## Instability of a Space Arc Frame

## Model Definition

In this example you study the lateral deflection of a space frame subjected to concentrated vertical loading at four different points. A small lateral load is applied to break the symmetry of the structure. The model is described in detail in section 6.3 of Ref. 1, where it is called "Space frame subjected to concentrated loading". A schematic description of the frame and loads are shown in Figure 1. There are two types of members used in the frame, marked as 1 and 2 respectively.


Figure 1: Space frame geometry.

## GEOMETRY

- Cross section properties of type 1 members are $A_{1}=0.5, I_{y 1}=0.4, I_{z 1}=0.133$.
- Cross section properties of type 2 members are $A_{2}=0.1, I_{y 2}=0.05, I_{z 2}=0.05$.

The local $y$ direction coincides with the global $y$ direction.
The torsional constant is not supplied in the reference, so the common approximation $\mathrm{J}=$ $I_{y}+I_{z}$ is used.

## MATERIAL

Linear elastic with $E=4.32 \cdot 10^{5}$ and $\mathrm{G}=1.66 \cdot 10^{5}$.

## CONSTRAINTS AND LOADS

- All the base points of the frame are pinned.
- The four corners at the top are subjected to vertical loads $P$, ranging from 0 to 8.65 , acting downward.
- The front two corners are subjected to lateral loads of $0.001 \cdot P$.


## Results and Discussion

With only vertical loads active on the frame this is a symmetric problem. Hence, it is necessary to perturb the symmetry somewhat to induce a controlled instability. The small lateral loads serve this purpose. As an alternative, you could introduce an initial imperfection in the geometry.

Figure 2 below shows the final state of the frame.

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P(113)=8.64 Line: Displacement magnitude (m)
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Figure 2: Final state of the deformed frame.
The horizontal displacement of point A on the frame versus the compressive load is shown in Figure 3. Data obtained from Ref. 1 is marked on the same curve. The agreement with the data from the reference is very good.


Figure 3: Load vs. displacement.
The plot of the lateral deflection shows that an instability occurs at a parameter value close to 8.0. In practice, the critical load of an imperfect structure is often significantly lower than that of the ideal structure.

Linear buckling analysis also gives the first critical buckling load as 8.67 which matches well with the critical load obtained from the above analysis. The corresponding buckling mode shape is shown in the Figure 4 below.


Figure 4: First buckling mode.

## Reference

1. Z.X. Li and L. Vu-Quoc, A Mixed Co-rotational 3D Beam Element for Arbitrarily Large Rotations, Advanced Steel Construction Vol. 6, No. 2, 767-787, 2010.

Application Library path: Structural_Mechanics_Module/
Verification_Examples/space_frame_instability

## Modeling Instructions

From the File menu, choose New.

## NEW

In the New window, click $\otimes$ Model Wizard.

## MODEL WIZARD

I In the Model Wizard window, click 3D.
2 In the Select Physics tree, select Structural Mechanics>Beam (beam).
3 Click Add.
4 Click $\rightarrow$ Study.
5 In the Select Study tree, select General Studies>Stationary.
6 Click $\sqrt{ }$ Done.

## GLOBAL DEFINITIONS

Define the load parameter as well as the geometric data.
Parameters I
I In the Model Builder window, under Global Definitions click Parameters I.
2 In the Settings window for Parameters, locate the Parameters section.

## 3 Click Load from File.

4 Browse to the model's Application Libraries folder and double-click the file space_frame_instability_parameters.txt.

## GEOMETRY I

Since the frame is symmetric, create only one quarter of the geometry and use two mirror operations to obtain the full geometry.

## Polygon I (poll)

I In the Geometry toolbar, click $\odot$ More Primitives and choose Polygon.
2 In the Settings window for Polygon, locate the Coordinates section.
3 From the Data source list, choose Vectors.
4 In the $\mathbf{x}$ text field, type $-11-12 / 2-12 / 2-12 / 20$.
5 In the $y$ text field, type $-b / 2-b / 2-b / 2-b / 2$.
6 In the $\mathbf{z}$ text field, type 0 h 1 h 1 h 1 .
Line Segment I (Is I)
I In the Geometry toolbar, click $\oslash$ More Primitives and choose Line Segment.
2 In the Settings window for Line Segment, locate the Starting Point section.
3 From the Specify list, choose Coordinates.
4 Locate the Endpoint section. From the Specify list, choose Coordinates.
5 Locate the Starting Point section. In the $\mathbf{x}$ text field, type $-12 / 2, \mathbf{y}$ to $-\mathrm{b} / 2$, and $\mathbf{z}$ to h 1 .

6 Locate the Endpoint section. In the $\mathbf{x}$ text field, type -12/2 and $\mathbf{z}$ to h 1 .

## Mirror I (mirl)

I In the Geometry toolbar, click ${ }_{k}^{\kappa} P_{y}^{x}$ Transforms and choose Mirror.
2 Click in the Graphics window and then press Ctrl+A to select both objects.
3 In the Settings window for Mirror, locate the Input section.
4 Select the Keep input objects check box.
5 Locate the Point on Plane of Reflection section. In the $\mathbf{x}$ text field, type -12/2.
6 In the $\mathbf{z}$ text field, type h1.
7 Locate the Normal Vector to Plane of Reflection section. In the $\boldsymbol{y}$ text field, type 1.
8 In the $\mathbf{z}$ text field, type 0 .
Mirror 2 (mir2)
I In the Geometry toolbar, click ${ }_{k \neq}^{\kappa} P_{y}^{\pi}$ Transforms and choose Mirror.
2 Click in the Graphics window and then press Ctrl+A to select all objects.
3 In the Settings window for Mirror, locate the Input section.
4 Select the Keep input objects check box.
5 Locate the Normal Vector to Plane of Reflection section. In the $\mathbf{x}$ text field, type 1.
6 In the $\mathbf{z}$ text field, type 0.
7 In the Geometry toolbar, click Build All.
8 Click the ${ }^{\uparrow}$ Go to Default View button in the Graphics toolbar.

## BEAM (BEAM)

## Linear Elastic Material I

I In the Model Builder window, under Component I (compl)>Beam (beam) click Linear Elastic Material I.

2 In the Settings window for Linear Elastic Material, locate the Linear Elastic Material section.

3 From the Specify list, choose Young's modulus and shear modulus.

## MATERIALS

## Material I (matl)

I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.

2 In the Settings window for Material, locate the Material Contents section.
3 In the table, enter the following settings:

| Property | Variable | Value | Unit | Property group |
| :--- | :--- | :--- | :--- | :--- |
| Young's modulus | E | 4.32 e 5 | Pa | Young's modulus and shear <br> modulus |
| Shear modulus | G | 1.66 e 5 | $\mathrm{~N} / \mathrm{m}^{2}$ | Young's modulus and shear <br> modulus |
| Density | rho | 0 | $\mathrm{~kg} / \mathrm{m}^{3}$ | Basic |

The density is set to zero since it is not used in the present analysis.

## BEAM (BEAM)

## Cross-Section Data I

I In the Model Builder window, under Component I (comp I)>Beam (beam) click CrossSection Data I.

2 In the Settings window for Cross-Section Data, locate the Basic Section Properties section.
3 In the $A$ text field, type A1.
4 In the $I_{z z}$ text field, type Iz1.
5 In the $I_{y y}$ text field, type Iy1.
6 In the $J$ text field, type $\mathrm{Iy} 1+\mathrm{Iz} 1$.

## Section Orientation I

I In the Model Builder window, click Section Orientation I.
2 In the Settings window for Section Orientation, locate the Section Orientation section.
3 From the Orientation method list, choose Orientation vector.
4 Specify the $V$ vector as
$0 \quad X$
1 Y
0 Z
Cross-Section Data 2
I In the Physics toolbar, click $\square$ Edges and choose Cross-Section Data.
2 Select Edges 3, 5, 9, and 11 only.
3 In the Settings window for Cross-Section Data, locate the Basic Section Properties section.

4 In the $A$ text field, type A2.
5 In the $I_{z z}$ text field, type Iz2.
6 In the $I_{y y}$ text field, type Iy2.
7 In the $J$ text field, type Iy2+Iz2.

## Section Orientation I

I In the Model Builder window, expand the Cross-Section Data 2 node, then click Section Orientation I.

2 In the Settings window for Section Orientation, locate the Section Orientation section.
3 From the Orientation method list, choose Orientation vector.
4 Specify the $V$ vector as
1 X
0 Y
0 Z
Pinned I
I In the Physics toolbar, click $\emptyset$ Points and choose Pinned.
2 Select Points 1, 2, 11, and 12 only.
Point Load I
I In the Physics toolbar, click $\forall$ Points and choose Point Load.
2 Select Points 3, 5, 8, and 10 only.
3 In the Settings window for Point Load, locate the Force section.
4 Specify the $\mathbf{F}_{\mathrm{P}}$ vector as

| 0 | $x$ |
| :--- | :--- |
| 0 | $y$ |
| $-P$ | $z$ |

Point Load 2
I In the Physics toolbar, click $\forall$ Points and choose Point Load.
2 Select Points 3 and 8 only.
3 In the Settings window for Point Load, locate the Force section.

4 Specify the $\mathbf{F}_{\mathrm{P}}$ vector as

| 0 | $x$ |
| :--- | :--- |
| $0.001 * P$ | $y$ |
| 0 | $z$ |

## MESH I

I In the Model Builder window, under Component I (compl) click Mesh I.
2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.
3 From the Element size list, choose Fine.

## STUDY I

## Step I: Stationary

Use geometric nonlinearity since the problem is expected to have an instability.
I In the Model Builder window, under Study I click Step I: Stationary.
2 In the Settings window for Stationary, locate the Study Settings section.
3 Select the Include geometric nonlinearity check box.
Set up parametric sweep for the load.
4 Click to expand the Study Extensions section. Select the Auxiliary sweep check box.
5 Click + Add.
Due to instability, the load increment for $\mathbf{P}>8$ is reduced.
6 In the table, enter the following settings:

| Parameter name | Parameter value list |
| :--- | :--- |
| $P$ (Load) | range $(0,0.1,8)$ range $(8.02,0.02,8.65)$ |

Solution I (soll)

Scale the dependent variables appropriately.
2 In the Model Builder window, expand the Solution I (soll) node.
3 In the Model Builder window, expand the Study I>Solver Configurations>
Solution I (sol I)>Dependent Variables I node, then click Displacement field (compl.beam.uLin).

4 In the Settings window for Field, locate the Scaling section.
5 From the Method list, choose Manual.

6 In the Model Builder window, under Study I>Solver Configurations>Solution I (soll)> Dependent Variables I click Rotation field (comp I.beam.thLin).

7 In the Settings window for Field, locate the Scaling section.
8 From the Method list, choose Manual.
9 In the Scale text field, type pi/10.
Increase the maximum allowed number of iterations due to the expected instability.
IO In the Model Builder window, expand the Study I>Solver Configurations> Solution I (solI)>Stationary Solver I node, then click Fully Coupled I.

II In the Settings window for Fully Coupled, click to expand the Method and Termination section.

I2 In the Maximum number of iterations text field, type 40.
I3 In the Study toolbar, click $\equiv$ Compute.

## RESULTS

Displacement (beam)
In the Settings window for 3D Plot Group, type Displacement (beam) in the Label text field.

Line I
I In the Model Builder window, expand the Displacement (beam) node, then click Line I.
2 In the Settings window for Line, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Beam>Displacement> beam.disp - Displacement magnitude - m.

3 In the Displacement (beam) toolbar, click 0 Plot.
4 Click the $\xrightarrow[H]{4}$ Zoom Extents button in the Graphics toolbar. Compare the load-displacement curve with the values from the reference.

## Load vs. Displacement

I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
2 In the Settings window for ID Plot Group, type Load vs. Displacement in the Label text field.

3 Locate the Plot Settings section.
4 Select the $\mathbf{x}$-axis label check box. In the associated text field, type $v$.
5 Select the $\mathbf{y}$-axis label check box. In the associated text field, type $P$.
6 Click to expand the Title section. From the Title type list, choose Manual.

7 In the Title text area, type Load vs. displacement.

## Point Graph I

I Right-click Load vs. Displacement and choose Point Graph.
2 Select Point 4 only.
3 In the Settings window for Point Graph, locate the $\mathbf{y}$-Axis Data section.
4 In the Expression text field, type $P$.
5 Locate the x-Axis Data section. From the Parameter list, choose Expression.
6 In the Expression text field, type beam. uLinY.
7 Click to expand the Legends section. Select the Show legends check box.
8 From the Legends list, choose Manual.
9 In the table, enter the following settings:

## Legends <br> COMSOL

$\mathbf{1 0}$ Click to expand the Coloring and Style section. From the Width list, choose 3.

## Reference Data

I In the Results toolbar, click Table.

2 In the Settings window for Table, locate the Data section.
3 Click Import.
4 Browse to the model's Application Libraries folder and double-click the file space_frame_instability_data.txt.

5 In the Label text field, type Reference Data.

## Table Graph I

I In the Model Builder window, right-click Load vs. Displacement and choose Table Graph.
2 In the Settings window for Table Graph, locate the Coloring and Style section.
3 Find the Line style subsection. From the Line list, choose None.
4 Find the Line markers subsection. From the Marker list, choose Cycle.
5 From the Positioning list, choose Interpolated.
6 In the Number text field, type 20.
7 Click to expand the Legends section. Select the Show legends check box.
8 From the Legends list, choose Manual.

9 In the table, enter the following settings:
$\frac{\frac{\text { Legends }}{\text { Ref. data }}}{\text { Load vs. Displacement }}$

I In the Model Builder window, click Load vs. Displacement.
2 In the Settings window for ID Plot Group, locate the Legend section.
3 From the Position list, choose Lower right.
4 In the Load vs. Displacement toolbar, click Plot.
5 Click the $\xrightarrow[H]{4}$ Zoom Extents button in the Graphics toolbar.
Next, you verify the critical buckling load by performing the linear buckling analysis.

## ADD STUDY

I In the Home toolbar, click ${ }^{\circ}$ Add Study to open the Add Study window.
2 Go to the Add Study window.
3 Find the Studies subsection. In the Select Study tree, select
Preset Studies for Selected Physics Interfaces>Linear Buckling.
4 Click Add Study in the window toolbar.
5 In the Home toolbar, click ${ }^{\circ}$ Add Study to close the Add Study window.
STUDY 2
In the Home toolbar, click $\equiv$ Compute.
The default plot from the buckling analysis shows the first buckling mode shape as shown in Figure 4.

## RESULTS

Mode Shape (beam)
I In the Mode Shape (beam) toolbar, click Plot.
2 Click the $\square$ Zoom Extents button in the Graphics toolbar.

