Created in COMSOL Multiphysics 6.1

Cloaking of a Cylindrical Scatterer with Graphene

Introduction

Electromagnetic cloaking has been an interesting research topic. Using an invisibility cloak to make an object seem transparent sounds like science fiction, but in fact it has been proven possible using different methods such as metasurfaces. Reducing the visibility and scattering of an object to practically zero is not only conceptually interesting, but also has many practical applications. For example, cloaking can be applied to aircraft to avoid detection by radar.

In this model, we introduce a cloaking method using an electrically tuned monolayer of graphene. We will show that when a cylindrical dielectric scatterer is covered in graphene, the scattering cross section is greatly reduced at the designated frequency, making it invisible. This cloak is atomically thin, offering advantages to other cloaking methods based on bulky layers.

Due to the axisymmetric nature of cylinder, the scattering simulation will be performed in 2D axisymmetry using the scattered field formulation. Compared to the 3D model, it is much more efficient. A linear polarized plane wave is used as the background excitation field. By computing the electric field distribution and the scattering cross section, we can assess the cloaking effect.

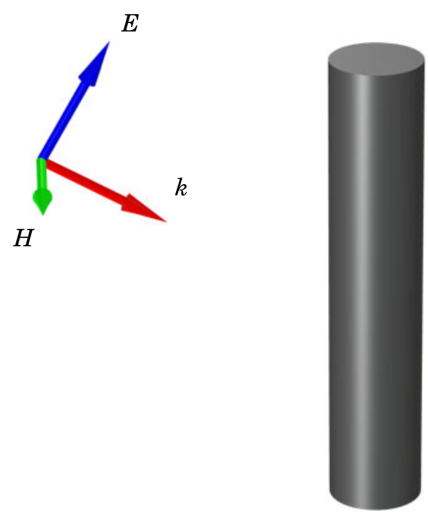


Figure 1: Linearly polarized plane wave incident on a dielectric cylinder covered by graphene. The conductivity of graphene is tuned to provide a good impedance matching such that the scattering is minimized.

Model Definition

This model demonstrates the use of the linearly polarized plane wave background field in 2D axisymmetry. Some care must be taken in the postprocessing to make sure we plot the correct results by suming over the expansion terms.

We attempt to cloak a cylindrical scatterer with permittivity $\varepsilon = 3.9$ at $f_0 = 3$ THz. The diameter of the cylinder is $D = \lambda_0/5$ and the length is $L = 3\lambda_0$, where λ_0 is the designated free space wavelength. The outer surface of the cylinder is covered by a monolayer of graphene. Cloaking at the designated frequency can be realized by electrically tuning the Fermi level of graphene to $E_F = 0.51$ eV such that graphene provides good impedance matching. It reduces scattering by functioning similarly to an anti-reflective coating. The Fermi level can be changed to achieve cloaking effect at a different frequency.

The conductivity of graphene is calculated using the well-established Kubo formula. Both the electronic intraband transitions and interband transitions contribute to the conductivity of graphene. The intraband and interband contributions are given by

$$\sigma_{\mathrm{intra}} = \frac{2k_B T e^2}{\pi \hbar} \ln\left(2\cosh\frac{E_F}{2k_B T}\right) \frac{-j}{\omega - j\tau^{-1}}, \tag{1}$$

$$\sigma_{\mathrm{inter}} = \frac{e^2}{4\hbar}\left[H\left(\frac{\omega}{2}\right) - j\frac{4\omega}{\pi}\int_0^\infty \frac{H(\Omega) - H\left(\frac{\omega}{2}\right)}{\omega^2 - 4\Omega^2} d\Omega\right], \tag{2}$$

where k_B is the Boltzmann constant, $\hbar$ is the reduced Planck constant, e is the electron charge, T is the temperature, E_F is the Fermi energy, τ is the relaxation time, and $\omega = 2\pi f$ is the angular frequency. The function $H(\Omega)$ is given by

$$H(\Omega) = \sinh\left(\frac{\hbar\Omega}{k_B T}\right) \Big/ \left(\left[\cosh\left(\frac{\hbar\Omega}{k_B T}\right) + \cosh\left(\frac{E_F}{k_B T}\right)\right]\right). \tag{3}$$

Finally, the total 2D sheet conductivity of graphene is given by $\sigma = \sigma_{\mathrm{intra}} + \sigma_{\mathrm{inter}}$. In this model, we consider $T = 300$ K and $\tau = 0.5$ ps. The integral in σ_{inter} can be performed using the built-in `integrate()` operator. Due to the atomic thickness of graphene, explicit volumetric modeling of it would be computationally expensive. We show that this can easily be avoided by using the Transition Boundary Condition, considering graphene as a 2D surface.

To investigate the performance of the graphene cloak, the scatterer is excited by a linearly polarized plane wave. The linearly polarized plane wave with arbitrary polarization and angle of incidence is not axisymmetric. Therefore, in 2D axisymmetry, the linearly polarized plane wave background field is formed using an expansion method. The key step is to expand a plane wave background field $\mathbf{E}_\mathrm{b} = \mathbf{E}_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}$ using an infinite sum of azimuthal modes with mode number `m`. The expansion in cylindrical coordinates yields

$$\mathbf{E}_\mathrm{b} = E_0 e^{i(\omega t + kz\cos\theta)} \left\{ \frac{1}{2}\cos\theta \sum_{m=-\infty}^{\infty} [(-i)^{m-1} J_{m-1}(kr\sin\theta) + \right.$$

$$(-i)^{m+1} J_{m+1}(kr\sin\theta)] e^{-im\phi} \hat{\mathbf{r}} - \frac{i}{2}\cos\theta \sum_{m=-\infty}^{\infty} [(-i)^{m-1} J_{m-1}(kr\sin\theta) -$$

$$\left. (-i)^{m+1} J_{m+1}(kr\sin\theta)] e^{-im\phi} \hat{\phi} + \sin\theta \sum_{m=-\infty}^{\infty} (-i)^m J_m(kr\sin\theta) e^{-im\phi} \hat{\mathbf{z}} \right\}, \qquad (4)$$

where J_m is the Bessel function of the first kind of order m. When choosing the linear polarized plane wave as the background field in a 2D axisymmetric model, an auxiliary sweep over the mode number will be added in the first Study. For a scatterer whose size is comparable to the wavelength, the infinite series can be truncated to only the first few terms. In this model, we set the highest mode number to 5.

Results and Discussion

The cloaking effect can be visualized in two ways. First, by plotting the electric field distribution around the cylinder without the graphene layer. Clearly, the wave front of the plane wave is strongly distorted due to scattering. By comparison, the field distribution with the graphene cloak shows minimal distortion, as if the scatterer is not present.

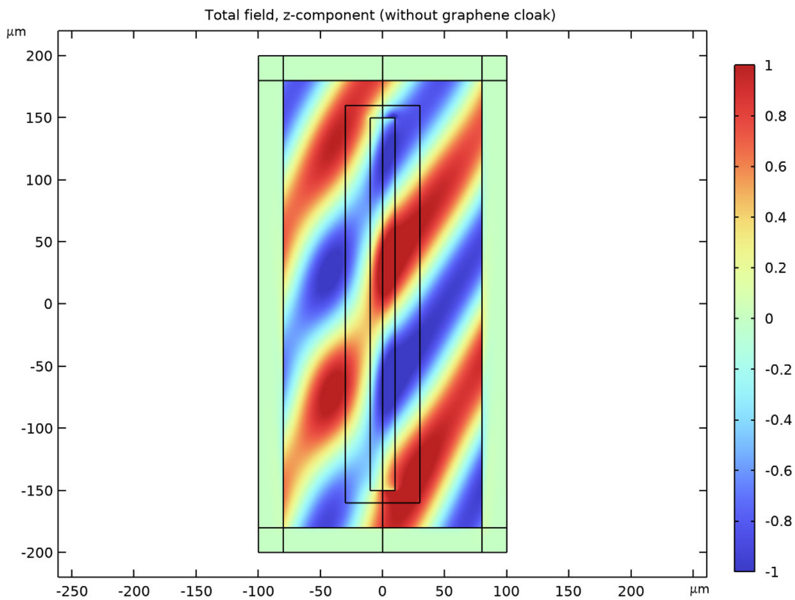


Figure 2: Electric field distribution around the dielectric cylinder without graphene cloak at 3 THz. The wave front is significantly distorted due to scattering.

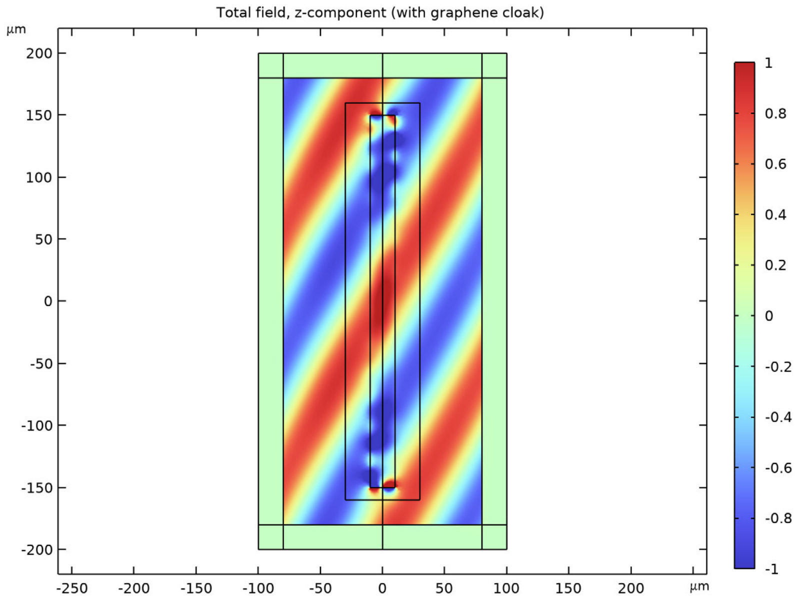


Figure 3: Electric field distribution around the dielectric cylinder with graphene cloak at 3 THz. The wave front is minimally distorted, indicating a good cloaking effect.

The second and more quantitative way of evaluating the cloaking effect is to compare the scattering cross section, which is calculated as the surface integral of the dot product of the Poynting vector and the surface normal, with and without graphene. As shown in the figure below, the scattering cross section is drastically reduced at the designated frequency of 3 THz when the graphene cloak is applied.

Furthermore, even though the simulation is performed in 2D axisymmetry, we can generate full 3D distribution of the physical quantities such as the electric field by utilizing the Revolution 2D dataset. The details on how to generate the plot is discussed below in the step-by-step instructions.

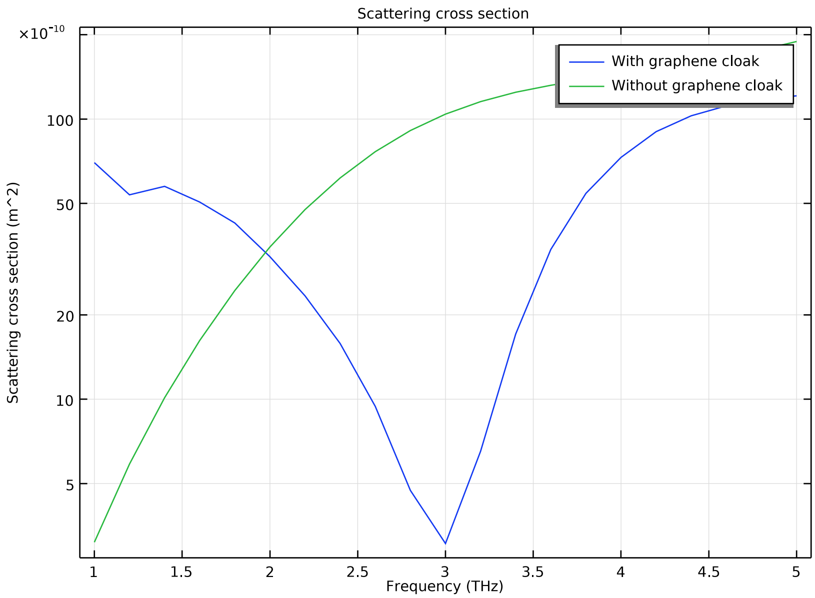


Figure 4: Comparison of the scattering cross section as a function of frequency with and without graphene. When the graphene cloak is applied, the scattering cross section is drastically reduced at the designated 3 THz, indicating a good cloaking performance.

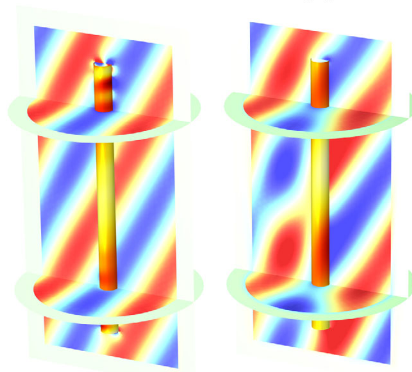


Figure 5: Comparison of the full 3D distributions of the electric field around the dielectric cylinder with and without the graphene cloak at 3 THz. With graphene cloak, the scattering cross section is reduced by over 97%.

Notes About the COMSOL Implementation

In postprocessing, we make frequent use of the `withsol()` operator to select results at specific mode number and frequency. The `sum()` operator is used to sum up the contributions from each mode. Information about these operators can be found in the COMSOL documentation under COMSOL Multiphysics > COMSOL Multiphysics Reference Manual > Global and Local Definitions > Operators, Functions, and Constants >Built-In Operators.

Reference

1. P. Chen and A. Alù, “Atomically thin surface cloak using graphene monolayers,” *ACS Nano*, vol. 5, no. 7, pp. 5855–5863, 2011.

Application Library path: `RF_Module/Scattering_and_RCS/cylinder_graphene_cloak`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **µm**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`f0`	`3[THz]`	3E12 Hz	`Designed cloaking frequency`
`lda0`	`c_const/f0`	9.9931E-5 m	`Designed cloaking wavelength`
`theta`	`pi/3`	1.0472	`Angle of incidence`
`D`	`lda0/5`	1.9986E-5 m	`Cylinder diameter`
`L`	`3*lda0`	2.9979E-4 m	`Cylinder length`
`eps_d`	`3.9`	3.9	`Cylinder dielectric constant`
`Ef`	`0.51[eV]`	8.1711E-20 J	`Fermi level of graphene`
`tau`	`0.5[ps]`	5E-13 s	`Scattering time`
`T`	`300[K]`	300 K	`Temperature`
`d_eff`	`1[nm]`	1E-9 m	`Effective graphene thickness`
`E0`	`1[V/m]`	1 V/m	`Electric field amplitude of the incident field`
`I0`	`E0^2/(2*Z0_const)`	0.0013272 W/m^2	`Incident power`

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `D/2`.

4 In the **Height** text field, type `L`.

5 Locate the **Position** section. In the **z** text field, type `-L/2`.

This rectangle represents the dielectric cylinder.

Rectangle 2 (r2)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `lda0`.

4 In the **Height** text field, type `L+lda0`.

5 Locate the **Position** section. In the **z** text field, type `-(L+lda0)/2`.

6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (µm)
Layer 1	lda0/5

7 Select the **Layers to the right** check box.

8 Select the **Layers on top** check box.

This rectangle represents the simulation domain. A layer on the top, bottom, and right is added for the PML.

Rectangle 3 (r3)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `D/2+lda0/5`.

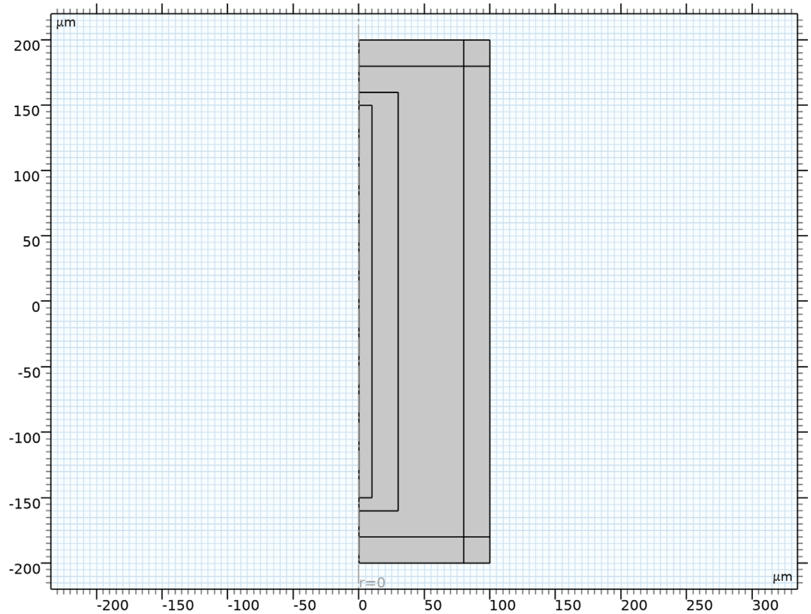
4 In the **Height** text field, type `L+lda0/5`.

5 Locate the **Position** section. In the **z** text field, type `-(L+lda0/5)/2`.

This rectangle represents a surface surrounding the scatterer. Perform the integration of the Poynting vector to calculate the scattering cross section on this surface.

6 Click **Build All Objects**.

7 Click the Zoom Extents button in the Graphics toolbar.



DEFINITIONS

Integration 1 (intop1)

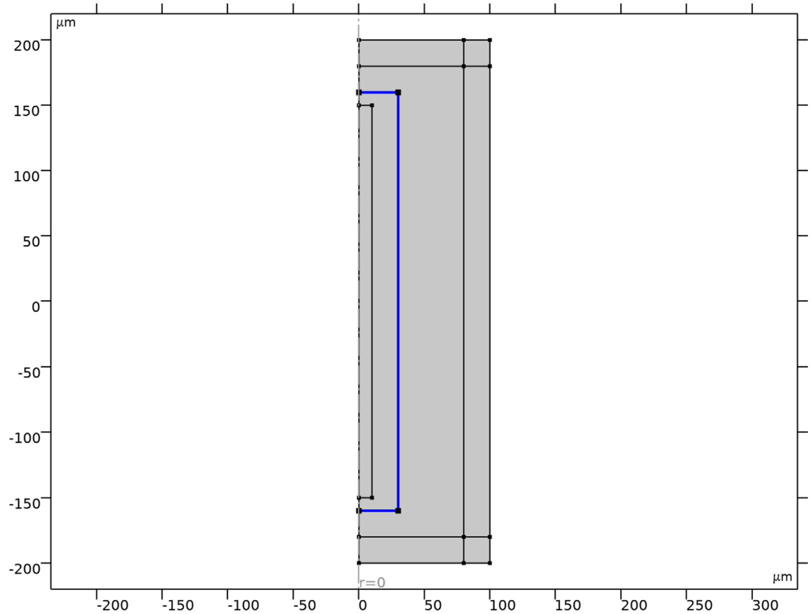
1 In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

2 Right-click **Definitions** and choose **Nonlocal Couplings>Integration**.

3 In the **Settings** window for **Integration**, locate the **Source Selection** section.

4 From the **Geometric entity level** list, choose **Boundary**.

5 Select Boundaries 6, 12, and 17 only.



The integration is performed on the boundaries of Rectangle 3.

Analytic 1 (an1)

1 In the **Definitions** toolbar, click **f(x) Analytic**.

2 In the **Settings** window for **Analytic**, type `H` in the **Function name** text field.

3 Locate the **Definition** section. In the **Expression** text field, type `sinh(hbar_const*x/(k_B_const*T))/(cosh(hbar_const*x/(k_B_const*T))+cosh(Ef/(k_B_const*T)))`.

4 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	rad/s

5 Locate the **Plot Parameters** section. In the table, enter the following settings:

Argument	Lower limit	Upper limit	Unit
x	0	1e16	rad/s

Variables 1

1 Right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Omega	1[rad/s]	rad/s	Variable of integral
integral	integrate((H(Omega)-H(emw.omega/2))/(emw.omega^2-4*Omega^2), Omega,0[rad/s],1e16[rad/s])	s	Integral in the interband conductivity equation
sigma_intra	((2*k_B_const*T*e_const^2)/(pi*hbar_const^2))*(log(2*cosh(Ef/(2*k_B_const*T)))*(-j/(emw.omega-j/tau)))	S	Intraband conductivity
sigma_inter	(e_const^2/(4*hbar_const))*(H(emw.omega/2)-(j*4*emw.omega/pi)*integral)	S	Interband conductivity
sigma	sigma_intra+sigma_inter	S	Total graphene conductivity
C_scatt	intop1(nr*emw.relPoavr+nz*emw.relPoavz)/I0		Scattering cross section

Next, set up the calculation for computing the graphene conductivity and the scattering cross section.

Perfectly Matched Layer 1 (pml1)

1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 Select Domains 1 and 5–8 only.

3 In the **Settings** window for **Perfectly Matched Layer**, locate the **Geometry** section.

4 From the **Type** list, choose **Cylindrical**.

The PML is used for absorbing outgoing radiation.

MATERIALS

Air

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

Dielectrics

1 Right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Dielectrics` in the **Label** text field.

3 Select Domain 4 only.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	`eps_d`	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (emw)**.

2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Formulation** section.

3 From the list, choose **Scattered field**.

4 From the **Background wave type** list, choose **Linearly polarized plane wave**.

5 In the θ text field, type `theta`.

6 In the **Highest mode number** text field, type `5`. This is sufficient for a scatterer whose size is comparable to the wavelength, where only a few terms in the expansion are needed to obtain an accurate result.

7 In the E_0 text field, type `E0`.

8 Click **Set up Sweep**. This generates two parameters: `modeNum` and `highestMode`. It also adds an auxiliary sweep in Study 1 that sweeps `modeNum` from `-highestMode` to `highestMode`.

Transition Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Transition Boundary Condition**.

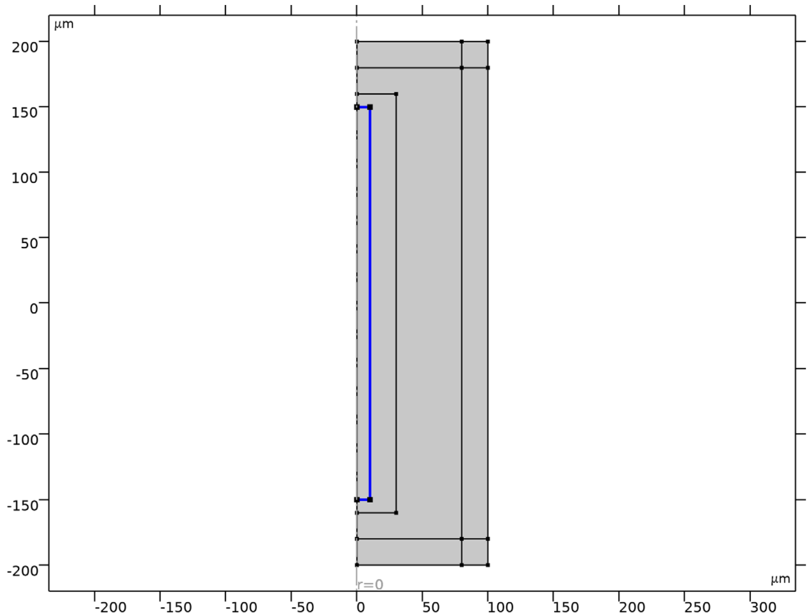
2 In the **Settings** window for **Transition Boundary Condition**, locate the **Transition Boundary Condition** section.

3 From the **Electric displacement field model** list, choose **Relative permittivity**.

4 Select Boundaries 8, 10, and 16 only.

5 From the ε_r list, choose **User defined**. From the μ_r list, choose **User defined**. From the σ list, choose **User defined**. In the associated text field, type `sigma/d_eff`.

6 In the d text field, type `d_eff`.



The Transition Boundary Condition is added on the boundary of the dielectric cylinder. The conductivity and effective thickness are set to that of graphene.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Extremely fine**.

Since the 2D axisymmetric model is very efficient to solve, you can use a very fine mesh to ensure accuracy.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 From the **Frequency unit** list, choose **THz**.

4 In the **Frequencies** text field, type `range(1,0.2,5)`.

Next, add a second study to simulate the case without graphene cloak. In this study, the Transition Boundary Condition is disabled. An auxiliary sweep that sweeps `modeNum` from `-highestMode` to `highestMode` is manually added.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Physics and Variables Selection** section.

2 Select the **Modify model configuration for study step** check box.

3 In the tree, select **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain (emw)>Transition Boundary Condition 1**.

4 Right-click and choose **Disable**.

5 Locate the **Study Settings** section. From the **Frequency unit** list, choose **THz**.

6 In the **Frequencies** text field, type `range(1, 0.2, 5)`.

7 Locate the **Study Extensions** section. Select the **Auxiliary sweep** check box.

8 Click **Add**.

9 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
modeNum (Azimuthal mode number)	`range(-highestMode,1, highestMode)`	

Now, run the two studies.

STUDY 1

In the **Home** toolbar, click **Compute**.

STUDY 2

Click **Compute**.

RESULTS

Add two Mirror 2D datasets for postprocessing the field distribution.

Mirror 2D 1

1 In the **Results** toolbar, click **More Datasets** and choose **Mirror 2D**.

2 In the **Settings** window for **Mirror 2D**, click to expand the **Advanced** section.

3 Select the **Define variables** check box.

Mirror 2D 2

1 In the **Results** toolbar, click **More Datasets** and choose **Mirror 2D**.

2 In the **Settings** window for **Mirror 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

4 Locate the **Advanced** section. Select the **Define variables** check box.

Total Field (With Graphene Cloak)

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Total Field (With Graphene Cloak)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

4 From the **Parameter value (freq (THz))** list, choose **3**.

5 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

6 In the **Title** text area, type `Total field, z-component (with graphene cloak)`.

7 Clear the **Parameter indicator** text field.

Surface 1

1 Right-click **Total Field (With Graphene Cloak)** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `sum(withsol('sol1',emw.Ez,setval(freq, freq),setind(modeNum,index)),index,1,2*highestMode+1)`.

4 Click to expand the **Range** section. Select the **Manual color range** check box.

5 In the **Minimum** text field, type `-1`.

6 In the **Maximum** text field, type `1`.

Filter 1

1 Right-click **Surface 1** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `mir1x>0`.

Surface 2

1 In the **Model Builder** window, right-click **Total Field (With Graphene Cloak)** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `sum(withsol('sol1',emw.Ez*exp(-j*modeNum*pi),setval(freq,freq),setind(modeNum,index)),index,1,2*highestMode+1)`.

4 Locate the **Coloring and Style** section. Clear the **Color legend** check box.

5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.

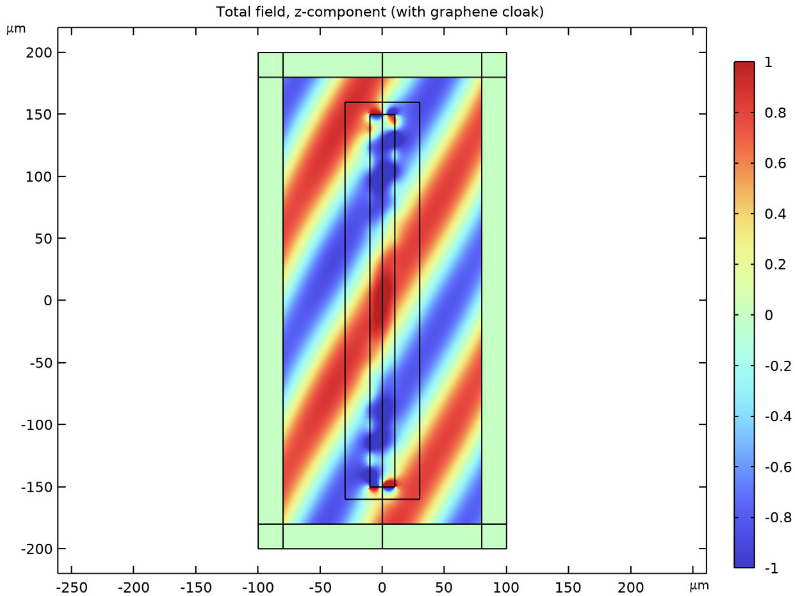
Filter 1

1 Right-click **Surface 2** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `mir1x<0`.

4 In the **Total Field (With Graphene Cloak)** toolbar, click **Plot**.



The default plots show the z-components of the scattered field and the background field. First, visualize the z-component of the total field in the case with graphene cloak. The wave front is only minimally distorted thanks to the graphene cloak.

Total Field (Without Graphene Cloak)

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Total Field (Without Graphene Cloak)` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 2**.

4 From the **Parameter value (freq (THz))** list, choose **3**.

5 Locate the **Title** section. From the **Title type** list, choose **Manual**.

6 In the **Title** text area, type `Total field, z-component (without graphene cloak)`.

7 Clear the **Parameter indicator** text field.

Surface 1

1 Right-click **Total Field (Without Graphene Cloak)** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `sum(withsol('sol2',emw.Ez,setval(freq,freq),setind(modeNum,index)),index,1,2*highestMode+1)`.
4 Locate the **Range** section. Select the **Manual color range** check box.
5 In the **Minimum** text field, type `-1`.
6 In the **Maximum** text field, type `1`.

Filter 1

1 Right-click **Surface 1** and choose **Filter**.
2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
3 In the **Logical expression for inclusion** text field, type `mir2x>0`.

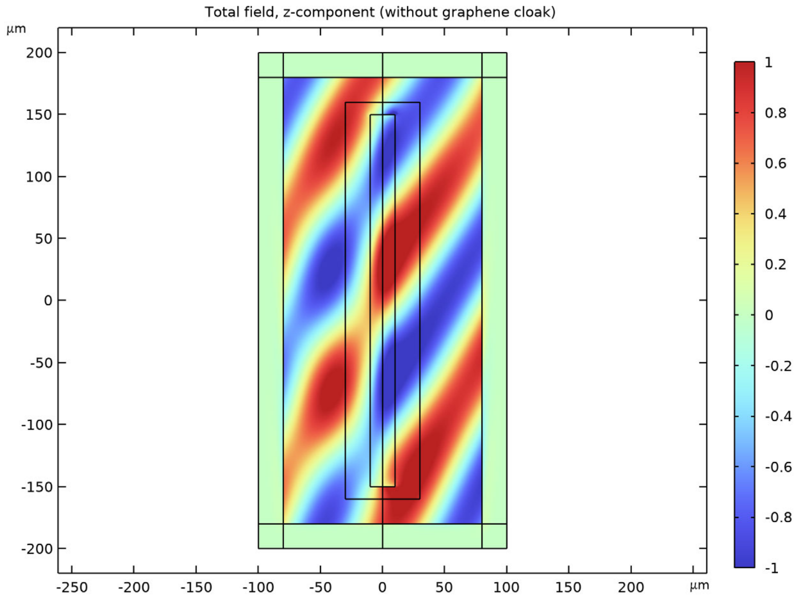
Surface 2

1 In the **Model Builder** window, right-click **Total Field (Without Graphene Cloak)** and choose **Surface**.
2 In the **Settings** window for **Surface**, locate the **Expression** section.
3 In the **Expression** text field, type `sum(withsol('sol2',emw.Ez*exp(-j*modeNum*pi),setval(freq,freq),setind(modeNum,index)),index,1,2*highestMode+1)`.
4 Locate the **Coloring and Style** section. Clear the **Color legend** check box.
5 Locate the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.

Filter 1

1 Right-click **Surface 2** and choose **Filter**.
2 In the **Settings** window for **Filter**, locate the **Element Selection** section.
3 In the **Logical expression for inclusion** text field, type `mir2x<0`.

4 In the **Total Field (Without Graphene Cloak)** toolbar, click **Plot**.



Similarly, visualize the *z*-component of the total field for the case without graphene cloak as a comparison. The field is noticeably distorted due to scattering.

Scattering Cross Section

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Scattering Cross Section` in the **Label** text field.

3 Locate the **Data** section. From the **Parameter selection (modeNum)** list, choose **First**.

4 Locate the **Plot Settings** section.

5 Select the **x-axis label** check box. In the associated text field, type `Frequency (THz)`.

6 Select the **y-axis label** check box. In the associated text field, type `Scattering cross section (m^2)`.

7 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

8 In the **Title** text area, type `Scattering cross section`.

Global 1

1 Right-click **Scattering Cross Section** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
sum(withsol('sol1',C_scatt, setval(freq,freq), setind(modeNum,index)),index, 1,2*highestMode+1)	1	With graphene cloak
sum(withsol('sol2',C_scatt, setval(freq,freq), setind(modeNum,index)),index, 1,2*highestMode+1)	1	Without graphene cloak

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **freq**.

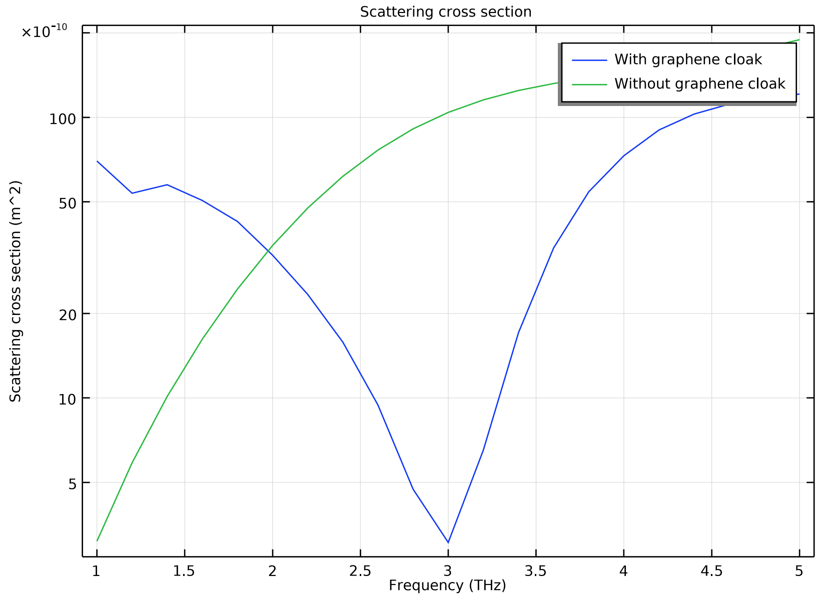
5 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

6 In the table, enter the following settings:

Legends
With graphene cloak
Without graphene cloak

7 In the **Scattering Cross Section** toolbar, click **Plot**.

8 Click the **y-Axis Log Scale** button in the **Graphics** toolbar.



The scattering cross section as a function of frequency is plotted for the cases with and without graphene. When the graphene cloak is applied, the scattering cross section is drastically reduced at the designated frequency of 3 THz as expected.

Next, visualize the field distribution in 3D by utilizing the Revolution 2D dataset.

Revolution 2D

1 In the **Model Builder** window, under **Results>Datasets** click **Revolution 2D**.

2 In the **Settings** window for **Revolution 2D**, click to expand the **Revolution Layers** section.

3 In the **Revolution angle** text field, type 360.

4 Click to expand the **Advanced** section. Select the **Define variables** check box.

Revolution 2D 1

1 In the **Model Builder** window, click **Revolution 2D 1**.

2 In the **Settings** window for **Revolution 2D**, locate the **Revolution Layers** section.

3 In the **Revolution angle** text field, type 360.

4 Locate the **Advanced** section. Select the **Define variables** check box.

Comparison of the Total Field in 3D

1 In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

2 In the **Settings** window for **3D Plot Group**, type Comparison of the Total Field in 3D in the **Label** text field.

3 Locate the **Data** section. From the **Parameter value (freq (THz))** list, choose **3**.

4 Click to expand the **Title** section. From the **Title type** list, choose **None**.

5 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

6 Locate the **Color Legend** section. Clear the **Show legends** check box.

Slice 1

1 Right-click **Comparison of the Total Field in 3D** and choose **Slice**.

2 In the **Settings** window for **Slice**, locate the **Expression** section.

3 In the **Expression** text field, type sum(withsol('sol1',emw.Ez*exp(-j*modeNum*rev1phi),setval(freq,freq),setind(modeNum,index)),index,1,2*highestMode+1).

4 Locate the **Plane Data** section. From the **Plane** list, choose **XY-planes**.

5 In the **Planes** text field, type 2.

6 Click to expand the **Range** section. Select the **Manual color range** check box.

7 In the **Minimum** text field, type -1.

8 In the **Maximum** text field, type 1.

Slice 2

1 Right-click **Slice 1** and choose **Duplicate**.

2 In the **Settings** window for **Slice**, locate the **Plane Data** section.

3 From the **Plane** list, choose **ZX-planes**.

4 In the **Planes** text field, type 1.

5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Slice 1**.

Surface 1

1 In the **Model Builder** window, right-click **Comparison of the Total Field in 3D** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type sum(withsol('sol1',emw.Ez*exp(-j*modeNum*rev1phi),setval(freq,freq),setind(modeNum,index)),index,1,2*highestMode+1).

4 Click to expand the **Range** section. Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Thermal>Thermal** in the tree.

6 Click **OK**.

Filter 1

1 Right-click **Surface 1** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `sqrt(rev1x^2+rev1y^2)<D/2 && abs(rev1z)<L/2`.

Comparison of the Total Field in 3D

1 In the **Model Builder** window, under **Results** click **Comparison of the Total Field in 3D**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Parameter value (freq (THz))** list, choose **3**.

Slice 3

1 Right-click **Comparison of the Total Field in 3D** and choose **Slice**.

2 In the **Settings** window for **Slice**, locate the **Data** section.

3 From the **Dataset** list, choose **Revolution 2D 1**.

4 From the **Parameter value (freq (THz))** list, choose **3**.

5 Locate the **Expression** section. In the **Expression** text field, type `sum(withsol('sol2', emw.Ez*exp(-j*modeNum*rev2phi),setval(freq,freq),setind(modeNum, index)),index,1,2*highestMode+1)`.

6 Locate the **Plane Data** section. From the **Plane** list, choose **XY-planes**.

7 In the **Planes** text field, type `2`.

8 Locate the **Inherit Style** section. From the **Plot** list, choose **Slice 1**.

Translation 1

1 Right-click **Slice 3** and choose **Translation**.

2 In the **Settings** window for **Translation**, locate the **Translation** section.

3 In the **x** text field, type `200`.

4 In the **y** text field, type `100`.

Slice 4

1 In the **Model Builder** window, right-click **Comparison of the Total Field in 3D** and choose **Slice**.

2 In the **Settings** window for **Slice**, locate the **Data** section.

3 From the **Dataset** list, choose **Revolution 2D 1**.

4 From the **Parameter value (freq (THz))** list, choose **3**.

5 Locate the **Expression** section. In the **Expression** text field, type `sum(withsol('sol2', emw.Ez*exp(-j*modeNum*rev2phi),setval(freq,freq),setind(modeNum, index)),index,1,2*highestMode+1)`.

6 Locate the **Plane Data** section. From the **Plane** list, choose **ZX-planes**.

7 In the **Planes** text field, type `1`.

8 Locate the **Inherit Style** section. From the **Plot** list, choose **Slice 1**.

Translation 1

1 Right-click **Slice 4** and choose **Translation**.

2 In the **Settings** window for **Translation**, locate the **Translation** section.

3 In the **x** text field, type `200`.

4 In the **y** text field, type `100`.

Surface 2

1 In the **Model Builder** window, right-click **Comparison of the Total Field in 3D** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Revolution 2D 1**.

4 From the **Parameter value (freq (THz))** list, choose **3**.

5 Locate the **Expression** section. In the **Expression** text field, type `sum(withsol('sol2', emw.Ez*exp(-j*modeNum*rev2phi),setval(freq,freq),setind(modeNum, index)),index,1,2*highestMode+1)`.

6 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Surface 1**.

Translation 1

1 Right-click **Surface 2** and choose **Translation**.

2 In the **Settings** window for **Translation**, locate the **Translation** section.

3 In the **x** text field, type `200`.

4 In the **y** text field, type `100`.

Filter 1

1 In the **Model Builder** window, right-click **Surface 2** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `sqrt(rev2x^2+rev2y^2)<D/2 && abs(rev2z)<L/2`.

Annotation 1

1 In the **Model Builder** window, right-click **Comparison of the Total Field in 3D** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `With graphene cloak`.

4 Locate the **Position** section. In the **X** text field, type `-100`.

5 In the **Z** text field, type `230`.

6 Locate the **Coloring and Style** section. Clear the **Show point** check box.

Annotation 2

1 Right-click **Annotation 1** and choose **Duplicate**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `Without graphene cloak`.

4 Locate the **Position** section. In the **X** text field, type `150`.

5 In the **Z** text field, type `260`.

Annotation 3

1 Right-click **Annotation 2** and choose **Duplicate**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `C_scatt = eval(sum(withsol('sol1',C_scatt, setval(freq,freq),setind(modeNum,index)),index,1,2*highestMode+1), um^2) um^2`.

4 Locate the **Position** section. In the **X** text field, type `0`.

5 In the **Z** text field, type `0`.

Annotation 4

Right-click **Annotation 3** and choose **Duplicate**.

Translation 1

1 In the **Model Builder** window, right-click **Annotation 3** and choose **Translation**.

2 In the **Settings** window for **Translation**, locate the **Translation** section.

3 In the **x** text field, type `-130`.

4 In the **y** text field, type `-20`.

5 In the **z** text field, type `-220`.

Annotation 4

1 In the **Model Builder** window, under **Results>Comparison of the Total Field in 3D** click **Annotation 4**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `C_scatt = eval(sum(withsol('sol2',C_scatt, setval(freq,3[THz]),setind(modeNum,index)),index,1,2*highestMode+1), um^2) um^2`.

Translation 1

1 Right-click **Annotation 4** and choose **Translation**.

2 In the **Settings** window for **Translation**, locate the **Translation** section.

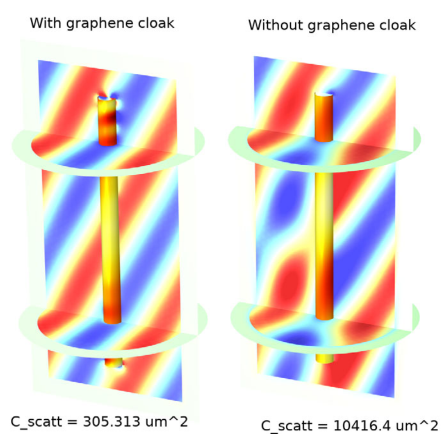
3 In the **x** text field, type `140`.

4 In the **y** text field, type `40`.

5 In the **z** text field, type `-200`.

6 Click the **Show Grid** button in the **Graphics** toolbar.

7 Click the **Show Axis Orientation** button in the **Graphics** toolbar.



At 3 THz, the scattering cross section is reduced by over 97% when the graphene cloak is applied.