

Optical Ring Resonator Notch Filter 3D

Introduction

The simplest optical ring resonator consists of a straight waveguide and a ring waveguide. The two waveguide cores are placed close to each other, so light couples from one waveguide to the other.

When the length of the ring waveguide is an integer number of wavelengths, the ring waveguide is resonant to the wavelength and the light power stored in the ring builds up.

The wave transmitted through the straight waveguide is the interference of the incident wave and the wave that couples over from the ring to the straight waveguide.

Schematically, you can think of the ring resonator as shown in [Figure 1](#page-1-0). A part of the incident wave E_{i1} is transmitted in the straight waveguide, whereas a fraction of that field couples over to the ring. Similarly, some of the light in the ring couples over to the straight waveguide, whereas the rest of that wave continues around the ring waveguide.

Figure 1: Schematic of an optical ring resonator, showing the incident fields Ei1 and Ei2 and the transmitted/coupled fields E_{t1} and E_{t2}. The transmission and coupling coefficients t and
κ are also indicated, as well as the round-trip loss L.

The transmitted fields are related to the incident fields through the matrix-vector relation

$$
\begin{bmatrix} E_{t1} \\ E_{t2} \end{bmatrix} = \begin{bmatrix} t & \kappa \\ -\kappa^* & t^* \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix} .
$$
 (1)

The matrix elements defined above assure that the total input power equals the total output power,

$$
|E_{t1}|^2 + |E_{t2}|^2 = |E_{i1}|^2 + |E_{i2}|^2,
$$
\n(2)

by assuming that the transmission and coupling coefficients are related by

$$
|t|^2 + |\kappa|^2 = 1.
$$
 (3)

Furthermore, as the wave propagates around the ring waveguide, one gets the relation

$$
E_{i2} = E_{t2} L \exp(-j\phi), \qquad (4)
$$

where L is the loss coefficient for the propagation around the ring and ϕ is the accumulated phase.

Combining [Equation 1,](#page-1-1) [Equation 3,](#page-2-0) and [Equation 4,](#page-2-1) the transmitted field can be written

$$
E_{t1} = \frac{|t| - L \exp(-j(\phi - \phi_t))}{1 - |t| L \exp(-j(\phi - \phi_t))} E_{i1} e^{-j\phi_t}.
$$
 (5)

Here the transmission coefficient is separated into the transmission loss $|t|$ and the $\text{corresponding phase } \phi_t,$

$$
t = |t|e^{-j\phi_t}.\tag{6}
$$

Notice that on resonance, when $\phi - \phi_t$ is an integer multiple of 2π , and when $|t| = L$, the transmitted field is zero. The condition $|t| = L$ is called critical coupling. Thus, when the coupler transmission loss balances the loss for the wave propagating around the ring waveguide, one gets the optimum condition for a bandstop filter, a notch filter.

The procedure to optimize the filter is as follows:

- **1** Calculate the transmittance $|t|^2$ for some values of the distance between the straight and the ring waveguide. Here, your should just include half (or a part) of the ring.
- **2** Calculate the loss coefficient *L* for some values of the ring radius. In this case, define a geometry with a short piece of straight waveguide, followed by half of the ring, and, finally, another short piece of straight waveguide. The short pieces of straight waveguide help to launch and properly absorb the propagating wave.
- **3** Find the geometry parameters where the transmittance and the loss coefficient are equal, $|t| = L$.
- **4** Make a wavelength sweep over a couple of free spectral ranges to find the resonances.
- **5** If the exact resonance wavelength is important, fine tune the ring radius to obtain the target resonance wavelength.

However, before starting a full 3D design, it is often good to begin with a 2D model, as described in the [Optical Ring Resonator Notch Filter](#page-0-0) model.

Model Definition

This application is set up using the Electromagnetic Waves, Beam Envelopes interface, to handle the propagation over distances that are many wavelengths long. Since the wave propagates in essentially one direction along the straight waveguide and along the waveguide ring, the unidirectional formulation is used. This assumes that the electric field for the wave can be written as

$$
\mathbf{E} = \mathbf{E}_1 \exp(-j\phi) \,,\tag{7}
$$

where \mathbf{E}_1 is a slowly varying field envelope function and ϕ is an approximation of the propagation phase for the wave. The definitions used for the phase in the straight and ring waveguide are shown in [Table 1](#page-3-0), [Table 2](#page-3-1), and [Table 3](#page-3-2).

NAME	EXPRESSION	UNIT	DESCRIPTION		
phi	ewbe.beta 1*y	rad	Phase		
	TABLE 2: PHASE DEFINITION IN RING WAVEGUIDE - LEFT DOMAIN.				
NAME	EXPRESSION			UNIT	DESCRIPTION
phi	ewbe.beta $1*rd*atan2(y,-(x-rd-dx))$			rad	Phase
	TABLE 3: PHASE DEFINITION IN RING WAVEGUIDE - RIGHT DOMAIN.				
NAME	EXPRESSION			UNIT	DESCRIPTION
phi	ewbe.beta $1*rd*atan2(-y,(x-rd-dx))$			rad	Phase

TABLE 1: PHASE DEFINITION IN STRAIGHT WAVEGUIDE DOMAINS.

The parameters r0 and dx correspond, respectively, to the curvature radius of the ring waveguide and to the separation between the straight and ring waveguide cores. The phase approximation defined in the tables above is discontinuous at the boundary between the straight waveguide and the ring waveguide as well as at the boundary between the left and the right ring waveguide domains. To handle this phase discontinuity and thereby the discontinuity in the field envelope, **E**1, a Field Continuity boundary condition is used at the aforementioned boundaries. The Field Continuity boundary condition ensures that the tangential components of the electric and the magnetic fields are continuous at the boundary, despite the phase jump.

In this model, not only the guided wave needs to be resolved. There is also coupling to radiating modes that needs to be resolved. Thus, the mesh needs resolve the beating

between these different waves. Instead of making a very fine mesh, cubic shape orders are used when solving for the electric field. However, when running this model on a Windows PC, approximately 24 GB of RAM is required.

Results and Discussion

[Figure 2](#page-4-0) shows the mode field at the launch port. As the height of the waveguide core is slightly larger than the width of the core, the lowest order mode is polarized in the *z* direction.

Figure 2: The mode field norm and polarization at the launch port.

[Figure 3](#page-5-0) shows that the transmittance at the resonance wavelength, 1.56 mm, is very small (below 5%), as the device was designed to approximately match the transmittance through the coupler with the loss coefficient in the ring (see the discussion in the [Introduction\)](#page-1-2).

Figure 3: The transmittance and loss spectra.

[Figure 4](#page-6-0) shows the *z*-component of the electric field at the resonance wavelength. Notice that the field in the straight waveguide is very weak after the coupler region, due to the destructive interference between the light passing straight through the coupler region and the light coupled back in from the ring. Furthermore, it is clear that there is a noticeable loss when the wave propagates around the ring.

Ida0(6)=1.56 um lambda0(1)=1.56 µm Multislice: Electric field, z-component (V/m)

Figure 4: The z-component of the electric field at the resonance wavelength 1.56 μ*m.*

Notes About the COMSOL Implementation

This model geometry is easily set up by importing a geometry part from the COMSOL Part Libraries. The rectangular waveguide coupling between a straight and a ring waveguide section, with the core embedded in a cladding domain, is available in the Wave Optics Module Part Library under Rectangular Waveguides.

Predefined geometry parts can be quickly modified by changing the default input parameters. Moreover, geometry parts provide targeted selections of domains and boundaries that greatly simplify the model building. As demonstrated in this model, these built-in selections are useful when adding materials, physics features and mesh sequences.

Application Library path: Wave_Optics_Module/Waveguides_and_Couplers/ optical_ring_resonator_3d

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

- **1** In the **Model Wizard** window, click **3D**.
- **2** In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Beam Envelopes (ewbe)**.
- **3** Click **Add**.
- **4** Click \rightarrow Study.
- **5** In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces> Wavelength Domain**.
- **6** Click $\boxed{\checkmark}$ **Done**.

GLOBAL DEFINITIONS

Parameters 1

- **1** In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- **2** In the **Settings** window for **Parameters**, locate the **Parameters** section.
- **3** Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file optical_ring_resonator_3d_parameters.txt.

GEOMETRY 1

- **1** In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- **2** In the **Settings** window for **Geometry**, locate the **Units** section.
- **3** From the **Length unit** list, choose **µm**.

PART LIBRARIES

- **1** In the **Home** toolbar, click **Windows** and choose **Part Libraries**.
- **2** In the **Part Libraries** window, select **Wave Optics Module>Rectangular Waveguides> rectangular_waveguide_straight_to_ring_coupler** in the tree.
- **3** Click \overline{A} Add to Geometry.

GEOMETRY 1

Rectangular Waveguide Straight-to-Ring Coupler 1 (pi1)

- **1** In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** click **Rectangular Waveguide Straight-to-Ring Coupler 1 (pi1)**.
- **2** In the **Settings** window for **Part Instance**, locate the **Input Parameters** section.
- **3** In the table, enter the following settings:

- **4** Locate the **Position and Orientation of Output** section. Find the **Displacement** subsection. In the **yw** text field, type -r0-w_clad/2.
- **5** Click to expand the **Domain Selections** section. Click the **Wireframe Rendering** button in the **Graphics** toolbar, to make it easier to see the selections.

6 In the table, enter the following settings:

7 Click to expand the **Boundary Selections** section. In the table, enter the following settings:

8 Click **Build All Objects**.

DEFINITIONS

First add a few selections that will be useful when defining the mesh.

Core Boundaries

- **1** In the **Definitions** toolbar, click **A**djacent.
- **2** In the **Settings** window for **Adjacent**, type Core Boundaries in the **Label** text field.
- **3** Locate the **Input Entities** section. Under **Input selections**, click $\mathbf{+}$ **Add**.
- **4** In the **Add** dialog box, select **Core (Rectangular Waveguide Straight-to-Ring Coupler 1)** in the **Input selections** list.
- **5** Click **OK**.
- **6** In the **Settings** window for **Adjacent**, locate the **Output Entities** section.

7 Select the **Interior boundaries** check box.

Triangular Mesh Core Boundaries

- **1** In the **Definitions** toolbar, click **Intersection**.
- **2** In the **Settings** window for **Intersection**, type Triangular Mesh Core Boundaries in the **Label** text field.
- **3** Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- **4** Locate the **Input Entities** section. Under **Selections to intersect**, click $+$ **Add**.
- **5** In the **Add** dialog box, in the **Selections to intersect** list, choose **Core Boundaries** and **Triangular mesh (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

6 Click **OK**.

MATERIALS

Cladding

- **1** In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- **2** In the **Settings** window for **Material**, type Cladding in the **Label** text field.
- **3** Locate the **Material Contents** section. In the table, enter the following settings:

Core

- **1** Right-click **Materials** and choose **Blank Material**.
- **2** In the **Settings** window for **Material**, type Core in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Selection** list, choose **Core (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

4 Locate the **Material Contents** section. In the table, enter the following settings:

DEFINITIONS

Before setting up the physics, first add the definition of the phase variable that will be used by the Electromagnetic Waves, Beam Envelopes interface.

Variables 1

- **1** In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
- **2** In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
- **3** From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Straight domain (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

5 Locate the **Variables** section. In the table, enter the following settings:

Here, ewbe.beta_1 is the propagation constant for the first port. This port will be defined when the physics is set up in the next steps. As the variable not yet exists, COMSOL warns about this condition by displaying the expression in orange.

Variables 2

- **1** Right-click **Variables 1** and choose **Duplicate**.
- **2** In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Ring domain 1 (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

4 Locate the **Variables** section. In the table, enter the following settings:

Variables 3

1 Right-click **Variables 2** and choose **Duplicate**.

2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Ring domain 2 (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

4 Locate the **Variables** section. In the table, enter the following settings:

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

Now, use the phase variables when configuring the physics interface.

- **1** In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.
- **2** In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Wave Vectors** section.
- **3** From the **Number of directions** list, choose **Unidirectional**.
- **4** From the **Type of phase specification** list, choose **User defined**.
- **5** In the ϕ_1 text field, type phi.
- **6** Click to expand the **Discretization** section. From the **Electric field envelopes** list, choose **Cubic**, to improve the spatial resolution.

Port 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Port**.

- In the **Settings** window for **Port**, locate the **Boundary Selection** section.
- From the **Selection** list, choose **Port 1 (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

Locate the **Port Properties** section. From the **Type of port** list, choose **Numeric**.

Port 2

- In the **Physics** toolbar, click **Boundaries** and choose **Port**.
- In the **Settings** window for **Port**, locate the **Boundary Selection** section.

From the **Selection** list, choose **Port 2 (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

Locate the **Port Properties** section. From the **Type of port** list, choose **Numeric**.

Scattering Boundary Condition 1

- In the **Physics** toolbar, click **Boundaries** and choose **Scattering Boundary Condition**.
- In the **Settings** window for **Scattering Boundary Condition**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Transverse perimeter (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

- 4 Click the **Show More Options** button in the **Model Builder** toolbar.
- **5** In the **Show More Options** dialog box, select **Physics>Advanced Physics Options** in the tree.
- **6** In the tree, select the check box for the node **Physics>Advanced Physics Options**.
- **7** Click **OK**.

Field Continuity 1

- **1** In the **Physics** toolbar, click **Boundaries** and choose **Field Continuity**.
- **2** In the **Settings** window for **Field Continuity**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Field continuity (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

MESH 1

Now, define the mesh. As in addition to the guided wave there is a fair amount of radiation loss in this model, the mesh must be rather fine.

Free Triangular 1

- 1 In the Mesh toolbar, click **A** Boundary and choose Free Triangular.
- **2** In the **Settings** window for **Free Triangular**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Triangular mesh (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

Size

- **1** In the **Model Builder** window, click **Size**.
- **2** In the **Settings** window for **Size**, locate the **Element Size** section.
- **3** Click the **Custom** button.
- **4** Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type w_clad/5.
- **5** In the **Maximum element growth rate** text field, type 2, to slightly reduce the number of mesh elements.

Size 1

- **1** In the **Model Builder** window, right-click **Free Triangular 1** and choose **Size**.
- **2** In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

From the **Selection** list, choose **Triangular Mesh Core Boundaries**.

- Locate the **Element Size** section. Click the **Custom** button.
- Locate the **Element Size Parameters** section.
- Select the **Maximum element size** check box. In the associated text field, type w_core/2.

Swept 1

- In the **Mesh** toolbar, click **Swept**.
- In the **Settings** window for **Swept**, locate the **Domain Selection** section.
- From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Mesh source domain (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

Size 1

- **1** Right-click **Swept 1** and choose **Size**.
- **2** In the **Settings** window for **Size**, locate the **Element Size** section.
- **3** Click the **Custom** button.
- **4** Locate the **Element Size Parameters** section.
- **5** Select the **Maximum element size** check box. In the associated text field, type wl0/2.

Copy Domain 1

- **1** In the **Model Builder** window, right-click **Mesh 1** and choose **Copying Operations> Copy Domain**.
- **2** In the **Settings** window for **Copy Domain**, locate the **Source Domains** section.
- **3** From the **Selection** list, choose **Mesh source domain (Rectangular Waveguide Straight-to-Ring Coupler 1)**.
- **4** Locate the **Destination Domains** section. From the **Selection** list, choose **Mesh destination domain (Rectangular Waveguide Straight-to-Ring Coupler 1)**.

Click **Build All.**

STUDY 1

Boundary Mode Analysis

- 1 In the **Study** toolbar, click **Full Study Steps** and choose Other>Boundary Mode Analysis.
- In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- In the **Mode analysis frequency** text field, type f0.
- In the **Search for modes around** text field, type n_core.

Step 3: Boundary Mode Analysis 1

- Right-click **Study 1>Step 2: Boundary Mode Analysis** and choose **Duplicate**.
- In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- In the **Port name** text field, type 2.

Step 1: Wavelength Domain

- In the **Model Builder** window, click **Step 1: Wavelength Domain**.
- In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.
- In the **Wavelengths** text field, type lda0.
- Right-click **Study 1>Step 1: Wavelength Domain** and choose **Move Down**. Repeat this command once, to move the **Wavelength Domain** study step to the last position in the study sequence.

Parametric Sweep

1 In the **Study** toolbar, click $\frac{128}{22}$ **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click $+$ **Add**.

4 In the table, enter the following settings:

5 In the **Study** toolbar, click **Compute**.

RESULTS

Electric Field (ewbe)

- **1** In the **Settings** window for **3D Plot Group**, locate the **Data** section.
- **2** From the **Parameter value (lda0 (um))** list, choose **1.56**, to select the resonance wavelength.

Now, make a few adjustments to the plot to get a more detailed view of the result.

- **3** Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.
- **4** Click the **Show Legends** button in the **Graphics** toolbar.
- **5** Click the **H** Show Grid button in the Graphics toolbar.
- **6** Click the $|\psi|$ **Show Axis Orientation** button in the **Graphics** toolbar.

Electric Field

- **1** In the **Model Builder** window, expand the **Electric Field (ewbe)** node, then click **Electric Field**.
- **2** In the **Settings** window for **Multislice**, locate the **Expression** section.
- **3** In the **Expression** text field, type ewbe.Ez.
- **4** Locate the **Multiplane Data** section. Find the **X-planes** subsection. In the **Planes** text field, type 0.
- **5** Find the **Y-planes** subsection. In the **Planes** text field, type 0.
- **6** Locate the **Coloring and Style** section. Click **Change Color Table**.
- **7** In the **Color Table** dialog box, select **Wave>WaveLight** in the tree.
- **8** Click **OK**.

Deformation 1

- **1** Right-click **Electric Field** and choose **Deformation**.
- **2** In the **Settings** window for **Deformation**, locate the **Expression** section.
- **3** In the **Z-component** text field, type ewbe.Ez.
- **4** Locate the **Scale** section.
- **5** Select the **Scale factor** check box. In the associated text field, type 5E-8.

Electric Field

- **1** In the **Model Builder** window, click **Electric Field**.
- **2** In the **Settings** window for **Multislice**, click to expand the **Quality** section.
- **3** From the **Resolution** list, choose **Extra fine**.
- **4** Click the **Zoom Extents** button in the Graphics toolbar.
- **5** In the **Electric Field (ewbe)** toolbar, click **Plot**.

Ida0(6)=1.56 um lambda0(1)=1.56 µm Multislice: Electric field, z-component (V/m)

At resonance, the waves in the straight waveguide and in the ring waveguide interfere to almost completely cancel out.

Transmittance and Loss (ewbe)

1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorptance (ewbe)**.

- **2** In the **Settings** window for **1D Plot Group**, type Transmittance and Loss (ewbe) in the **Label** text field.
- **3** Locate the **Plot Settings** section. In the **y-axis label** text field, type Transmittance and loss (1).

Global 1

- **1** In the **Model Builder** window, expand the **Transmittance and Loss (ewbe)** node, then click **Global 1**.
- **2** In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- **3** Ctrl-click to select table rows 1 and 3.
- **4** Click \equiv **Delete.**
- **5** Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.
- **6** In the table, enter the following settings:

Legends

Transmittance

Loss

7 In the **Transmittance and Loss (ewbe)** toolbar, click **Plot**.

It is clear that at the resonance wavelength the transmission is very small (below 5 %), making the device behave as a notch filter.

Arrow Surface 1

- **1** In the **Model Builder** window, right-click **Electric Mode Field, Port 1 (ewbe)** and choose **Arrow Surface**.
- **2** In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Beam Envelopes>Ports>ewbe.tEmodex_1,...,ewbe.tEmodez_1 - Port tangential electric mode field**.

3 In the **Electric Mode Field, Port 1 (ewbe)** toolbar, click **Plot**.

Ida0(11)=1.57 um lambda0(1)=1.57 µm Surface: Tangential boundary mode electric field norm (V/m) Arrow Surface: Port tangential electric mode field

The mode field is localized in and around the core, with a polarization in the *z*direction.

Arrow Surface 1

- **1** In the **Model Builder** window, right-click **Electric Mode Field, Port 2 (ewbe)** and choose **Arrow Surface**.
- **2** In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Beam Envelopes>Ports>ewbe.tEmodex_2,...,ewbe.tEmodez_2 - Port tangential electric mode field**.

3 In the **Electric Mode Field, Port 2 (ewbe)** toolbar, click **P** Plot.

 $\frac{1}{1}$ lda0(11)=1.57 um lambda0(1)=1.57 µm Surface: Tangential boundary mode electric field norm (V/m)
Arrow Surface: Port tangential electric mode field

As expected, the mode field for the second port is also polarized in the *z*-direction.