Optically Anisotropic Waveguide

Introduction

Optical waveguides consist of a core with a high refractive index, surrounded by a cladding with a low refractive index. Thereby, due to total internal reflection, the mode is guided by the high-index core. The smaller the refractive index difference is between the core and the cladding, the more the mode extends into the cladding. For example, the tutorial called Directional Coupler demonstrates how to model a GaAs-based waveguide structure consisting of two closely spaced waveguide cores surrounded by a cladding.

For most waveguide structures, the optical properties for the core and the cladding are different. However, within each domain, the material properties are uniform and isotropic. This model demonstrates what happens when the material in the core exhibit anisotropy. Specifically, in this case, the electric displacement field is no longer parallel to the electric field. Instead, the electric displacement field **D** relates to the electric field **E** by a matrix-vector expression

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \varepsilon_{rxx} & \varepsilon_{rxy} & \varepsilon_{rxz} \\ \varepsilon_{ryx} & \varepsilon_{ryy} & \varepsilon_{ryz} \\ \varepsilon_{rzx} & \varepsilon_{rzy} & \varepsilon_{rzz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}, \tag{1}$$

where D_x, D_y, and D_z are the components of the electric displacement field **D**, ε_0 is the vacuum permittivity, ε_{rij} is the relative permittivity tensor element for row index i and column index j, and E_x, E_y, and E_z are the components of the electric field **E**.

However, for all materials, you can find a coordinate system in which you only have nonzero diagonal elements in the permittivity tensor, whereas the off-diagonal elements are all zero. The three coordinate axes in this rotated coordinate system are the *principal axes* of the material and, correspondingly, the three values for the diagonal elements in the permittivity tensor are called the *principal permittivities* of the material.

There are basically two kinds of anisotropic crystals: *uniaxial* and *biaxial* crystals. With a suitable choice of coordinate system, where only the diagonal elements of the permittivity tensor are nonzero, uniaxial crystals are characterized by an *extraordinary refractive index* n_e for waves polarized along the crystal's *optical axis* and an *ordinary refractive index* n_o for waves polarized orthogonally to the crystal's optical axis. So the diagonal elements of the relative permittivity in the rotated coordinate system are given by $\varepsilon_{rxx} = \varepsilon_{ryy} = (n_o)^2$, $\varepsilon_{rzz} = (n_e)^2$. However, for a biaxial crystal, all three principal permittivities are different.

Model Definition

In this model a uniaxial material will be used. Two cases will be considered — *diagonal transverse anisotropy*, where the crystal's optical axis is oriented in an orthogonal Cartesian direction to the propagation direction of the waveguide, and *off-diagonal longitudinal anisotropy*, where the optical axis has a components both in the propagation direction and in the transverse direction. Here, for the diagonal transverse anisotropy, the extraordinary refractive index is defined in the x direction. For the off-diagonal longitudinal anisotropy case, the optical axis is defined in a direction 45 degrees from the z-axis in the yz-plane.

The ordinary refractive index is 1.52 and the extraordinary refractive index is 1.48.

A mode analysis is performed where the width and the thickness is parametrically swept from 0.5 μm to 4 μm. For each geometry, the effective index is solved for and plotted in a dispersion diagram. The results agree well with the results reported in Ref. 1.

Results and Discussion

Figure 1 shows the power in the dominant mode $E^y{}_{11}$ for diagonal transverse anisotropy. Here, the superscript denotes the dominant polarization direction and the first and second subscripts denote the number of peaks in the x and y direction, respectively.
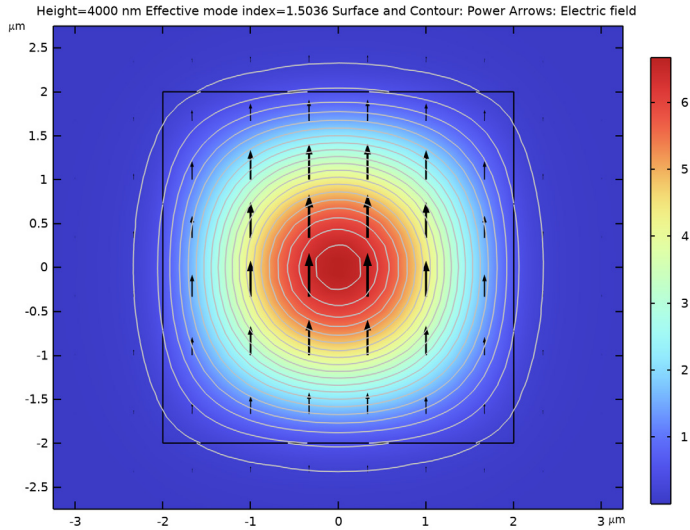

Figure 1: The power in the propagation direction (out-of-plane) for the dominant mode, $E^y{}_{11}$. The arrows show that the mode is polarized in the y direction.

Figure 2 shows the dispersion curves for the lowest order modes for the case of diagonal transverse anisotropy. The dispersion curves show good agreement with those in Fig. 3 of Ref. 1.
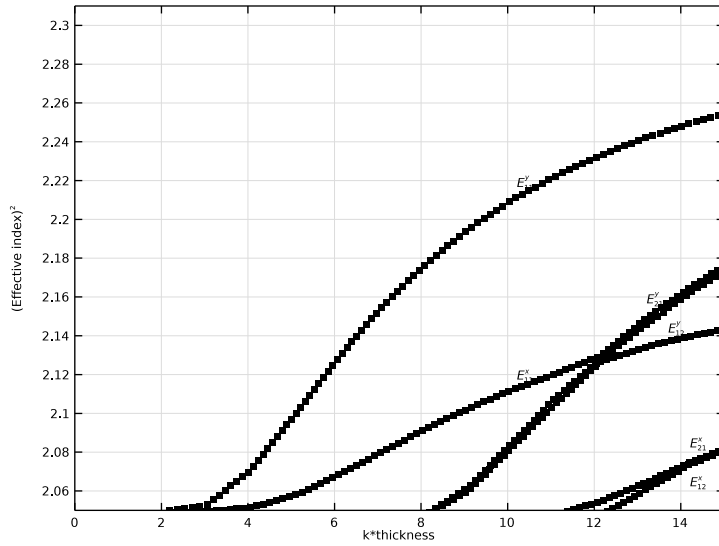

Figure 2: Dispersion curves for the lowest order modes for the diagonal transverse anisotropic case.

Figure 3 shows the power in the dominant mode $E^x{}_{11}$ for off-diagonal longitudinal anisotropy. As seen from the plot, the polarization is not uniform for this type of anisotropy.

Figure 4 shows the dispersion curves for off-diagonal longitudinal anisotropy. These results show good agreement with Fig. 8 in Ref. 1.
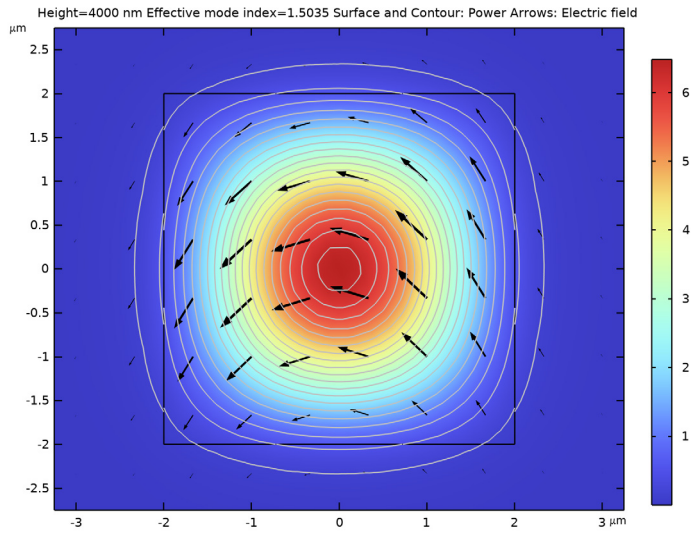
Height=4000 nm Effective mode index=1.5035 Surface and Contour: Power Arrows: Electric field

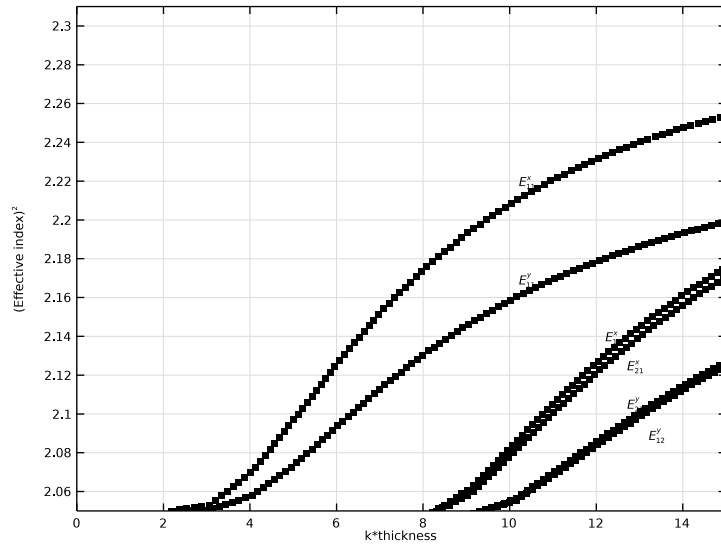Figure 3: The lowest order mode, E^x_{11}, for off-diagonal longitudinal anisotropy.

Figure 4: Dispersion curves for the lowest order modes for off-diagonal longitudinal anisotropy.

Reference

1. M. Koshiba, K. Hayata, and M. Suzuki, "Finite-element solution of anisotropic waveguides with arbitrary tensor permittivity," *J. Lightwave Technol.*, vol. LT-4, p. 121, 1986.

Application Library path: Wave_Optics_Module/Verification_Examples/optically_anisotropic_waveguide

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click ⚛ **2D**.

2 In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Frequency Domain (ewfd)**.

3 Click **Add**.

4 Click ➜ **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces> Mode Analysis**.

6 Click ☑ **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click 📂 **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file optically_anisotropic_waveguide_parameters.txt.
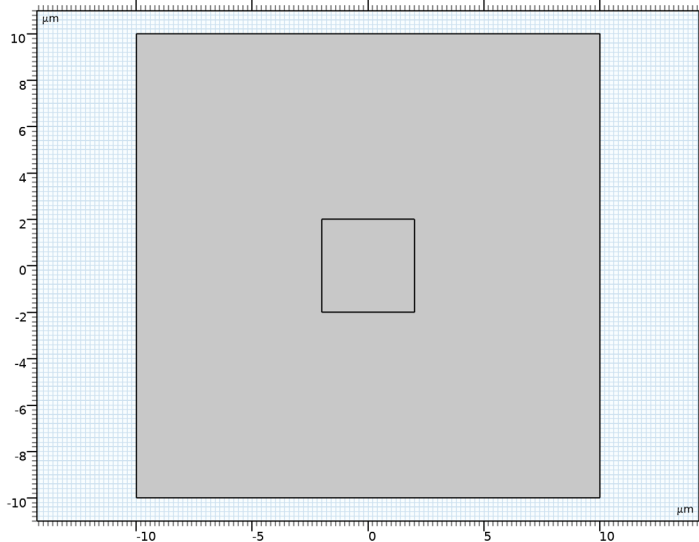
1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **µm**.

Square 1 (sq1)

1 In the **Geometry** toolbar, click ▢ **Square**.

2 In the **Settings** window for **Square**, locate the **Size** section.

3 In the **Side length** text field, type 20[um].

4 Locate the **Position** section. From the **Base** list, choose **Center**.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click ▢ **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type height.

4 In the **Height** text field, type height.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 Click ▦ **Build All Objects**.

7 Click the ⊹ **Zoom Extents** button in the **Graphics** toolbar.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

Wave Equation, Electric 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain (ewfd)** click **Wave Equation, Electric 1**.

2 In the **Settings** window for **Wave Equation, Electric**, locate the **Electric Displacement Field** section.

3 From the **Electric displacement field model** list, choose **Relative permittivity**.
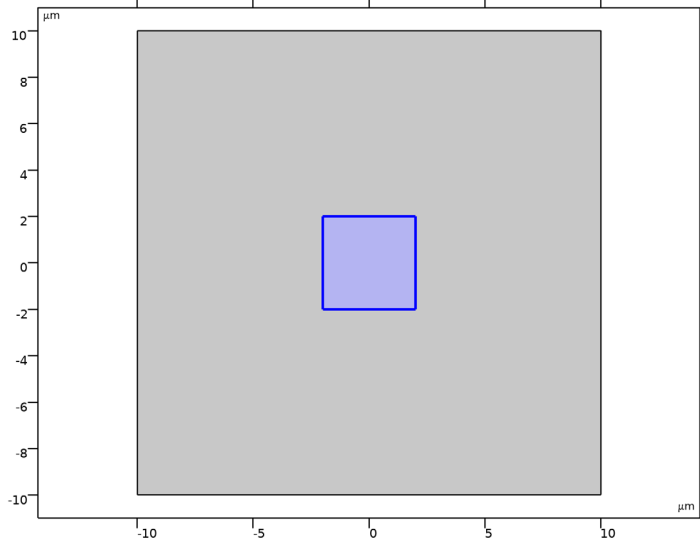
MATERIALS

Cladding

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type Cladding in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	eps_clad	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

Core

1 Right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type Core in the **Label** text field.

3 Select Domain 2 only.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	{epsilonr11, epsilonr22, epsilonr33} ; epsilonrij = 0	{epsr_eo, epsr_o, epsr_o}	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.

3 From the list, choose **User-controlled mesh**.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Predefined** button.

Size 1

1 In the **Model Builder** window, click **Size 1**.

2 In the **Settings** window for **Size**, locate the **Element Size Parameters** section.

3 In the **Maximum element size** text field, type height/15.

4 Click ⊞ **Build All**.

STUDY - DIAGONAL TRANSVERSE ANISOTROPY

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type Study - Diagonal Transverse Anisotropy in the **Label** text field.

Parametric Sweep

1 In the **Study** toolbar, click 〓 **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click ＋ **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
height (Height of waveguide)	range(500[nm],250[nm], 4000[nm])	nm

Step 1: Mode Analysis

1 In the **Model Builder** window, click **Step 1: Mode Analysis**.

2 In the **Settings** window for **Mode Analysis**, locate the **Study Settings** section.

3 In the **Mode analysis frequency** text field, type f0.

4 Select the **Desired number of modes** check box. In the associated text field, type 7.

5 Select the **Search for modes around** check box. In the associated text field, type n_core_o.

6 In the **Study** toolbar, click 〓 **Compute**.

Power and Electric Field - Diagonal Transverse Anisotropy

1 In the **Model Builder** window, expand the **Results>Electric Field (ewfd)** node, then click **Electric Field (ewfd)**.

2 In the **Settings** window for **2D Plot Group**, type Power and Electric Field - Diagonal Transverse Anisotropy in the **Label** text field.

3 Locate the **Data** section. From the **Effective mode index** list, choose **1.5036**.

Surface 1

1 In the **Model Builder** window, click **Surface 1**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Frequency Domain>Energy and power>Power flow, time average - W/m²>ewfd.Poavz - Power flow, time average, z-component**.

Arrow Surface 1

1 In the **Model Builder** window, right-click **Power and Electric Field - Diagonal Transverse Anisotropy** and choose **Arrow Surface**.

2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Frequency Domain>Electric>ewfd.Ex,ewfd.Ey - Electric field**.

3 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type 30.

4 Find the **Y grid points** subsection. In the **Points** text field, type 30.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Black**.

Contour 1

1 Right-click **Power and Electric Field - Diagonal Transverse Anisotropy** and choose **Contour**.

2 In the **Settings** window for **Contour**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **ewfd.Poavz - Power flow, time average, z-component - W/m²**.

3 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

4 From the **Color** list, choose **Gray**.

5 Clear the **Color legend** check box.

Power and Electric Field - Diagonal Transverse Anisotropy

1 In the **Model Builder** window, click **Power and Electric Field - Diagonal Transverse Anisotropy**.

2 In the **Settings** window for **2D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **Manual**.

4 From the **Number format** list, choose **Automatic**.

5 In the **Precision** text field, type 5.

6 In the **Title** text area, type Height=eval(height,nm) nm Effective mode index=eval(ewfd.neff) Surface and Contour: Power Arrows: Electric field.

7 Clear the **Parameter indicator** text field.

8 In the **Power and Electric Field - Diagonal Transverse Anisotropy** toolbar, click 🔲 **Plot**.

9 Click the ⊕ **Zoom In** button in the **Graphics** toolbar twice.

Height=4000 nm Effective mode index=1.5036 Surface and Contour: Power Arrows: Electric field
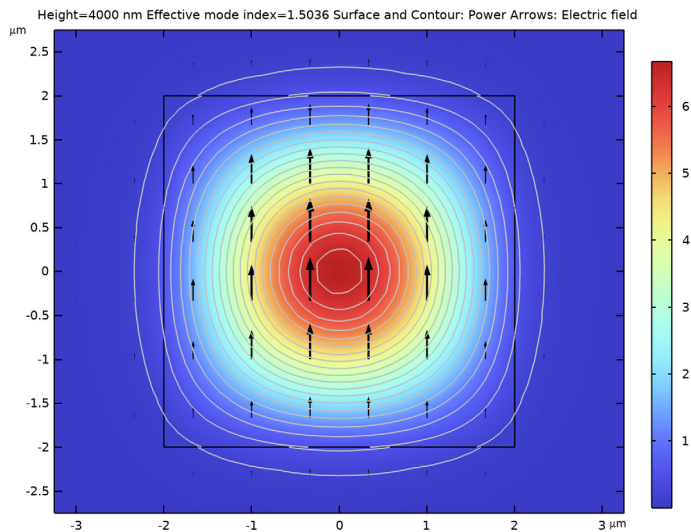
Dispersion Curves - Diagonal Transverse Anisotropy

1 In the **Home** toolbar, click 🖼 **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type Dispersion Curves - Diagonal Transverse Anisotropy in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study - Diagonal Transverse Anisotropy/Parametric Solutions 1 (sol2)**.

4 Locate the **Legend** section. Clear the **Show legends** check box.

Global 1

1 Right-click **Dispersion Curves - Diagonal Transverse Anisotropy** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
ewfd.neff^2	1	

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **Outer solutions**.

5 From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type ewfd.k0*height.

7 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

8 From the **Color** list, choose **Black**.

9 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

10 From the **Positioning** list, choose **Interpolated**.

11 In the **Number** text field, type 100.

12 In the **Dispersion Curves - Diagonal Transverse Anisotropy** toolbar, click 🔲 **Plot**.

Dispersion Curves - Diagonal Transverse Anisotropy

Remove solution 3, which represents the $E^y{}_{22}$ mode, as this mode was not included in
Ref. 1.

1 In the **Model Builder** window, click **Dispersion Curves - Diagonal Transverse Anisotropy**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Effective mode index selection** list, choose **Manual**.

4 In the **Effective mode index indices (1-7)** text field, type 1 2 4 5 6 7.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** check box. In the associated text field, type k*thickness.

7 Select the **y-axis label** check box. In the associated text field, type (Effective index)².

Adjust the axis limits to be the same as in Ref. 1.

8 Locate the **Axis** section. Select the **Manual axis limits** check box.

9 In the **x minimum** text field, type 0.

10 In the **x maximum** text field, type 15.

11 In the **y minimum** text field, type 2.05.

12 In the **y maximum** text field, type 2.31.

Annotation 1

1 Right-click **Dispersion Curves - Diagonal Transverse Anisotropy** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{y}_{11}\].

4 Locate the **Position** section. In the **X** text field, type 10.

5 In the **Y** text field, type 2.225.

6 Locate the **Annotation** section. Select the **LaTeX markup** check box.

7 Locate the **Coloring and Style** section. Clear the **Show point** check box.

Annotation 2

1 Right-click **Annotation 1** and choose **Duplicate**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{x}_{11}\].

4 Locate the **Position** section. In the **Y** text field, type 2.125.

Annotation 3

1 Right-click **Annotation 2** and choose **Duplicate**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{y}_{21}\].

4 Locate the **Position** section. In the **X** text field, type 13.

5 In the **Y** text field, type 2.165.

Annotation 4

1 Right-click **Annotation 3** and choose **Duplicate**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{y}_{12}\].

4 Locate the **Position** section. In the **X** text field, type 13.5.

5 In the **Y** text field, type 2.15.

Annotation 5

1 Right-click **Annotation 4** and choose **Duplicate**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `\[E^{x}_{21}\]`.

4 Locate the **Position** section. In the **X** text field, type 14.

5 In the **Y** text field, type 2.09.

Annotation 6

1 Right-click **Annotation 5** and choose **Duplicate**.

**2** In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `\[E^{x}_{12}\]`.

4 Locate the **Position** section. In the **Y** text field, type 2.07.

Dispersion Curves - Diagonal Transverse Anisotropy

1 In the **Model Builder** window, click **Dispersion Curves - Diagonal Transverse Anisotropy**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.
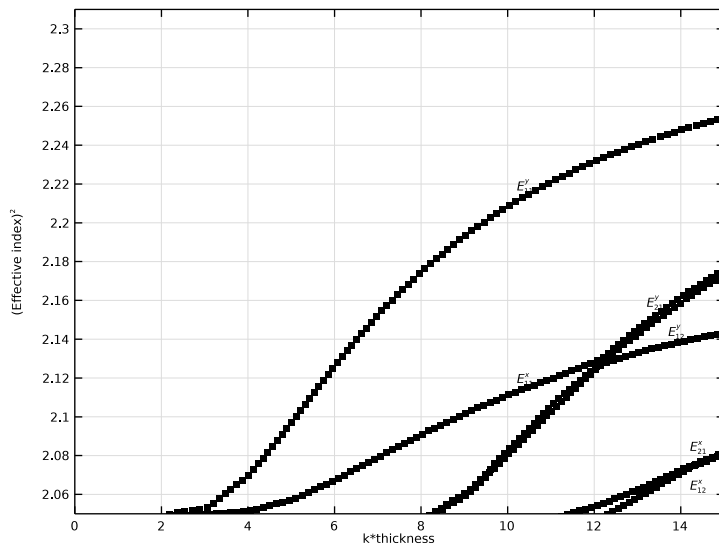
DEFINITIONS

Now, redo the simulation for off-diagonal longitudinal anisotropy. That is, in this case the optical axis is directed at 45-degree angle from the *y*-axis in the *yz*-plane.

Rotated System 2 (sys2)

1 In the **Definitions** toolbar, click ⤢ **Coordinate Systems** and choose **Rotated System**.

2 In the **Settings** window for **Rotated System**, locate the **Rotation** section.

3 From the **Input method** list, choose **General rotation**.

4 Find the **Euler angles (Z-X-Z)** subsection. In the β text field, type phi.

5 In the γ text field, type zeta.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

Use **Rotated System 2** in the **Wave Equation, Electric 1** feature, to rotate the optical axis.

Wave Equation, Electric 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain (ewfd)** click **Wave Equation, Electric 1**.

2 In the **Settings** window for **Wave Equation, Electric**, locate the **Coordinate System Selection** section.

3 From the **Coordinate system** list, choose **Rotated System 2 (sys2)**.

ADD STUDY

1 In the **Home** toolbar, click ⬡ **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Mode Analysis**.

4 Click **Add Study** in the window toolbar.

STUDY - OFF-DIAGONAL LONGITUDINAL ANISOTROPY

1 In the **Model Builder** window, click **Study 2**.

2 In the **Settings** window for **Study**, type Study - Off-Diagonal Longitudinal Anisotropy in the **Label** text field.

STUDY - DIAGONAL TRANSVERSE ANISOTROPY

Parametric Sweep

In the **Model Builder** window, under **Study - Diagonal Transverse Anisotropy** right-click **Parametric Sweep** and choose **Copy**.

STUDY - OFF-DIAGONAL LONGITUDINAL ANISOTROPY

In the **Model Builder** window, right-click **Study - Off-Diagonal Longitudinal Anisotropy** and choose **Paste Parametric Sweep**.

Step 1: Mode Analysis

1 In the **Settings** window for **Mode Analysis**, locate the **Study Settings** section.

2 In the **Mode analysis frequency** text field, type f0.

3 Select the **Search for modes around** check box. In the associated text field, type n_core_o.

4 In the **Home** toolbar, click ⬡ **Add Study** to close the **Add Study** window.

5 In the **Home** toolbar, click ▬ **Compute**.

RESULTS

Power and Electric Field - Off-Diagonal Longitudinal Anisotropy

1 In the **Settings** window for **2D Plot Group**, type Power and Electric Field - Off-Diagonal Longitudinal Anisotropy in the **Label** text field.

2 Locate the **Data** section. From the **Effective mode index** list, choose **1.5035**.

Arrow Surface 1, Contour 1

1 In the **Model Builder** window, under **Results>Power and Electric Field - Diagonal Transverse Anisotropy**, Ctrl-click to select **Arrow Surface 1** and **Contour 1**.

2 Right-click and choose **Copy**.

Arrow Surface 1

In the **Model Builder** window, right-click **Power and Electric Field - Off-Diagonal Longitudinal Anisotropy** and choose **Paste Multiple Items**.

Surface 1

In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **ewfd.Poavz - Power flow, time average, z-component - W/m²**.

Power and Electric Field - Off-Diagonal Longitudinal Anisotropy

1 In the **Model Builder** window, click **Power and Electric Field - Off-Diagonal Longitudinal Anisotropy**.

2 In the **Settings** window for **2D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **Manual**.

4 From the **Number format** list, choose **Automatic**.

5 In the **Precision** text field, type 5.

6 In the **Title** text area, type Height=eval(height,nm) nm Effective mode index=eval(ewfd.neff) Surface and Contour: Power Arrows: Electric field.

7 Clear the **Parameter indicator** text field.

Height=4000 nm Effective mode index=1.5035 Surface and Contour: Power Arrows: Electric field
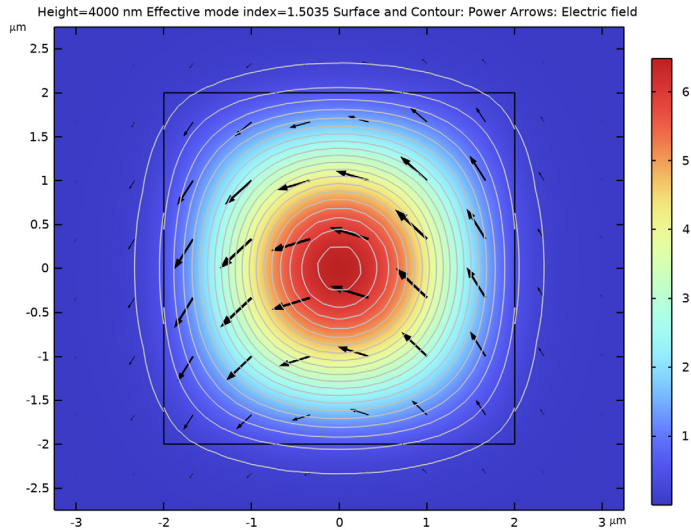
Dispersion Curves - Off-Diagonal Longitudinal Anisotropy

1 In the **Model Builder** window, right-click **Dispersion Curves - Diagonal Transverse Anisotropy** and choose **Duplicate**.

2 In the **Settings** window for **ID Plot Group**, type Dispersion Curves - Off-Diagonal Longitudinal Anisotropy in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study - Off-Diagonal Longitudinal Anisotropy/Parametric Solutions 2 (sol19)**.

4 From the **Effective mode index selection** list, choose **All**.

Annotation 1

1 In the **Model Builder** window, expand the **Dispersion Curves - Off-Diagonal Longitudinal Anisotropy** node, then click **Annotation 1**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{x}_{11}\].

Annotation 2

1 In the **Model Builder** window, click **Annotation 2**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{y}_{11}\].

4 Locate the **Position** section. In the **Y** text field, type 2.175.

1 In the **Model Builder** window, click **Annotation 3**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{x}_{12}\].

4 Locate the **Position** section. In the **X** text field, type 12.

5 In the **Y** text field, type 2.145.

Annotation 4

1 In the **Model Builder** window, click **Annotation 4**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{x}_{21}\].

4 Locate the **Position** section. In the **X** text field, type 12.5.

5 In the **Y** text field, type 2.13.

Annotation 5

1 In the **Model Builder** window, click **Annotation 5**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{y}_{21}\].

4 Locate the **Position** section. In the **X** text field, type 12.5.

5 In the **Y** text field, type 2.111.

Annotation 6

1 In the **Model Builder** window, click **Annotation 6**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type \[E^{y}_{12}\].

4 Locate the **Position** section. In the **X** text field, type 13.

5 In the **Y** text field, type 2.095.

6 In the **Dispersion Curves - Off-Diagonal Longitudinal Anisotropy** toolbar, click ▣ **Plot**.
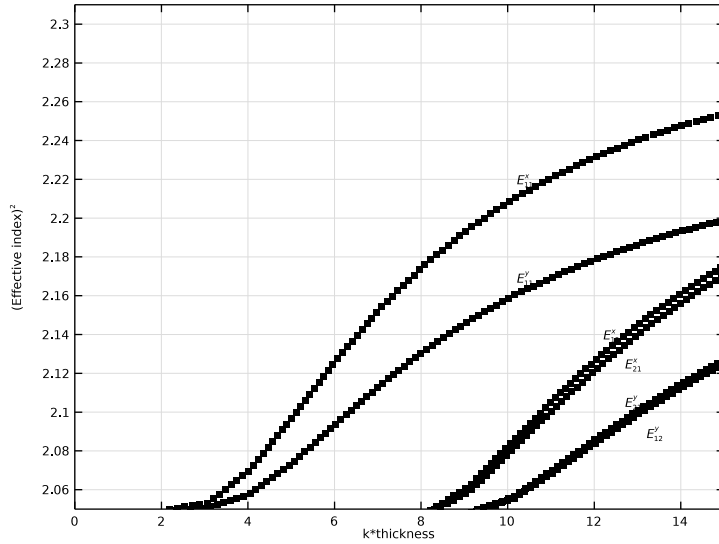
Now, that **Rotated System 2** controls the direction of the optical axis, to rerun **Study - Diagonal Transverse Anisotropy** first set phi = zeta = 0.