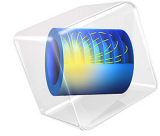# Small-Signal Analysis of a MOSFET

This model shows how to compute the AC characteristics of a MOSFET. Both the output conductance and the transconductance are computed as a function of the drain current.

Introduction

The small-signal analysis is a mathematical approximation that allows to see the effect of incremental changes on the inputs of an electrical device on its output characteristics. For the MOSFET, two small-signal outputs are particularly interesting to get as they provide information on the device performance. Those are the transconductance (g_m) and the output conductance (g_0).

The MOSFET's transconductance is defined as the ratio of the current change at the drain port to the voltage change at the gate port. The output conductance is defined as the ratio of the current change at the drain port to the voltage change at the drain port. The larger the transconductance, the greater the gain (amplification) the MOSFET is capable of delivering for a given gate voltage, when all other factors are held constant. Similarly, the larger the output conductance, the larger the drain current for a given drain voltage. Using small-signal analysis, an expression for the MOSFET's output current (drain current) can be obtained showing the influence of these two variables:

$$i_d = g_m v_{gs} + g_0 v_{ds}$$

Where i_d is the small-signal drain current, v_{gs} the small-signal gate to source voltage, and v_{ds} the small-signal drain to source voltage.

The present model used the small-signal analysis to compute both g_m and g_0. To achieve this, the COMSOL Multiphysics model uses two study steps: a stationary step followed by a frequency domain perturbation study. The Stationary study is used to compute the linearization point for the equation system, which is the solution for the DC operating point in the absence of any AC signals. The Frequency Domain Perturbation study step then computes the response of the system to small AC deviations from the linearization point, correctly accounting for nonlinearities in the equation system.

Model Definition

The geometry and operation of the device are discussed in the DC Characteristics of a MOS Transistor (MOSFET) model.

This model adds a small AC voltage to the DC drain and gate voltage using the harmonic perturbation sub-feature. The model adds two studies to the existing MOSFET model.

In the first study the DC gate voltage is varied from 0 to 8 V under a constant drain voltage of 2 V using a stationary step. Using the same parameters, a 1 mV, 10 MHz AC signal is superposed to the gate voltage using a frequency-domain perturbation step. This first study is performed to determine the transconductance (g_m) of the device as a function of the drain current.

In the second study it is the DC drain voltage that is varied from 0 to 4 V under a constant gate voltage of 2 V. The frequency domain perturbation step used the same 1 mV, 10 MHz AC signal but this time the signal is superposed to the drain voltage. The second study is performed to determine the output conductance (g_0) of the device as a function of the drain current.

Results and Discussion

When evaluating the Harmonic Perturbation it is important that the **Compute differential** check box is selected so that the COMSOL Multiphysics software differentiates the solution at the linearization point when evaluating the expression. The solutions to Harmonic Perturbation studies are in general complex valued, with the argument representing the phase of the signal. By default, the real part of the solution is plotted in postprocessing. Note that evaluating the global terminal current `semi.I0_2` for the Harmonic perturbation (with the **Compute differential** check box selected) gives the (complex) perturbation in the drain current, δI_{drain}.

The transconductance is the ratio of the current change at the drain port to the voltage change at the gate port. The transconductance (dI_{drain}/dV_{gate}) is obtained by evaluating the quantity `-semi.I0_2/Vac` (in COMSOL Multiphysics, the current at the boundary is positive when directed outward of the semiconductor material). It is not possible to evaluate the transconductance by evaluating the perturbation part of the expression `-semi.I0_2/abs(semi.V0_4)` because COMSOL linearizes the expression entered, computing the quantity $dI_{drain}/V_{gate}-(I_{drain}/V_{gate}^2)dV_{gate}$. A similar argument applies to the output conductance.

Figure 1 shows the transconductance as a function of the drain current. The transconductance increases rapidly with the drain current and reaches its maximum at about 80 μA. Figure 2 shows the output conductance as a function of the drain current. The output conductance decreases rapidly with the drain current.
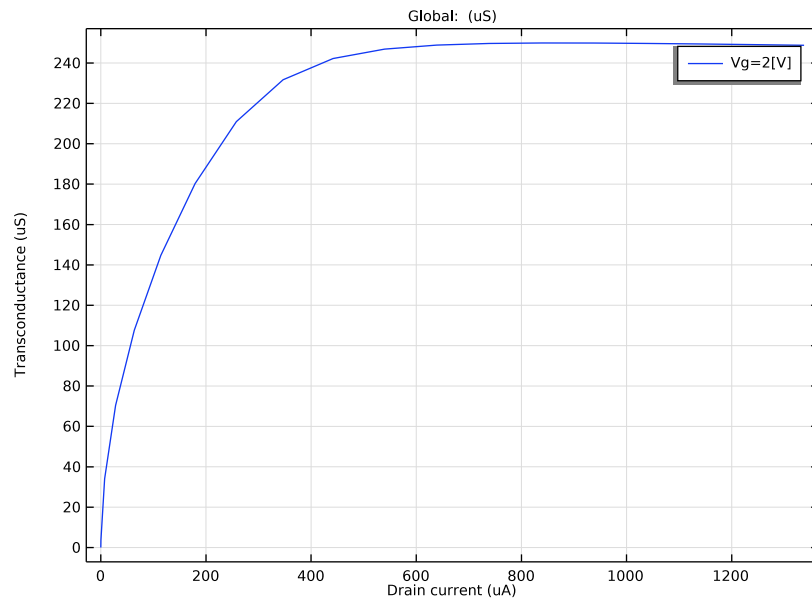
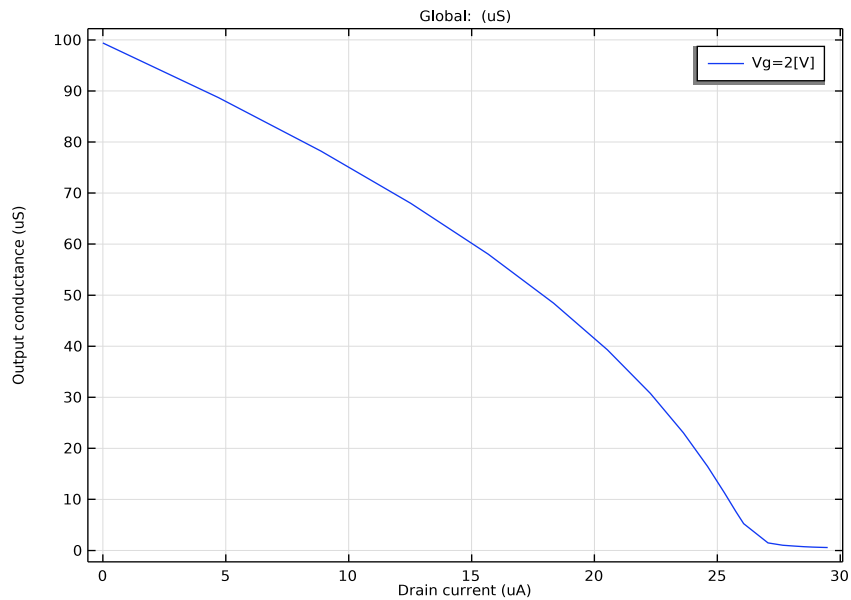*Figure 1: Transconductance as a function of the drain current under a gate voltage of 2 V.*

Figure 2: Output conductance as a function of the drain current under a gate voltage of 2 V.

Application Library path: Semiconductor_Module/Transistors/
mosfet_small_signal

Modeling Instructions

ROOT

Load the mosfet model.

APPLICATION LIBRARIES

1 From the **File** menu, choose **Application Libraries**.

2 In the **Application Libraries** window, select **Semiconductor Module>Transistors>mosfet** in the tree.

3 Click ◈ **Open**.

Add the small signal analysis parameters.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
Vac	1[mV]	0.001 V	Small signal voltage
freq	10[MHz]	1E7 Hz	Small signal frequency

COMPONENT 1 (COMP1)

In the **Model Builder** window, expand the **Component 1 (comp1)** node.

SEMICONDUCTOR (SEMI)

Add the harmonic perturbation to the drain and to the gate. They will be enabled/
disabled in the study settings later.

Metal Contact 2

1 Click the ⟨+⟩ **Zoom Extents** button in the **Graphics** toolbar.

2 In the **Model Builder** window, expand the **Component 1 (comp1)>Semiconductor (semi)**
node, then click **Metal Contact 2**.

Harmonic Perturbation 1

1 In the **Physics** toolbar, click 🔧 **Attributes** and choose **Harmonic Perturbation**.

2 In the **Settings** window for **Harmonic Perturbation**, locate the **Terminal** section.

3 In the V_0 text field, type Vac.

Thin Insulator Gate 1

In the **Model Builder** window, under **Component 1 (comp1)>Semiconductor (semi)** click
Thin Insulator Gate 1.

Harmonic Perturbation 1

1 In the **Physics** toolbar, click 🔧 **Attributes** and choose **Harmonic Perturbation**.

2 In the **Settings** window for **Harmonic Perturbation**, locate the **Terminal** section.

3 In the V_0 text field, type Vac.

Rename study 1 and study 2.

ID VS. VG

1 In the **Model Builder** window, right-click **Study 1** and choose **Rename**.

2 In the **Rename Study** dialog box, type Id vs. Vg in the **New label** text field.

3 Click **OK**.

ID VS. VD

1 In the **Model Builder** window, right-click **Study 2** and choose **Rename**.

2 In the **Rename Study** dialog box, type Id vs. Vd in the **New label** text field.

3 Click **OK**.

Change the value for the Vd parameter to 2V.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
Vd	2[V]	2 V	Drain voltage

Add a new study to compute the transconductance, gm. The study will consist of a stationary step followed by a frequency-domain, perturbation step. Both steps will compute values for a range of Vg potentials and a set Vd potential of 2V.

ADD STUDY

1 In the **Home** toolbar, click ⬡ **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click ⬡ **Add Study** to close the **Add Study** window.

Step 1: Stationary

In order to get the solution to converge it is often useful to provide a good initial starting point for the solver.

1 In the **Settings** window for **Stationary**, click to expand the **Values of Dependent Variables** section.

2 Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.

3 From the **Method** list, choose **Solution**.

4 From the **Study** list, choose **Id vs. Vg, Stationary**.

5 From the **Parameter value (Vg (V),Vd (V))** list, choose **1: Vg=0 V, Vd=0.01 V**.

6 In the **Model Builder** window, click **Study 3**.

7 In the **Settings** window for **Study**, type `gm vs. Id` in the **Label** text field.

8 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Set up an auxiliary continuation sweep for the 'Vg' parameter.

9 In the **Model Builder** window, click **Step 1: Stationary**.

10 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

11 Select the **Auxiliary sweep** check box.

12 Click ╋ **Add**.

13 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Vg (Gate voltage)	`range(0,0.4,8)`	V

Frequency Domain Perturbation

1 In the **Study** toolbar, click ⟋ **Study Steps** and choose **Frequency Domain> Frequency Domain Perturbation**.

2 In the **Settings** window for **Frequency Domain Perturbation**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `freq`.

4 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** check box.

5 Click ╋ **Add**.

6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Vg (Gate voltage)	range(0,0.4,8)	V

7 Locate the **Physics and Variables Selection** section. Select the
  Modify model configuration for study step check box.

8 In the tree, select **Component 1 (comp1)>Semiconductor (semi)>Metal Contact 2>
  Harmonic Perturbation 1**.

9 Right-click and choose **Disable**.

10 In the **Study** toolbar, click ▬ **Compute**.

Add a 1d plot to show the transconductance.

RESULTS

gm vs. Id

1 In the **Home** toolbar, click ⧉ **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type gm vs. Id in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **gm vs. Id/Solution 3 (sol3)**.

4 Locate the **Plot Settings** section.

5 Select the **x-axis label** check box. In the associated text field, type Drain current (uA).

6 Select the **y-axis label** check box. In the associated text field, type Transconductance
  (uS).

Global 1

1 Right-click **gm vs. Id** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
semi.I0_2/Vac	uS	

4 Select the **Compute differential** check box.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 Click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the
  menu, choose **Component 1 (comp1)>Semiconductor>Terminals>semi.I0_2 -
  Terminal current - A**.

7 Locate the **x-Axis Data** section. From the **Expression evaluated for** list, choose **Static solution**.

8 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
Vg=2[V]

10 Locate the **x-Axis Data** section. From the **Unit** list, choose **µA**.

11 In the **gm vs. Id** toolbar, click ⬛ **Plot**.

Add a new study to compute the output conductance, g0. The study will consist of a stationary step followed by a frequency-domain, perturbation step. Both steps will compute values for a range of Vd potentials and a set Vg potential of 2V.

ADD STUDY

1 In the **Home** toolbar, click ⬵ **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click ⬵ **Add Study** to close the **Add Study** window.

G0 VS. ID

1 In the **Model Builder** window, click **Study 4**.

2 In the **Settings** window for **Study**, type g0 vs. Id in the **Label** text field.

3 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

Step 1: Stationary

Set up an auxiliary continuation sweep for the 'Vd' parameter.

1 In the **Model Builder** window, under **g0 vs. Id** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Study Extensions** section.

3 Select the **Auxiliary sweep** check box.

4 Click ＋ **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Vd (Drain voltage)	`range(0,0.05,0.6) range(1, 0.5,4)`	V

Frequency Domain Perturbation

I In the **Study** toolbar, click ⟋ **Study Steps** and choose **Frequency Domain>
Frequency Domain Perturbation**.

2 In the **Settings** window for **Frequency Domain Perturbation**, locate the **Study Settings**
section.

3 In the **Frequencies** text field, type `freq`.

4 Locate the **Study Extensions** section. Select the **Auxiliary sweep** check box.

5 Click ╋ **Add**.

6 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Vd (Drain voltage)	`range(0,0.05,0.6) range(1, 0.5,4)`	V

7 Locate the **Physics and Variables Selection** section. Select the
Modify model configuration for study step check box.

8 In the tree, select **Component 1 (comp1)>Semiconductor (semi)>Thin Insulator Gate 1>
Harmonic Perturbation 1**.

9 Right-click and choose **Disable**.

Add a 1d plot to show the output conductance.

I0 In the **Study** toolbar, click ▭ **Compute**.

RESULTS

g0 vs. Id

I In the **Home** toolbar, click ▤ **Add Plot Group** and choose **ID Plot Group**.

2 In the **Settings** window for **ID Plot Group**, type g0 vs. Id in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **g0 vs. Id/Solution 5 (sol5)**.

4 Locate the **Plot Settings** section.

5 Select the **x-axis label** check box. In the associated text field, type Drain current (uA).

6 Select the **y-axis label** check box. In the associated text field, type Output conductance (uS).

Global 1

1 Right-click **g0 vs. Id** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
semi.I0_2/Vac	uS	

4 Select the **Compute differential** check box.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 Click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)>Semiconductor>Terminals>semi.I0_2 - Terminal current - A**.

7 Locate the **x-Axis Data** section. From the **Unit** list, choose **µA**.

8 From the **Expression evaluated for** list, choose **Static solution**.

9 Locate the **Legends** section. From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
Vg=2[V]

11 In the **g0 vs. Id** toolbar, click **Plot**.