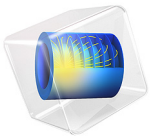
Created in COMSOL Multiphysics 6.1



Czerny-Turner Monochromator

Introduction

This model simulates a grating spectrometer in a crossed Czerny-Turner configuration. The model uses the Geometrical Optics interface to compute the positions of incident rays on the detector plane, from which the instrument's spectral resolution can be derived.

A Czerny-Turner spectrometer spatially separates polychromatic light into a series of monochromatic rays. The configuration includes a slit source, a spherical collimating mirror, a planar diffraction grating, a spherical imaging mirror, and an array charge coupled device (CCD) detector, see Figure 1.

With a given optical component arrangement and with the knowledge of the detector properties it is possible to determine the wavelength calibration as well as the spectral resolution of the instrument.

Model Definition

A Czerny-Turner spectrometer is usually designed with F-number > 3 to avoid large aberrations of the image at the detector. Accordingly, this model treats the polychromatic light source as a cone-based release of rays with F-number = 10. Note that the system's F-number is related to its numerical aperture NA by:

$$\mathrm{F-number} = \frac{1}{2\mathrm{NA}}$$

Where the numerical aperture NA is related to the half-angle of the maximum cone of light that can enter the system Θ:

$$\mathrm{NA} = n\sin(\Theta)$$

Where n is the refractive index of the propagation medium (air, $n = 1$).

The polychromatic source is simulated using twenty frequency values that are sampled from a uniform distribution. The corresponding free-space wavelength distribution has a minimum of λ=460 nm and a maximum of λ=840 nm.

The geometry of the spectrometer is shown in Figure 1. The rays are released from the front focal point of the collimating mirror. The collimating mirror is tilted with an angle θ_c to direct the collimated light toward the diffraction grating. The rays of diffraction order $m = 1$ are then directed toward the focusing mirror; because the ray trajectories of nonzero diffraction order are frequency-dependent, the rays of different frequency arrive at the focusing mirror at different positions and with different angles of incidence.

The focusing mirror reflects the rays onto the detector. The detector is composed of an array of $N = 3648$ pixels of width $w_p = 8$ µm. The geometrical parameters used in the model are listed in Table 1.

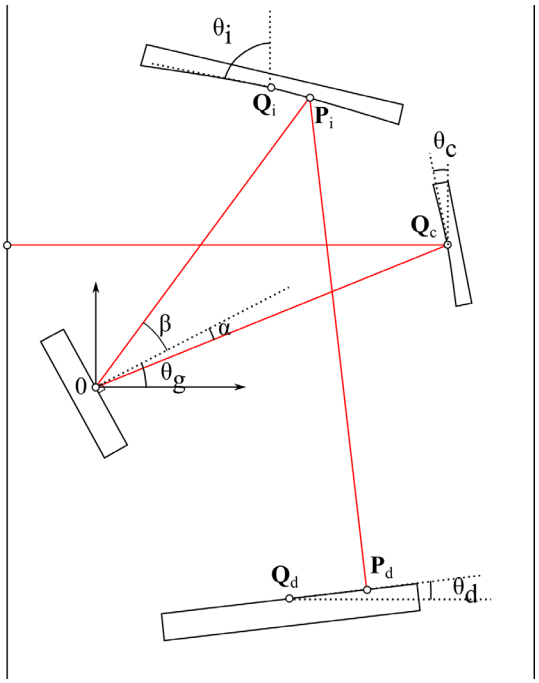


Figure 1: Typical crossed Czerny-Turner configuration. Numerical values are displayed in Table 1

TABLE 1: DEFINITION OF THE DESIGN PARAMETERS.

PARAMETERS	VALUE	DESCRIPTION
θ_g (deg)	28.76	Grating's angle
θ_c (deg)	11.0	Collimating mirror's angle
θ_i (deg)	77	Imaging mirror's angle
θ_d (deg)	6.76	Detector's angle
Q_i (mm)	(20.0,34.0)	Coordinates, imaging mirror
Q_c (mm)	(40,16.161)	Coordinates, collimating mirror
Q_d (mm)	(22.08,-24.12)	Coordinates, detector

TABLE 1: DEFINITION OF THE DESIGN PARAMETERS.

PARAMETERS	VALUE	DESCRIPTION
R_i (mm)	130	`Radius of curvature, imaging mirror`
R_c (mm)	100	`Radius of curvature, collimating mirror`

The ray trajectories can be used to compute the spectrometer's resolution. The pixel number p_{num} of a ray on the collector is

$$p_{\mathrm{num}} = \mathrm{ceil}\left(\frac{N}{2} - \frac{q_x - Q_{dx}}{w_p \cos(\theta_d)}\right)$$

Where N is the pixel number of the CCD, w_p the pixel's width, and q_x the x-coordinate of the particle hitting the detector. Q_{dx} and θ_d are respectively the detector's center x-coordinate and the angle of the CCD. The spectral (optical) resolution $\delta\lambda$ of the element can then be estimated by

$$\delta\lambda \propto \frac{\Delta\lambda}{N}\frac{w_i}{w_p}$$

Where $\Delta\lambda = 650$ nm is the spectral range of the detector, and w_i is the width of the slit's image on the CCD.

The knowledge of the image width on the detector is given by the distance between the two rays delimiting a pencil of rays of uniform wavelength on the detector. Figure 2 shows how this distance is evaluated.

It is possible to evaluate the image width using the following expression:

```
gop.gopmaxop1(if(samefreq,gop.gopmaxop1(if(samefreq,distance,0)),0))
```

where `gop.gopmaxop1` is a nonlocal coupling that returns the maximum value of an expression over all rays. Two nested maximum couplings are used because the resulting width should be the maximum distance from any ray to any other ray of the same frequency. The logical expression `samefreq` is used to evaluate the maximum only over rays of a single frequency. This variable is defined as

```
samefreq=abs(gop.nu-dest(gop.nu))<1[Hz]
```

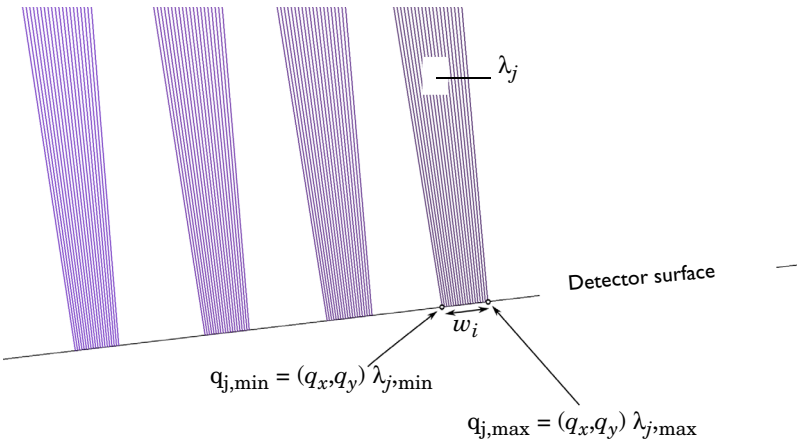


Figure 2: Determination of the image width for a given wavelength.

Results and Discussion

Figure 3 shows the ray trajectories in the spectrometer for an F-number of 10. The free-space wavelength is indicated by the color expression. After the rays are reflected by the grating, rays of different frequency propagate in different directions and arrive at different locations on the CCD. The pixel numbers corresponding to incident rays of each wavelength are plotted in Figure 4. The spectral resolution of the image is shown in Figure 5. The spectral resolution of a system determines the maximum number of spectral peaks that the spectrometer can resolve.

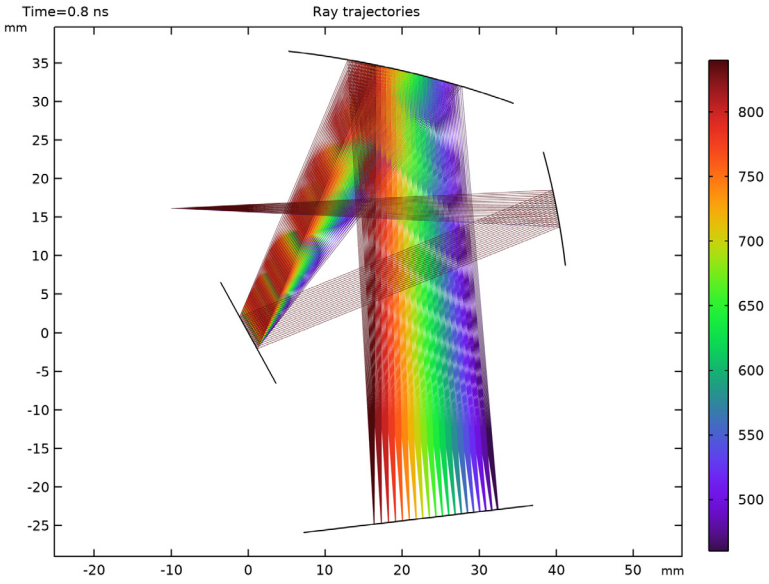


Figure 3: Wavelength separation on the CCD detector.

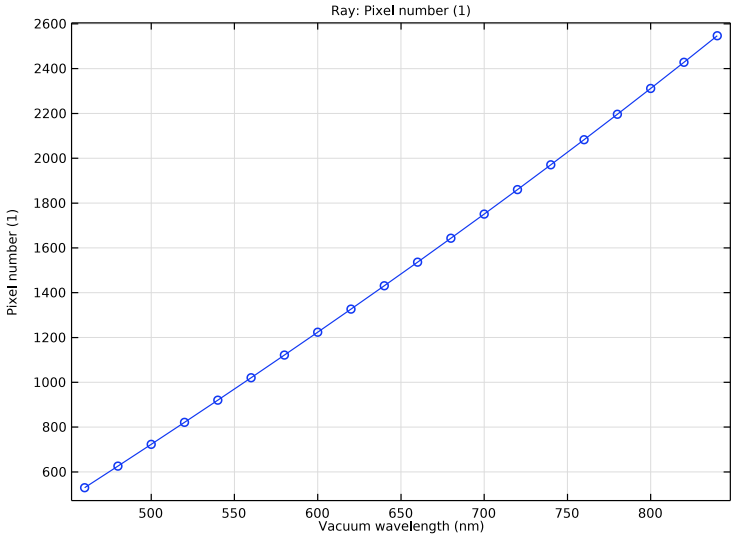


Figure 4: Wavelength calibration.

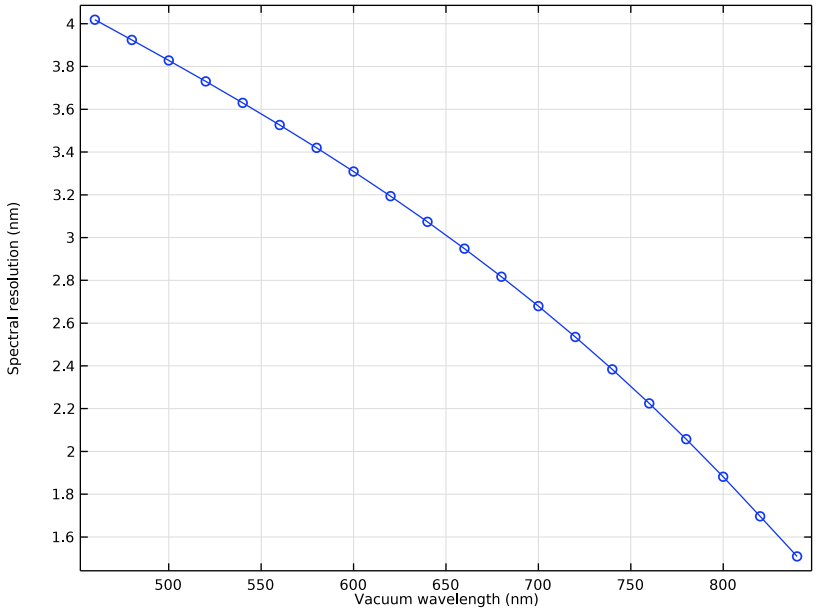


Figure 5: Spectral resolution as a function of the wavelength.

Reference

1. K. Liu and F. Yu, "Accurate wavelength calibration method using system parameters for grating spectrometers," *Opt. Eng.*, vol. 52, no. 1, pp. 013603-1–013603-6, 2013.

Application Library path: `Ray_Optics_Module/Spectrometers_and_Monochromators/czerny_turner_monochromator`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Ray Tracing**.

6 Click **Done**.

ROOT

Insert the prepared geometry sequence from file. You can read the instructions for creating the geometry in the appendix.

GEOMETRY 1

1 In the **Geometry** toolbar, click **Insert Sequence** and choose **Insert Sequence**.

2 Browse to the model's Application Libraries folder and double-click the file `czerny_turner_monochromator_geom_sequence.mph`.

3 In the **Geometry** toolbar, click **Build All**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
lambda0min	450[nm]	4.5E-7 m	Minimum vacuum wavelength
lambda0max	850[nm]	8.5E-7 m	Maximum vacuum wavelength
N	3648	3648	Number of pixels
wp	8[um]	8E-6 m	Pixel width
Fnum	10	10	F-number
NA	1/(2*Fnum)	0.05	Numerical aperture
Srange	650[nm]	6.5E-7 m	Spectral range

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Domain Selection** section.

3 Click **Clear Selection**.

4 Locate the **Ray Release and Propagation** section. From the **Wavelength distribution of released rays** list, choose **Polychromatic, specify vacuum wavelength**.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

Release rays from a grid point located at the focus of the collimating mirror. Use a cone angle that corresponds to the numerical aperture of the system and select a uniform frequency distribution for the initial rays.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 In the $q_{x,0}$ text field, type `-10`.

4 In the $q_{y,0}$ text field, type `16.16104903340627`.

5 Locate the **Ray Direction Vector** section. From the **Ray direction vector** list, choose **Conical**.

6 In the N_w text field, type `20`.

7 In the α text field, type `asin(NA)`.

8 Locate the **Vacuum Wavelength** section. From the **Distribution function** list, choose **Uniform**.

9 In the N text field, type `20`.

10 In the $\lambda_{0,\min}$ text field, type `lambda0min`.

11 In the $\lambda_{0,\max}$ text field, type `lambda0max`.

Wall 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Wall**.

2 Select Boundary 8 only.

Mirror 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Mirror**.

2 Select Boundaries 15 and 16 only.

Grating 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Grating**.

Add a grating with a groove density of 600 lines/mm and include the first diffraction order. Note that in grating spectrometer systems, the first order spectra usually carry the primary diffraction intensity of the grating.

2 Select Boundary 3 only.

3 In the **Settings** window for **Grating**, locate the **Device Properties** section.

4 In the d text field, type `1[mm]/600`.

5 From the **Rays to release** list, choose **Reflected**.

6 Locate the **Grating Orientation** section. From the **Direction of periodicity** list, choose **Reverse**.

Diffraction Order (m = 0)

1 In the **Model Builder** window, expand the **Grating 1** node, then click **Diffraction Order (m = 0)**.

2 In the **Settings** window for **Diffraction Order**, locate the **Device Properties** section.

3 In the m text field, type `1`.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.

3 From the list, choose **User-controlled mesh**.

Refine the mesh around the curved surfaces.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Minimum element size** text field, type `0.002`.

5 In the **Curvature factor** text field, type `0.002`.

6 Click **Build All**.

DEFINITIONS

Add the definition for the pixel number and for the image width of the entrance slit. The latter expression uses the `gopmaxop` nonlocal coupling, `if` conditions and the `dest` operator to determine the difference in position of extremum rays for a given wavelength.

Variables 1

1. In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
2. In the **Settings** window for **Variables**, locate the **Variables** section.
3. In the table, enter the following settings:

Name	Expression	Unit	Description
pnum	`ceil(N/2-(qx-Qdx)/(wp*cos(theta_d)))`		Pixel number
samefreq	`abs(gop.nu-dest(gop.nu))<1[Hz]`		Logical expression for rays on the detector with the same frequency
distance	`sqrt((qx-dest(qx))^2+(qy-dest(qy))^2)`	m	Distance between the rays
distance2	`gop.max(if(samefreq,distance,0))`	m	Maximum distance to given ray
wi	`gop.max(if(samefreq,distance2,0))`	m	Image width of the entrance slit

STUDY 1

Step 1: Ray Tracing

1. In the **Model Builder** window, under **Study 1** click **Step 1: Ray Tracing**.
2. In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.
3. In the **Output times** text field, type `range(0,0.01,0.8)`.
4. In the **Home** toolbar, click **Compute**.

RESULTS

Ray Bin 1

1 In the **Model Builder** window, expand the **Results>Datasets** node.

2 Right-click **Results>Datasets** and choose **More Datasets>Ray Bin**.

3 In the **Settings** window for **Ray Bin**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Geometrical Optics>Ray properties>gop.lambda0 - Vacuum wavelength - m**.

4 Locate the **Bins** section. From the **Entry method** list, choose **Tolerance**.

5 In the **Tolerance** text field, type `1[nm]`.

Ray Trajectories 1

1 In the **Model Builder** window, expand the **Results>Ray Trajectories (gop)** node, then click **Ray Trajectories 1**.

2 In the **Settings** window for **Ray Trajectories**, locate the **Coloring and Style** section.

3 Find the **Line style** subsection. From the **Type** list, choose **Tube**.

4 Select the **Radius scale factor** check box. In the associated text field, type `0.025`.

Color Expression 1

1 In the **Model Builder** window, expand the **Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Geometrical Optics>Ray properties>gop.lambda0 - Vacuum wavelength - m**.

3 Locate the **Coloring and Style** section. Click **Change Color Table**.

4 In the **Color Table** dialog box, select **Rainbow>Spectrum** in the tree.

5 Click **OK**.

6 In the **Settings** window for **Color Expression**, locate the **Expression** section.

7 From the **Unit** list, choose **nm**.

8 In the **Ray Trajectories (gop)** toolbar, click **Plot**.

9 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting plot to Figure 3.

Pixel Number

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Pixel Number` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Ray Bin 1**.

4 From the **Time selection** list, choose **Last**.

Ray 1

1 In the **Pixel Number** toolbar, click **More Plots** and choose **Ray**.

2 In the **Settings** window for **Ray**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `pnum`.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `gop.lambda0`.

6 From the **Unit** list, choose **nm**.

7 Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

8 In the **Pixel Number** toolbar, click **Plot**. Compare the resulting plot to Figure 4.

Device Resolution

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Device Resolution` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Ray Bin 1**.

4 From the **Time selection** list, choose **Last**.

5 Click to expand the **Title** section. From the **Title type** list, choose **None**.

Ray 1

1 In the **Device Resolution** toolbar, click **More Plots** and choose **Ray**.

2 In the **Settings** window for **Ray**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `Srange/N*wi/wp`.

4 From the **Unit** list, choose **nm**.

5 Select the **Description** check box. In the associated text field, type `Spectral resolution`.

6 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

7 In the **Expression** text field, type `gop.lambda0`.

8 From the **Unit** list, choose **nm**.

9 Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

10 In the **Device Resolution** toolbar, click **Plot**. Compare the resulting plot to Figure 5.

ADD COMPONENT

In the **Home** toolbar, click **Add Component** and choose **2D**.

GEOMETRY 1

1 In the **Settings** window for **Geometry**, locate the **Units** section.

2 From the **Length unit** list, choose **mm**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
theta_g	28.76[deg]	0.50196 rad	Grating's angle
theta_c	11[deg]	0.19199 rad	Collimating mirror's angle
theta_i	77[deg]	1.3439 rad	Imaging mirror's angle
theta_d	6.76[deg]	0.11798 rad	Detector's angle
Qix	20[mm]	0.02 m	x coordinate Qi
Qiy	34[mm]	0.034 m	y coordinate Qi
Qcx	40[mm]	0.04 m	x coordinate Qc
Qcy	40*tan(2*theta_c)	16.161	y coordinate Qc
Qdx	22.08[mm]	0.02208 m	x coordinate Qd
Qdy	-24.12[mm]	-0.02412 m	y coordinate Qd
Ri	130[mm]	0.13 m	Imaging mirror's radius of curvature
Rc	100[mm]	0.1 m	Collimating mirror's radius of curvature

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `3`.

4 In the **Height** text field, type `15`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `-1.5*cos(theta_g)`.

7 In the **y** text field, type `-1.5*sin(theta_g)`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `theta_g`.

Rectangle 2 (r2)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `3`.

4 In the **Height** text field, type `15`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `Qcx`.

7 In the **y** text field, type `Qcy`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `theta_c`.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Rc`.

4 Locate the **Position** section. In the **x** text field, type `Qcx-Rc*cos(theta_c)`.

5 In the **y** text field, type `Qcy-Rc*sin(theta_c)`.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

3 Select the object **r2** only.

4 In the **Settings** window for **Difference**, locate the **Difference** section.

5 Find the **Objects to subtract** subsection. Click to select the **Activate Selection** toggle button.

6 Select the object **c1** only.

Rectangle 3 (r3)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `3`.

4 In the **Height** text field, type `30`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `Qix`.

7 In the **y** text field, type `Qiy`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `theta_i`.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `Ri`.

4 Locate the **Position** section. In the **x** text field, type `Qix-Ri*cos(theta_i)`.

5 In the **y** text field, type `Qiy-Ri*sin(theta_i)`.

Difference 2 (dif2)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **r3** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Find the **Objects to subtract** subsection. Click to select the **Activate Selection** toggle button.

5 Select the object **c2** only.

Rectangle 4 (r4)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `30`.

4 In the **Height** text field, type `3`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `Qdx+1.5*sin(theta_d)`.

7 In the **y** text field, type `Qdy-1.5*cos(theta_d)`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `theta_d`.

9 Click **Build All Objects**.

10 Click the  **Zoom Extents** button in the **Graphics** toolbar.