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Two-Mirror Laser Cavity

Introduction

Lasers are ubiquitous in application areas such as cutting, ablation, telecommunication, and spectroscopy, among others. Typically, lasers are produced by a *laser cavity* or *optical cavity* containing a set of mirrors, a gain medium, and possibly some other optical components such as prisms or lenses.

Stability analysis of the laser cavity ensures that light remains confined in the cavity, allowing the laser to operate reliably. If the laser cavity is not stable, laser production may abruptly stop as light escapes from the cavity into the surroundings. The stability of the laser cavity can be analyzed by the standard ABCD matrix analysis based on the paraxial approximation, or alternatively by geometrical optics simulation.

In this model, two concave mirrors are placed at opposite ends of a laser cavity. This two-mirror cavity is the most basic configuration for studying laser stability. A single ray is released from a point within the cavity, initially with a very small angle relative to the optical axis. Then the ray is traced for a predefined time period that is sufficiently long for many reflections to occur. Ray tracing continues until the predefined computation time has passed if the laser cavity is stable, whereas the time-dependent study terminates earlier if the ray escapes from the cavity. A **Parametric Sweep** demonstrates the effect of cavity length on stability and compares the result with the ABCD matrix theory.

Model Definition

The laser cavity consists of two spherical end mirrors with radii of curvature both equal to $R = 1$ m. The mirrors are separated by distance L (SI unit: m) as shown in Figure 1. A ray is released from the center of one of the mirrors, at a very small angle to the optical axis. The ray is traced for a predefined total computation time, T_0 (SI unit: s), which is sufficient for it to be reflected a large number of times, at least several hundred reflections for the largest value of L.

The **Ray Termination** feature is used to end the time-dependent study early if the ray gets out of the cavity; in that case, the last computation time, T_1 (SI unit: s) is stored. The cavity stability is represented by the ratio T_1/T_0, with a value of 1 indicating that the ray is still inside the cavity and the configuration is stable.

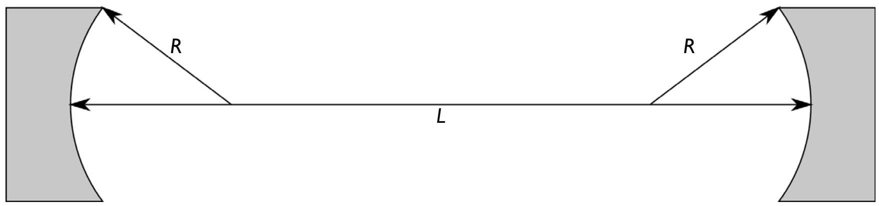

Figure 1: Optical layout of the laser cavity.

ABCD MATRIX THEORY

The result of the ray tracing analysis can be compared to an analytic solution based on ABCD matrix theory, as long as the paraxial approximation holds. In ABCD matrix theory, while following Hecht's notation (Ref. 1), a ray is characterized by the ray angle θ (SI unit: rad) and the ray position y (SI unit: m) relative to the optical axis in a 2-by-1 column vector as

$$\begin{bmatrix} \theta \\ y \end{bmatrix}$$

Elements of the optical system are represented as 2-by-2 matrices that are multiplied by this vector. Propagation through a distance L is denoted by the matrix

$$\begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix}$$

and reflection in a mirror with the radius of curvature R in the air by

$$\begin{bmatrix} -1 & \frac{2}{R} \\ 0 & 1 \end{bmatrix}$$

After the ray is reflected by both mirrors once and returns to its original position, the new angle θ and position y of the ray can be described by the matrix product of each propagation or reflection in the sequence, multiplied by the initial angle θ_0 and position y_0,

$$\begin{bmatrix} \theta \\ y \end{bmatrix} = \mathrm{T}\begin{bmatrix} \theta_0 \\ y_0 \end{bmatrix}$$

where

$$\mathrm{T} = \begin{bmatrix} -1 & -\frac{2}{R} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -L & 1 \end{bmatrix} \begin{bmatrix} -1 & \frac{2}{R} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix}$$

According to Kogelnik's stability theory (Ref. 2), the system is stable if the initial angle and position give bounded values when multiplied by an arbitrarily high power of the matrix **T**; this stability criterion can also be written as

$$-1 \le \frac{1}{2}\mathrm{Tr}(\mathrm{T}) \le 1$$

where **Tr** stands for the trace of a matrix. Some arithmetic reduces these inequalities to

$$0 \le \left(1 - \frac{L}{R}\right)^2 \le 1 \qquad (1)$$

which predicts that the stable range is $0 \le L \le 2R$. Note that the computation may result in unstable results for $L = R$ and $L = 2R,$ where the cavity is *marginally stable*.

Results and Discussion

Figure 2 shows the ray propagation when the distance between the mirrors is $L = 0.2$ m, the smallest parameter value used. The ray remains confined inside the cavity for the full duration of the study. If the total computation time is sufficiently long, the result means the cavity is stable for this particular parameter value.

Figure 3 is a 1D plot of the stability versus the cavity length, which shows good agreement between the computed results and the ABCD matrix theory. The two results differ when the value of L is slightly outside the region of stability, for example at $L = 2.2$ m. This is because the stability criterion derived from the ABCD matrix theory holds for an arbitrarily large number of reflections, whereas in the ray optics simulation the maximum number of reflections is finite.

At points of marginal stability, such as $L = 2.0$ m, the two results disagree because the analytic result from ABCD matrix theory is entered into the model as a smoothed step function.

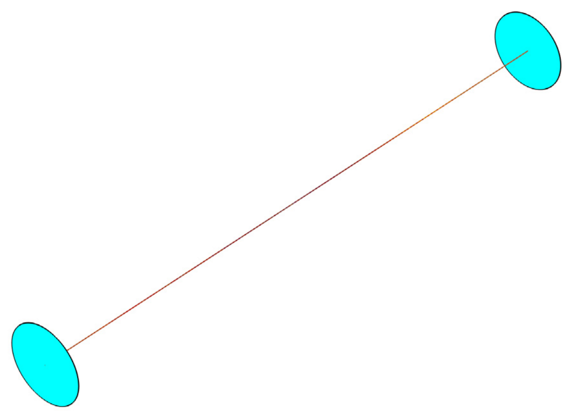

Figure 2: Ray tracing result for L = 0.2 m. The ray is confined in the cavity after the total computation time T0 = 1 µs.

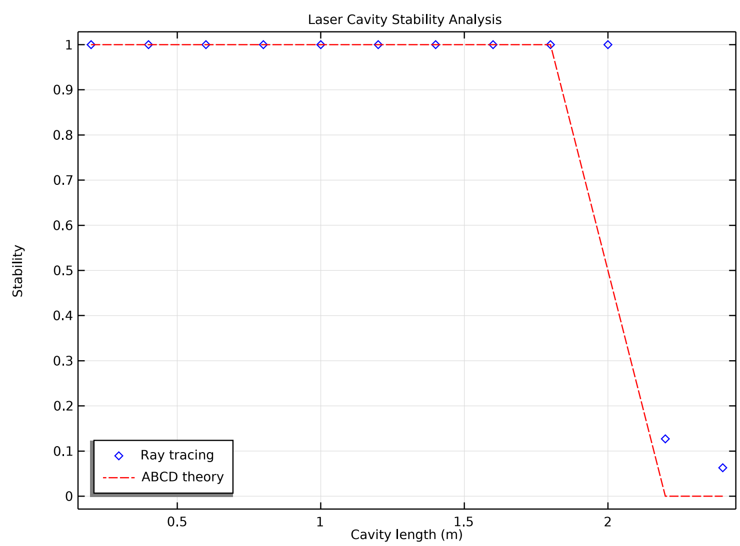

Figure 3: Stability plot as a function of the cavity length showing a good agreement with the ABCD matrix theory.

References

1. E. Hecht, *Optics*, 4th ed., Addison-Wesley, 1998.

2. H. Kogelnik and T. Li, “Laser beams and resonators,” *Applied Optics*, vol. 5, no. 10, pp. 1550–1567, 1966.

Application Library path: `Ray_Optics_Module/Laser_Cavities/two_mirror_laser_cavity`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

Load the global parameters for the laser cavity from a text file.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model’s Application Libraries folder and double-click the file `two_mirror_laser_cavity_parameters.txt`.

DEFINITIONS

Create a **Rectangle** function. This function will be used during postprocessing to define the theoretical stability criterion.

Rectangle 1 (rect1)

1 In the **Home** toolbar, click f(x) **Functions** and choose **Global>Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Parameters** section.

3 In the **Lower limit** text field, type 0.

4 In the **Upper limit** text field, type 2*R.

5 Click to expand the **Smoothing** section. In the **Size of transition zone** text field, type 0.001.

GEOMETRY 1

Create a laser cavity geometry.

PART LIBRARIES

1 In the **Home** toolbar, click **Windows** and choose **Part Libraries**.

2 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

3 In the **Part Libraries** window, select **Ray Optics Module>3D>Mirrors>spherical_mirror_3d** in the tree.

4 Click **Add to Geometry**.

5 In the **Select Part Variant** dialog box, select **Specify clear aperture diameter** in the **Select part variant** list.

6 Click **OK**.

GEOMETRY 1

Spherical Mirror 3D 1 (pi1)

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** click **Spherical Mirror 3D 1 (pi1)**.

2 In the **Settings** window for **Part Instance**, locate the **Input Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
R	-R	-1 m	Radius of curvature (+convex/-concave)
Tc	12[mm]	0.012 m	Center thickness

Name	Expression	Value	Description
d0	D	0.025 m	Mirror full diameter
d1	0	0 m	Mirror surface diameter
d_clear	0	0 m	Clear aperture diameter
d_hole	0	0 m	Center hole diameter
nix	0	0	Local optical axis, x-component
niy	0	0	Local optical axis, y-component
niz	-1	-1	Local optical axis, z-component
n_extra_r	0	0	Number of extra radial points
n_extra_a	0	0	Number of extra azimuthal points

4 Click to expand the **Boundary Selections** section. Click to select row number 2 in the table.

5 Click **New Cumulative Selection**.

6 In the **New Cumulative Selection** dialog box, type `Mirrors` in the **Name** text field.

7 Click **OK**.

Spherical Mirror 3D 2 (pi2)

1 Right-click **Component 1 (comp1)>Geometry 1>Spherical Mirror 3D 1 (pi1)** and choose **Duplicate**.

2 In the **Settings** window for **Part Instance**, locate the **Input Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
niz	1	1	Local optical axis, z-component

4 Locate the **Position and Orientation of Output** section. Find the **Displacement** subsection. In the **zw** text field, type `L`.

5 Locate the **Boundary Selections** section. In the table, enter the following settings:

Name	Keep	Physics	Contribute to
Mirror surface		√	Mirrors

6 Click **Build All Objects**. Compare the resulting geometry to Figure 1.

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Domain Selection** section.

3 Click **Clear Selection**. The mirror domains can be excluded because rays never actually pass through them in this model.

4 Locate the **Ray Release and Propagation** section. In the **Maximum number of secondary rays** text field, type `0`.

Mirror 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Mirror**.

2 In the **Settings** window for **Mirror**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Mirrors**. These are the mirror surfaces that face the inside of the cavity.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Ray Direction Vector** section.

3 Specify the $\mathbf{L}_0$ vector as

sin(th)	x
0	y
cos(th)	z

Ray Termination 1

1 In the **Physics** toolbar, click **Global** and choose **Ray Termination**.

2 In the **Settings** window for **Ray Termination**, locate the **Termination Criteria** section.

3 From the **Spatial extents of ray propagation** list, choose **Bounding box, from geometry**.

MESH 1

Adjust the default mesh to improve the resolution of the curved mirror surfaces.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Finer**.

4 Locate the **Sequence Type** section. From the list, choose **User-controlled mesh**.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, click to expand the **Element Size Parameters** section.

3 In the **Maximum element size** text field, type `D/20`.

4 Click Build All.

STUDY 1

Add a **Parametric Sweep** to vary the cavity length to see the effect on the stability.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, click to select the cell at row number 1 and column number 2.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
L (Cavity length)		m

6 Click **Range**.

7 In the **Range** dialog box, type `0.2` in the **Start** text field.

8 In the **Step** text field, type `0.2`.

9 In the **Stop** text field, type `2.4`.

10 Click **Replace**.

Step 1: Ray Tracing

Add a **Stop condition** to end the simulation if the ray gets out of the cavity.

1 In the **Model Builder** window, click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 In the **Output times** text field, type `0 T0`.

4 From the **Stop condition** list, choose **No active rays remaining**.

5 In the **Study** toolbar, click **Compute**.

RESULTS

Ray Trajectories (gop)

1 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

2 From the **Parameter value (L)** list, choose **0.2**.

3 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

4 In the **Title** text area, type `Ray trajectory, L=0.2`.

Surface 1

1 Right-click **Ray Trajectories (gop)** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Coloring and Style** section.

3 From the **Coloring** list, choose **Uniform**.

4 From the **Color** list, choose **Cyan**.

Ray Trajectories 1

1 In the **Model Builder** window, click **Ray Trajectories 1**.

2 In the **Settings** window for **Ray Trajectories**, locate the **Extra Time Steps** section.

3 From the **Maximum number of extra time steps rendered** list, choose **All**. This ensures that the ray path is rendered correctly even for the smallest distance between the mirrors, when there are several thousand reflections.

Color Expression 1

1 In the **Model Builder** window, expand the **Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Coloring and Style** section.

3 Clear the **Color legend** check box.

4 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

For values of L that satisfy the stability criterion (Equation 1), the plot should look like Figure 2. Otherwise, the ray eventually escapes from the cavity.

1D Plot Group 2

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Ray 1**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Plot Settings** section.

6 Select the **y-axis label** check box. In the associated text field, type `Stability`.

7 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

8 In the **Title** text area, type `Laser Cavity Stability Analysis`.

9 Locate the **Legend** section. From the **Position** list, choose **Lower left**.

Global 1

1 Right-click **1D Plot Group 2** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`t/T0`	`1`	`Ray tracing`

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **All solutions**.

5 From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `L`.

7 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

8 From the **Color** list, choose **Blue**.

9 Find the **Line markers** subsection. From the **Marker** list, choose **Diamond**.

10 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

11 In the table, enter the following settings:

Legends
`Ray tracing`

Global 2

1 In the **Model Builder** window, right-click **1D Plot Group 2** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`rect1(L)`	`1`	`ABCD theory`

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **All solutions**.

5 From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `L`.

7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dashed**.

8 From the **Color** list, choose **Red**.

9 Locate the **Legends** section. From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
`ABCD theory`

11 In the **1D Plot Group 2** toolbar, click **Plot**. The plot should look like Figure 3.