Created in COMSOL Multiphysics 6.1

Fabry–Perot Resonator

Introduction

By properly arranging a set of mirrors, we can create passive optical resonators that are of great use to probe and manipulate light or to build lasers. The textbook example of a free space optical resonator is the combination of two spherical mirrors along the optical axis – a Fabry–Perot resonator. A peculiar property of such resonators is the fact that a light beam of defined shape and frequency can be transmitted with high transmission through the resonator even though it consists of two highly reflecting mirrors. In this model, we will study the transmission properties of a Fabry–Perot resonator and compare this to the analytic solution.

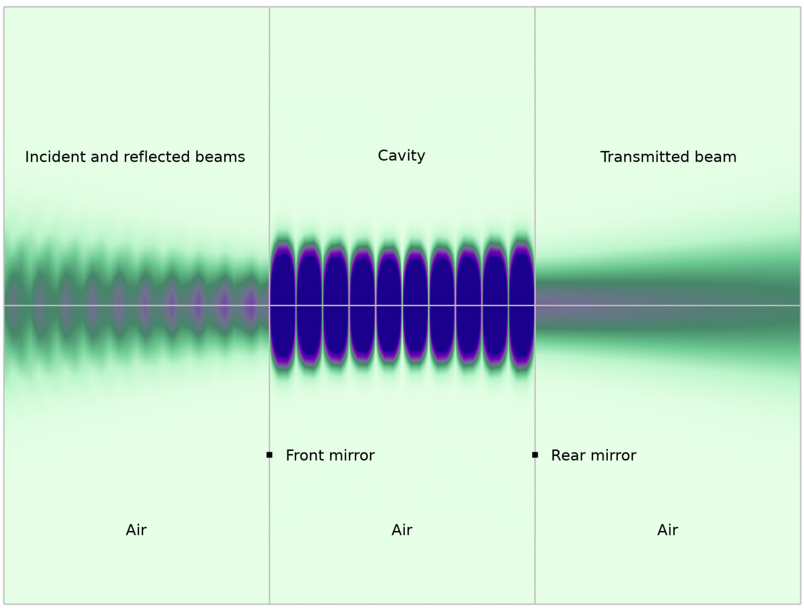


Figure 1: Geometry configuration of a Fabry–Perot resonator. The resonator consists of curved front and rear mirrors. Inside the cavity (located between the mirrors) and outside the cavity, there is air. The incident Gaussian beam propagates from left to right. Thus, to the left of the cavity there is both the incident and the reflected beam. Inside the cavity there is both a right- and left-propagating beam. To the right of the cavity, there is only the transmitted beam.

Two mirrors with radii ρ_1 and ρ_2, separated a distance L, can create a stable optical resonator, if they fulfill the stability criterion $0 < g_1 g_2 < 1$, where $g_{1,2} = 1 - L/\rho_{1,2}$ are the stability parameters of the two mirrors. A stable optical resonator defines a set of spatial modes, each one with a defined mode shape. If we want to excite a spatial mode of the resonator, we also have to consider that the distance of the two mirrors creates a resonance condition that needs to be fulfilled for a certain spatial mode. While it is possible to have high transmission on resonance, the resonator is highly reflective for off resonant frequencies.

We can define the free spectral range of the resonator $\Delta\nu_{FSR}$ and the finesse F as

$$\Delta\nu_{FSR} = \frac{c}{2L}$$

and

$$F = \frac{\pi(R_1 R_2)^{\frac{1}{4}}}{1-\sqrt{R_1 R_2}},$$

where c is the velocity of light and R_1 and R_2 are the respective mirror reflectance. The bandwidth of the resonance is then defined as

$$\delta\nu = \frac{\Delta\nu_{FSR}}{F}.$$

Please note that, depending on the cavity configuration, different spatial modes can exhibit different resonance frequencies. Any input field can be decomposed into the spatial modes of the resonator. Therefore, the resonator will act as a spatial as well as a frequency filter. We will focus here on efficiently exciting the fundamental mode of the resonator.

The fundamental mode of the resonator can be described by the waist,

$$w_0 = \sqrt{\frac{L\lambda}{\pi}}\left(\frac{g_1 g_2(1-g_1 g_2)}{(g_1+g_2-2g_1 g_2)^2}\right)^{\frac{1}{4}},$$

where λ is the wavelength.

For the case of equal radii of curvature for the mirrors, $\rho_1 = \rho_2$, the waist is located at the center of the resonator.

Model Definition

To efficiently model the full wave solution of the 50 mm long Fabry–Perot resonator, the Electromagnetic Waves, Beam Envelopes interface in the bi-directional formulation is used. The first and second waves are two counter-propagating plane waves in vacuum. The geometry consists of the cavity separating the two mirrors and two exterior domains, one in front and one behind the cavity.

The mirrors are approximated as highly reflecting thin dielectric layers, modeled using the Transition Boundary Condition. To compute the reflectivity R of the thin dielectric layer (Transition Boundary Condition – TBC), we assume that the layer has a refractive index n and the surrounding layers have the refractive indices n_1 and n_2, respectively.

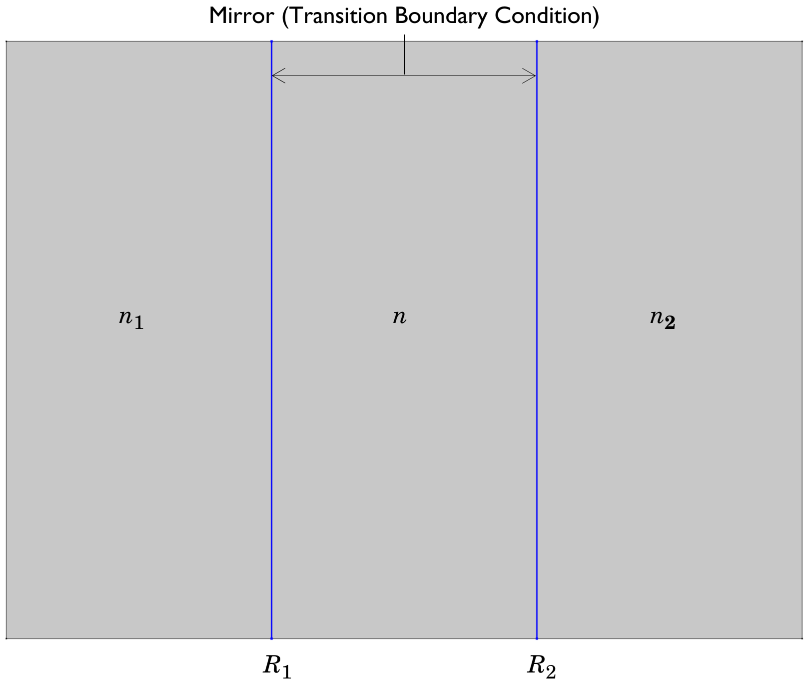


Figure 2: A mirror modeled as a thin dielectric layer with refractive index n*, surrounded by materials with refractive index* n_1 *and* n_2*, respectively. The reflectances* R_1 *and* R_2*, for each interface, are indicated at the bottom of the picture. Each mirror is modeled using a Transition Boundary Condition feature.*

The two material interfaces, depicted in Figure 2, have the amplitude reflectivities

$$r_1 = \frac{n_1 - n}{n_1 + n}$$

and

$$r_2 = \frac{n - n_2}{n + n_2},$$

respectively.

The corresponding power reflectances are then

$$R_i = r_i^2, \; i = 1, 2.$$

The reflectance R of the film due to interference is

$$R = \left| \frac{\sqrt{R_1} - \sqrt{R_2}\exp(-2i\phi)}{1 - \sqrt{R_1 R_2}\exp(-2i\phi)} \right|^2,$$

where ϕ is the accumulated phase when passing through the layer

$$\phi = 2\pi n d / \lambda.$$

Using a thickness $d = \lambda/100$ and refractive indices $n = 15$ and $n_1 = n_2 = 1$, both mirrors have a reflectance of 0.973. This corresponds to a finesse of $F = 116.2$.

A Gaussian beam is launched using the Scattering Boundary Condition. The input beam waist corresponds to the analytical solution of the cavity mode and the waist is located at the center of the cavity.

Results and Discussion

The sweep over one free spectral range shows only one resonance (Figure 3). This indicates that only one spatial mode is excited.

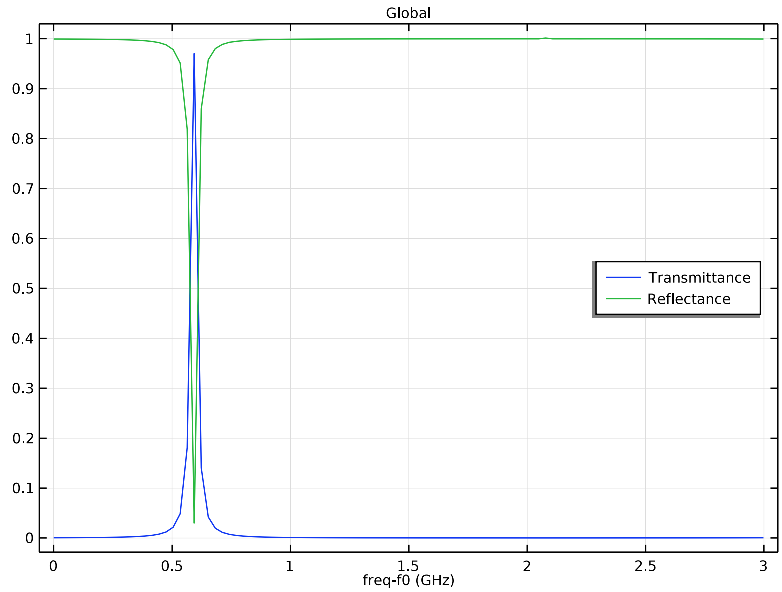


Figure 3: A frequency sweep over one free spectral range.

Figure 4 shows the norm of the electric field for the first and second waves at resonance. The first wave propagates from left to right and the second wave propagates from right to left. It is clear that at resonance, the field in the cavity is much larger than outside the cavity. Furthermore, there is almost no reflected wave at resonance and all power is transmitted.

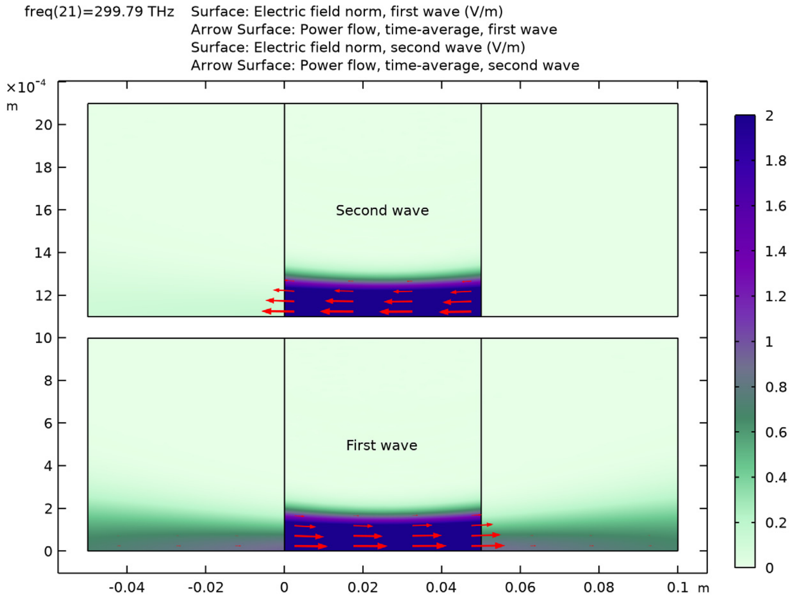


Figure 4: The norm of the electric field for the first wave (bottom) and the second wave (top) at the resonance frequency. The arrows show the power flow for the two waves.

The second study (see Figure 5) carries out a finer sweep over the cavity resonance. On resonance, all light is transmitted through the cavity as is expected for good mode match.

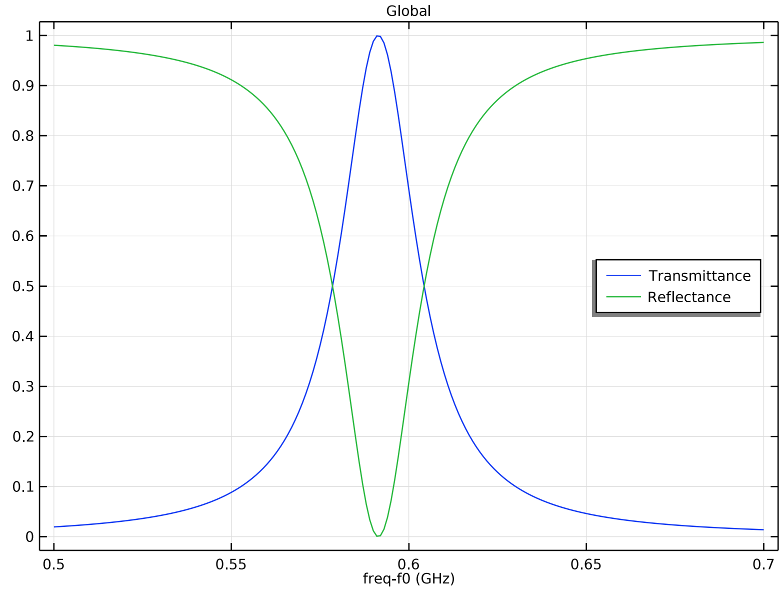


Figure 5: A frequency sweep over the resonance peak.

Figure 6 shows the electric field norm for the first wave at resonance. It confirms that the field inside the cavity is much higher than outside the cavity at resonance.

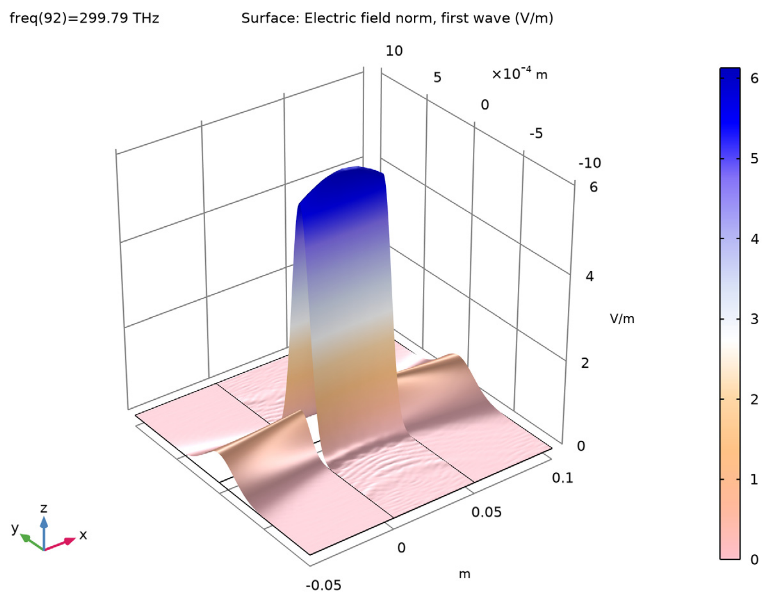


Figure 6: The norm of the electric field for the first wave at resonance.

Application Library path: `Wave_Optics_Module/Verification_Examples/fabry_perot_resonator`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Beam Envelopes (ewbe)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

Geometry Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Geometry Parameters` in the **Label** text field.

3 Locate the **Parameters** section. Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `fabry_perot_resonator_geometry_parameters.txt`.

Cavity Parameters

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Cavity Parameters` in the **Label** text field.

3 Locate the **Parameters** section. Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `fabry_perot_resonator_cavity_parameters.txt`.

GEOMETRY 1

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `rho1`.

4 Locate the **Position** section. In the **x** text field, type `rho1`.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

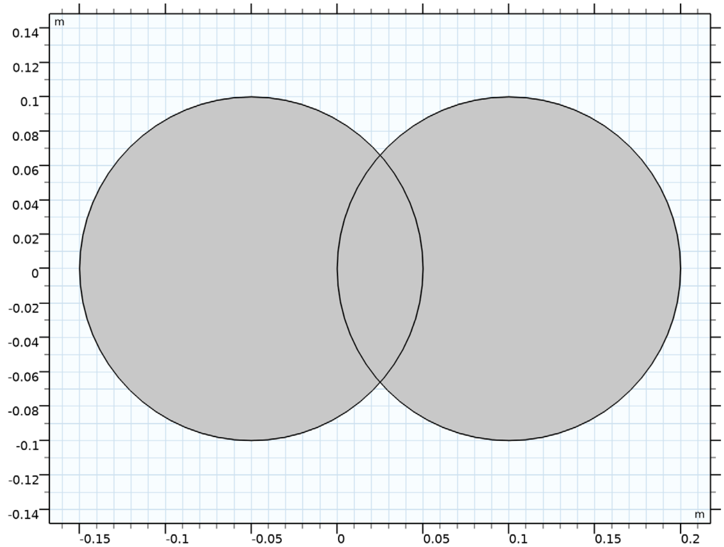
2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `rho2`.

4 Locate the **Position** section. In the **x** text field, type `l_cav-rho2`.

5 Click **Build Selected**.

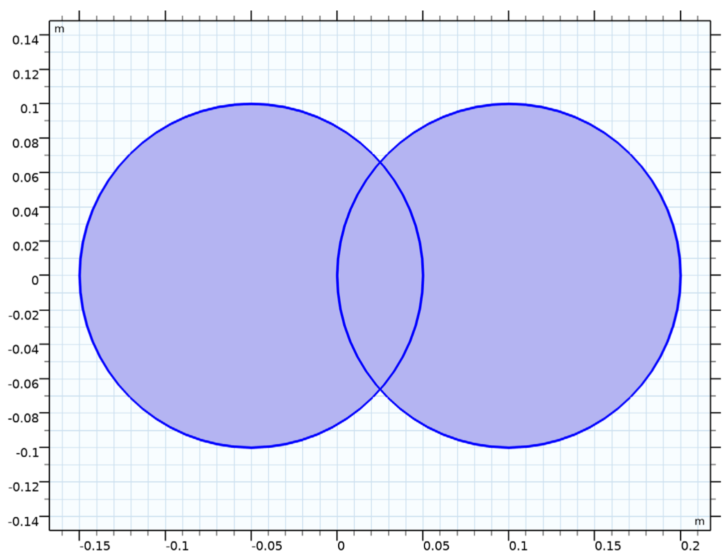
6 Click the **Zoom Extents** button in the **Graphics** toolbar.



Union 1 (uni1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Union**.

2 Click in the **Graphics** window and then press Ctrl+A to select both objects.



Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `l_total`.

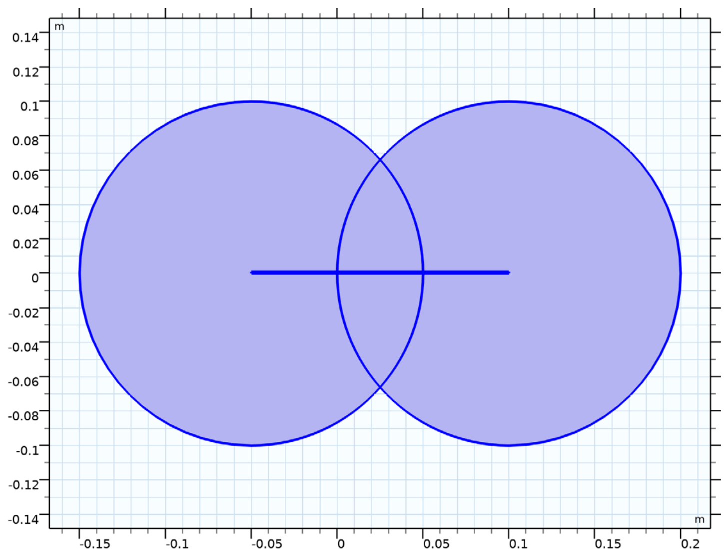
4 In the **Height** text field, type `h_cav/2`.

5 Locate the **Position** section. In the **x** text field, type `-l_in`.

Intersection 1 (int1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Intersection**.

2 Click in the **Graphics** window and then press Ctrl+A to select both objects.



3 In the **Settings** window for **Intersection**, click **Build All Objects**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

DEFINITIONS

In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

Axis

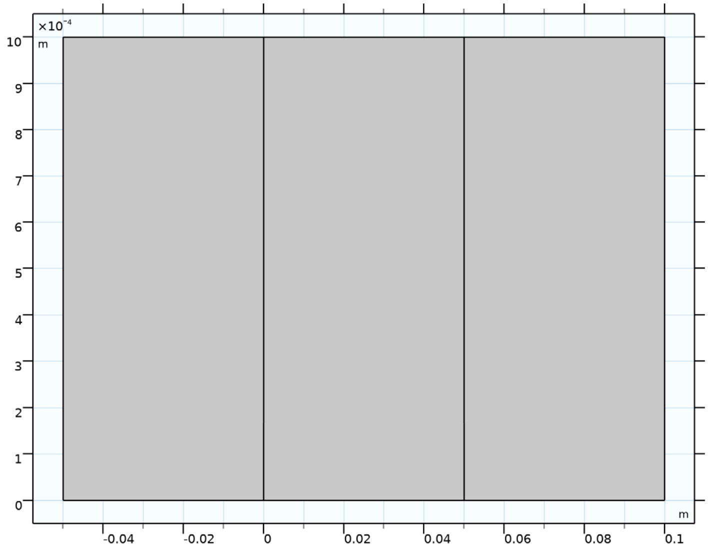
1 In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions>View 1** node, then click **Axis**.

2 In the **Settings** window for **Axis**, locate the **Axis** section.

3 From the **View scale** list, choose **Automatic**.

4 Click **Update**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



MATERIALS

Air

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	n1	1	Refractive index
Refractive index, imaginary part	ki_iso ; kiii = ki_iso, kiij = 0	0	1	Refractive index

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.

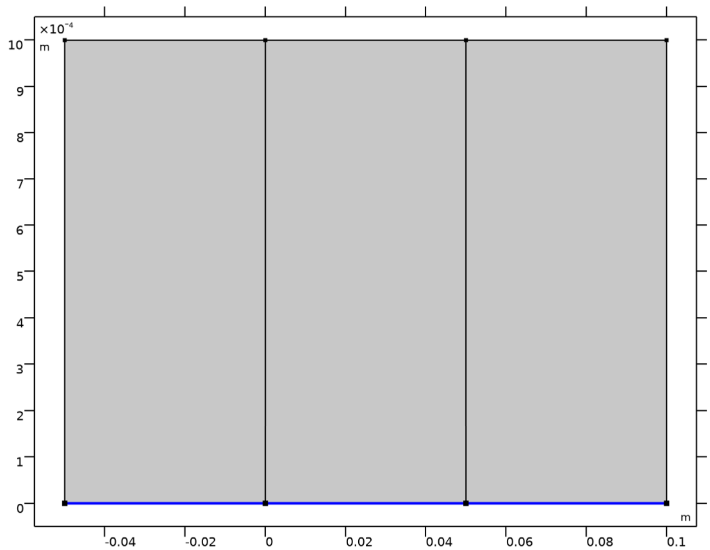
2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **Out-of-plane vector**. This reduces the degrees of freedom for the model and restricts the solution to out-of-plane TE waves.

Symmetry Plane 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Symmetry Plane** condition to model even symmetry on the optical axis.

2 Select Boundaries 2, 4, and 7 only.

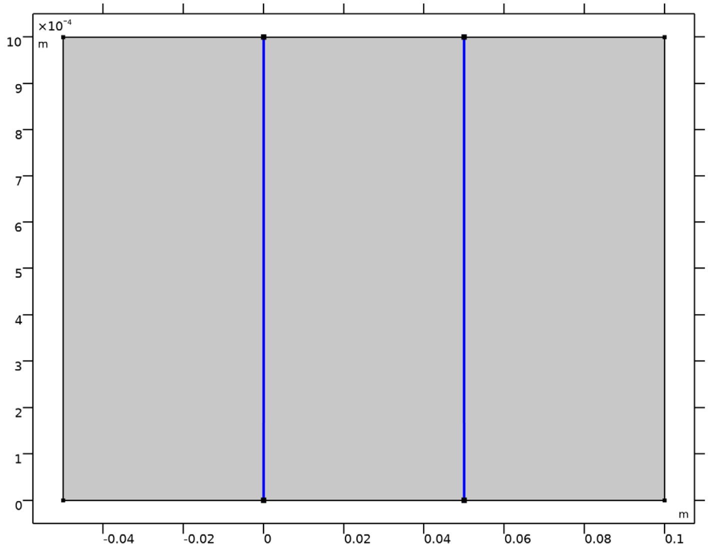


Transition Boundary Condition 1

Now, add a **Transition Boundary Condition** to model the mirrors. Assign a refractive index that is much larger than the refractive index of air, to create a high reflectivity for the mirrors.

1 In the **Physics** toolbar, click **Boundaries** and choose **Transition Boundary Condition**.

2 Select Boundaries 9 and 10 only.



3 In the **Settings** window for **Transition Boundary Condition**, locate the **Transition Boundary Condition** section.

4 From the n list, choose **User defined**. In the associated text field, type `n`.

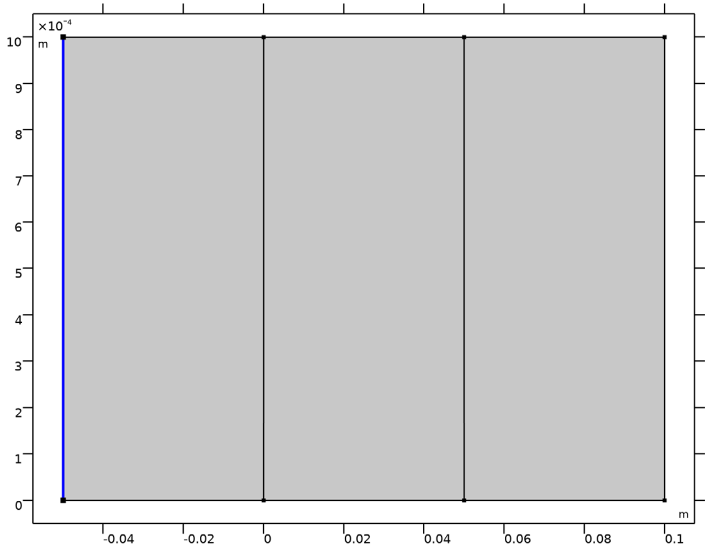
5 From the k list, choose **User defined**. In the d text field, type `d`.

Scattering Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Scattering Boundary Condition**.

Use a **Scattering Boundary Condition** to launch an incident Gaussian beam polarized in the z direction. This is compatible with the out-of plane setting for the interface.

2 Select Boundary 1 only.



3 In the **Settings** window for **Scattering Boundary Condition**, locate the **Scattering Boundary Condition** section.

4 From the **Incident field** list, choose **Gaussian beam**.

5 In the w_0 text field, type `w0`.

6 In the p_0 text field, type `l_in+l_cav/2`, which places the focal plane for the Gaussian beam at the center of the cavity.

7 Specify the $\mathbf{E}_{g0}$ vector as

0	x
0	y
1	z

Reference Point 1

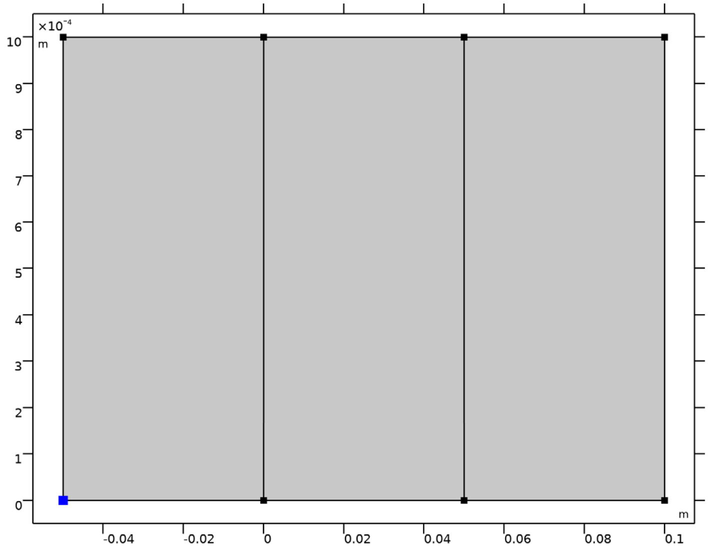
1 In the **Physics** toolbar, click **Attributes** and choose **Reference Point**.

If no **Reference Point** subfeature is added to the **Scattering Boundary Condition**, the reference point will appear at the center point of the boundary. Add a **Reference Point** subfeature to make sure the reference point appears on the optical axis.

2 In the **Settings** window for **Reference Point**, locate the **Point Selection** section.

3 Click **Clear Selection**.

4 Select Point 1 only.



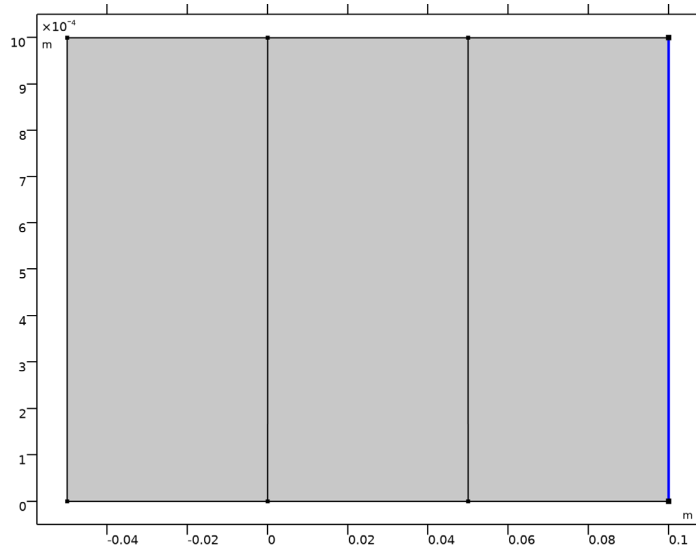
Scattering Boundary Condition 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Scattering Boundary Condition**.

2 Select Boundary 8 only.

3 In the **Settings** window for **Scattering Boundary Condition**, locate the **Scattering Boundary Condition** section.

4 From the **Input wave** list, choose **Second wave**. This makes the boundary transparent for the transmitted field.



DEFINITIONS

Create integral operators for use in reflectance and transmittance calculation.

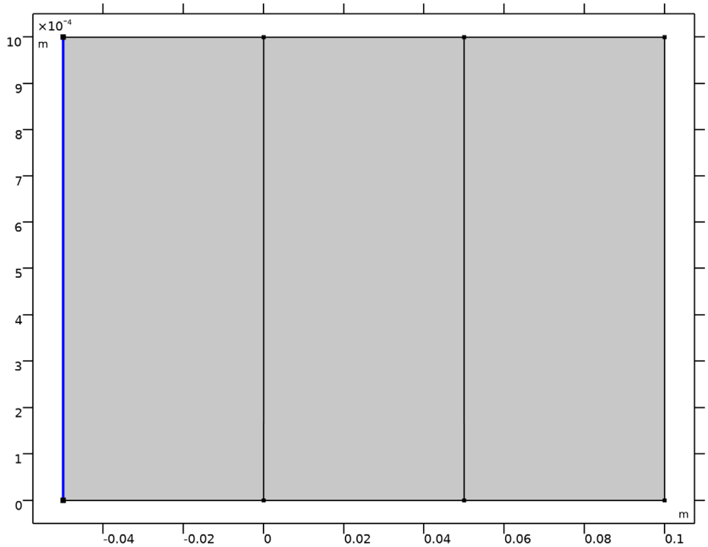
Integration 1 (intop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 1 only.



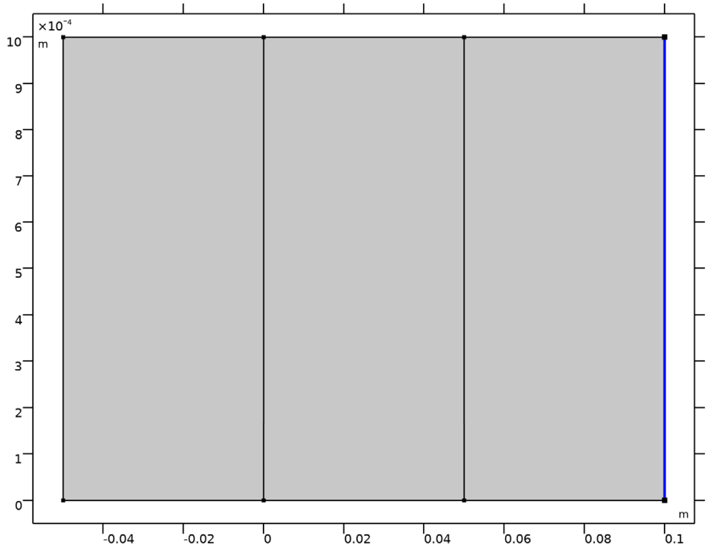
Integration 2 (intop2)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 8 only.



Variables 1

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
`R`	`intop1(ewbe.nPoav2)/intop1(-ewbe.nPoav1)`		`Reflectance`
`T`	`intop2(ewbe.nPoav1)/intop1(-ewbe.nPoav1)`		`Transmittance`

MESH 1

The beam envelopes interface allows us to resolve only the resulting field envelope after demodulation with the prescribed phase function. Here, we adjust the number of elements perpendicular and along the optical axis to resolve this envelope properly.

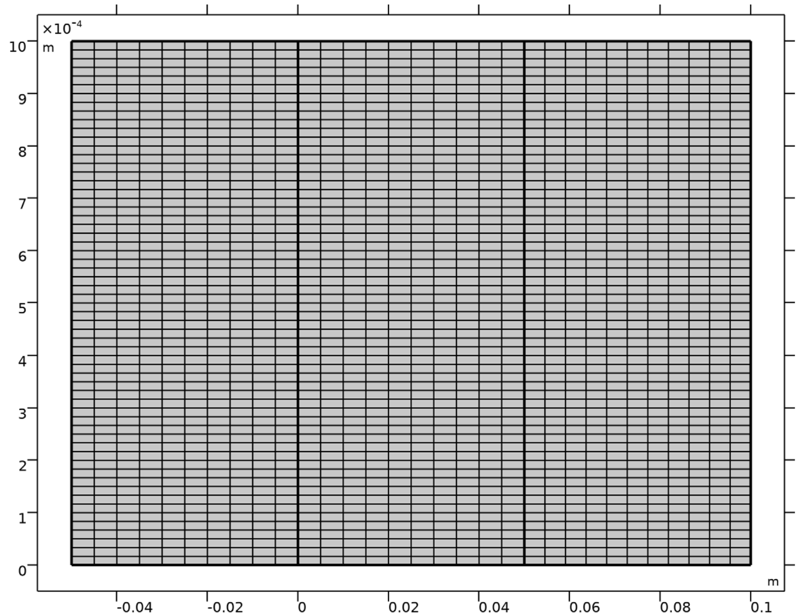
1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Beam Envelopes (ewbe)** section.

3 In the N_T text field, type `60`. This will create the mapped mesh with sixty elements over the wavefront.

4 In the N_L text field, type `30`. This will create the mapped mesh with thirty elements along the simulation domain. This will be sufficient to study the Fabry-Perot cavity.

5 Click **Build All**.



STUDY 1 - FSR SWEEP

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type `Study 1 - FSR Sweep` in the **Label** text field.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1 - FSR Sweep** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `range(f0,FSR/101,f0+FSR)`.

4 In the **Home** toolbar, click **Compute**.

RESULTS

Electric Field (ewbe)

1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

2 From the **Parameter value (freq (THz))** list, choose **299.79 (21)**, which corresponds to the resonance frequency.

Electric Field

1 In the **Model Builder** window, expand the **Electric Field (ewbe)** node, then click **Electric Field**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewbe.normE1` to display the norm of the electric field for the first wave.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Aurora>AuroraBorealis** in the tree.

6 Click **OK**.

7 In the **Settings** window for **Surface**, click to expand the **Range** section.

8 Select the **Manual color range** check box.

9 In the **Maximum** text field, type `2`.

Arrow Surface 1

1 In the **Model Builder** window, right-click **Electric Field (ewbe)** and choose **Arrow Surface**.

2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Beam Envelopes>Energy and power>ewbe.Poav1x,ewbe.Poav1y - Power flow, time-average, first wave**.

3 Locate the **Arrow Positioning** section. Find the **X grid points** subsection. In the **Points** text field, type `10`.

4 Find the **Y grid points** subsection. In the **Points** text field, type `21`.

5 Locate the **Coloring and Style** section. From the **Arrow length** list, choose **Logarithmic**.

6 Select the **Scale factor** check box. In the associated text field, type `0.2`.

Electric Field 1

1 In the **Model Builder** window, right-click **Electric Field** and choose **Duplicate**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewbe.normE2` to display the norm of the electric field for the second wave.

4 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Electric Field**.

Translation 1

1 Right-click **Electric Field 1** and choose **Translation**.

2 In the **Settings** window for **Translation**, locate the **Translation** section.

3 In the **y** text field, type `h_cav*0.55`.

Arrow Surface 2

1 In the **Model Builder** window, under **Results>Electric Field (ewbe)** right-click **Arrow Surface 1** and choose **Duplicate**.
2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.
3 In the **X-component** text field, type `ewbe.Poav2x`.
4 In the **Y-component** text field, type `ewbe.Poav2y`.
5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Arrow Surface 1**.

Translation 1

1 Right-click **Arrow Surface 2** and choose **Translation**.
2 In the **Settings** window for **Translation**, locate the **Translation** section.
3 In the **y** text field, type `h_cav*0.55`.

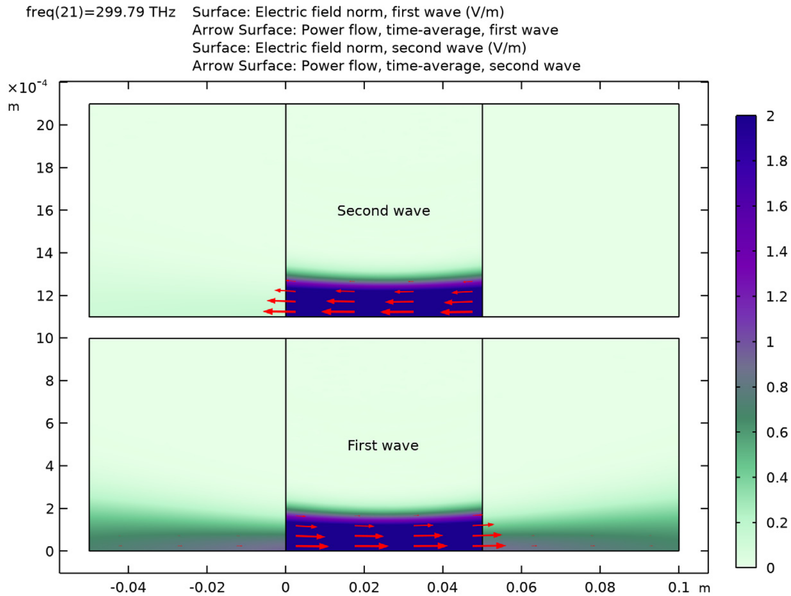
Annotation 1

1 In the **Model Builder** window, right-click **Electric Field (ewbe)** and choose **Annotation**.
2 In the **Settings** window for **Annotation**, locate the **Annotation** section.
3 In the **Text** text field, type `First wave`.
4 Locate the **Position** section. In the **X** text field, type `l_cav/2`.
5 In the **Y** text field, type `h_cav/4`.
6 Locate the **Coloring and Style** section. From the **Anchor point** list, choose **Center**.
7 Clear the **Show point** check box.

Annotation 2

1 Right-click **Annotation 1** and choose **Duplicate**.
2 In the **Settings** window for **Annotation**, locate the **Annotation** section.
3 In the **Text** text field, type `Second wave`.
4 Locate the **Position** section. In the **Y** text field, type `h_cav*0.55+h_cav/4`.
5 In the **Electric Field (ewbe)** toolbar, click **Plot**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.



Sweep Over One FSR

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Sweep Over One FSR` in the **Label** text field.

Global 1

1 Right-click **Sweep Over One FSR** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
T	1	Transmittance
R	1	Reflectance

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `freq-f0`.

6 From the **Unit** list, choose **GHz**.

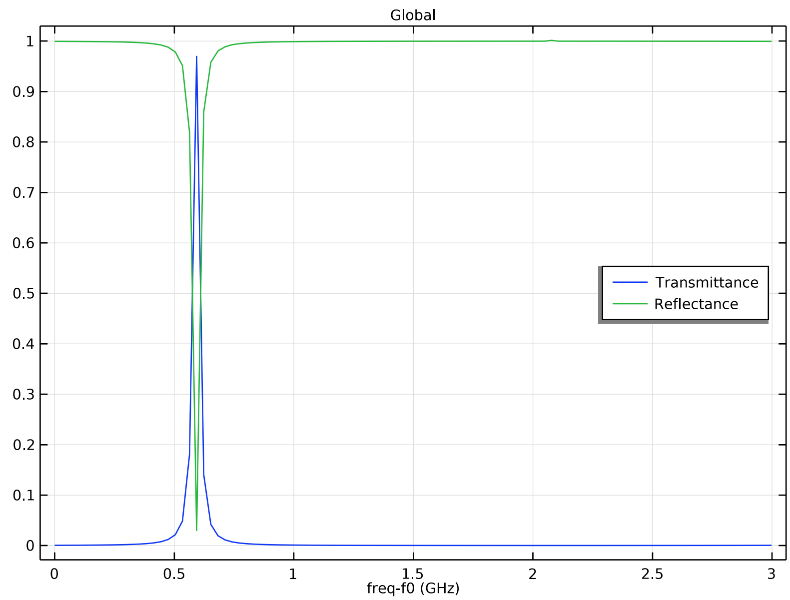
7 In the **Sweep Over One FSR** toolbar, click **Plot**.

Sweep Over One FSR

1 In the **Model Builder** window, click **Sweep Over One FSR**.

2 In the **Settings** window for **1D Plot Group**, locate the **Legend** section.

3 From the **Position** list, choose **Middle right**.



ADD STUDY

Now, create a study to only sweep over the resonance peak.

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2 - RESONANCE SWEEP

1 In the **Model Builder** window, click **Study 2**.

2 In the **Settings** window for **Study**, type `Study 2 - Resonance Sweep` in the **Label** text field.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 2 - Resonance Sweep** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `range(f0+0.5[GHz],0.001[GHz],f0+0.7[GHz])`.

4 In the **Home** toolbar, click **Compute**.

RESULTS

Create a mirror dataset to be able to plot the full beam.

Mirror 2D 1

1 In the **Results** toolbar, click **More Datasets** and choose **Mirror 2D**.

2 In the **Settings** window for **Mirror 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2 - Resonance Sweep/Solution 2 (sol2)**.

4 Locate the **Axis Data** section. In row **Point 2**, set **X** to `1`.

5 In row **Point 2**, set **Y** to `0`.

Electric Field (ewbe) 1

1 In the **Model Builder** window, under **Results** click **Electric Field (ewbe) 1**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Mirror 2D 1**.

4 From the **Parameter value (freq (THz))** list, choose **299.79 (92)**, which corresponds to the resonance frequency.

Electric Field

1 In the **Model Builder** window, expand the **Electric Field (ewbe) 1** node, then click **Electric Field**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewbe.normE1`.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Aurora>Twilight** in the tree.

6 Click **OK**.

Height Expression 1

1 Right-click **Electric Field** and choose **Height Expression**.

2 In the **Settings** window for **Height Expression**, locate the **Axis** section.

3 Select the **Scale factor** check box. In the associated text field, type `0.001`.

4 In the **Electric Field (ewbe) 1** toolbar, click **Plot**.

5 In the **Model Builder** window, expand the **Results>Views** node.

Camera

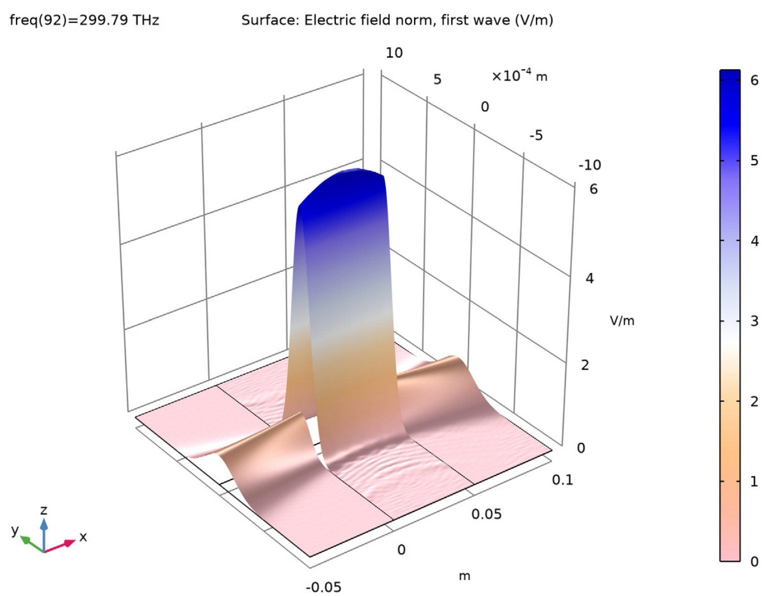
1 In the **Model Builder** window, expand the **Results>Views>View 3D 3** node, then click **Camera**.

2 In the **Settings** window for **Camera**, locate the **Camera** section.

3 From the **View scale** list, choose **Automatic**.

4 Click **Update**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



Resonance Shape

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Resonance Shape` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Study 2 - Resonance Sweep/ Solution 2 (sol2)**.

4 Locate the **Legend** section. From the **Position** list, choose **Middle right**.

Global 1

1 Right-click **Resonance Shape** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
T	1	Transmittance
R	1	Reflectance

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `freq-f0`.

6 From the **Unit** list, choose **GHz**.