**.
- 4 In the **Height** text field, type 14.
- 5 Locate the **Position** section. In the x text field, type -160.
- 6 In the y text field, type -17.

# Rectangle 3 (r3) I In the Geometri

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type **320**.
- 4 In the Height text field, type 16.
- **5** Locate the **Position** section. In the **x** text field, type -160.
- 6 In the y text field, type -3.

# Rectangle 4 (r4)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 6.
- 4 In the Height text field, type 6.
- **5** Locate the **Position** section. In the **x** text field, type -3.
- 6 In the y text field, type -3.

# Rectangle 5 (r5)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 20.
- 4 In the Height text field, type 20.
- **5** Locate the **Position** section. In the **x** text field, type -10.
- **6** In the **y** text field, type -10.
- 7 Click Pauld Selected.

The last rectangular region encloses the optical computational domain. It can be enlarged if needed for validating the results. The region should be chosen large enough so that the computed propagation constants do not change significantly if the region is enlarged.

#### SOLID MECHANICS (SOLID)

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 In the Settings window for Solid Mechanics, locate the Thickness section.
- **3** In the d text field, type 2\*d.

#### Linear Elastic Material I

In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Linear Elastic Material I.

#### Thermal Expansion 1

- I In the Physics toolbar, click Attributes and choose Thermal Expansion.
- 2 In the Settings window for Thermal Expansion, locate the Model Input section.
- **3** From the T list, choose **User defined**. In the associated text field, type T1.
- 4 Click of Go to Source for Volume reference temperature.

#### **GLOBAL DEFINITIONS**

#### Default Model Inputs

- I In the Model Builder window, under Global Definitions click Default Model Inputs.
- 2 In the Settings window for Default Model Inputs, locate the Browse Model Inputs section.
- 3 Find the Expression for remaining selection subsection. In the Volume reference temperature text field, type T0.

#### DEFINITIONS

#### Variables 1

- I In the Home toolbar, click a= Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

| Name | Expression | Unit | Description                               |
|------|------------|------|---|
| N    | nCore      |      | Refractive index for stress-free material |

- 4 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- **5** Select Domain 6 only.

# Variables 2

- I In the Home toolbar, click a= Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

| Name | Expression | Unit | Description                               |
|------|------------|------|---|
| N    | nSiO2      |      | Refractive index for stress-free material |

- 4 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- **5** Select Domains 4 and 5 only.

#### Variables 3

- I In the Home toolbar, click a= Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

| Name | Expression                                 | Unit | Description                   |
|------|--|------|-------------------------------|
| Nx   | N-(B1*solid.sx+B2*<br>(solid.sy+solid.sz)) |      | Refractive index, x-component |
| Ny   | N-(B1*solid.sy+B2*<br>(solid.sx+solid.sz)) |      | Refractive index, y-component |
| Nz   | N-(B1*solid.sz+B2*<br>(solid.sx+solid.sy)) |      | Refractive index, z-component |

- 4 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- **5** Select Domains 4–6 only.

#### SOLID MECHANICS (SOLID)

Rigid Motion Suppression I

- I In the Physics toolbar, click **Domains** and choose Rigid Motion Suppression.
- **2** Select Domain 1 only.

#### ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

The computational domain is reduced significantly for the optical mode analysis.

- I In the Model Builder window, under Component I (compl) click Electromagnetic Waves, Frequency Domain (ewfd).
- **2** Select Domains 4–6 only.

Wave Equation, Electric 1

- I In the Model Builder window, under Component I (compl)>Electromagnetic Waves, Frequency Domain (ewfd) click Wave Equation, Electric I.
- 2 In the Settings window for Wave Equation, Electric, locate the Electric Displacement Field section.
- **3** From the n list, choose **User defined**. From the list, choose **Diagonal**.

**4** In the *n* table, enter the following settings:

| Nx | 0  | 0  |
|----|----|----|
| 0  | Ny | 0  |
| 0  | 0  | Nz |

**5** From the k list, choose **User defined**.

#### MATERIALS

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Si in the Label text field.
- **3** Select Domain 1 only.
- **4** Locate the **Material Contents** section. In the table, enter the following settings:

| Property                         | Variable   | Value   | Unit  | Property group                               |
|----------------------------------|--|---------|-------|--|
| Young's modulus                  | E  | ESi     | Pa    | Young's<br>modulus and<br>Poisson's<br>ratio |
| Poisson's ratio                  | nu   | nuSi    | I     | Young's<br>modulus and<br>Poisson's<br>ratio |
| Density                          | rho  | rhoSi   | kg/m³ | Basic  |
| Coefficient of thermal expansion | alpha_iso;<br>alphaii =<br>alpha_iso,<br>alphaij = 0 | alphaSi | I/K   | Basic  |

#### SiO2

- I Right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type SiO2 in the Label text field.
- **3** Select Domains 2–6 only.

**4** Locate the **Material Contents** section. In the table, enter the following settings:

| Property                         | Variable   | Value     | Unit  | Property group                               |
|----------------------------------|--|-----------|-------|--|
| Young's modulus                  | E  | ESiO2     | Pa    | Young's modulus and Poisson's ratio          |
| Poisson's ratio                  | nu   | nuSiO2    | I     | Young's<br>modulus and<br>Poisson's<br>ratio |
| Density                          | rho  | rhoSiO2   | kg/m³ | Basic  |
| Coefficient of thermal expansion | alpha_iso;<br>alphaii =<br>alpha_iso,<br>alphaij = 0 | alphaSiO2 | I/K   | Basic  |

#### MESH I

Free Triangular 1

In the Mesh toolbar, click Free Triangular.

#### Size 1

- I Right-click Free Triangular I and choose Size.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domains 4–6 only.
- **5** Locate the **Element Size** section. Click the **Custom** button.
- 6 Locate the Element Size Parameters section.
- 7 Select the Maximum element size check box. In the associated text field, type 0.2.

#### Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Extra fine.
- 4 Click Build All.

#### STUDY I

Mode Analysis

- I In the Study toolbar, click Study Steps and choose Other>Mode Analysis.
- 2 In the Settings window for Mode Analysis, locate the Study Settings section.
- 3 From the Transform list, choose Effective mode index.
- 4 In the Search for modes around text field, type 1.46.
- **5** Select the **Desired number of modes** check box. In the associated text field, type 4.
- 6 In the Mode analysis frequency text field, type c const/lambda0 ewfd.

These settings make the eigenmode solver search for the 4 eigenmodes with effective mode indices closest to the value 1.46. This value is an estimate of the effective mode index for the fundamental mode.

Exclude Solid Mechanics from the Mode Analysis step.

7 Locate the Physics and Variables Selection section. In the table, clear the Solve for check box for Solid Mechanics (solid).

Steb 1: Stationary

- I In the Model Builder window, click Step I: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Electromagnetic Waves, Frequency Domain (ewfd).
- 4 In the Study toolbar, click **Compute**.

#### RESULTS

Stress (solid)

- I In the Settings window for 2D Plot Group, locate the Data section.
- 2 From the Dataset list, choose Study I/Solution Store I (sol2).
- 3 In the Stress (solid) toolbar, click Plot.
- **4** Click the **Go to Default View** button in the **Graphics** toolbar.
- **5** Click the **Zoom Extents** button in the **Graphics** toolbar.
- 6 In the Study toolbar, click Create Solution Copy.

Study I/Solution I - Plane Strain

I In the Model Builder window, expand the Results>Datasets node, then click Study I/ Solution I - Copy I (sol3).

2 In the **Settings** window for **Solution**, type Study 1/Solution 1 - Plane Strain in the **Label** text field.

#### SOLID MECHANICS (SOLID)

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 In the Settings window for Solid Mechanics, locate the 2D Approximation section.
- 3 From the list, choose Generalized plane strain.

#### DEFINITIONS

#### Variables 4

- I In the Model Builder window, under Component I (compl) right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

| Name | Expression  | Unit | Description               |
|------|---|------|---------------------------|
| u1   | u-solid.wxZ/2*Z^2                                 | m    | Displacement, x-component |
| v1   | v-solid.wyZ/2*Z^2                                 | m    | Displacement, y-component |
| w1   | <pre>(solid.wxZ*X+solid.wyZ* Y+solid.wOZ)*Z</pre> | m    | Displacement, z-component |

#### STUDY I

In the **Study** toolbar, click **Compute**.

#### RESULTS

Stress (solid)

- I In the Model Builder window, under Results click Stress (solid).
- 2 In the Stress (solid) toolbar, click Plot.
- 3 Click the Go to Default View button in the Graphics toolbar.
- 4 Click the **Zoom Extents** button in the **Graphics** toolbar.

#### Strain level

To visualize the area, where the relative difference between the x and z strain components is within 5%, follow these steps:

- I In the Model Builder window, under Results click Electric Field (ewfd).
- 2 In the Settings window for 2D Plot Group, type Strain level in the Label text field.

3 Locate the Data section. From the Dataset list, choose Study I/Solution Store I (sol2).

Surface 1

- I In the Model Builder window, expand the Strain level node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type abs((solid.eX-solid.eZ)/solid.eZ)<0.05.
- 4 Locate the Coloring and Style section. Clear the Color legend check box.
- 5 Click to expand the Quality section. From the Resolution list, choose Extra fine.
- 6 In the Strain level toolbar, click Plot.
- 7 Click the **Zoom Extents** button in the **Graphics** toolbar.

Cut Line 2D I

Next, plot the stress-induced birefringence along the symmetry line within the waveguide.

- I In the Results toolbar, click Cut Line 2D.
- 2 In the Settings window for Cut Line 2D, locate the Line Data section.
- **3** In row **Point I**, set **Y** to -10.
- 4 In row Point 2, set X to 0 and Y to 10.
- 5 Click Plot.

Cut Line 2D 2

- I Right-click Cut Line 2D I and choose Duplicate.
- 2 In the Settings window for Cut Line 2D, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I Plane Strain (sol3).

ID Plot Group 3

In the **Results** toolbar, click  $\sim$  **ID Plot Group**.

Line Graph 1

- I Right-click ID Plot Group 3 and choose Line Graph.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Dataset list, choose Cut Line 2D 1.
- 4 Locate the y-Axis Data section. In the Expression text field, type Nx-Ny.
- 5 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **6** In the **Expression** text field, type Y.

Line Grabh 2

I Right-click Line Graph I and choose Duplicate.

- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Dataset list, choose Cut Line 2D 2.

#### ID Plot Group 3

- I In the Model Builder window, click ID Plot Group 3.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- 3 From the Title type list, choose Manual.
- 4 In the Title text area, type Birefringence.
- 5 Locate the Plot Settings section.
- 6 Select the x-axis label check box. In the associated text field, type Y.
- 7 Select the y-axis label check box. In the associated text field, type Nx-Ny.
- 8 In the ID Plot Group 3 toolbar, click Plot.

To visualize the details of the eigenmode with the lowest effective mode index, you first set up a view that includes the optical computation domain only.

- **9** Click the **Show More Options** button in the **Model Builder** toolbar.
- 10 In the Show More Options dialog box, in the tree, select the check box for the node Results>Views.
- II Click OK.

#### View 2D 2

- I In the Model Builder window, under Results right-click Views and choose View 2D.
- 2 In the Settings window for View 2D, locate the View section.
- 3 Select the **Lock axis** check box.

#### Axis

- I In the Model Builder window, expand the View 2D 2 node, then click Axis.
- 2 In the Settings window for Axis, locate the Axis section.
- 3 In the x minimum text field, type -10.
- 4 In the x maximum text field, type 10.
- 5 In the y minimum text field, type -10.
- 6 In the y maximum text field, type 10.

#### 2D Plot Group 4

- I In the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, locate the Data section.

- **3** From the **Effective mode index** list, choose **1.4514**.
- 4 Locate the Plot Settings section. From the View list, choose View 2D 2.

#### Surface I

- I Right-click 2D Plot Group 4 and choose Surface.
- 2 In the Settings window for Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Electromagnetic Waves, Frequency Domain>Energy and power>Power flow, time average (spatial frame) - W/m2>ewfd.Poavz - Power flow, time average, zcomponent.
- 3 In the 2D Plot Group 4 toolbar, click Plot. This creates a visualization of the power flow, also called optical intensity or the Poynting vector, in the z direction (out-of-plane direction).

#### Global Evaluation 1

To collect all computed effective mode indices in a table, follow these steps:

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Electromagnetic Waves, Frequency Domain>Global>ewfd.neff - Effective mode index.
- **3** Locate the **Expressions** section. In the table, enter the following settings:

| Expression | Unit | Description                                     |
|------------|------|---|
| ewfd.neff  | 1    | Effective mode index (Generalized plane strain) |

4 Click **= Evaluate**.

#### TABLE

I Go to the **Table** window.

If you see too few digits in the table, click the **Full Precision** toolbar button.

#### Global Evaluation 2

- I Right-click Global Evaluation I and choose Duplicate.
- 2 In the Settings window for Global Evaluation, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I Plane Strain (sol3).

**4** Locate the **Expressions** section. In the table, enter the following settings:

| Expression | Unit | Description                         |
|------------|------|-------------------------------------|
| ewfd.neff  | 1    | Effective mode index (Plane strain) |

5 Click **= Evaluate**.

Create a table plot to visualize the shift of the effective mode indices.

# ID Plot Group 5

In the Results toolbar, click  $\sim$  ID Plot Group.

#### Table Graph 1

- I Right-click ID Plot Group 5 and choose Table Graph.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 From the Plot columns list, choose Manual.
- 4 From the x-axis data list, choose Row index.
- 5 In the Columns list, choose Effective mode index (Generalized plane strain) (1) and Effective mode index (Plane strain) (1).
- 6 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose None.
- 7 Find the Line markers subsection. From the Marker list, choose Circle.
- **8** Click to expand the **Legends** section. Select the **Show legends** check box.

#### ID Plot Group 5

- I In the Model Builder window, click ID Plot Group 5.
- 2 In the Settings window for ID Plot Group, locate the Title section.
- 3 From the Title type list, choose Manual.
- 4 In the Title text area, type Eigenvalue shift.
- **5** Locate the **Plot Settings** section. Select the **x-axis label** check box.
- 6 Select the y-axis label check box.
- 7 In the x-axis label text field, type Mode number.
- 8 In the y-axis label text field, type Effective mode index.
- 9 Locate the Legend section. From the Layout list, choose Outside graph axis area.
- **10** From the **Position** list, choose **Bottom**.
- II In the Number of rows text field, type 2.

12 In the 1D Plot Group 5 toolbar, click Plot.

Finally, visualize the out-of-plane warping of the waveguide.

#### Extrusion 2D I

- I In the Results toolbar, click More Datasets and choose Extrusion 2D.
- 2 In the Settings window for Extrusion 2D, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution Store I (sol2).
- **4** Locate the **Extrusion** section. In the **z maximum** text field, type d.
- 5 In the z variable text field, type Z.

# 3D Plot Group 6

- I In the Results toolbar, click **3D Plot Group**.
- 2 In the Settings window for 3D Plot Group, click to expand the Number Format section.
- 3 Select the Manual grid settings check box.
- 4 In the Precision text field, type 4.

## Surface I

- I Right-click **3D Plot Group 6** and choose **Surface**.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type v1.

#### Deformation I

- I Right-click Surface I and choose Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the x-component text field, type u1.
- **4** In the **y-component** text field, type v1.
- **5** In the **z-component** text field, type w1.
- 6 Locate the Scale section.
- 7 Select the Scale factor check box. In the associated text field, type 1.
- 8 In the 3D Plot Group 6 toolbar, click Plot.
- **9** Click the **Go to Default View** button in the **Graphics** toolbar.
- **10** Click the **Zoom Extents** button in the **Graphics** toolbar.