Created in COMSOL Multiphysics 6.1

Critical Speed of a Dual Rotor System

Introduction

Dual shaft systems with intershaft bearings are becoming a standard configuration for gas turbine engines, where high power output is required. These systems consist of two coaxial rotors (shafts) running at different speeds, and interlinked through a multi-spool bearing. In this example, an eigenfrequency analysis is performed for such a dual rotor system to determine critical speeds. Cross exciting vibrations through the multi-spool bearing couple the dynamic behavior of the two rotors.

Model Definition

The model consists of two coaxial rotors connected through an intershaft bearing. The solid inner rotor is supported by bearings at both ends, at station 1 and station 6. The left end of the hollow outer rotor (station 7) is supported by a bearing. At the right end of the outer rotor (station 10), a multi-spool bearing provides mutual support between the inner and outer rotors. The rotor configuration is shown in Figure 1.

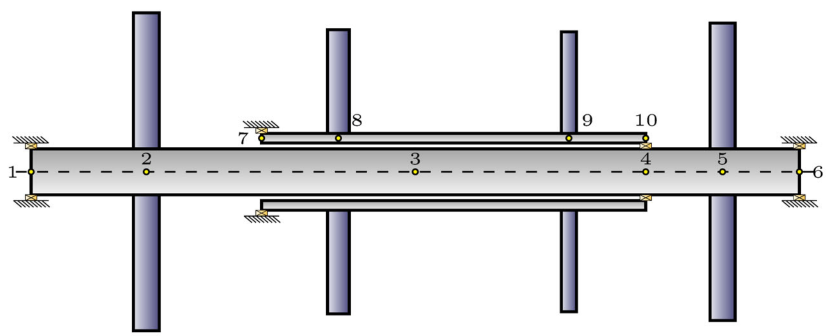

Figure 1: Rotor configuration.

Two disks are mounted on the inner rotor, at station 2 and station 5, and two disks are also mounted on the outer rotor, at station 8 and station 9. The positions of the stations, with station 1 as reference, are given in Table 1.

TABLE 1: POSITIONS OF THE STATIONS.

STATION	POSITION
1	0 cm
2	7.62 cm
3	25.4 cm
4	40.64 cm

TABLE 1: POSITIONS OF THE STATIONS.

STATION	POSITION
5	45.72 cm
6	50.8 cm
7	15.24 cm
8	20.32 cm
9	35.56 cm
10	40.64 cm

The properties for the rotors are given in the Table 2.

TABLE 2: PROPERTIES OF THE ROTORS.

PROPERTY	VALUE
Density ρ	8304 kg/m^3
Young's modulus E	206.9 GPa
Poisson's ratio ν	0.3
Inner rotor radius r_1	1.52 cm
Outer rotor inner radius r_{2i}	1.905 cm
Outer rotor outer radius r_{2o}	2.54 cm

The properties of the mounted disks are given in the Table 3.

TABLE 3: PROPERTIES OF THE DISKS.

STATION	MASS	POLAR MOMENT OF INERTIA	DIAMETRAL MOMENT OF INERTIA
2	4.904 kg	0.02712 kg/m^2	0.01356 kg/m^2
5	4.203 kg	0.02034 kg/m^2	0.01017 kg/m^2
8	3.327 kg	0.01469 kg/m^2	0.007345 kg/m^2
9	2.227 kg	0.00972 kg/m^2	0.00486 kg/m^2

All the bearings are isotropic. The stiffnesses of the bearings at different stations are given in Table 4.

TABLE 4: BEARING STIFFNESS.

STATION	STIFFNESS
1	27.95 MN/m
4–10	8.7598 MN/m
6	17.519 MN/m
7	17.519 MN/m

Figure 2 shows the whirl plot for the first mode (backward whirl, $f = 70.061$ Hz) at 25,000 rpm. To relate this plot to the rotor system, the rotors, bearings and disks are shown beneath it.

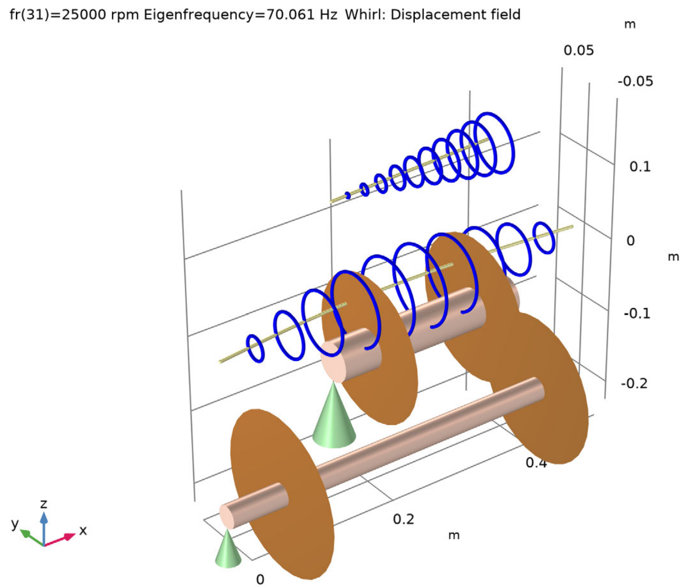

Figure 2: Whirl plot for the first mode.

Whirl plots for other modes are shown in Figure 3. The figure shows that the inner rotor exhibits bending modes, while the outer rotor exhibits rigid body modes.

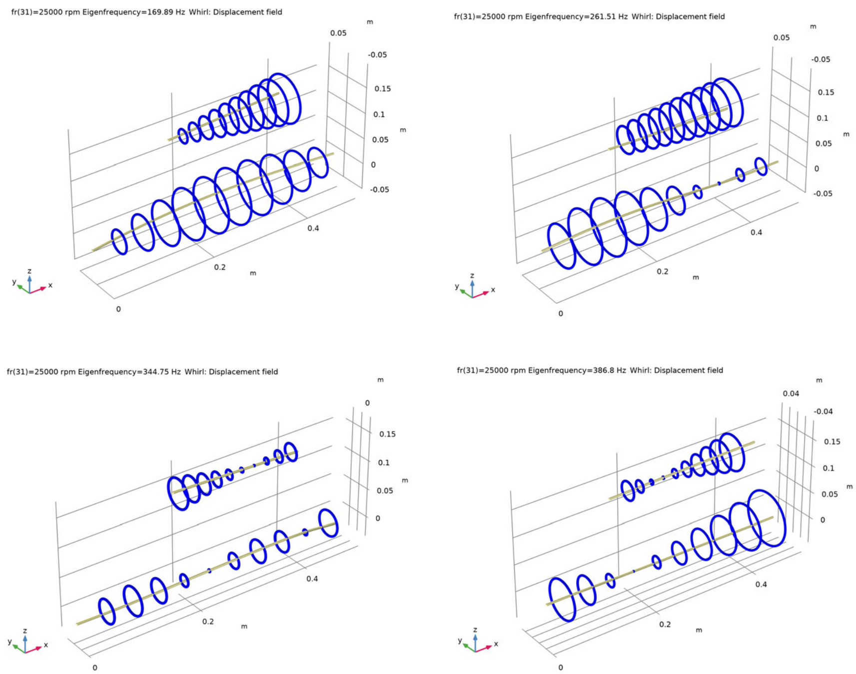

Figure 3: Mode shapes for different frequencies.

A Campbell plot for the dual rotor system is shown in Figure 4. The two lines Ω_1 and Ω_2 represent the speeds of rotors 1 and 2 (inner and outer rotors), and the dashed and dotted blue lines represent natural frequencies due to forward and backward whirl, respectively. The critical speeds of the inner rotor are compared in Table 5 to critical speeds of Ref. 1.

TABLE 5: COMPARISON OF CRITICAL SPEEDS FOR THE INNER ROTOR.

MODE	COMSOL (RAD/S)	REF. 1 (RAD/S)
First (backward)	658	660
First (forward)	866	863
Second (backward)	1430	1423
Second (forward)	1612	1606
Third (backward)(	2118	2125
Third (forward)	2270	2283

.

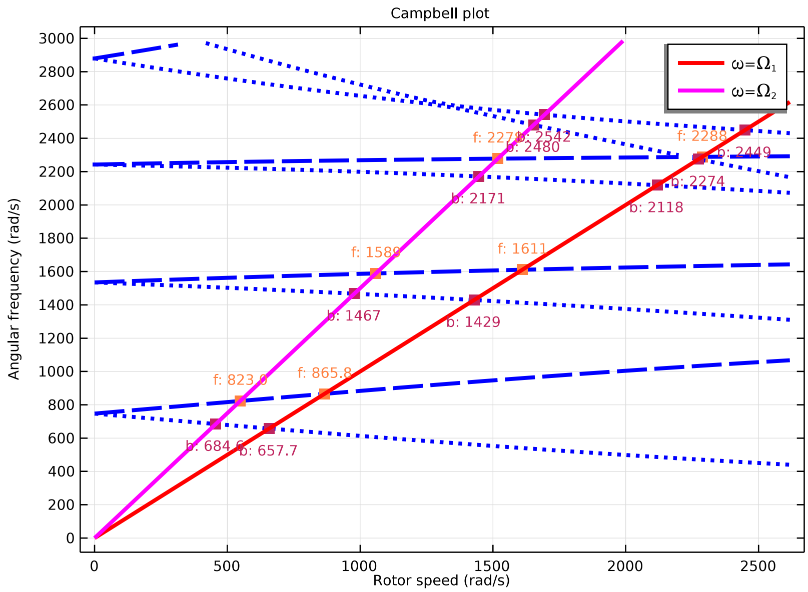

Figure 4: Campbell plot for the rotor system.

The critical speeds of the outer rotor are compared in Table 6 to critical speeds of Ref. 1.

TABLE 6: COMPARISON OF CRITICAL SPEEDS FOR THE OUTER ROTOR.

MODE	COMSOL (RAD/S)	REF. 1 (RAD/S)
First (backward)	685	687
First (forward)	823	822
Second (backward)	1468	1462
Second (forward)	1589	1584
Third (backward)	2171	2175
Third (forward)	2278	2274

Reference

1. J.S. Rao., *Rotor Dynamics*, example 8.11, pp. 266–269, New Age International (P) Limited, 2014.

Application Library path: `Rotordynamics_Module/Verification_Examples/dual_rotors`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1. In the **Model Wizard** window, click **3D**.
2. In the **Select Physics** tree, select **Structural Mechanics>Rotordynamics>Beam Rotor (rotbm)**.
3. Click **Add**.
4. Click **Study**.
5. In the **Select Study** tree, select **General Studies>Eigenfrequency**.
6. Click **Done**.

Create a list of parameters for the geometry of the rotors.

GLOBAL DEFINITIONS

Parameters: Geometry

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, type `Parameters: Geometry` in the **Label** text field.
3. Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
x1	0[cm]	0 m	Position of station 1
x2	7.62[cm]	0.0762 m	Position of station 2
x3	25.4[cm]	0.254 m	Position of station 3
x4	40.64[cm]	0.4064 m	Position of station 4
x5	45.72[cm]	0.4572 m	Position of station 5

Name	Expression	Value	Description
x6	50.8[cm]	0.508 m	Position of station 6
x7	15.24[cm]	0.1524 m	Position of station 7
x8	20.32[cm]	0.2032 m	Position of station 8
x9	35.56[cm]	0.3556 m	Position of station 9
x10	40.64[cm]	0.4064 m	Position of station 10
r1	1.52[cm]	0.0152 m	Radius of the inner rotor
r2i	1.905[cm]	0.01905 m	Inner radius of the outer rotor
r2o	2.54[cm]	0.0254 m	Outer radius of the outer rotor

Create a list of parameters for the bearing properties.

Parameters: Bearing

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Bearing` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
k1	27.95e6[N/m]	2.795E7 N/m	Stiffness, bearing at station 1
k4	8.7598e6[N/m]	8.7598E6 N/m	Stiffness, bearing between stations 4 and 10
k6	17.519e6[N/m]	1.7519E7 N/m	Stiffness, bearing at station 6
k7	17.519e6[N/m]	1.7519E7 N/m	Stiffness, bearing at station 7

Create a list of parameters for the material properties.

Parameters: Material

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Material` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
Es	206.9[GPa]	2.069E11 Pa	Young's modulus of the rotors
rhos	8304[kg/m^3]	8304 kg/m³	Density of the rotors
nus	0.3	0.3	Poisson's ratio of the rotors
m2	4.904[kg]	4.904 kg	Mass at station 2
m5	4.203[kg]	4.203 kg	Mass at station 5
m8	3.327[kg]	3.327 kg	Mass at station 8
m9	2.227[kg]	2.227 kg	Mass at station 9
Ip2	0.02712[kg*m^2]	0.02712 kg·m²	Polar moment of inertia at station 2
Ip5	0.02034[kg*m^2]	0.02034 kg·m²	Polar moment of inertia at station 5
Ip8	0.01469[kg*m^2]	0.01469 kg·m²	Polar moment of inertia at station 8
Ip9	0.00972[kg*m^2]	0.00972 kg·m²	Polar moment of inertia at station 9
Id2	Ip2/2	0.01356 kg·m²	Diametral moment of inertia at station 2
Id5	Ip5/2	0.01017 kg·m²	Diametral moment of inertia at station 5
Id8	Ip8/2	0.007345 kg·m²	Diametral moment of inertia at station 8
Id9	Ip9/2	0.00486 kg·m²	Diametral moment of inertia at station 9

Finally, create a list of parameters for the angular speeds of the rotors.

Parameters: Angular speed

1 In the **Home** toolbar, click Pi **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, type `Parameters: Angular speed` in the **Label** text field.

3 Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
fr	1000[rpm]	16.667 1/s	Angular speed of the inner rotor
fr2	1.5*fr	25 1/s	Angular speed of the outer rotor

Now, create the lines (polygons) representing the axles of the rotors. For coaxial rotors, these lines would overlap. Here, create the lines with an offset for clarity and to facilitate making selections for various features in the instructions that follow.

GEOMETRY 1

Polygon 1 (pol1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 In the table, enter the following settings:

x (m)	y (m)	z (m)
x1	0	0
x2	0	0
x3	0	0
x4	0	0
x5	0	0
x6	0	0

Add the selection as **Inner Rotor** for later use.

4 Locate the **Selections of Resulting Entities** section. Find the **Cumulative selection** subsection. Click **New**.

5 In the **New Cumulative Selection** dialog box, type `Inner Rotor` in the **Name** text field.

6 Click **OK**.

Polygon 2 (pol2)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 In the table, enter the following settings:

x (m)	y (m)	z (m)
x7	0	6*r2o
x8	0	6*r2o
x9	0	6*r2o
x10	0	6*r2o

Add the selection as **Outer Rotor** for later use.

4 Locate the **Selections of Resulting Entities** section. Find the **Cumulative selection** subsection. Click **New**.

5 In the **New Cumulative Selection** dialog box, type `Outer Rotor` in the **Name** text field.

6 Click **OK**.

7 In the **Settings** window for **Polygon**, click **Build All Objects**.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	Es	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	nus	1	Young's modulus and Poisson's ratio
Density	rho	rhos	kg/m³	Basic

BEAM ROTOR (ROTBM)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Beam Rotor (rotbm)**.

2 In the **Settings** window for **Beam Rotor**, locate the **Rotor Speed** section.

3 In the text field, type `fr`.

Rotor Cross Section 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Beam Rotor (rotbm)** click **Rotor Cross Section 1**.

2 In the **Settings** window for **Rotor Cross Section**, locate the **Cross-Section Definition** section.

3 In the d_o text field, type `2*r1`.

Rotor Cross Section 2

1 In the **Physics** toolbar, click **Edges** and choose **Rotor Cross Section**.

2 In the **Settings** window for **Rotor Cross Section**, locate the **Edge Selection** section.

3 From the **Selection** list, choose **Outer Rotor**.

4 Locate the **Cross-Section Definition** section. From the **Section type** list, choose **Pipe**.

5 In the d_o text field, type `2*r2o`.

6 In the d_i text field, type `2*r2i`.

Change Rotor Speed 1

1 In the **Physics** toolbar, click **Edges** and choose **Change Rotor Speed**.

2 In the **Settings** window for **Change Rotor Speed**, locate the **Edge Selection** section.

3 From the **Selection** list, choose **Outer Rotor**.

4 Locate the **Rotor Speed** section. In the text field, type `fr2`.

Disk 1

1 In the **Physics** toolbar, click **Points** and choose **Disk**.

2 Select Point 2 only.

3 In the **Settings** window for **Disk**, locate the **Disk Properties** section.

4 In the m text field, type `m2`.

5 In the I_p text field, type `Ip2`.

6 In the I_d text field, type `Id2`.

Disk 2

1 In the **Physics** toolbar, click **Points** and choose **Disk**.

2 Select Point 9 only.

3 In the **Settings** window for **Disk**, locate the **Disk Properties** section.

4 In the m text field, type `m5`.

5 In the I_p text field, type `Ip5`.

6 In the I_d text field, type `Id5`.

Disk 3

1 In the **Physics** toolbar, click **Points** and choose **Disk**.

2 Select Point 4 only.

3 In the **Settings** window for **Disk**, locate the **Disk Properties** section.

4 In the m text field, type `m8`.

5 In the I_p text field, type `Ip8`.

6 In the I_d text field, type `Id8`.

Disk 4

1 In the **Physics** toolbar, click **Points** and choose **Disk**.

2 Select Point 6 only.

3 In the **Settings** window for **Disk**, locate the **Disk Properties** section.

4 In the m text field, type `m9`.

5 In the I_p text field, type `Ip9`.

6 In the I_d text field, type `Id9`.

Journal Bearing 1

1 In the **Physics** toolbar, click **Points** and choose **Journal Bearing**.

2 Select Point 1 only.

3 In the **Settings** window for **Journal Bearing**, locate the **Bearing Properties** section.

4 From the **Bearing model** list, choose **Total spring and damping constant**.

5 In the $\mathbf{k}_u$ table, enter the following settings:

k1	0
0	k1

Multi-Spool Bearing 1

1 In the **Physics** toolbar, click **Points** and choose **Multi-Spool Bearing**.

2 Select Point 7 only.

3 In the **Settings** window for **Multi-Spool Bearing**, locate the **Destination Point Selection** section.

4 Click to select the **Activate Selection** toggle button.

5 Select Point 8 only.

6 Locate the **Bearing Properties** section. From the **Displacement connection** list, choose **Flexible**.

7 In the $\mathbf{k}_u$ table, enter the following settings:

k4	0
0	k4

Journal Bearing 2

1 In the **Physics** toolbar, click **Points** and choose **Journal Bearing**.

2 Select Point 10 only.

3 In the **Settings** window for **Journal Bearing**, locate the **Bearing Properties** section.

4 From the **Bearing model** list, choose **Total spring and damping constant**.

5 In the $\mathbf{k}_u$ table, enter the following settings:

k6	0
0	k6

Journal Bearing 3

1 In the **Physics** toolbar, click **Points** and choose **Journal Bearing**.

2 Select Point 3 only.

3 In the **Settings** window for **Journal Bearing**, locate the **Bearing Properties** section.

4 From the **Bearing model** list, choose **Total spring and damping constant**.

5 In the $\mathbf{k}_u$ table, enter the following settings:

k7	0
0	k7

STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
fr (Angular speed of the inner rotor)	`range(0,1000,20000) range(20500,500,25000)`	`rpm`

Step 1: Eigenfrequency

1 In the **Model Builder** window, click **Step 1: Eigenfrequency**.

2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.

3 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type 8.

4 In the **Study** toolbar, click **= Compute**.

A result parameter **scale** is used to create and offset between the undeformed geometry and the whirl plot. Increase the value to make them sufficiently separated.

RESULTS

Parameters

1 In the **Results** toolbar, click Pi **Parameters**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
	1.4	1.4	

The default whirl plot is shown in Figure 2.

Whirl (rotbm)

1 In the **Model Builder** window, click **Whirl (rotbm)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.

3 From the **View** list, choose **New view**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

5 In the **Whirl (rotbm)** toolbar, click **Plot**.

Now, disable the nodes corresponding to the geometry in the **Whirl Plot** group to plot only the mode shapes. These plots are shown in Figure 3.

6 In the **Model Builder** window, expand the **Whirl (rotbm)** node.

Disk 1, Disk 2, Disk 3, Disk 4, Journal Bearing 1, Journal Bearing 2, Journal Bearing 3, Rotor

1 In the **Model Builder** window, under **Results>Whirl (rotbm)**, Ctrl-click to select **Rotor**, **Disk 1**, **Disk 2**, **Disk 3**, **Disk 4**, **Journal Bearing 1**, **Journal Bearing 2**, and **Journal Bearing 3**.

2 Right-click and choose **Disable**.

Whirl (rotbm)

1 In the **Model Builder** window, click **Whirl (rotbm)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Eigenfrequency (Hz)** list, choose **169.89**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

5 In the **Whirl (rotbm)** toolbar, click **Plot**.

6 From the **Eigenfrequency (Hz)** list, choose **261.51**.

7 In the **Whirl (rotbm)** toolbar, click **Plot**.

8 From the **Eigenfrequency (Hz)** list, choose **344.75**.

9 In the **Whirl (rotbm)** toolbar, click **Plot**.

10 From the **Eigenfrequency (Hz)** list, choose **386.8**.

11 In the **Whirl (rotbm)** toolbar, click **Plot**.

You can now enable the nodes corresponding to the geometry to revert the plot to the default state.

Disk 1, Disk 2, Disk 3, Disk 4, Journal Bearing 1, Journal Bearing 2, Journal Bearing 3, Rotor

1 In the **Model Builder** window, under **Results>Whirl (rotbm)**, Ctrl-click to select **Rotor**, **Disk 1**, **Disk 2**, **Disk 3**, **Disk 4**, **Journal Bearing 1**, **Journal Bearing 2**, and **Journal Bearing 3**.

2 Right-click and choose **Enable**.

Whirl (rotbm)

1 Click the **Zoom Extents** button in the **Graphics** toolbar.

The predefined Campbell plot only shows the ω=Ω curve for the primary rotor (inner rotor). Modify the **Graph Marker** settings to add a similar curve for the outer rotor. The Campbell plot is shown in Figure 4.

2 In the **Home** toolbar, click **Add Predefined Plot**.

ADD PREDEFINED PLOT

1 Go to the **Add Predefined Plot** window.

2 In the tree, select **Study 1/Parametric Solutions 1 (sol2)>Beam Rotor>Campbell Plot (rotbm)**.

3 Click **Add Plot** in the window toolbar.

4 In the **Home** toolbar, click **Add Predefined Plot**.

RESULTS

Campbell Plot (rotbm)

1 In the **Model Builder** window, under **Results** click **Campbell Plot (rotbm)**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **x-axis label** check box. In the associated text field, type `Rotor speed (rad/s)`.
4 In the **Model Builder** window, expand the **Campbell Plot (rotbm)** node.

Graph Marker 1

1 In the **Model Builder** window, expand the **Results>Campbell Plot (rotbm)>Forward Whirl Mode** node, then click **Graph Marker 1**.
2 In the **Settings** window for **Graph Marker**, locate the **Display** section.
3 From the **Line type** list, choose **General**.
4 In the A text field, type `1 1.5`.
5 In the B text field, type `-1`.
6 In the C text field, type `0`.
7 Locate the **Text Format** section. Clear the **Show x-coordinate** check box.

Graph Marker 1

1 In the **Model Builder** window, expand the **Results>Campbell Plot (rotbm)>Backward Whirl Mode** node, then click **Graph Marker 1**.
2 In the **Settings** window for **Graph Marker**, locate the **Display** section.
3 From the **Line type** list, choose **General**.
4 In the A text field, type `1 1.5`.
5 In the B text field, type `-1`.
6 In the C text field, type `0`.
7 Locate the **Text Format** section. Clear the **Show x-coordinate** check box.

omega=Omega

1 In the **Model Builder** window, under **Results>Campbell Plot (rotbm)** click **omega=Omega**.
2 In the **Settings** window for **Global**, click to expand the **Legends** section.
3 In the table, enter the following settings:

Legends
`\omega=\Omega<sub>1</sub>`

omega=Omega 1

1 Right-click **omega=Omega** and choose **Duplicate**.
2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`if(rotbm.Ovg<=0.8*rotbm.omega,2*pi[rad]*fr2, NaN)`		

4 Locate the **Legends** section. In the table, enter the following settings:

Legends
`\omega=\Omega<sub>2</sub>`

5 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Magenta**.

Campbell Plot (rotbm)

1 In the **Model Builder** window, click **Campbell Plot (rotbm)**.

2 In the **Campbell Plot (rotbm)** toolbar, click **Plot**.

Whirl (rotbm)

Click the **Zoom Extents** button in the **Graphics** toolbar.